Letter

Logical and thermodynamical reversibility: Optimized experimental implementation of the NOT operation

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The NOT operation is a reversible transformation acting on a 1-bit logical state and should be achievable in a physically reversible manner at no energetic cost. We experimentally demonstrate a bit-flip protocol based on the momentum of an underdamped oscillator confined in a double-well potential. The protocol is designed to be reversible in the ideal dissipationless case, and the thermodynamic work required is inversely proportional to the quality factor of the system. Our implementation demonstrates an energy dissipation significantly lower than the minimal cost of information processing in logically irreversible operations. It is, moreover, performed at high speed: A fully equilibrated final state is reached in only half a period of the oscillator. The results are supported by an analytical model that takes into account the presence of irreversibility. This Research Letter concludes with a discussion of optimization strategies.

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Starting with 1 bit of information $b \in (0, 1)$, only four deterministic information-processing operations are possible Table I). Their outcomes are as follows: initial value b (HOLD), 0 (RESET to 0), 1 (RESET to 1), and opposite value $\bar{b}$ (NOT). The first one is straightforward (do nothing), the second and the third are 1-bit erasures, and the last one corresponds to a bit flip. It has been shown theoretically [1] and experimentally [2–10] that erasing a 1-bit memory at temperature T_0 requires at least work $\mathcal{W}_{\rm LB} = k_B T_0 \ln 2$, with k_B being Boltzmann's constant. This intrinsic and universal minimal energetic cost is known as Landauer's bound (LB) and comes from the logical irreversibility of a RESET operation, or in other words from the entropic loss caused by the reduction of the states available to the system (from two initial states to a single reset state). Indeed, the work required to proceed and the heat released during the operation equalize with the entropic loss according to the second law of thermodynamics. In contrast, HOLD (or COPY [4]) and NOT are fully reversible logical operations: They do not come with any information loss. This logical reversibility implies that there is no fundamental minimal bound to the work required to operate. Reference [4] demonstrated a COPY operation with very low cost, below $0.01 k_B T$. We explore in this Research Letter the NOT operation, that is to say, the feasibility of performing a bit flip in a physically reversible fashion, without spending energy.

A bit-flip operation in stochastic but Markovian one-dimensional (1D) memories, whose dynamics only depends on the current state, is impossible [11]. Indeed, as sketched in Fig. 1, a protocol using only one degree of freedom (DOF) has to pass through the same state in the phase space, whatever the initial state (0 or 1): The information is lost, and so the output is random. A second degree of freedom is therefore required to proceed: It can be a second spatial dimension y, but if we stick to a 1D memory, observing non-Markovian dynamics requires the use of the velocity $v = \dot{x}$. Operating in the underdamped regime (quality factor $Q \gg 1$), where the inertia allows control of the speed [bit flip conducted in the (x, v) plane], is therefore a mandatory requirement.

In this Research Letter, we experimentally implement a bit-flip protocol based on the momentum of an underdamped system, proposed in Ref. [11] and designed to be reversible at very low damping. We are able to perform a fast and cheap NOT operation: The protocol is performed in the smallest of the typical timescales of the system, and the work required scales as $1/Q$, the prefactor depending only on the memory reliability requirements. Using as 1-bit memory an oscillator of period $\mathcal{T}_0 = 0.68$ ms and quality factor $Q = 100$, we perform a bit flip complying with high-standard reliability requirements in only half a period (0.34 ms) for an average energetic cost $\langle \mathcal{W} \rangle = 0.46 k_B T_0$, significantly below the minimal cost of information processing in logically irreversible operations, $k_B T_0 \ln 2$. This Research Letter is organized as follows: First, after defining the reliability criteria, we detail the bit-flip protocol designed to be reversible in the ideal dissipationless case. Second, we implement the protocol experimentally and measure the thermodynamic cost. The experimental results are then supported by a theoretical model that takes into account the presence of irreversibility: The model perfectly matches the experimental results. Finally, we conclude and discuss optimization strategies.

The memory is modeled by a single DOF x evolving in a double-well potential $U(x, x_1(t)) = \frac{1}{2}k[|x| - x_1(t)]^2$, where $\pm x_1(t)$ set the center ofthe two quadratic wells controlled by the operator. At rest (before or after the logical operation), we set $x_1 = X_1$, and the potential is $U_1(x) = \frac{1}{2}k(|x| - X_1)^2$. The memory states 0 and 1 correspond to the left- and right-hand-side wells, respectively, thus to the sign $S(x)$ of x. The stiffness of the oscillator k in both wells leads to the position

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TABLE I. The four deterministic 1-bit operations.

Initial	Final state			
state	HOLD	RESET to 0	RESET to 1	NOT
0	0	0	1	1
1	1	0	1	0

and speed standard deviation at equilibrium: $\sigma_x = \sqrt{k_B T_0/k}$ and $\sigma_v = \omega_0\sigma_x$, with $\omega_0 = 2\pi/\mathcal{T}_0$ being the angular resonance frequency in a single well. The reliability of the memory depends on the barrier height $\mathcal{B} = \frac{1}{2}kX_1^2$ between the two states: The higher $\mathcal{B}$ is, the less probable is a thermal fluctuation high enough to spontaneously flip from one well to the other. We choose $X_1 \sim 5\sigma_x$, leading to $\mathcal{B} \sim 12.5k_BT_0$: Memory losses occur only once every $e^{12.5} \sim 3\times10^5$ relaxation times $\tau_{\mathrm{relax}} = Q\mathcal{T}_0/\pi$ of the system (∼1.6 h for our experiment), far beyond any relevant timescale we probe.

To achieve a high success rate, the bit-flip protocol has to be designed to avoid the overlap of the two possible pieces of information in the phase space as illustrated in Fig. 1. Indeed, a full overlap results in the impossibility mentioned for the single-degree-of-freedom case, while a partial overlap (when the speed is bounded by a moderate damping, $Q \sim 1$) decreases the success rate, since the information is likely to slip to the wrong state due to thermal noise. In accordance with the reliability criterion for the static memory, we impose as safety criterion that the centers of the two states' probability distribution functions (PDFs) must be separated at all times by 10 times their characteristic spreading in the phase space (2D Gaussian in Fig. 1), that is to say, $10\sigma_x$ and $10\sigma_v$ along the (x, v) axes. This implies that a minimum speed is also imposed to safely convey the information: When $\langle x\rangle = 0$, the criterion translates into

$$\langle |v|\rangle_{x=0} > 5\sigma_v. \tag{1}$$

We therefore need to work in the underdamped regime to allow such high values of the system momentum without having a prohibitive damping cost.

As there is no entropic cost associated with the bit flip (logical reversibility), the energetic cost of the operation can only come from dissipation during the procedure. There are two strategies to reduce dissipation costs: proceeding at low speed in a quasistatic fashion or working at very low damping. In 1D, the first strategy has to be eliminated to meet the reliability criterion of Eq. (1). Hence the only strategy left to maintain physical reversibility without hampering the success rate consists in lowering the viscous damping from the

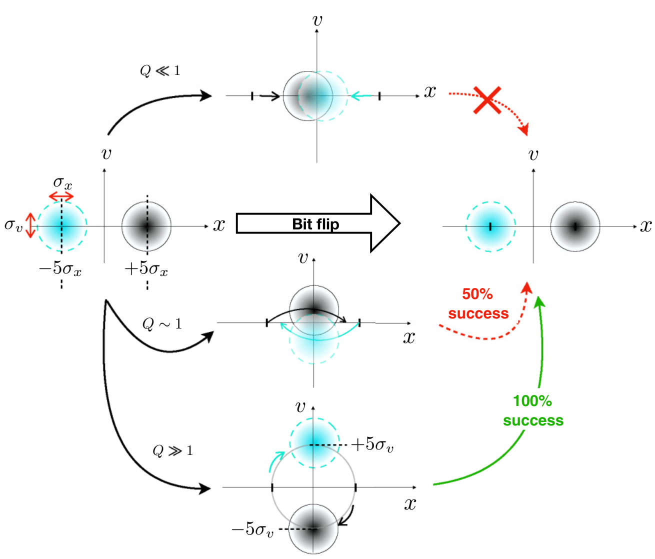

FIG. 1. Schematic overview of the bit-flip success requirements. The system two-dimensional PDFs are sketched by 2D Gaussians in the phase space (x, v) with light blue dashed circles (initial state 0) and black solid circles (initial state 1). Using a single DOF (if $Q \ll 1$, only the position can be driven) makes the bit-flip operation impossible: When the system passes through the phase space origin, the Markovian dynamics makes the initial information indistinguishable. The underdamped regime opens a second DOF to process the information: The speed $v = \dot{x}$. Moderate damping ($Q \sim 1$) limits the velocity range that is accessible and results in a partial overlap of the memory PDFs in the two different states: The operation can fail. To prevent the overlap and ensure a 100% success rate, we impose the requirement that the two states be separated by a minimal distance: We choose $(\langle x\rangle/\sigma_x)^2 + (\langle v\rangle/\sigma_v)^2 > 25$ as a safety criterion, with σ_x^2 and σ_v^2 being the position and velocity variance at equilibrium. Bit-flip protocols allowing such velocities require a high enough quality factor of the memory ($Q \gg 1$).

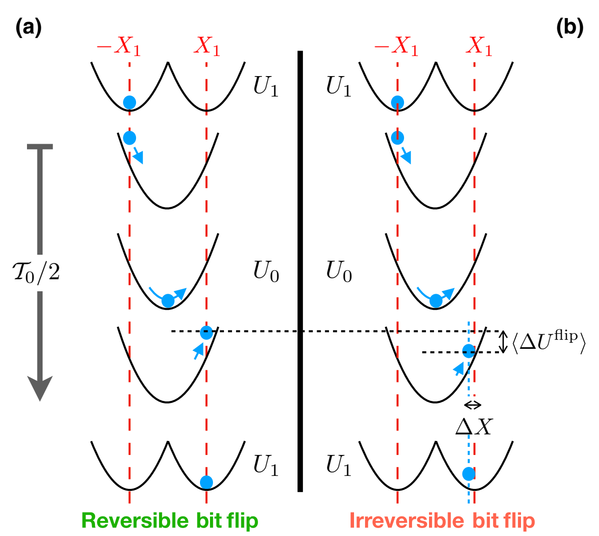

FIG. 2. Schematic overview of the bit-flip protocol. (a) Reversible operation (no dissipation). The systems starts in state 0 in the encoding potential U_1. The operation starts with a sudden change of the potential into a single well U_0 centered in 0. The system without velocity on average therefore initiates an oscillation of period $\mathcal{T}_0$ from the average position $-X_1$. After half a period $\mathcal{T}_0/2$, the trajectory reaches on average the opposite position $+X_1$ without velocity. At this exact moment, the potential U_1 is restored, so that the system ends up at equilibrium in state 1. (b) Origin of the irreversibility. When the system oscillation is damped by the viscous force, the system cannot reach $+X_1$ and culminates at $X_1 - \Delta X$. Therefore the operator has to pay for the potential energy difference $\langle\Delta U^{\mathrm{flip}}\rangle$. Besides, the system does not finish in perfect equilibrium in state 1 and has to relax to the well center, only reaching equilibrium in the typical relaxation time $\tau_{\mathrm{relax}} = Q\mathcal{T}_0/\pi$.

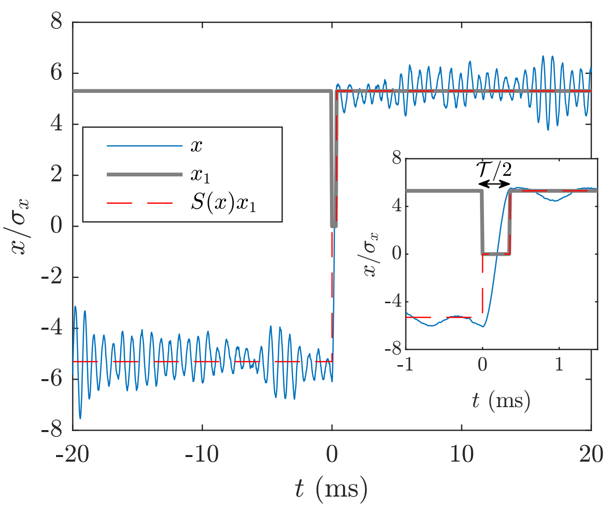

FIG. 3. Experimental response to the bit-flip protocol. The bit flip successfully drives the system from its initial state 0 to state 1 in half a period. The protocol consists in suddenly changing the well center position x_1 from X_1 to 0 at $t_i = 0$, and changing it back to X_1 at $t_f = \mathcal{T}/2 = 0.34$ ms (thick gray line). The oscillator trajectory (thin blue line) starts in equilibrium at $\langle x\rangle_i = -X_1$, naturally evolves in the transient single well, and ends up at $\langle x\rangle_f = +X_1$. The center of the well in which the cantilever is trapped is plotted with a dashed red line. The main panel illustrates the initial and final equilibria, while the inset presents a zoom focused on the protocol itself.

environment: $Q \gg 1$. Within this context, we implement here an innovative bit-flip protocol relying on the system momentum in a nonviscous environment to reach physical reversibility while complying with the reliability criterion.

Following Refs. [11,12], the operation consists in suddenly moving both wells' centers to $x_1(t_i) = 0$ at the initial time $t_i = 0$, as sketched in Fig. 2: The potential becomes a single harmonic well $U_0(x) = \frac{1}{2}kx^2$. After half the oscillator period, at time $t_f = \mathcal{T}_0/2$, the wells' centers are brought back to $\pm x_1(t_f) = \pm X_1$ to rebuild $U_1(x)$. We report in Fig. 3 the protocol $x_1(t)$ in gray, with an example of a single trajectory in blue, and the corresponding trapping-well center $[S(x)x_1(t)$, shown with a dashed red line]. It demonstrates the success of the NOT operation (here $0 \to 1$).

This bit-flip protocol, illustrated in Fig. 2, has been designed to be physically reversible when the dissipation can be neglected ($Q \to \infty$). For clarity purposes, let us consider that the system is initially in state 0 (the symmetric case is equivalent): $\langle x\rangle_i = -X_1$ and $\langle v\rangle_i = 0$. At t_i, the center of the well is suddenly changed from $x_1 = -X_1$ to $x_1 = 0$, and the cantilever starts an oscillation into the single-well potential. After half a period it reaches on average the opposite maximal position without speed: $\langle x\rangle_f = +X_1$, $\langle v\rangle_f = 0$. The second change of the potential at this exact moment therefore does not affect the average position of the system or its velocity: The memory is immediately in equilibrium. Let us point out that between the two changes, the velocity reaches $\langle |v|\rangle = 5\sigma_v$ when $\langle x\rangle = 0$ as required by the safety criterion recalled in Fig. 1. The operation results in changing the position of the oscillator from $\langle x\rangle_i = -X_1$ to $\langle x\rangle_f = +X_1$ using only the free evolution of the system inside the potential U_0: It is a reversible bit flip.

In U_1 the second well is statistically inaccessible; hence the potential remains in practice quadratic with a constant stiffness during the operation. The Fokker-Planck equation ruling the stochastic dynamic is thus linear: The system response is at all times the sum of the deterministic contribution $x_D = \langle x\rangle$ and of the thermal stochastic one x_{th}, i.e., $x = x_D + x_{\text{th}}$. The latter is not impacted by the bit-flip protocol and remains at equilibrium: $\langle x_{\text{th}}^2\rangle = \sigma_x^2 = k_B T_0/k$. Therefore the dynamics is ruled by the deterministic trajectory of the oscillator.

In the ideal case without any dissipation, the energy given to the system at the first potential change is fully recovered when U_1 is restored: The operation is reversible, and no work is required for the process. Formally, as the changes are instantaneous, the work corresponds to the potential loss ΔU^{flip} during the flip:

$$\mathcal{W} = U_0(t_i) - U_1(t_i) + U_1(t_f) - U_0(t_f), \tag{2a}$$

$$\langle \mathcal{W}\rangle = -\langle \Delta U^{\text{flip}}\rangle. \tag{2b}$$

Since $\langle U(t)\rangle = \frac{1}{2}k[(x_D(t) - x_1(t))^2 + \langle x_{\text{th}}^2\rangle]$, without dissipation we have

$$\langle U_1(t_i)\rangle = \langle U_1(t_f)\rangle_{Q=\infty} = \tfrac{1}{2}k_B T_0, \tag{3}$$

$$\langle U_0(t_i)\rangle = \langle U_0(t_f)\rangle_{Q=\infty} = \tfrac{1}{2}\left(kX_1^2 + k_B T_0\right), \tag{4}$$

so that $\langle \mathcal{W}\rangle_{Q=\infty} = -\langle \Delta U^{\text{flip}}\rangle_{Q=\infty} = 0$.

In our experiment, the 1-bit information is encoded into the position x of an underdamped micromechanical oscillator of effective mass m, in the form of a micrometric cantilever [10,13–15]. The natural angular resonance frequency of the oscillator is $\omega_0 = 2\pi \times 1.39$ kHz, and its low stiffness $k = m\omega_0^2$ results in $\sigma_x \lesssim 1$ nm at room temperature $T_0 = 295$ K. The quality factor is tuned to $Q = 100 \pm 5$ using a low-vacuum environment (pressure 1 mbar). The deflection x is precisely measured by interferometry [16], and the

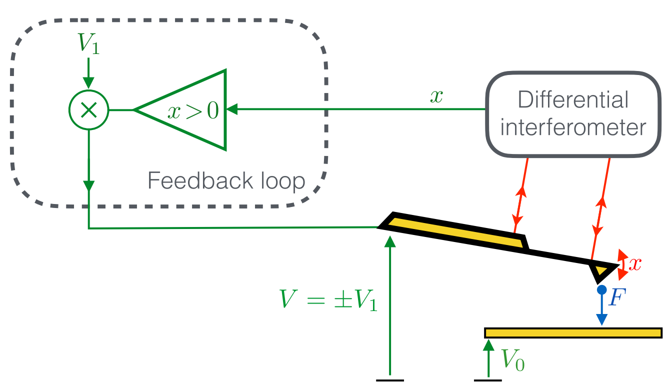

FIG. 4. Experimental setup. The deflection x of a conductive cantilever is measured with high precision by a differential interferometer. A voltage $V_0 \pm V_1$ (with $V_0 \gg V_1$) is applied between the cantilever and a facing electrode by a fast feedback loop; it centers the oscillator around $\pm X_1$ according to the sign of x [13,15]. The protocol is performed by turning off the feedback (setting $V_1 = 0$) during half an oscillation period.

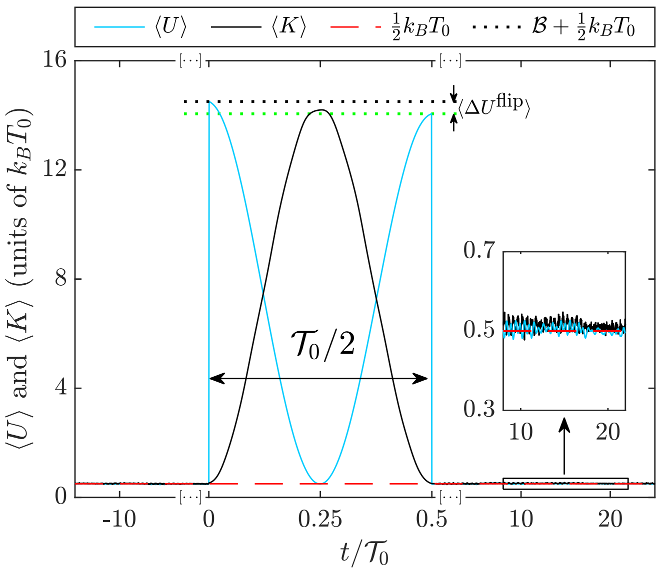

FIG. 5. Average potential and kinetic energies. $\langle U\rangle$ (light blue) and $\langle K\rangle$ (black) are averaged on $N = 2000$ trajectories. The bit-flip protocol takes place between $t_i = 0$ ms and $t_f = \mathcal{T}_0/2 = 0.34$ ms. Before and after the protocol (note the nonlinear scale for these time intervals, displaying a long time trace, with a vertical zoom in the inset), the oscillator is at equilibrium and equipartition applies: $\langle U\rangle = \langle K\rangle = \frac{1}{2}k_BT_0$ (red dashed line). During the protocol, the potential energy gains the barrier height $\mathcal{B} = 14k_BT_0$ at $t_i = 0$ ms when the potential is changed from U_1 to U_0. The oscillator then evolves in a harmonic well for half a period, and the energies display the deterministic evolution $U_D = \frac{1}{2}kx_D^2$ and $K_D = \frac{1}{2}mv_D^2$: The potential energy reaches its minimum when the kinetic is maximum and increases again till the second potential peak (light green dotted line) as the system reaches the opposite position. At t_f, the average speed is 0; thus the kinetic energy recovers its average equilibrium value $\frac{1}{2}k_BT_0$ (red dashed line). The potential energy does not perfectly recover its initial value, as the small damping removes an energy $\langle \Delta U^{\rm flip}_{\rm meas}\rangle = (-0.450 \pm 0.002)k_BT_0$ between the two extreme potential values.

double-well potential is created by applying an electrostatic force driven by a fast feedback loop based on the comparison of x with 0: $x > 0$ ($x < 0$) results in a constant force centering the well in $+x_1$ ($-x_1$). The setup is sketched in Fig. 4 and described in greater detail in Refs. [13,15].

The initial distance is calibrated using the position signal during the equilibrium steps: $X_1 = 5.3\,\sigma_x$, so that $\mathcal{B} = (14.00 \pm 0.05)k_BT_0$. We record $N = 2000$ trajectories, alternating between $0 \to 1$ and $1 \to 0$ operations. The protocol success rate is 100%: None of the 2000 trajectories ended in the wrong final state. We use the experimental data to compute the average potential and kinetic energies $\langle U\rangle$ and $\langle K\rangle = \frac{1}{2}m\langle v^2\rangle$ displayed in Fig. 5. Both quantities present a one-cycle oscillation of amplitude $\mathcal{B}$ during the protocol from $t_i = 0$ to $t_f = \mathcal{T}_0/2$ and immediately go back to their equilibrium value $\frac{1}{2}k_BT_0$ prescribed by the equipartition at t_f.

We compute the work and heat in the stochastic thermodynamic framework [10,17] and extract their distributions on the N trajectories. In the Supplemental Material [18], we plot these distributions and study the contributions of the intrinsic thermal noise and the extrinsic measurement noise. We measure

$$\langle \mathcal{W}\rangle = (0.46 \pm 0.04)k_BT_0, \tag{5a}$$

$$\langle \mathcal{Q}\rangle = (0.43 \pm 0.04)k_BT_0. \tag{5b}$$

In the data analysis, we use measured quantities for all variables, including $x_1(t)$: We take into account the finite speed $\dot{x}_1$ when switching back and forth between X_1 and 0. Let us point out that our work measurement is independent of the potential energy computation of Eq. (2). Note that the average stochastic heat is computed on a time window spanning several relaxation times after the end of the protocol, to allow the memory to return to equilibrium. Shorter integration times (limited to t_f, for example) would bias the result, as heat exchanges are slow for underdamped systems [14]. As the system initial and final states are at equilibrium at temperature T in the same potential U_1, the initial and final potential and kinetic energies are equal: $\langle\Delta U\rangle = \langle\Delta K\rangle = 0$. The first law of thermodynamics thus implies that the average heat dissipated at the end of the procedure is equal to the average work required: $\langle\mathcal{Q}\rangle = \langle\mathcal{W}\rangle$, as measured experimentally. These non-null values can be explained by the small residual damping at the origin of the irreversibility, as detailed in the following paragraph. Nevertheless, our experimental implementation of the bit flip already requires less energy than the landmark cost for irreversible operations on a 1-bit memory, Landauer's bound $\mathcal{W}_{\rm LB} \sim 0.69k_BT_0$. Furthermore, this logically reversible operation is performed in a very short time (0.34 ms here). Carrying out an irreversible operation such as an erasure on a similar duration would lead to the LB being exceeded by several k_BT_0 [10,14]. Finally, the equilibrium is restored just after the procedure, so that bit flips can be repeated successively without altering the memory reliability.

We tackle in this paragraph the origin of the irreversibility detected through the work and heat mean values: It is the residual damping in the vacuum in which evolves the cantilever. Indeed, because of the dissipation during the half oscillation, the potential energy given back by the system is lower than the one initially given by the operator, so that in total, work is required to proceed. To phrase it differently, the damped oscillator launched in $-X_1$ stops at zero speed after half a period a little bit before the exact opposite position $+X_1$ as sketched in Fig. 2(b). To provide a quantitative description, let us express the deterministic term of trajectory x_D during a $0 \to 1$ operation. The oscillation initiated in $x_D(0) = -X_1$ and $v_D(0) = 0$ obeys

$$x_D(t) = X_1 e^{\frac{-t\omega_0}{2Q}}\left(\frac{\omega_0}{2Q\Omega}\sin\Omega t - \cos\Omega t\right), \tag{6}$$

where $\Omega = \omega_0\sqrt{1 - 1/(4Q^2)}$ is approximately ω_0 at high quality factor. After half a period, at $t_f = \mathcal{T}/2 = \pi/\Omega$ the cantilever reaches on average the extreme position:

$$\langle x(t_f)\rangle = x_D(\mathcal{T}/2) = X_1 e^{-\pi/\sqrt{4Q^2-1}}. \tag{7}$$

Without damping, $Q \to \infty$, so that we recover $\langle x\rangle_f = X_1$ and consequently a reversible behavior. Meanwhile, in a viscous environment, the cantilever undershoots the targeted position by $\Delta X = X_1(1 - e^{-\pi/\sqrt{4Q^2-1}}) \simeq \frac{\pi}{2Q}X_1$. For $Q = 100$, we have $\Delta X/X_1 = 1.56\%$. As a consequence, there is a potential energy loss $\langle \Delta U^{\text{flip}} \rangle$ that we compute using Eq. (2) as

$$\langle \Delta U^{\text{flip}} \rangle = -\tfrac{1}{2}k\big[X_1^2 + (\langle x(t_f)\rangle - X_1)^2 - \langle x(t_f)\rangle^2\big] \quad (8)$$

$$= -kX_1^2(1 - e^{-\pi/\sqrt{4Q^2-1}}) \simeq -\frac{\pi}{Q}\mathcal{B} \quad (9)$$

$$= (-0.44 \pm 0.02)k_B T_0, \quad (10)$$

where the approximation in Eq. (9) is true for $Q \gg 1$. The value in Eq. (10) corresponds to the theoretical prediction knowing the parameters Q and $\mathcal{B}$ from calibration, and matches the measured stochastic work and heat reported in Eq. (5). We also compare it with the experimental value measured of the experimental potential energy evolution displayed in Fig. 5:

$$\big\langle \Delta U_{\text{meas}}^{\text{flip}} \big\rangle = (-0.450 \pm 0.002)k_B T_0. \quad (11)$$

The errors are inferred from the error on σ_x calibrated before each of the N operations. The theory and the experiment are in very good agreement. Besides, the first peak value in Fig. 5 is also consistent with the model, being worth the thermal energy plus the barrier energy (deterministic contribution): $\mathcal{B} + \frac{1}{2}k_B T_0 = 14.5 k_B T_0$.

It can be noted that when Q is not very large, the undershoot ΔX can be significant: It diverges for $Q = \frac{1}{2}$, when the motion is not underdamped anymore. Moreover, the protocol takes longer, as the effective period $\mathcal{T}$ increases when Q decreases. At low Q values, meeting the safety criteria at t_f implies choosing $X_1 \geqslant 5\sigma_x e^{\pi/\sqrt{4Q^2-1}}$ to compensate for the decrease in amplitude. From Eq. (9), we compute the minimum value for the mean work:

$$\mathcal{W}_{xv}^{\min} = 25k_B T_0 e^{2\pi/\sqrt{4Q^2-1}}(1 - e^{-\pi/\sqrt{4Q^2-1}}). \quad (12)$$

In agreement with Eq. (12), the best way to cut the bit-flip cost is to enhance the quality factor as displayed in Fig. 6. In particular, to ensure less than 5% of the LB, the quality factor has to exceed $Q = 1000$.

As mentioned in the introduction, using a second spatial DOF [19] is an alternative to the speed DOF. Maintaining the bit-flip success rate ensured by the safety criterion ($2X_1 = 10\sigma_x$ between the two states at all times) would in this case cost at least the work required to proceed along a circle in the (x, y) 2D plane, thus covering a distance πX_1 in time τ. Bypassing transients using optimized protocols [20] and moving at constant speed, the work implied is

$$\mathcal{W}_{xy}^{\min} = k\frac{(\pi 5\sigma_x)^2}{Q\tau\omega_0} = 25k_B T_0 \frac{\mathcal{T}_0}{\tau}\frac{\pi}{2Q}. \quad (13)$$

As illustrated in Fig. 6, for operations as fast as $\tau = \mathcal{T}/2$, the protocol based on momentum is better than its spatial (x, y) counterpart, as long as $Q > 3.6$. However, the bit flip in the (x, y) plane allows one to reduce the operation speed and get close to a quasistatic motion of the system in the viscous bath. As shown in Fig. 6, if one accepts extending the duration to $\tau = 10\mathcal{T}$, then the (x, y) bit-flip protocol is better. In particular, such a slow process would reach the quasireversibility

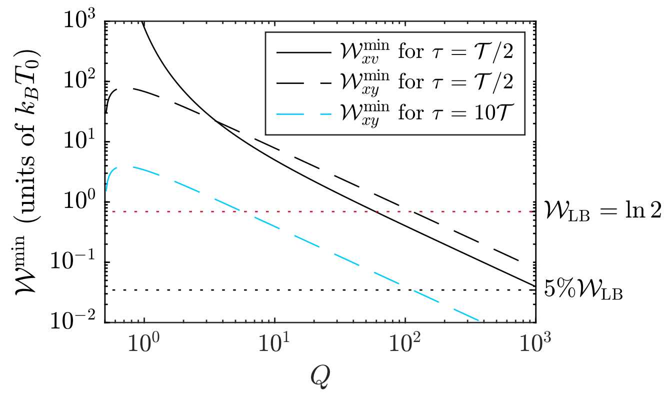

FIG. 6. Work of a bit-flip protocol depending on the quality factor Q and the operation duration τ. Our protocol (solid black line) allows very fast erasures: $\tau = \mathcal{T}/2$. It corresponds to a bit flip in the (x, v) plane, whose cost to ensure the safety criterion is given by Eq. (12). Increasing the quality factor reduces the bit-flip cost: The quasireversibility ($\langle \mathcal{W} \rangle < 5\%\mathcal{W}_{\text{LB}}$ shown with the black, lower short-dashed line) is reached for $Q > 1000$. Proceeding at the same speed using the 2D spatial alternative [bit flip in the (x, y) plane, long-dashed black line] requires twice as much energy at large Q, as expressed in Eq. (13). In contrast, at small Q or lower speed (for example, for $\tau = 10\mathcal{T}$, long-dashed light blue line) the (x, y) bit flip is cheaper than the momentum-based protocol.

($\langle \mathcal{W} \rangle < 5\%$ LB) under our experimental conditions, i.e., $Q = 100$.

To summarize, we have experimentally illustrated the connection between physical reversibility and logical reversibility in information processing. The bit-flip protocol, designed to be secure and cost-free [11,12], has been tested experimentally: It successfully performs the NOT operation in $\mathcal{T}_0/2 = 0.34$ ms for a very small amount of work, for example, significantly below the landmark of the LB of irreversible operations. The deviation from the desired zero-work operation is fully explained by the coupling of the memory to the surrounding bath: Even very low damping introduces irreversibility. The theoretical description has proven reliable to quantify the remaining irreversibility, evaluated with high accuracy in our experiment with four independent measurements, all in agreement [Eqs. (5), (10), and (11)]. Even faster and cheaper NOT operations could be achieved with oscillators having a higher resonance frequency and a larger quality factor. After the reset operation [10,14,15], this Research Letter demonstrates the last logical operation on single-bit underdamped memories and further highlights their small energy footprint and their interest.

The data that support the findings of this study are openly available [21].

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