


Interplay of decoherence and relaxation in a two-level system interacting with an infinite-temperature reservoir

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We study the time evolution of a single qubit in contact with a bath, within the framework of projection operator methods. Employing the so-called modified Redfield theory, which also treats energy conserving interactions nonperturbatively, we are able to study the regime beyond the scope of the ordinary approach. Reduced equations of motion for the qubit are derived in an idealistic system where both the bath and system-bath interactions are modeled by Gaussian distributed random matrices. In the strong decoherence regime, a simple relation between the bath correlation function and the decoherence process induced by the energy conserving interaction is found. It implies that energy conserving interactions slow down the relaxation process, which leads to a Zeno freezing if they are sufficiently strong. Furthermore, our results are also confirmed in numerical simulations.

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I. INTRODUCTION

In the field of open quantum systems, the question of whether or how a small quantum system evolves to a steady state, when coupled to a large bath, has attracted significant attention and has been studied extensively in recent decades in various fields of physics [1–14].

On the route of the system evolving towards an equilibrium state, decoherence and relaxation are two fundamental processes which often coexist and may, in general, be correlated to each other. It is natural to ask in which exact way decoherence and relaxation processes are related. Or, more specifically, as focused on in this paper: What is the impact of decoherence on the relaxation process? The question has been discussed in the weak-coupling regime [15], as well as in the case of the pure-dephasing interaction [16,17]. The effect of spatial decoherence on the transport properties of particle(s) is investigated in ordered [15] and disordered [17] tight-binding lattices. In Ref. [16], based on the memory kernel approach, the dynamics of the system is found to be slowed down by decoherence, which leads to the transition towards Zeno freezing [18] in the strong decoherence regime. However, not as much is known for more generic spin-bath coupling.

The answer to this question relies on the knowledge of the time evolution of the reduced density matrix (RDM) of the system of interest. However, for most open quantum systems, which are not exactly solvable, it is too complicated to obtain it without making approximations. The standard procedure in theoretical treatments is to derive closed approximate equations of motion of the system, i.e., the quantum master equation (QMEQ) [1,3,19], from the underlying

time-dependent Schrödinger equation (TDSE) by eliminating the environmental degrees of freedom.

One of the most important and commonly used methods to derive the QMEQ is the projection method (such as the Nakajima-Zwanzig method [20,21] or the time-convolutionless method [22–25]). Another foundational method is the Hilbert space average method [8], which estimates conditional quantum expectation values through an appropriate average over a constraint region in Hilbert space. Besides the usual master equation approach, there are also other methods aiming to derive closed equations of the system, e.g., the approach based on resonance theory [26,27], linear-response theory [28], and the Dyson Brownian technique [29].

For weak system-bath coupling, the Redfield equation [30] can be derived by keeping the perturbations to second order and making use of the Born-Markov approximation. In this case, exact equations for the relaxation and decoherence process can be derived. To better understand the relation between these two processes, it is desirable to go beyond the weak system-bath coupling regime, which is always a challenging task. One possible method to achieve this is to straightforwardly extend the perturbation theory to higher orders. Another possible method is to treat a certain part of the interaction Hamiltonian nonperturbatively, as done, e.g., in the Förster theory [31,32] and the modified Redfield theory [33].

In our paper, we study the problem in an idealistic system where a single qubit is coupled to a bath modeled by a random matrix [7,28,29,34] via system-bath interaction consisting of both energy-conserving (EC) and energy-exchange (EX) parts (with respect to system energy). We employ the modified Redfield theory [33,35–38], where the EC interaction is also treated nonperturbatively. Reduced equations of motion for the system in the interaction picture are derived, which are valid for arbitrary large EC interaction whenever the EX interaction is weak compared to the unperturbed Hamiltonian. Employing further assumptions, we also derive the

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equations for the time evolution of the system's RDM in the Schrödinger picture. A simple relation is found between the bath correlation function and the decoherence process induced by the EC interaction, which implies that the relaxation process is slowed down by the EC interaction, leading to a Zeno freezing if it is sufficiently strong. The paper is organized as follows. In Sec. II, we introduce the general setup, and in Sec. III, the reduced equations of motion for the system are derived. In Sec. IV, we apply our analytical results to a spin random-matrix model, while the results are checked numerically in Sec. V. Conclusions and outlook are given in Sec. VI.

II. GENERAL SETUP

We consider a model where a single qubit is coupled to a bath, the Hamiltonian of which reads

$$H = H_S + H_B + V, \quad (1)$$

$$V = \sum_{m,n=1}^2 \lambda_{mn} |m\rangle\langle n| \otimes H_{mn},$$

where $H_S = \omega \sigma_z^S$ (σ_z^S denotes the z -direction Pauli operator of the system), H_B , and V indicate system, bath, and interaction Hamiltonian, respectively. Eigenstates of H_S and H_B are denoted by $|m\rangle$ and $|E_k\rangle$,

$$H_S |m\rangle = e_m |m\rangle, \quad H_B |E_k\rangle = E_k |E_k\rangle, \quad (2)$$

where $m = 1, 2$ correspond to spin down and up. H_{mn} are some generic operators in the Hilbert space of the bath satisfying

$$\|H_{mn}\| = \|H_B\|. \quad (3)$$

$\|\cdot\|$ indicates the norm of the operator, which is defined as (for a generic operator \mathcal{O})

$$\|\mathcal{O}\| \equiv \text{Tr}\{\mathcal{O}\mathcal{O}^\dagger\}. \quad (4)$$

The Hermiticity of the interaction Hamiltonian requires that

$$(\lambda_{mn} H_{mn})^\dagger = \lambda_{nm} H_{nm} \quad (5)$$

holds for $m, n = 1, 2$. The interaction Hamiltonian can be divided into an energy-conserving (EC) part (denoted by V_{ec}) and an energy-exchange (EX) part (denoted by V_{ex}),

$$V = V_{\text{ec}} + V_{\text{ex}}, \quad (6)$$

where

$$V_{\text{ec}} = \lambda_{11} |1\rangle\langle 1| \otimes H_{11} + \lambda_{22} |2\rangle\langle 2| \otimes H_{22},$$

$$V_{\text{ex}} = \lambda_{12} |1\rangle\langle 2| \otimes H_{12} + \lambda_{12}^* |2\rangle\langle 1| \otimes H_{12}^\dagger. \quad (7)$$

The initial state considered here is a product state, written as

$$\rho(0) = \rho^S(0) \otimes \rho^B(0), \quad (8)$$

where $\rho^S(0) = |\psi_S(0)\rangle\langle\psi_S(0)|$ and

$$\rho^B(0) = \frac{1}{Z} \exp(-\beta H_B), \quad Z = \text{Tr}\{\exp(-\beta H_B)\}. \quad (9)$$

In our paper, we only consider the simplest case where the bath is at infinite temperature, $\beta = 0$. $\rho^B(0)$ reads

$$\rho^B(0) = \frac{\mathbb{1}_B}{d}, \quad (10)$$

where d denotes the Hilbert space dimension of the bath. To study the time evolution of the RDM of the system, $\rho^S(t) = \text{Tr}_B[\rho(t)]$, we divide the Hamiltonian into an unperturbed part H_0 and a perturbation part H^{int} ,

$$H = H_0 + H^{\text{int}}. \quad (11)$$

As the key ingredient in the modified Redfield theory [33], the unperturbed Hamiltonian,

$$H_0 = H_S + H_B + V_{\text{ec}}, \quad (12)$$

also comprises the EC interaction V_{ec} , while the perturbation,

$$H^{\text{int}} = V_{\text{ex}}, \quad (13)$$

only consists of the EX interaction. In our paper, we only consider the situation where $\|H^{\text{int}}\| \ll \|H_0\|$. In this case, the system's eigenbasis $|m\rangle$ usually forms a good preferred basis [4,39,40] or, e.g., the stationary RDM is approximately diagonal in $|m\rangle$. Thus, in the rest of the paper, whenever talking about decoherence, we refer to decoherence in the eigenbasis of the system.

In the interaction picture, the density matrix of the composite system at time t is written as

$$\rho_I(t) = \exp(iH_0 t) \rho(t) \exp(-iH_0 t), \quad (14)$$

where

$$\exp(-iH_0 t) = |m\rangle\langle m| \otimes U_m(t), \quad (15)$$

$$U_m(t) \equiv \exp[-i(H_B^{(m)} + e_m)t]. \quad (16)$$

$H_B^{(m)}$ can be regarded as an effective bath Hamiltonian with respect to system state $|m\rangle$, defined as

$$H_B^{(m)} = H_B + \lambda_{mm} H_{mm}. \quad (17)$$

The spectral density of $H_B^{(m)}$ is denoted by $\Omega_m(E)$, which will be used later in Sec. IV. Denoting the eigenvalues and eigenstates of $H_B^{(m)}$ by $E_k^{(m)}$ and $|E_k^{(m)}\rangle$, one gets

$$H_0 |m\rangle |E_k^{(m)}\rangle = (E_k^{(m)} + e_m) |m\rangle |E_k^{(m)}\rangle. \quad (18)$$

Similarly, one has

$$H_I^{\text{int}}(t) = \exp(iH_0 t) H_I^{\text{int}} \exp(-iH_0 t),$$

$$= \lambda_{12} |1\rangle\langle 2| \otimes U_1^\dagger(t) H_{12} U_2(t) + \text{H.c.} \quad (19)$$

The RDM of the qubit in the interaction picture can be written as

$$\rho_I^S(t) = \text{Tr}_B\{\rho_I(t)\}, \quad (20)$$

where the matrix elements $[\rho_I^S(t)]_{mn} = \langle m | \rho_I^S(t) | n \rangle$ can be written as

$$[\rho_I^S(t)]_{mn} = \begin{cases} \rho_{mn}^S(t), & m = n \\ \langle m | \text{Tr}_B\{U_m^\dagger(t) \rho(t) U_n(t)\} | n \rangle, & m \neq n. \end{cases} \quad (21)$$

Here, $\rho_{mn}^S(t) \equiv \langle m | \rho^S(t) | n \rangle$ indicate the matrix elements of RDM in the Schrödinger picture. One can see that in the

modified Redfield approach, the diagonal elements of RDM in the interaction picture are the same as in the Schrödinger picture. Note that due to $[H_0, H_S + H_B] \neq 0$, which is different from the usual approaches, there is no simple relation between off-diagonal elements in the interaction and Schrödinger picture. This is known to be a main drawback of this method, as the decoherence dynamics in the Schrödinger picture cannot be straightforwardly studied [35,37,38,41–43]. In this paper, we will tackle this problem by employing further assumptions.

III. REDUCED EQUATIONS OF MOTION

In this section, we are going to derive the reduced equations of motion for the system in both the interaction and Schrödinger pictures.

A. Time evolution of the system in the interaction picture

In this section, we derive reduced equations of motion for the system in the interaction picture using modified Redfield theory. Different from the derivations shown in Refs. [37,38], which only consider the diagonal elements of the RDM of the system, we also study the time evolution of off-diagonal elements of RDM as well. Here we only focus on the simplest case where the interaction Hamiltonians H_{mn} are uncorrelated with each other. As a guideline of this section, we would like to mention here that the derivations given below are almost the same as the standard derivations of the quantum master equations based on the Born-Markov approach, but with a different unperturbed Hamiltonian given in Eq. (12). More detailed derivations can be found in Appendix A.

Let us consider a projection superoperator \mathcal{P} , defined as

$$\mathcal{P}\rho = \text{Tr}_B\{\rho\} \otimes \frac{\mathbb{1}_B}{d}. \quad (22)$$

Applying \mathcal{P} to the density matrix in the interaction picture yields

$$\mathcal{P}\rho_I(t) = \text{Tr}_B\{\rho_I(t)\} \otimes \frac{\mathbb{1}_B}{d} = \rho_I^S(t) \otimes \frac{\mathbb{1}_B}{d}. \quad (23)$$

Using the general method of projection operator technique and keeping perturbation terms up to second order, for initial states satisfying $\mathcal{P}\rho_I(0) = \rho_I(0)$, one has

$$\frac{d}{dt}\rho_I^S(t) = -\frac{1}{d} \int_0^t ds \text{Tr}_B\{[H_I^{\text{int}}(t), [H_I^{\text{int}}(s), \rho_I^S(s) \otimes \mathbb{1}_B]]\}. \quad (24)$$

To study the right-hand side of Eq. (24), it is useful to introduce a bath correlation function $F_{mn}(\tau)$ defined as

$$F_{mn}(\tau) = \frac{1}{d} \text{Tr}_B\{U_m^\dagger(\tau) H_{mn} U_n(\tau) H_{mn}^\dagger\}. \quad (25)$$

With straightforward derivations (see Appendix A for details), by employing the Markovian approximation, in the case where the interaction Hamiltonians H_{mn} are uncorrelated with each other, the time evolution of the RDM of the system can

be written as

$$\begin{aligned} \frac{d[\rho_I^S(t)]_{11}}{dt} &= -\Gamma_r \{[\rho_I^S(t)]_{11} - [\rho_I^S(\text{eq})]_{11}\}, \\ \frac{d[\rho_I^S(t)]_{22}}{dt} &= -\Gamma_r \{[\rho_I^S(t)]_{22} - [\rho_I^S(\text{eq})]_{22}\}, \\ \frac{d[\rho_I^S(t)]_{21}}{dt} &= -\frac{\Gamma_r}{2} [\rho_I^S(t)]_{21}, \end{aligned} \quad (26)$$

where

$$[\rho_I^S(\text{eq})]_{11} = [\rho_I^S(\text{eq})]_{22} = \frac{1}{2}. \quad (27)$$

Γ_r indicates the relaxation rate, which can be written as

$$\begin{aligned} \Gamma_r &= 4|\lambda_{12}|^2 \int_0^\infty \text{Re}[F_{12}(\tau)] d\tau \\ &= 2|\lambda_{12}|^2 \int_{-\infty}^\infty F_{12}(\tau) d\tau. \end{aligned} \quad (28)$$

It should be mentioned here that the Markovian approximation can only be applied under the condition that the correlation function $F_{12}(\tau)$ decays sufficiently fast on a time τ_B (correlation time) compared to the relaxation time of the system τ_R , that is,

$$\tau_B \ll \tau_R. \quad (29)$$

From Eq. (26), one obtains the solution of reduced equations of motion of the system in the interaction picture,

$$\begin{aligned} [\rho_I^S(t)]_{11} - [\rho_I^S(\text{eq})]_{11} &= e^{-\Gamma_r t} \{[\rho_I^S(0)]_{11} - [\rho_I^S(\text{eq})]_{11}\}, \\ [\rho_I^S(t)]_{22} - [\rho_I^S(\text{eq})]_{22} &= e^{-\Gamma_r t} \{[\rho_I^S(0)]_{22} - [\rho_I^S(\text{eq})]_{22}\}, \\ [\rho_I^S(t)]_{21} &= e^{-\frac{\Gamma_r}{2} t} [\rho_I^S(0)]_{21}. \end{aligned} \quad (30)$$

One can see that in the interaction picture, the resulting equations of motion for the diagonal and off-diagonal elements of RDM are decoupled, which is due to the second-order approximation used in Eq. (24).

B. Time evolution of RDM in the Schrödinger picture

After having derived the reduced equations of motion for the system in the interaction picture, we continue to consider the Schrödinger picture. As has already been shown in Eq. (21), the diagonal elements of the RDM in the Schrödinger picture are the same as in the interaction picture, so we only need to study the off-diagonal elements.

To this end, instead of the mixed state in Eq. (8), it is more convenient to consider a pure state $|\psi(0)\rangle$ as an initial state, where

$$|\psi(0)\rangle = |\psi^S(0)\rangle \otimes |\psi^B(0)\rangle. \quad (31)$$

The bath initial state is written as

$$|\psi^B(0)\rangle = \sum_k c_k |E_k\rangle, \quad (32)$$

where c_k are complex numbers, the real and imaginary parts of which are drawn independently from a Gaussian distribution. Based on the idea of dynamical quantum typicality [44–47], the dynamics of the system starting from the pure initial state employed in Eq. (32) is almost the same as that of the mixed

initial state in Eq. (8), if the dimension of the bath Hilbert space is large enough. At time t , the state of the composite system in the interaction picture can always be written in the following form:

$$|\psi_I(t)\rangle = |1\rangle|\psi_I^1(t)\rangle + |2\rangle|\psi_I^2(t)\rangle. \quad (33)$$

The RDM of the system in the interaction picture can then be written as

$$[\rho_I^S(t)]_{mn} = \langle\psi_I^n(t)|\psi_I^m(t)\rangle. \quad (34)$$

Now we switch to the Schrödinger picture. At time t , one has

$$|\psi(t)\rangle = e^{-iH_0 t}|\psi_I(t)\rangle = |1\rangle|\psi^1(t)\rangle + |2\rangle|\psi^2(t)\rangle, \quad (35)$$

where

$$|\psi^m(t)\rangle = e^{-i(H_B^{(m)} + e_m)t}|\psi_I^m(t)\rangle. \quad (36)$$

Then, the RDM of the system in the Schrödinger picture can be written by making use of $|\psi_I^m(t)\rangle$ as

$$[\rho^S(t)]_{mn} = \langle\psi_I^n(t)|e^{i(H_B^{(n)} + e_n)t}e^{-i(H_B^{(m)} + e_m)t}|\psi_I^m(t)\rangle. \quad (37)$$

As we consider a two-level system here, there is only one independent term in the off-diagonal elements,

$$[\rho^S(t)]_{21} = e^{-2i\omega t}\langle\psi_I^1(t)|e^{iH_B^{(1)}t}e^{-iH_B^{(2)}t}|\psi_I^2(t)\rangle. \quad (38)$$

At any time t , one can always divide $|\psi_I^2(t)\rangle$ into a branch which is parallel to $|\psi_I^1(t)\rangle$ and the other which is vertical to $|\psi_I^1(t)\rangle$, as

$$|\psi_I^2(t)\rangle = c(t)|\psi_I^1(t)\rangle + |\psi_I^{\perp}(t)\rangle, \quad (39)$$

where

$$c(t) = \frac{\langle\psi_I^1(t)|\psi_I^2(t)\rangle}{\langle\psi_I^1(t)|\psi_I^1(t)\rangle} = \frac{[\rho_I^S(t)]_{21}}{[\rho_I^S(t)]_{11}} = \frac{[\rho_I^S(t)]_{21}}{[\rho_I^S(t)]_{11}}. \quad (40)$$

Inserting Eqs. (39) and (40) into Eq. (38), one gets

$$[\rho^S(t)]_{21} = \frac{[\rho_I^S(t)]_{21}e^{-2i\omega t}}{\langle\psi_I^1(t)|\psi_I^1(t)\rangle}\langle\psi_I^1(t)|e^{iH_B^{(1)}t}e^{-iH_B^{(2)}t}|\psi_I^1(t)\rangle + e^{-2i\omega t}\langle\psi_I^1(t)|e^{iH_B^{(1)}t}e^{-iH_B^{(2)}t}|\psi_I^{\perp}(t)\rangle. \quad (41)$$

If we make the assumption that

$$|\langle\psi_I^1(t)|e^{iH_B^{(1)}t}e^{-iH_B^{(2)}t}|\psi_I^{\perp}(t)\rangle| \approx 0, \quad (42)$$

then $[\rho^S(t)]_{21}$ can be related to $[\rho_I^S(t)]_{21}$ as

$$[\rho^S(t)]_{21} = [\rho_I^S(t)]_{21}e^{-2i\omega t}L_{12}(t), \quad (43)$$

where

$$L_{12}(t) = \frac{1}{\langle\psi_I^1(t)|\psi_I^1(t)\rangle}\langle\psi_I^1(t)|e^{iH_B^{(1)}t}e^{-iH_B^{(2)}t}|\psi_I^1(t)\rangle, \quad (44)$$

which can be regarded as a kind of Loschmidt echo (LE). We employ a further assumption, which is

$$L_{12}(t) \approx L_{12}^{\text{typ}}(t) \equiv \langle\psi_{\text{typ}}|e^{iH_B^{(1)}t}e^{-iH_B^{(2)}t}|\psi_{\text{typ}}\rangle \simeq \frac{1}{d}\text{Tr}_B\{e^{iH_B^{(1)}t}e^{-iH_B^{(2)}t}\}, \quad (45)$$

where $|\psi_{\text{typ}}\rangle$ is a typical state in the Hilbert space of the bath. In this way, we arrive at the solution of the equation of motion

for the off-diagonal elements of the RDM in the Schrödinger picture, which reads

$$[\rho^S(t)]_{21} \simeq [\rho_I^S(t)]_{21}e^{-2i\omega t}L_{12}^{\text{typ}}(t). \quad (46)$$

In generic systems, the decay of the LE is a complicated question, which has been investigated in many works during recent decades [14,48–57]. However, answers to some of the questions, especially for the LE decay in an intermediate perturbation regime, still remain unclear. These are complicated questions and is not our intention to give exhaustive answers here. At present, we only focus on the weak ($\lambda_{mm} \ll 1$) and strong ($\lambda_{mm} \gg 1$) perturbation limits. In the weak perturbation limit $\lambda_{mm} \ll 1$, it is found that after a Gaussian decay at initial times, the LE follows an exponential decay (see [14] and Refs. therein),

$$|L_{12}^{\text{typ}}(t)| = \exp(-\Gamma_L t), \quad (47)$$

where

$$\Gamma_L = \int_0^\infty \text{Tr}_B\{e^{iH_B\tau}\tilde{V}e^{-iH_B\tau}\tilde{V}\}d\tau, \quad (48)$$

with

$$\tilde{V} = \lambda_{11}H_{11} - \lambda_{22}H_{22}. \quad (49)$$

In the case of very strong perturbation ($\lambda_{mm} \gg 1$), the LE can be written as

$$L_{12}^{\text{typ}}(t) \simeq \langle\psi^{\text{typ}}|e^{i\lambda_{11}H_{11}t}e^{-i\lambda_{22}H_{22}t}|\psi^{\text{typ}}\rangle \simeq \frac{1}{d}\text{Tr}_B\{e^{i\lambda_{11}H_{11}t}e^{-i\lambda_{22}H_{22}t}\}. \quad (50)$$

Making use of the eigenstates and eigenvalues of H_{mm} , denoted by $|E_k^{mm}\rangle$ and E_k^{mm} , respectively, $L_{12}^{\text{typ}}(t)$ is written as

$$L_{12}^{\text{typ}}(t) = \frac{1}{d}\sum_k e^{i\lambda_{11}E_k^{11}t}\langle E_k^{11}|e^{-i\lambda_{22}H_{22}t}|E_k^{11}\rangle. \quad (51)$$

As the interaction Hamiltonians H_{11} and H_{22} are uncorrelated, based on quantum typicality, one has

$$\langle E_k^{11}|e^{-i\lambda_{22}H_{22}t}|E_k^{11}\rangle \simeq \frac{1}{d}\text{Tr}\{e^{-i\lambda_{22}H_{22}t}\}, \quad (52)$$

which leads to

$$L_{12}^{\text{typ}}(t) \simeq \frac{1}{d^2}\text{Tr}\{e^{i\lambda_{11}H_{11}t}\}\text{Tr}\{e^{-i\lambda_{22}H_{22}t}\}. \quad (53)$$

C. Relation between relaxation and decoherence process

In open systems, for generic system-bath coupling, decoherence is induced by both EC and EX interactions. The behavior of these two different kinds of decoherence processes is usually different [27]. So we will study their relation to the relaxation process separately. In our case, the decoherence process induced by the EX interaction is described by decoherence in the interaction picture, while the decoherence process induced by the EC interaction is described by the Loschmidt echo introduced in Eq. (44). Decoherence in the Schrödinger picture is a joint effect of these two processes, which is described by Eq. (46).

The relation between decoherence induced by the EX interaction and relaxation is quite simple, which can be seen from Eq. (30): the decoherence rate induced by the EX interaction

is half of the relaxation rate. However, the relation between decoherence induced by the EC interaction and relaxation is not easily seen. Here we only consider the case of very strong EC interaction, $\lambda_{mm} \gg 1$. In this case, the bath correlation function $F_{12}(\tau)$ can be written as

$$F_{12}(\tau) = \frac{1}{d} \text{Tr}_B \{ e^{iH_{11}\tau} H_{12} e^{-iH_{22}\tau} H_{12}^\dagger \}, \quad (54)$$

which, in eigenstates of H_{mm} , reads

$$F_{12}(\tau) = \frac{1}{d} \sum_{kl} e^{i(E_k^{11} - E_l^{22})\tau} |\langle E_k^{11} | H_{12} | E_l^{22} \rangle|^2. \quad (55)$$

If H_{12} is traceless and uncorrelated with H_{11} and H_{22} , $\langle E_k^{11} | H_{12} | E_l^{22} \rangle$ can be regarded as Gaussian random numbers with mean zero. Then one can replace $|\langle E_k^{11} | H_{12} | E_l^{22} \rangle|^2$ by its variance, denoted by σ_{12} , which can be estimated in the following way:

$$\text{Tr}_B(H_{12}H_{12}^\dagger) = \sum_{kl} |\langle E_k^{11} | H_{12} | E_l^{22} \rangle|^2 \simeq d^2 \sigma_{12}, \quad (56)$$

yielding

$$\sigma_{12} \simeq \frac{1}{d^2} \text{Tr}_B \{ H_{12} H_{12}^\dagger \}. \quad (57)$$

As a result, one has

$$F_{12}(\tau) = \frac{1}{d^2} \text{Tr}_B \{ H_{12} H_{12}^\dagger \} \frac{1}{d} \text{Tr} \{ e^{i\lambda_{11}H_{11}\tau} \} \text{Tr} \{ e^{-i\lambda_{22}H_{22}\tau} \}. \quad (58)$$

Comparing with Eq. (53), one can see that the bath correlation function $F_{12}(t)$ is totally determined by the decoherence process induced by the EC interaction, as

$$F_{12}(\tau) = \mathcal{K} L_{12}^{\text{lyp}}(\tau), \quad (59)$$

where

$$\mathcal{K} = \frac{1}{d} \text{Tr}_B \{ H_{12} H_{12}^\dagger \}, \quad (60)$$

which is the second moment of H_{12} . Straightforwardly, one gets

$$\Gamma_r = 4\mathcal{K} |\lambda_{12}|^2 \int_0^{+\infty} \text{Re} [L_{12}^{\text{lyp}}(\tau)] d\tau. \quad (61)$$

Under a corresponding ‘‘uncorrelatedness assumption’’ (i.e., that H_{mm} are uncorrelated), the result of Eq. (59) can also be generated to a N -level system (see Appendix B for more details). Generally speaking, a larger decay rate of $L_{12}^{\text{lyp}}(\tau)$ results in a smaller time integral, which in turn yields a smaller relaxation rate Γ_r . So it can be expected that the relaxation process will be slowed down by decoherence induced by the EC interaction in the case of $\lambda_{mm} \gg 1$. At the same time, decoherence induced by the EX interaction is also suppressed by decoherence induced by the EC interaction.

Before ending the section, it should be mentioned here that the simple relation between the relaxation and decoherence process induced by the EC interaction (at large λ_{mm}) shown in Eq. (61) only exists under the assumption of the interaction Hamiltonians being uncorrelated, which can hardly be the case in realistic systems. In realistic systems, we expect that a certain relation between those two processes still exists, but it may take a more complicated form.

IV. RESULTS IN THE SPIN RANDOM-MATRIX MODEL

In this section we are going to apply the results obtained in Sect. III to a spin random-matrix model. The Hamiltonian is written as

$$H = \omega \sigma_z^S + H_B + \lambda_d V_d + \lambda_r V_r, \quad (62)$$

where

$$\begin{aligned} V_d &= |1\rangle\langle 1| \otimes H_{11} + |2\rangle\langle 2| \otimes H_{22}, \\ V_r &= |1\rangle\langle 2| \otimes H_{12} + |2\rangle\langle 1| \otimes H_{12}^\dagger, \end{aligned} \quad (63)$$

and H_{mm} and H_B are modeled by random matrices, the elements of which are drawn from Gaussian distribution with zero mean and variance $\sigma_0^2 = \frac{1}{4d}$. Due to the Hermiticity of the total Hamiltonian, H_B and H_{mm} should also be Hermitian, which means that they are actually drawn from a Gaussian orthogonal ensemble (GOE). Moreover, λ_d and λ_r should be real. One can see that the Hamiltonian we consider here is a special case of the Hamiltonian defined in Eq. (1), where $\lambda_{11} = \lambda_{22} = \lambda_d$, $\lambda_{12} = \lambda_{12}^* = \lambda_r$, and the EC and EX interactions are given by

$$V_{\text{ec}} = \lambda_d V_d, \quad V_{\text{ex}} = \lambda_r V_r. \quad (64)$$

It should be mentioned here that this model is also very similar to the spin-Gaussian orthogonal random matrices model introduced in Ref. [34], except that H_{12} considered here is not Hermitian.

A. Time evolution of diagonal elements of RDM

As shown in Eq. (28), the relaxation rate Γ_r is determined by the bath correlation function,

$$F_{12}(\tau) = \frac{1}{d} \text{Tr}_B \{ e^{i(-\omega + H_B^{(1)})\tau} H_{12} e^{-i(\omega + H_B^{(2)})\tau} H_{12}^\dagger \}. \quad (65)$$

Expanding $F_{12}(\tau)$ in the eigenbasis of $H_B^{(m)}$ yields

$$F_{12}(\tau) = \frac{1}{d} \sum_{kl} e^{i(E_k^{(1)} - E_l^{(2)})\tau} e^{-2i\omega\tau} |\langle E_k^{(1)} | H_{12} | E_l^{(2)} \rangle|^2. \quad (66)$$

As H_{12} is uncorrelated with H_B and H_{mm} , the $\langle E_k^{(1)} | H_{12} | E_l^{(2)} \rangle$ can also be regarded as Gaussian random numbers with variance $\frac{1}{4d}$. As a result,

$$\begin{aligned} F_{12}(\tau) &\simeq \frac{1}{4d^2} e^{-2i\omega\tau} \text{Tr}_B \{ e^{iH_B^{(1)}\tau} \} \text{Tr}_B \{ e^{-iH_B^{(2)}\tau} \} \\ &\simeq \frac{1}{4d^2} e^{-2i\omega\tau} \int dE_1 \Omega_1(E_1) e^{-iE_1\tau} \int dE_2 \Omega_2(E_2) e^{-iE_2\tau}, \end{aligned} \quad (67)$$

where, as has already been defined, $\Omega_m(E)$ indicates the density of states of $H_B^{(m)}$. Recalling the definition of $H_B^{(m)}$ in Eq. (17), one has

$$H_B^{(1)} = H_B + \lambda_d H_{11}, \quad H_B^{(2)} = H_B + \lambda_d H_{22}. \quad (68)$$

As H_B and H_{mm} are uncorrelated, $H_B^{(1)}$ and $H_B^{(2)}$ can both be regarded as GOE random matrices, the elements of which are drawn from the Gaussian distribution with mean zero and variance $\sigma_1 = \sqrt{1 + \lambda_d^2} \sigma_0$. Thus, one has

$$\Omega_1(E) \simeq \Omega_2(E) \simeq \Omega_0(E), \quad (69)$$

where $\Omega_0(E)$ has a semicircle distribution [58],

$$\Omega_0(E) = \begin{cases} \frac{2d}{\pi\sqrt{1+\lambda_d^2}} \sqrt{1 - \frac{E^2}{1+\lambda_d^2}} & \text{for } |E| < \sqrt{1+\lambda_d^2} \\ 0 & \text{otherwise.} \end{cases} \quad (70)$$

Substituting Eq. (70) into Eq. (67) and carrying out the integral, the bath correlation function can be written as

$$F_{12}(\tau) = \frac{[\mathcal{J}_1(\alpha\tau)]^2}{(\alpha\tau)^2} e^{-2i\omega\tau}, \quad (71)$$

where $\alpha = \sqrt{1+\lambda_d^2}$ and $\mathcal{J}_1(\tau)$ indicates the first-order Bessel function of the first kind. In the case of $\omega \ll \alpha$ (which means the energy scale of the system is much smaller compared to that of the effective bath $H_B^{(m)}$), one can employ the rotating-wave approximation (RWA). The phase factor $e^{-2i\omega\tau}$ can be approximated by 1 (for time $\tau \lesssim \tau_B$) and $F_{12}(\tau)$ is only dependent on the rescaled time $\alpha\tau$ as

$$F_{12}(\tau) = \frac{[\mathcal{J}_1(\alpha\tau)]^2}{(\alpha\tau)^2}. \quad (72)$$

Inserting Eq. (72) to Eq. (28) and carrying out the integral, one has

$$\int_{-\infty}^{\infty} F_{12}(\tau) d\tau = \frac{8}{3\alpha\pi} = \frac{8}{3\sqrt{1+\lambda_d^2}\pi}, \quad (73)$$

which leads to

$$\Gamma_r \approx \frac{16\lambda_r^2}{3\pi\sqrt{1+\lambda_d^2}} \propto \frac{\lambda_r^2}{\sqrt{1+\lambda_d^2}}. \quad (74)$$

Thus one has

$$\rho_{11}^S(t) - \rho_{11}^S(\text{eq}) = e^{-\frac{16\lambda_r^2}{3\pi\sqrt{1+\lambda_d^2}}t} [\rho_{11}^S(0) - \rho_{11}^S(\text{eq})]. \quad (75)$$

It implies that relaxation is boosted by the EX interaction and suppressed by the EC interaction.

It should be mentioned here that with the traditional approach where only the noninteracting part ($H_S + H_B$) of the Hamiltonian is treated nonperturbatively, a similar exponential decay $\rho_{11}^S(t) - \rho_{11}^S(\text{eq}) \propto e^{-\Gamma_r^0 t}$ can be derived. The difference is that in this case, one has $\Gamma_r^0 = -\frac{16\lambda_r^2}{3\pi}$, which is independent of the EC interaction strength λ_d . This indicates that the traditional approach is unable to account for the impact of decoherence on the relaxation process. It is not surprising, as the traditional approach is only supposed to work in the weak-coupling regime where not only the EX interaction but also the EC interaction should be weak. In this regime ($\lambda_d \ll 1$, $\lambda_r \ll 1$), one can easily see that $\Gamma_r \approx \Gamma_r^0$ and the results of the two approaches agree with each other.

After having derived the relaxation rate, we come back to Eq. (29) to check under what condition it is fulfilled. Here, for example, we can define τ_B and τ_R to be the time at which $\text{Re}[F_{12}(\tau)]$ and $|\rho_{11}^S(t) - \rho_{11}^S(\text{eq})|$ decay to 1% of its initial value and never exceed that value afterwards. Then, with straightforward calculations, one has

$$\tau_B \approx \frac{5.9}{\sqrt{1+\lambda_d^2}}, \quad \tau_R = \frac{2 \ln 10}{\Gamma_r} \approx \frac{2.7\sqrt{1+\lambda_d^2}}{\lambda_r^2}. \quad (76)$$

As a result, Eq. (29) can be approximately rewritten as

$$\lambda_r \ll \sqrt{\frac{1+\lambda_d^2}{2}}, \quad (77)$$

indicating that Eq. (29) would be better fulfilled for larger λ_d , if λ_r is fixed.

B. Time evolution of off-diagonal elements of RDM

As Γ_r has already been derived in Eq. (74), the time evolution of the off-diagonal elements of RDM in the interaction picture can be written as

$$|[\rho_I^S(t)]_{21}| = e^{-\frac{8\lambda_r^2}{3\pi\sqrt{1+\lambda_d^2}}t} |[\rho_I^S(0)]_{21}|, \quad (78)$$

from which one can see that decoherence in the interaction picture (or decoherence induced by the EX interaction) is also suppressed by the EC interaction.

In the Schrödinger picture, the time evolution of off-diagonal elements of the RDM is given in Eq. (46). As $|[\rho_I^S(t)]_{21}|$ has already been given above, one only needs to study the LE term $L_{12}^{\text{yp}}(t)$, which characterizes the decoherence process induced by the EC interaction. In the weak perturbation limit $\lambda_{mm} \ll 1$, as has already been discussed in Sec. III B, after a Gaussian decay at initial times, the LE decays exponentially,

$$|L_{12}^{\text{yp}}(t)| = \exp(-\Gamma_L t), \quad (79)$$

where

$$\Gamma_L = \lambda_d^2 \int_0^{\infty} d\tau F_{dd}(\tau) \quad (80)$$

and

$$F_{dd}(\tau) \equiv \frac{1}{d} \text{Tr}_B \{ e^{iH_B\tau} H_d e^{-iH_B\tau} H_d \}, \quad (81)$$

$$H_d \equiv H_{11} - H_{22}. \quad (82)$$

Expanding H_d in the eigenbasis of H_B , one has

$$F_{dd}(\tau) = \frac{1}{d} \sum_{kl} e^{i(E_k - E_l)\tau} |\langle E_k | H_d | E_l \rangle|^2. \quad (83)$$

As H_d is also a (GOE) random matrix, $|\langle E_k | H_d | E_l \rangle|^2$ can be replaced by its variance $\frac{1}{2d}$, which yields

$$F_{dd}(\tau) = \frac{1}{2d^2} |\text{Tr}_B \{ e^{-iH_B\tau} \}|^2 = \frac{2[\mathcal{J}_1(\tau)]^2}{\tau^2}. \quad (84)$$

Carrying out the integral in Eq. (80) exactly, one gets

$$\Gamma_L = \frac{8\lambda_d^2}{3\pi}. \quad (85)$$

In the case of $\lambda_d \gg 1$, one has

$$\begin{aligned} L_{12}^{\text{yp}}(t) &\simeq \frac{1}{d} \text{Tr}_B \{ e^{i\lambda_d H_{11}t} e^{-i\lambda_d H_{22}t} \} \\ &\simeq \frac{1}{d^2} \text{Tr}_B \{ e^{i\lambda_d H_{11}t} \} \text{Tr}_B \{ e^{-i\lambda_d H_{22}t} \} \\ &\simeq \frac{1}{d^2} |\text{Tr}_B \{ e^{i\lambda_d H_{11}t} \}|^2 \simeq \frac{4[\mathcal{J}_1(\lambda_d t)]^2}{(\lambda_d t)^2}, \end{aligned} \quad (86)$$

where the second line is obtained due to the fact that H_{11} and H_{22} are uncorrelated. As we have discussed in Sec. III C, by comparing Eq. (86) with Eq. (72), one finds

$$F_{12}(t) = \frac{1}{4}L_{12}^{\text{typ}}(t) \text{ for } \lambda_d \gg 1, \quad (87)$$

indicating that the bath correlation function $F_{12}(t)$ is determined by the decoherence process induced by the EC interaction. We stress here again that the simple relation between $F_{12}(t)$ and $L_{12}^{\text{typ}}(t)$ in Eq. (87) for $\lambda_d \gg 1$ relies on the assumption that H_{12} , H_{22} , and H_{11} are all uncorrelated. To see what would happen if they are not fully uncorrelated, here as an example one can relax one of the restrictions, by considering the case $H_{22} = -H_{11}$, while still assuming they are uncorrelated with H_{12} . With similar derivations, one can see that $F_{12}(t) \simeq (\frac{\mathcal{J}_1(\lambda_d t)}{\lambda_d t})^2$, which remains the same, while $L_{12}^{\text{typ}}(t) \propto \frac{\mathcal{J}_1(2\lambda_d t)}{\lambda_d t}$, different from the fully uncorrelated case. In this case, although $F_{12}(t)$ cannot be directly written as a function of $L_{12}^{\text{typ}}(t)$, they are still strongly correlated. It is also to be expected that a faster decay of $L_{12}^{\text{typ}}(t)$ results in a smaller relaxation rate Γ_r .

In summary, one has

$$|L_{12}^{\text{typ}}(t)| \simeq \begin{cases} \exp(-\frac{8}{3\pi}\lambda_d^2 t), & \lambda_d \ll 1 \\ \frac{4[\mathcal{J}_1(\lambda_d t)]^2}{(\lambda_d t)^2}, & \lambda_d \gg 1. \end{cases} \quad (88)$$

From the result in Eq. (88), one can see that in both cases, the decay of $L_{12}^{\text{typ}}(t)$, which characterize the decoherence process induced by the EC interaction, becomes faster for larger λ_d . Substituting Eq. (88) into Eq. (46), one derives the equation of motion for the off-diagonal elements of the RDM in the Schrödinger picture,

$$|\rho_{12}^S(t)| \simeq \begin{cases} \exp[-\frac{8}{3\pi}(\frac{\lambda_r^2}{\sqrt{1+\lambda_d^2}} + \lambda_d^2)t] |\rho_{12}^S(0)|, & \lambda_d \ll 1 \\ \exp(-\frac{8}{3\pi}\frac{\lambda_r^2}{\sqrt{1+\lambda_d^2}}t) \frac{4[\mathcal{J}_1(\lambda_d t)]^2}{(\lambda_d t)^2} |\rho_{12}^S(0)|, & \lambda_d \gg 1. \end{cases} \quad (89)$$

In both the $\lambda_d \ll 1$ and $\lambda_d \gg 1$ cases, it can be seen that the decoherence time τ_D decreases with increasing EX coupling strength λ_r . For $\lambda_d \gg 1$, one can actually approximate $\exp(-\frac{8}{3\pi}\frac{\lambda_r^2}{\sqrt{1+\lambda_d^2}}t)$ by 1, and in this case

$$|\rho_{21}^S(t)| \simeq \frac{4[\mathcal{J}_1(\lambda_d t)]^2}{(\lambda_d t)^2} |\rho_{21}^S(0)|, \quad (90)$$

which is a function of $\lambda_d t$. One can conclude that $\tau_D \propto \frac{1}{\lambda_d}$. For $\lambda_d \ll 1$, $|\rho_{12}^S(t)|$ follows an exponential decay, with a decoherence rate Γ_d given by

$$\Gamma_d = -\frac{8}{3\pi} \left(\frac{\lambda_r^2}{\sqrt{1+\lambda_d^2}} + \lambda_d^2 \right). \quad (91)$$

Expanding $\frac{1}{\sqrt{1+\lambda_d^2}}$ to the second order of λ_d , one finds that $\Gamma_d \propto \lambda_d^2(1 - \frac{\lambda_d^2}{2})$, which is a monotonically increasing function when $\lambda_r < \sqrt{2}$, and becomes a monotonically decreasing function of λ_d when $\lambda_r > \sqrt{2}$. Combining with the condition for the Markov approximation $\lambda_r \ll \sqrt{\frac{1+\lambda_d^2}{2}}$ in Eq. (29), one

expects that in situations where the analytical result in Eq. (89) is applicable, Γ_d is a monotonically increasing function of λ_d . In summary, we find that in the case of $\lambda_d \ll 1$ and $\lambda_d \gg 1$, decoherence in the Schrödinger picture is boosted by both EX and EC interactions.

V. NUMERICAL RESULTS

A. Main results on relaxation and decoherence dynamics

In this section, we numerically checked our analytical results on the relaxation and decoherence processes in the spin random-matrix model, which are given in Eqs. (75) and (89). In the numerical simulations, as in Eq. (32), we consider a pure state as the initial state, which is written as

$$|\psi(0)\rangle = |\psi^S(0)\rangle \otimes |\psi^B(0)\rangle. \quad (92)$$

Here, $|\psi^S(0)\rangle = \frac{\sqrt{3}}{2}|1\rangle + \frac{1}{2}|2\rangle$, and $|\psi^B(0)\rangle$ is a random state in the bath Hilbert space,

$$|\psi^B(0)\rangle = \sum_{k=1}^d c_k |E_k\rangle. \quad (93)$$

Here, c_k are complex numbers, the real and imaginary parts of which are drawn independently from the Gaussian distribution, and due to the normalization of $|\psi^B(0)\rangle$, $\sum_{k=1}^d |c_k|^2 = 1$.

The time evolution of the diagonal elements of the RDM are studied in Figs. 1 and 2, where the results are shown for varying λ_d with a fixed λ_r , and varying λ_r with a fixed λ_d , respectively. One can see that in both figures, the resulting curves approximately overlap as functions of $\Gamma_r t$ [Γ_r is given in Eq. (74)], which also agrees with the analytical prediction in Eq. (75), at least up to a certain timescale. The relaxation rate is found to increase with λ_r , while it decreases with λ_d [Figs. 2(a) and 1(a)]. It indicates that the relaxation process is boosted by the EX interaction, and suppressed by the EC interaction at the same time.

We notice that for extremely strong λ_d , e.g., $\lambda_d = 100$, the numerical results only follow the analytical prediction for a very short time and then go to a nonthermal steady value. This is due to a finite-size effect. In systems with finite Hilbert space dimension d , for sufficiently large λ_d , the off-diagonal elements of the EC interaction $\lambda_r V_r$ in the eigenbasis of the unperturbed Hamiltonian $H_0 = H_S + H_B + \lambda_d V_d$ are not much larger than, or even smaller than, the average level spacing of H_0 . As a result, the system cannot thermalize to the infinite-temperature Gibbs state (which is just the unitary matrix), but to a nonthermal steady state which usually depends on the initial state. But as the off-diagonal elements of $\lambda_r V_r$ scale as $\frac{1}{\sqrt{d}}$ while the mean level spacing of H_0 scales as $\frac{1}{d}$, such finite-size effect will vanish if d is large enough. Thus we expect that if we continued to increase d , the result for $\lambda_d = 100$ would follow the analytical prediction for a longer time.

Results for the decoherence process in the Schrödinger picture are shown in Fig. 3. One can see from Figs. 3(a) and 3(c) that our semianalytical predictions in Eq. (46) work quite well in both cases. It implies that the two assumptions that we made in Eqs. (42) and (45) are reasonable. Moreover, the analytical prediction for $\lambda_d \ll 1$ and $\lambda_d \gg 1$ in Eq. (89) is found to agree quite well with the numerical results. Surprisingly, the

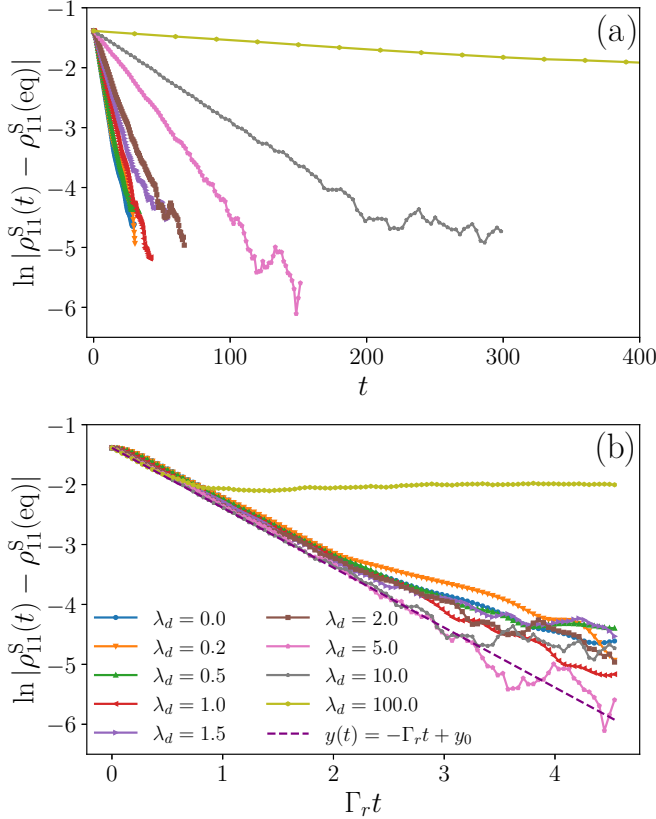


FIG. 1. (a) $\ln |\rho_{11}^S(t) - \rho_{11}^S(\text{eq})|$ as a function of (a) t and (b) $\Gamma_r t$, for different λ_d , where Γ_r is given in Eq. (74), for $\lambda_r = 0.3$, $\omega = 0.05$, $d = 2^{14}$. The dashed line indicates the analytical prediction $\ln |\rho_{11}^S(t) - \rho_{11}^S(\text{eq})| = -\Gamma_r t + y_0$, where $y_0 = \ln |\rho_{11}^S(0) - \rho_{11}^S(\text{eq})|$.

analytical prediction for weak λ_d is even valid for intermediate $\lambda_d \approx 1.0$. For all values of λ_d that we consider here, one can see that decoherence becomes faster if one increases λ_d , indicating that the decoherence process is always boosted by the EC interaction, which is just what one would expect.

B. Additional results on the bath correlation function and decoherence induced by EX and EC interactions

Additionally, we study the decoherence process induced by EX and EC interactions separately; the results are shown in Figs. 4 and 5. Results for decoherence induced by the EX interaction (which is described by decoherence in the interaction picture) are shown in Fig. 4. A good agreement with the analytical prediction $\ln |[\rho_I^S(t)]_{21}| = -\frac{\Gamma_r}{2}t$ up to a certain timescale can be seen. Similar to the relaxation process, one finds in Fig. 4(a) that the decoherence process induced by the EX interaction is also suppressed by the EC interaction.

In Fig. 5, we show results for the decoherence process induced by the EC interaction, which is described by $L_{12}^{\text{yp}}(t)$, for a wide range of λ_d . A good agreement with the analytical prediction in Eq. (88) can be found, for both $\lambda_d \ll 1$ and $\lambda_d \gg 1$. Based on the results shown in Fig. 5, one finds that $L_{12}^{\text{yp}}(t)$ decays faster for larger λ_d , indicating that the decoherence process induced by the EC interaction is always boosted by the EC interaction, just as one expects.

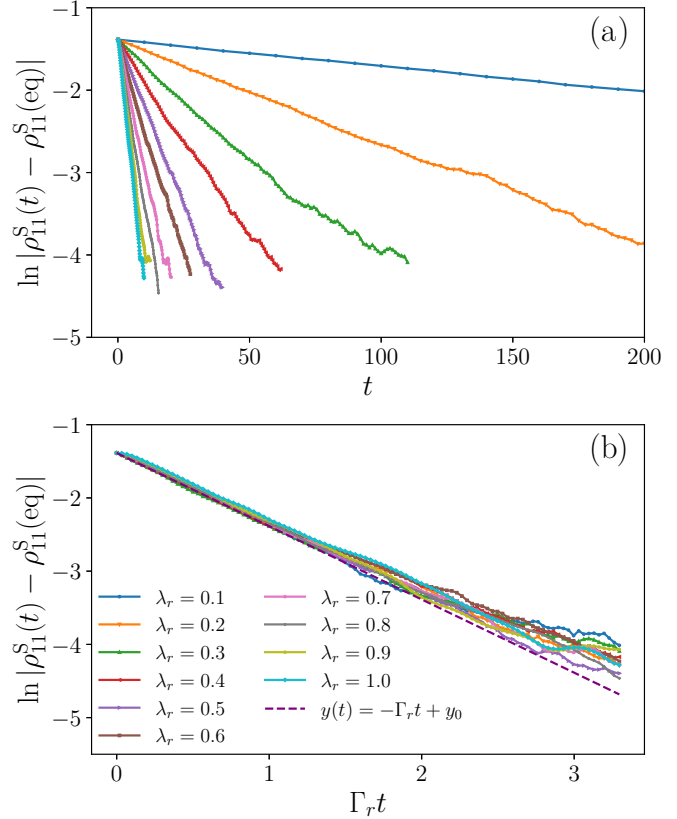


FIG. 2. Similar to Fig. 1, but results for varying λ_r with fixed λ_d , where $\lambda_d = 5.0$, $\omega = 0.05$, $d = 2^{14}$.

The bath correlation function $F_{12}(\tau)$ is also calculated, and the results are shown in Fig. 6. $F_{12}(\tau)$ for different λ_d overlap as functions of $\sqrt{1 + \lambda_d^2}$, which agrees with the analytical prediction in Eq. (71) as well. A good agreement between $F_{12}(\tau)$ and $\frac{1}{4}L_{12}^{\text{yp}}(\tau)$ (for $\lambda_d = 5$) can also be seen, which confirms the analytical result in Eq. (87). It indicates that in the spin random-matrix model that we consider here, the correlation function $F_{12}(\tau)$ is determined by the decoherence process induced by the EC interaction in the case of $\lambda_d \gg 1$. In the inset, the numerical results of the infinite time integral of $F_{12}(\tau)$ fit perfectly with the analytical prediction in Eq. (73), from which one concludes that the relaxation rate Γ_r becomes smaller for larger λ_d .

In summary, our main results for the reduced equations of motion for the qubit in the spin random-matrix model are confirmed in a wide range of coupling strengths λ_r and λ_d . A simple relation between the bath correlation function and the decoherence process induced by the EC interaction can be seen for $\lambda_d \gg 1$. Moreover, we find that if one increases λ_d , decoherence induced by the EC interaction becomes faster, while relaxation as well as decoherence induced by the EX interaction become slower.

VI. CONCLUSIONS AND OUTLOOK

In this paper, by employing the modified Redfield theory, we derived reduced equations of motion for the qubit in a spin random-matrix model, which is also valid for large EC interactions. The relation between the relaxation and

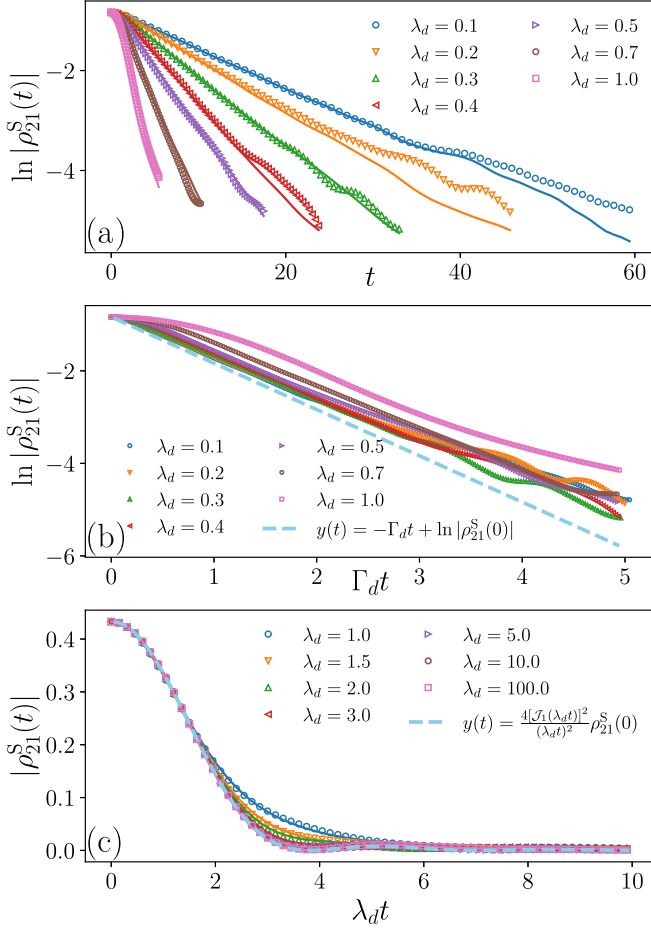


FIG. 3. Decoherence in the Schrödinger picture: (a) $\ln |\rho_{21}^S(t)|$ vs t , (b) $\ln |\rho_{21}^S(t)|$ vs $\Gamma_d t$, and (c) $|\rho_{21}^S(t)|$ vs $\lambda_d t$, for different λ_d . The dashed line in (b) shows the analytical prediction for $\lambda_d \ll 1$, $\ln |\rho_{21}^S(t)| = -\Gamma_d t + \ln |\rho_{21}^S(0)|$, where Γ_d is given in Eq. (91). The solid line in (a) and (c) indicates the semianalytical prediction in Eq. (46), where both $[\rho_I^S(t)]_{21}$ and $L_{12}^{\text{yp}}(t)$ are calculated numerically as well (shown in Figs. 4 and 5). The dashed line in (c) indicates the analytical prediction for $\lambda_d \gg 1$, $|\rho_{21}^S(t)| \simeq \frac{4|J_1(\lambda_d t)|^2}{(\lambda_d t)^2} |\rho_{21}^S(0)|$.

decoherence process is discussed. We find a simple relation between the bath correlation function and the decoherence process induced by the EC interaction in the strong decoherence regime. It implies that the relaxation process is suppressed by decoherence induced by the EC interaction. The relaxation rate goes to zero, if the EC interaction is sufficiently strong, which coincides with the quantum Zeno effect. Furthermore, decoherence induced by the EX interaction is also found to be suppressed by decoherence induced by the EC interaction.

As our main results are derived in an idealistic model where all the interaction Hamiltonians are assumed to be uncorrelated, it is interesting to ask whether or to what extent our finding could be applied to realistic systems, which will be investigated in our future work.

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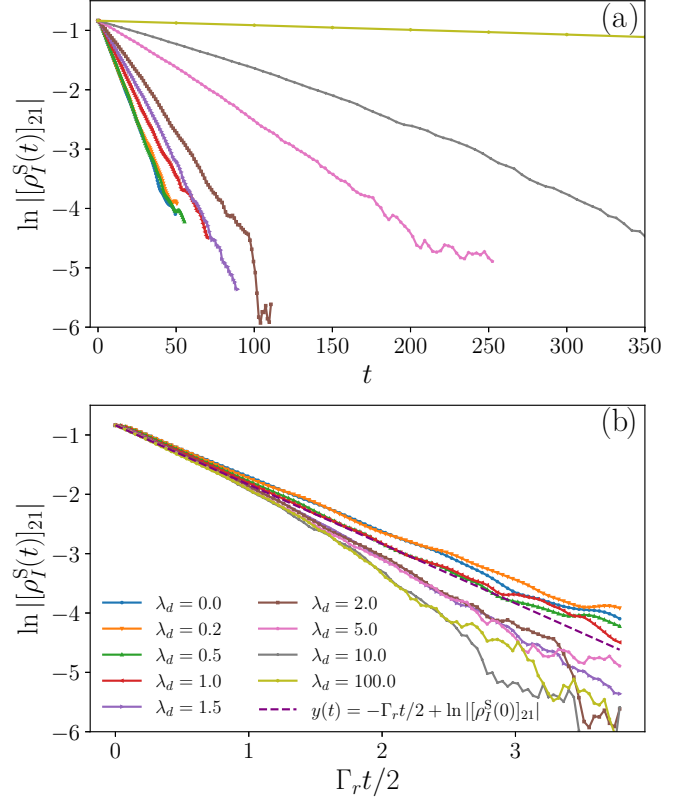


FIG. 4. Decoherence induced by the EX interaction (decoherence in the interaction picture): $\ln |[\rho_I^S(t)]_{21}|$ as a function of (a) t and (b) $\Gamma_r t/2$, for different λ_d , where Γ_r is given in Eq. (74). The dashed line in (b) indicates the analytical prediction $\ln |[\rho_I^S(t)]_{21}| = -\Gamma_r t/2 + \ln |[\rho_I^S(0)]_{21}|$. Here, $\lambda_r = 0.3$, $\omega = 0.05$, $d = 2^{14}$.

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APPENDIX A: DERIVATION OF REDUCED MOTION OF RDM IN THE INTERACTION PICTURE

In this Appendix, we show detailed derivations of the reduced equations of motion for the system in the interaction picture, where we start from Eq. (24) in the main text,

$$\frac{d}{dt} \rho_I^S(t) = -\frac{1}{d} \int_0^t ds \text{Tr}_B(\{H_I^{\text{int}}(t), [H_I^{\text{int}}(s), \rho_I^S(s)]\}). \quad (\text{A1})$$

For the convenience of the discussions below, we divide $\frac{d}{dt} \rho^S(t)$ into four terms as

$$\frac{d}{dt} \rho_I^S(t) = S_1 + S_2 + S_1^\dagger + S_2^\dagger, \quad (\text{A2})$$

where

$$S_1 = -\frac{1}{d} \int_0^t ds \text{Tr}_B\{H_I^{\text{int}}(t) H_I^{\text{int}}(s) \rho_I^S(s)\}, \quad (\text{A3})$$

$$S_2 = \frac{1}{d} \int_0^t ds \text{Tr}_B\{H_I^{\text{int}}(t) \rho_I^S(s) H_I^{\text{int}}(s)\}. \quad (\text{A4})$$

Here and in the rest of the Appendix, we omit the subscript I for simplicity.

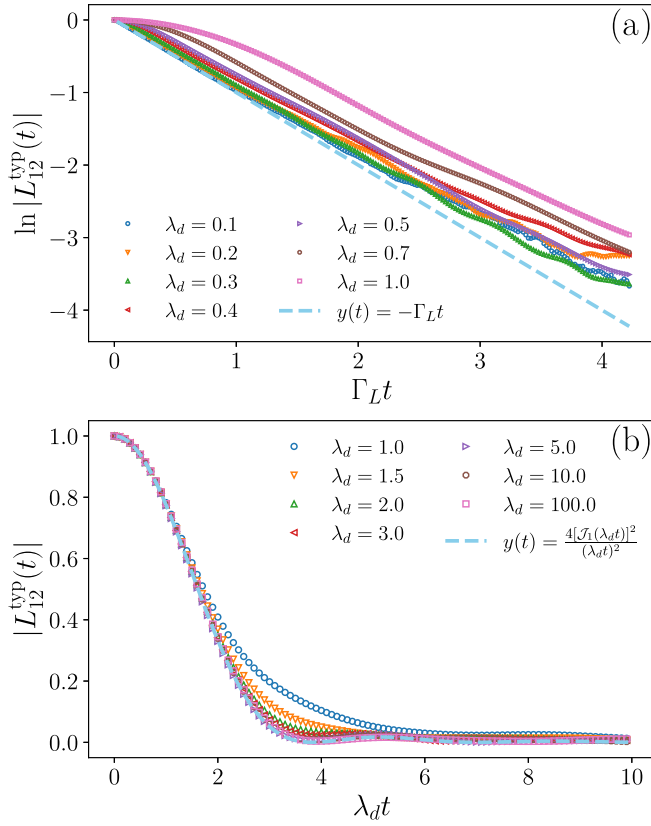


FIG. 5. Decoherence induced by the EC interaction: (a) $\ln|L_{12}^{\text{typ}}(t)|$ vs $\Gamma_L t$, (b) $|L_{12}^{\text{typ}}(t)|$ vs $\lambda_d t$, for different λ_d . The solid line in (a) shows the analytical prediction for $\lambda_d \ll 1$, $\ln(|L_{12}^{\text{typ}}(t)|) = -\Gamma_L t$, where $\Gamma_L = \frac{8}{3\pi}\lambda_d^2$. The dashed line in (b) indicates the analytical prediction for $\lambda_d \gg 1$, $|L_{12}^{\text{typ}}(t)| = \frac{4[\mathcal{J}_1(\lambda_d t)]^2}{(\lambda_d t)^2}$. Here, $\lambda_r = 0.3$, $\omega = 0.05$, $d = 2^{14}$.

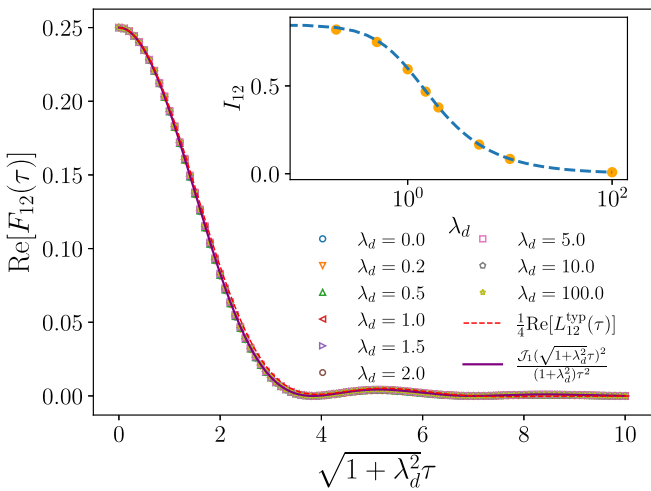


FIG. 6. The real part of correlation function $F_{12}(\tau)$ vs rescaled time $\sqrt{1 + \lambda_d^2} \tau$ for different λ_d for $\omega = 0.05$ and $d = 2^{14}$. The solid line indicates the analytical prediction in Eq. (71), and the dashed line shows the real part of the Loschmidt echo, $\frac{1}{4}\text{Re}[L_{12}^{\text{typ}}(t)]$, for $\lambda_d = 5$. The inset shows the infinite time integral $I_{12} = \int_{-\infty}^{\infty} F_{12}(\tau) d\tau$ where the dashed line indicates the analytical prediction in Eq. (73).

First we start from S_1 , which can be written as

$$S_1 = -\frac{1}{d} \int_0^t ds \text{Tr}_B \{H^{\text{int}}(t)H^{\text{int}}(s)\} \rho^S(s). \quad (\text{A5})$$

Inserting Eq. (19), one has

$$\begin{aligned} S_1 &= -\frac{1}{d} |\lambda_{12}|^2 |1\rangle\langle 1| \int_0^t ds \text{Tr}_B \\ &\quad \times \{U_1^\dagger(t-s)H_{12}U_2(t-s)H_{12}^\dagger\} \rho^S(s) \\ &\quad - \frac{1}{d} |\lambda_{12}|^2 |2\rangle\langle 2| \int_0^t ds \text{Tr}_B \\ &\quad \times \{U_2^\dagger(t-s)H_{12}^\dagger U_1(t-s)H_{12}\} \rho^S(s) \\ &= -|\lambda_{12}|^2 |1\rangle\langle 1| \int_0^t F_{12}(t-s) \rho^S(s) ds \\ &\quad - |\lambda_{12}|^2 |2\rangle\langle 2| \int_0^t F_{21}(t-s) \rho^S(s) ds, \end{aligned} \quad (\text{A6})$$

where $U_m(\tau) = \exp[-i(e_m + H_B + \lambda_{mm}H_{mm})\tau]$ and

$$F_{mn}(\tau) = \frac{1}{d} \text{Tr}_B \{U_m^\dagger(\tau)H_{mn}U_n(\tau)H_{mn}^\dagger\}. \quad (\text{A7})$$

Based on the definition of $F_{mn}(t)$, it is easy to see that

$$F_{mn}(\tau) = F_{nm}^*(\tau) = F_{nm}(-\tau). \quad (\text{A8})$$

Expanding $\rho^S(s)$ as

$$\rho^S(s) = \sum_{m,n=1}^2 \rho_{mn}^S(s) |m\rangle\langle n|, \quad (\text{A9})$$

and rewriting S_1 in a more concrete form, one gets

$$\begin{aligned} S_1 &= -|\lambda_{12}|^2 \\ &\quad \times \begin{bmatrix} \int_0^t F_{12}(t-s) \rho_{11}^S(s) ds & \int_0^t F_{12}(t-s) \rho_{12}^S(s) ds \\ \int_0^t F_{21}(t-s) \rho_{21}^S(s) ds & \int_0^t F_{21}(t-s) \rho_{22}^S(s) ds \end{bmatrix}, \end{aligned} \quad (\text{A10})$$

where $[S_1]_{mn} = \langle m|S_1|n\rangle$. Similarly, one has

$$\begin{aligned} S_2 &= |\lambda_{12}|^2 \\ &\quad \times \begin{bmatrix} \int_0^t F_{12}(t-s) \rho_{22}^S(s) ds & \int_0^t G_{12}(t,s) \rho_{21}^S(s) ds \\ \int_0^t G_{21}(t,s) \rho_{12}^S(s) ds & \int_0^t F_{21}(t-s) \rho_{11}^S(s) ds \end{bmatrix}, \end{aligned} \quad (\text{A11})$$

where

$$G_{mn}(t,s) = \frac{1}{d} \text{Tr}_B \{U_m^\dagger(t)H_{mn}U_n(t)U_m^\dagger(s)H_{mn}U_n(s)\}. \quad (\text{A12})$$

Before moving forward, one needs to estimate the correlation function $G_{mn}(t,s)$, where we consider $G_{21}(t,s)$ as an example, which can be written as

$$G_{21}(t,s) = \frac{1}{d} \text{Tr}_B \{U_{12}(s,t)H_{21}U_{12}(t,s)H_{21}\}, \quad (\text{A13})$$

where

$$U_{mn}(s,t) \equiv U_m(s)U_n^\dagger(t). \quad (\text{A14})$$

Denoting the eigenstate of $U_{12}(s, t)$ and $U_{12}(t, s)$ by $|k\rangle$ and $|k'\rangle$, respectively, $G_{21}(t, s)$ can be further rewritten as

$$G_{21}(t, s) = \frac{1}{d} \sum_{kk'} \langle k|U_{12}(s, t)|k\rangle \langle k'|U_{12}(s, t)|k'\rangle \times [H_{21}]_{kk'} [H_{21}]_{k'k}, \quad (\text{A15})$$

where

$$[H_{21}]_{kk'} \equiv \langle k|H_{21}|k'\rangle. \quad (\text{A16})$$

As H_{21} is not Hermitian, so in general cases $[H_{21}]_{kk'}$ and $[H_{21}]_{k'k}$ do not have strong correlations, and thus $R_{kk'} \equiv [H_{21}]_{kk'} [H_{21}]_{k'k}$ can be taken as random numbers with mean zero and variance $\frac{1}{d^2}$ for $k \neq k'$. At the same time, the diagonal elements R_{kk} scale as $R_{kk} \sim \frac{1}{d}$. Combining with the fact that the diagonal elements of $U_{12}(t, s)$ and $U_{12}(s, t)$ are of the order of 1, one has the following estimation for $G_{21}(t, s)$:

$$G_{21}(t, s) \sim \frac{1}{d} \left(\sum_k R_{kk} + \sum_{\substack{k, k' \\ k \neq k'}} R_{kk'} \right) \sim \frac{1}{d}. \quad (\text{A17})$$

Similarly, one has

$$G_{12}(t, s) \sim \frac{1}{d}. \quad (\text{A18})$$

If the Hilbert space dimension of the bath is sufficient large, the off-diagonal part of S_2 in Eq. (A11) can be neglected, which yields

$$S_2 = |\lambda_{12}|^2 \begin{bmatrix} \int_0^t F_{12}(t-s) \rho_{22}^S(s) ds & 0 \\ 0 & \int_0^t F_{21}(t-s) \rho_{11}^S(s) ds \end{bmatrix}. \quad (\text{A19})$$

Inserting Eqs. (A10) and (A19) into Eq. (A2), one has, for the diagonal elements,

$$\begin{aligned} \frac{d\rho_{11}^S(t)}{dt} &= -4|\lambda_{12}|^2 \int_0^t \text{Re}[F_{12}(t-s)] [\rho_{11}^S(s) - \rho_{11}^S(\text{eq})] ds, \\ \frac{d\rho_{22}^S(t)}{dt} &= -\frac{d\rho_{11}^S(t)}{dt}, \end{aligned} \quad (\text{A20})$$

as well as for off-diagonal elements,

$$\frac{d\rho_{21}^S(t)}{dt} = -2|\lambda_{12}|^2 \int_0^t \text{Re}[F_{12}(t-s)] \rho_{21}^S(s) ds, \quad (\text{A21})$$

where

$$\rho_{11}^S(\text{eq}) = \rho_{22}^S(\text{eq}) = \frac{1}{2}. \quad (\text{A22})$$

Under the condition that the correlation function $F_{12}(\tau)$ decays sufficiently fast on a time τ_B (correlation time), which is small compared to the relaxation time of the system τ_R , that is,

$$\tau_B \ll \tau_R, \quad (\text{A23})$$

one can employ the Markov approximation. As a result, one obtains Eq. (26), which is the Markovian master equation in the interaction picture.

APPENDIX B: GENERALIZATION OF EQ. (59) TO A N -LEVEL SYSTEM

In this Appendix, we discuss the relation between the Loschmidt echo $L_{mn}^{\text{typ}}(t)$ [Eq. (45)] and bath correlation function $F_{mn}(t)$ [Eq. (25)] in a more general setup, where a N -level system is coupled to a bath. The Hamiltonian reads

$$H = H_S + H_B + V, \quad (\text{B1})$$

$$V = \sum_{m,n=1}^N \lambda_{mn} |m\rangle \langle n| \otimes H_{mn}.$$

We employ the same assumption that H_{mn} are uncorrelated with each other. Eigenstates of H_S and H_B are denoted by $|m\rangle$ and $|E_k\rangle$,

$$H_S |m\rangle = e_m |m\rangle, \quad H_B |E_k\rangle = E_k |E_k\rangle. \quad (\text{B2})$$

Employing the modified Redfield theory, the unperturbed Hamiltonian and the perturbation are written as

$$H_0 = H_S + H_B + \sum_{m=1}^N |m\rangle \langle m| \otimes H_{mm},$$

$$H^{\text{int}} = \sum_{\substack{m,n=1 \\ m \neq n}}^N |m\rangle \langle n| \otimes H_{mn}. \quad (\text{B3})$$

The bath correlation function reads

$$F_{mn}(\tau) = \frac{1}{d} \text{Tr}_B \{ U_m^\dagger(\tau) H_{mn} U_n(\tau) H_{mn}^\dagger \}, \quad (\text{B4})$$

where

$$U_m(\tau) = e^{-i(e_m + H_B + \lambda_{mm} H_{mm})\tau}. \quad (\text{B5})$$

In the case of $\lambda_{mm} \gg 1$, the LE term $L_{mn}^{\text{typ}}(t)$ (which characterizes the decoherence process induced by EC interaction) can be written as

$$L_{mn}^{\text{typ}}(t) \simeq \frac{1}{d} \text{Tr}_B \{ e^{i\lambda_{mm} H_{mm} t} e^{-i\lambda_{nn} H_{nn} t} \}. \quad (\text{B6})$$

If H_{mn} are uncorrelated, following similar derivations as in Secs. III B and III C, one gets that

$$F_{mn}(t) \simeq \frac{1}{d^2} \text{Tr}_B \{ H_{mn} H_{mn}^\dagger \} \frac{1}{d} \text{Tr}_B \{ e^{i\lambda_{mm} H_{mm} t} \} \text{Tr}_B \{ e^{-i\lambda_{nn} H_{nn} t} \},$$

$$L_{mn}^{\text{typ}}(t) \simeq \frac{1}{d^2} \text{Tr} \{ e^{i\lambda_{mm} H_{mm} t} \} \text{Tr} \{ e^{-i\lambda_{nn} H_{nn} t} \}. \quad (\text{B7})$$

From Eq. (B7), one can see that

$$F_{mn}(t) \simeq \frac{1}{d^2} \text{Tr}_B \{ H_{mn} H_{mn}^\dagger \} L_{mn}^{\text{typ}}(t), \quad (\text{B8})$$

which is a generalization of Eq. (59) to a N -level system.

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