

Reply to “Comment on ‘Microscopic kinetic theory of the mean collision force of a particle moving in rarefied gases’ ”

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The relative speed distribution function [Eq. (2)] in the Comment is discussed. It shows that Eq. (2) in the Comment is not the distribution function that should be explored in our work and is therefore not applicable to our research.

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In his Comment on our paper, the author introduced a relative velocity distribution function which is different than ours, consequently, a different collision force of a particle moving through a thermal equilibrium rarefied gas was given [1,2]. However, we do not think the distribution functional given in the Comment is applicable to our work.

We agree with the Comment that the difference is derived from the *relative velocity distribution function of field atoms*, i.e., Eq. (2) in the Comment [1],

$$f_M(\mathbf{v} + \mathbf{v}_1) = \left(\frac{\mu}{2\pi T}\right)^{3/2} \exp\left[-\frac{\mu(\mathbf{v} + \mathbf{v}_1)^2}{2T}\right], \quad (1)$$

where $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass (the test particle and a gas molecule, with mass m_1 and m_2), and T is the gas temperature (energy units are used so that $k_B T \rightarrow T$). In our work, the mean collision force of a particle with a given velocity moving through the thermodynamic equilibrium gas was studied, then the velocity distribution function of the gas molecules relative to the test particle was investigated. If Eq. (1) is really “the relative velocity distribution function of field atoms” depicted in the Comment, it is not applicable to our work.

Additionally, there are several other issues in the Comment. The author deemed Eq. (1) can be obtained by shifting the following Eq. (1) in his Comment,

$$\begin{aligned} \frac{dN(v)}{N} &= \iint_{|\mathbf{v}_1 - \mathbf{v}_2| = \text{const}} f_1(\mathbf{v}_1) d\mathbf{v}_1 f_2(\mathbf{v}_2) d\mathbf{v}_2 \\ &= \int_{|\mathbf{v}| = \text{const}} \left(\frac{\mu}{2\pi T}\right)^{3/2} e^{-\frac{\mu \mathbf{v}^2}{2T}} d\mathbf{v} \int \left(\frac{m_1 + m_2}{2\pi T}\right)^{3/2} \\ &\quad \times e^{-\frac{(m_1 + m_2) \mathbf{V}^2}{2T}} d\mathbf{V} = 4\pi v^2 \left(\frac{\mu}{2\pi T}\right)^{3/2} e^{-\frac{\mu v^2}{2T}} dv. \quad (2) \end{aligned}$$

In his description, Eq. (2) is *the distribution of the relative velocities when a test particle is in thermal equilibrium with the gas* [1]. However, we do not think the shifting is tenable and some concerns are stated below. First, the physical meanings of some symbols used in the Comment are unclear: (i) \mathbf{v}_1 was depicted as the drifting velocity of the test particle in Eq. (1), whereas in Eq. (2) it was used to denote a velocity of the test particle with mass m_1 that is in thermal equilibrium with the gas. (ii) The physical meaning of $\mathbf{v} + \mathbf{v}_1$ in Eq. (1) is unclear. If $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$ as defined in the Comment, then $\mathbf{v} + \mathbf{v}_1 = 2\mathbf{v}_1 - \mathbf{v}_2$, which cannot be the relative velocity. (iii) \mathbf{v}_2 , N , and $dN(v)$ were not defined throughout the Comment. Second, Eq. (1) was depicted as the *relative velocity distribution function of field atoms*. It should be noted, however, that the relative velocity distribution function of field atoms is independent of the chosen reference frame under the Galilean transformation, and reads

$$f'_M(\mathbf{v}') = \left(\frac{\mu'}{2\pi T}\right)^{3/2} \exp\left[-\frac{\mu' \mathbf{v}'^2}{2T}\right], \quad (3)$$

where \mathbf{v}' is the relative velocity between two field atoms, and μ' is the reduced mass of the two field atoms. Note that the test particle is not involved here, then $\mu' = m_2^2 / (m_2 + m_2) = m_2/2$ is not μ that was used in Eq. (1). It easy to see that Eq. (1) is in conflict with Eq. (3), and the shifting from Eq. (2) to Eq. (1) is questionable.

Essentially, Eq. (1) is a distribution of two types of molecules in thermal equilibrium. However, the study in our paper is about the nonequilibrium case where the test particle is moving with a given velocity. So, Eq. (1) cannot be the distribution function we needed.

The relative velocity distribution studied in our paper can be derived as follows. As shown in Fig. 1, the relative velocity between the molecule and the test particle is $\mathbf{v}_r = \mathbf{v}_2 - \mathbf{V}_1$, where \mathbf{V}_1 is the velocity of the test particle (uppercase letters are used to avoid misunderstandings regarding the Comment),

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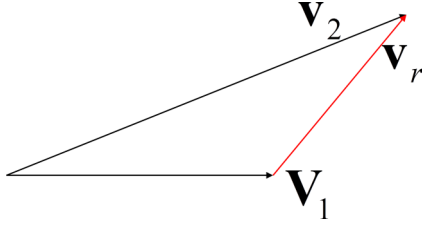


FIG. 1. Relations of \mathbf{V}_1 , \mathbf{v}_2 , and \mathbf{v}_r , where \mathbf{V}_1 is the velocity of the test particle in the laboratory frame, \mathbf{v}_r is the velocity of the molecule relative to the particle, and $\mathbf{v}_2 = \mathbf{V}_1 + \mathbf{v}_r$ is the velocity of the molecule in the laboratory frame.

and \mathbf{v}_2 is the velocity of the molecule with $\mathbf{v}_2 = \mathbf{v}_r + \mathbf{V}_1$. In the laboratory frame, the velocity distribution of the gas molecules is the Maxwell distribution

$$f(\mathbf{v}_2) = \left(\frac{m_2}{2\pi T}\right)^{\frac{3}{2}} \exp\left[-\frac{m_2 \mathbf{v}_2^2}{2T}\right] = \left(\frac{m_2}{2\pi T}\right)^{\frac{3}{2}} \exp\left[-\frac{m_2(\mathbf{v}_r + \mathbf{V}_1)^2}{2T}\right], \quad (4)$$

which is independent of the mass and the velocity of the test particle. In the nonrelativistic regime, we are going to assume that the number of background molecules in a velocity volume keeps the same value in all frames. If dN is the differential number of the molecules in differential volume $dv_x dv_y dv_z$ in velocity frame I (the laboratory frame), $dv'_x dv'_y dv'_z$ is the differential volume in frame II (the \mathbf{v}_1 frame) transformed from frame I, and dN' is the molecule number in $dv'_x dv'_y dv'_z$, then we have $dv'_x = dv_x$, $dv'_y = dv_y$, $dv'_z = dv_z$, and $dN' = dN$. The velocity distribution of the background molecule m_2 in the \mathbf{v}_1 frame is given by

$$f^{(t)}(\mathbf{v}_r) = \frac{dN'}{N dv'_x dv'_y dv'_z} = \frac{dN}{N dv_x dv_y dv_z} = f(\mathbf{v}_2) = \left(\frac{m_2}{2\pi T}\right)^{\frac{3}{2}} \exp\left[-\frac{m_2(\mathbf{v}_r + \mathbf{V}_1)^2}{2T}\right], \quad (5)$$

where N is the total molecule number. We can see that $f^{(t)}(\mathbf{v}_r)$ is independent with the mass of the reference object. Integrat-

ing $f^{(t)}(\mathbf{v}_r)$ over the solid angle, the speed distribution of the background molecules relative to the particle can be given as

$$\rho_{v_1}(v_r) = \frac{v_r}{V_1} \left(\frac{m_2}{2\pi T}\right)^{\frac{3}{2}} \left[e^{-\frac{m_2(V_1 - v_r)^2}{2T}} - e^{-\frac{m_2(V_1 + v_r)^2}{2T}} \right]. \quad (6)$$

It is Eq. (15) in our paper [2]. Based on Eq. (6), the general mean collision force in the present symbols and unit can be obtained as

$$\bar{f}_r = -n_2 \pi D^2 \mu \times \left\{ \left(V_1 + \frac{1}{V_1} \frac{T}{m_2} \right) \sqrt{\frac{2T}{\pi m_2}} e^{-\frac{m_2 V_1^2}{2T}} + \left[V_1^2 + \frac{2T}{m_2} - \frac{1}{V_1^2} \left(\frac{T}{m_2} \right)^2 \right] \operatorname{erf} \left(\sqrt{\frac{m_2}{2T}} V_1 \right) \right\}, \quad (7)$$

which is the result in our paper [Eq. (21)] [2], where D is the radius of the scattering cross section, and n_2 is the number density of the background molecules. The low- and high-speed limiting friction equations in our paper can be derived from the above general equation.

We must declare that the mean collision force Eq. (7) obtained in our paper [2] is a statistical result, and is the mean term of the random dynamics of the particle that is depicted by the Langevin equation. In fact, the test particle will acquire a thermal velocity (random) component during the collision process, so in this sense, Eq. (7) is not a complete derivation of the collision force. In the study of the dynamics of particles of large masses, our derivation can be used directly where the random component can be neglected; but in the study of the dynamics of small masses, the random force must be investigated and a full form Langevin equation is needed. In our next work on this subject, some statistical properties of the random collision force will be contributed.

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