Electrically driven kinklike distorting waves in microsized liquid crystals

S. S. Kharlamov,[*] D. V. Shmeliova,[†] and S. V. Pasechnik[‡]
Moscow Technological University (MIREA), Moscow 119454, Russia

P. V. Maslennikov[§]
Immanuel Kant Baltic Federal University, Kaliningrad 236040, Str. Universitetskaya 2, Russia

A. V. Zakharov[‖]
Saint Petersburg Institute for Machine Sciences, The Russian Academy of Sciences, Saint Petersburg 199178, Russia

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Electrically driven kinklike distortion regimes in a microsized liquid crystal channel have been investigated both experimentally and analytically. Kinklike distortion waves were excited by the interaction between the electric field **E** and the gradient $\nabla\hat{\mathbf{n}}$ of the director field in a homogeneously aligned liquid crystal (HALC) channel. Having obtained the evolution of the normalized light intensity, which was recorded by the high-speed camera, the process of excitation and evolution of the traveling wave in the HALC channel was visualized for the first time. It was shown, based on a nonlinear extension of the classical Ericksen-Leslie theory, that in the case when the electric field $E \gg E_{\mathrm{th}}$, the flow of liquid crystal material completely stops and a new mechanism for converting the electric field arises in the form of the electrically driven distorting traveling kinklike wave, which can be excited in the LC channel, composed of 4-n-pentyl-4′-cyanobiphenyl molecules.

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I. INTRODUCTION

In a large number of studies of liquid crystals (LCs), both experimental and theoretical, some features of deformations of these LC materials under the effect of a strong electric field have been demonstrated [1–11]. Some researchers have studied the processes of excitation of structured periodic domains in initially homogeneous LC systems under the effect of strong electric or magnetic fields [2,3,6–8]. Other researchers, on the contrary, excluded the possibility of the formation of periodic domains in initially homogeneous systems and considered the further evolution of these systems in the form of kink- and π-like running fronts in the homogeneously aligned LC volume [5,10,11]. Whatever the actual purpose, the condition for initiating of the excitation of periodic structures or kink- and π-like running fronts under the influence of strong external fields is important information for this LC system. It is necessary to understand the conditions that define the boundary between the excitation of a periodic pattern and the state when the excitation of this periodic structure is impossible. Knowing this, it is possible to predict the further behavior of this LC system under the influence of a strong electric or magnetic field. In turn, it is necessary to understand how the externally applied electric field can be converted to the running kinklike distortion wave $\theta(z - vt)$. Recently, the description of a new mechanism for converting the electric field **E** into additional pressure **P** has been proposed [10,12]. It was shown that the torques acting on the director $\hat{\mathbf{n}}$ may excite the kinklike pressure $P(z - vt)$ traveling wave spreading along the normal to both boundaries, whose resemblance to a kinklike wave increases with an increase of the applied electric field. It is shown that the kinklike deformation wave $\theta(z - vt)$ is responsible for the excitation of the kinklike pressure wave $P(z - vt)$ propagating along the normal to both boundaries under the effect of the external electric field **E** moving from an unstable to a stable state in the LC cell [5,10,12].

It should be noted that these results concerning the excitation of periodic structures or kink- and π-like running fronts in the LC microvolumes under the influence of strong external electric field were obtained numerically, based on the classical Ericksen-Leslie approach.

In turn, the main purpose of our article is to show experimentally how the externally applied electric field can be converted into the traveling kinklike distortion wave $\theta(z - vt)$, which was predicted analytically. Taking into account that the director reorientation in the homogeneously aligned nematic cell under the effect of the strong electric field takes place in the narrow area of the LC sample (the width of the traveling wave), the running wave will be visualized in polarized white light. Having obtained the evolution of the normalized light intensity, which was recorded by a high-speed camera capable of capturing grayscale video, the process of excitation and evolution of the traveling wave in

[*]semen.kharlamov.95@mail.ru
[†]shmeliova@mail.ru
[‡]s-p-a-s-m@mail.ru
[§]pashamaslennikov@mail.ru
[‖]Author to whom correspondence should be addressed: alexandre.zakharov@yahoo.ca; www.ipme.ru

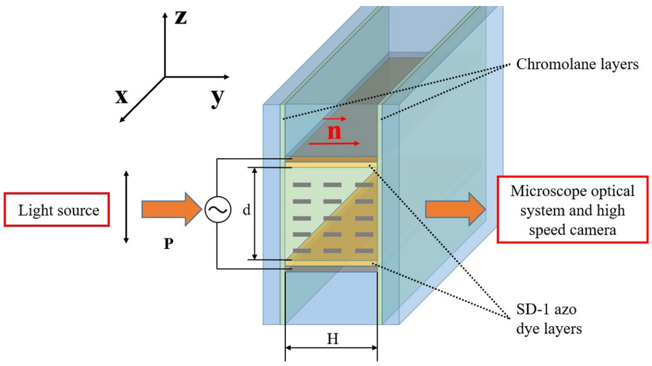

FIG. 1. A schematic view of the HALC cell.

the homogeneously aligned liquid crystal (HALC) film was visualized.

The outline of this article is as follows: the experimental study of the excitation of the traveling wave in the HALC cell under the effect of the externally applied electric field is given in Sec. II. The analytical approach to the description of the excitation and evolution of the traveling wave in a HALC cell is given in Sec. III. Experimentally and numerically obtained results for the relaxation regimes for the distortion field, in the form of the running kinklike wave is given in Sec. IV. Discussions and conclusions are summarized in Sec. V.

II. EXPERIMENTAL STUDY OF ELECTRICALLY DRIVEN DISTORTING TRAVELING WAVE IN HALC CELL

The main idea of the experimental study of the excitation and evolution of the electrically driven distorting traveling wave in a HALC cell is that the reorientation of the director field $\hat{\mathbf{n}}$ should occur in the narrow area of the microsized LC volume. Thus our attention will be focused on the description of the possibilities of formation of the traveling distortion wave in the microsized nematic cell, composed of 4-n-pentyl-4′-cyanobiphenyl (5CB) molecules, under the effect of the electric field $\mathbf{E}$, directed along the z axis (see Fig. 1). According to this geometry the director is maintained within the yz plane defined by the electric field and the unit vector $\hat{\mathbf{j}}$ directed parallel to the electrodes, while $\hat{\mathbf{k}}$ is a unit vector directed normally to the electrodes, respectively.

In this case, the uniform textures of nematic LCs are produced by orientation of a drop of bulk material in between two conveniently treated bounding surfaces, which defines usually a fixed orientation for the boundary molecules. Application of the electric field $\mathbf{E}$ perpendicularly to a uniformly (homogeneously) aligned nematic microvolume can distort the molecular orientation $\hat{\mathbf{a}}$ with respect to director $\hat{\mathbf{n}}$, at a critical threshold field E_{th} given by [13]

$$E_{\text{th}} = \frac{\pi}{d}\sqrt{\frac{K_1}{\epsilon_0\epsilon_a}}, \tag{1}$$

where d is the thickness of the microsized LC channel, K_1 is the splay elastic constant, ϵ_0 is the absolute dielectric permittivity of free space, and ϵ_a is the dielectric anisotropy of the nematic phase. This form for the critical field is based upon the assumption that the director remains strongly anchored (in our case, homogeneously) at the two electrodes and that the physical properties of the LC are uniform over the entire sample for $E < E_{\text{th}}$. When the electric field is switched on with a magnitude E greater than E_{th}, the director $\hat{\mathbf{n}}$, in the "splay" geometry, reorients as a simple monodomain [14], and excitation of the electrically driven nematic flow in the microfluidic channel is a question of great fundamental interest, as well as an essential piece of knowledge in soft material science [15].

In order to carry out the experimental study of the excitation and evolution of the traveling wave in the HALC cell under the effect of the electric field, the experimental setup was assembled, including a polarizing microscope, the experimental HALC cell, and a high-speed camera capable of capturing grayscale video at 20 000 frames per second. The upper and lower substrates were made of display glasses of thickness 1.1 mm with a spin-coated layer of 0.5% solution of chromium stearyl chloride (chromolane) in isopropanole with further heating at 90 °C for 1 h, which provides a homeotropic boundary orientation.

The spacers acting as electrodes were made of glass 350 μm thick (H in Fig. 1) with polished edges, on which a conductive coating was deposited, as well as a layer providing planar boundary orientation of the LC director along the y axis [it was made by spin coating 0.1% solution of azo dye (SD-1) in dimethyl formamide (DMF), baked at 90 °C for 1 h and treated by UV linear-polarized irradiation ($\lambda = 345$ nm) of exposure dose $D = 0.45\ \text{W/cm}^2$]. The distance d between spacers (see Fig. 1) was equal to 270 μm. Thus, the uniform orientation of the LC director $\hat{\mathbf{n}}_{z=0,d} \parallel \hat{\mathbf{j}}$ in the direction along the y axis was realized in the LC sample. The direction of light propagation coincides with the orientation of the LC director $\hat{\mathbf{n}}$. The experimental cell was filled with nematic liquid crystal 5CB in isotropic phase and then cooled. The experiment was carried out at room temperature 26 ± 0.5 °C. Thus, the threshold voltage $U_{\text{th}} = dE_{\text{th}}$ for our case is equal to ∼1.0 V. In order to cause the reorientation of the director $\hat{\mathbf{n}}$, an alternating voltage with an amplitude value of 250 V and a frequency of 50 kHz was applied along the z direction. Thus, one deals with the case when $U \gg U_{\text{th}}$ (or $E \gg E_{\text{th}}$). The HALC sample was illuminated with polarized white light. The transmission plane of the polarizer was codirected with the z axis, i.e., along the direction of the electric field $\mathbf{E}$.

In order to take into account the possible influence of the light irradiation on the observed process of fast reorientation of the LC director and the caused change in the transmitted light intensity, the experimental LC cell was preliminarily illuminated by the optical system of the microscope under conditions similar to those of the experiment. An analysis of the images obtained for about an hour showed no effect of light irradiation on the orientational structure of the liquid crystal.

In Figs. 2 and 3 are shown images of the LC channel after illumination with polarized white light corresponding to the different times $t = 0.05$ ms [Fig. 2(a)], $t = 0.10$ ms [Fig. 2(b)], $t = 0.20$ ms [Fig. 2(c)], $t = 0.30$ ms [Fig. 3(a)], $t = 0.40$ ms [Fig. 3(b)], and $t = 0.50$ ms [Fig. 3(c)], respectively.

In turn, in Figs. 4 and 5 are shown the normalized light intensity $\Omega(L,t) = [I(L,t) - I_0]/I_0$ in the vicinity of the

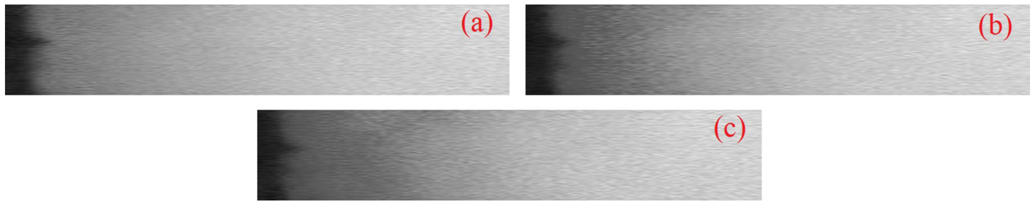

FIG. 2. Images of the LC channel after illumination with polarized white light at different times: (a) $t = 0.05$ ms, (b) $t = 0.10$ ms, and (c) $t = 0.20$ ms, respectively.

bounding surfaces, where L is the distance (in micrometers) counted from the right electrode, while t is time after turning on the electric field $\mathbf{E}$. As seen in Figs. 4 and 5, after turning on the electric field, there is a strong decrease in the normalized light intensity $\Omega(L, t)$ in the vicinity of the bounding surfaces after turning on the electric field $\mathbf{E}$. Indeed, at time of $t = 0.30$ ms [see Fig. 5(a)] the narrow domain with the sharp change in intensity $\Omega(L \approx 12\ \mu\text{m}, t = 0.3\ \text{ms})$ is clearly visible, which subsequently moves to the center of the LC channel. With the further increase in time, up to $t = 0.40$ ms [see Fig. 5(b)], the narrow area with the sharp change in intensity $\Omega(L \approx 14\ \mu\text{m}, t = 0.40\ \text{ms})$ is also clearly visible. This situation repeats after 0.50, 0.60, and 0.70 ms, respectively. Such an oscillation in the intensity of the transmitted light $\Omega(L, t)$ can be interpreted as the sharp reorientation of the director field $\hat{\mathbf{n}}$ in the narrow area of the LC volume. The processing of the obtained data allows us to determine the velocity of movement of this narrow area, which can be correlated with the movement of the kink-like traveling wave, under the effect of the electric field $\mathbf{E}$. The time dependence of the velocity w of the movement of the narrow area (traveling kinklike wave) (in μm/ms) under the effect of the voltage with the amplitude value of 250 V and frequency 50 kHz applied across the HALC cell is shown in Fig. 6. The measurement results show that over time, the velocity value w decreased from the value $w(t = 0.30\ \text{ms}) = 40\ \mu\text{m/ms}$ to $w(t = 0.70\ \text{ms}) = 10\ \mu\text{m/ms}$, respectively.

III. A THEORETICAL TREATMENT

We are primarily interested in the description of the physical mechanism responsible for the electrically driven excitation and evolution of the distorting traveling wave in the microsized HALC cell, composed of 5CB molecules, under the effect of the electric field $\mathbf{E}$, directed along the z axis (see Fig. 1). This description is fixed between the planar upper and lower electrodes. As a result, one arrives at the picture where there is a balance between the applied electric, viscous, elastic, and anchoring forces, and, in general, the LC fluid settles down to a stationary flow regime in the horizontal direction [7,12].

Assuming the incompressibility of the fluid, the hydrodynamic equations describing the orientational dynamics induced by electric field $\mathbf{E} = E\hat{\mathbf{k}}$ can be derived from the torque and linear momentum equations for such LC system. We consider a homogeneously aligned nematic system such as cyanobiphenyls, which is delimited by two electrodes at distance d on a scale in the order of tens of micrometers.

According to this geometry, the director is maintained within the yz plane defined by the electric field and the unit vector $\hat{\mathbf{j}}$ directed parallel to the electrodes, $\hat{\mathbf{k}}$ is a unit vector directed normal to these electrodes, and $\hat{\mathbf{i}} = \hat{\mathbf{k}} \times \hat{\mathbf{j}}$. Because we deal with the HALC channel under the influence of the electric field $\mathbf{E} = E(z)\hat{\mathbf{k}}$ directed perpendicular to the HALC channel, and taking into account that the length of the channel L is much bigger than the thickness d, we can suppose that the component of the director $\hat{\mathbf{n}} = n_y\hat{\mathbf{j}} + n_z\hat{\mathbf{k}} = \sin\theta(z,t)\hat{\mathbf{j}} + \cos\theta(z,t)\hat{\mathbf{k}}$ as well as the rest of the physical quantities also depend only on the z coordinate and time t. Here θ denotes the angle between the director $\hat{\mathbf{n}}$ and the unit vector $\hat{\mathbf{k}}$.

Our main aim is to investigate the influence of the external electric field $\mathbf{E}$ on the process of the director reorientation $\hat{\mathbf{n}}$ and electrically driven nematic flow $\mathbf{v}$ in the microfluidic HALC channel. In order to elucidate the role of the electric field $\mathbf{E}$ in the process of reorientation in the microsized HALC channel, we consider a regime when the director $\hat{\mathbf{n}}$ is strongly anchored to both electrodes, planarly to the upper and lower bounding surfaces, where

$$\theta(z)_{z=0} = \frac{\pi}{2}, \quad \theta(z)_{z=d} = \frac{\pi}{2}. \tag{2}$$

If the director is disturbed by the electric field $\mathbf{E}$, the relaxation of $\hat{\mathbf{n}}(z,t)$ to its stationary orientation $\hat{\mathbf{n}}_{eq}(z)$ in the HALC channel is governed by electric $\mathbf{T}_{\rm el} = \frac{\delta\psi_{\rm el}}{\delta\hat{\mathbf{n}}} \times \hat{\mathbf{n}}$, elastic $\mathbf{T}_{\rm elast} = \frac{\delta\mathcal{W}_{\rm F}}{\delta\hat{\mathbf{n}}} \times \hat{\mathbf{n}}$, and viscous $\mathbf{T}_{\rm vis} = \frac{\delta\mathcal{R}^{\rm vis}}{\delta\hat{\mathbf{n}}_{,t}} \times \hat{\mathbf{n}}$ torques exerted per unit LC's volume. Here $\mathcal{R} = \mathcal{R}^{\rm vis}$ is the viscous contribution to $\mathcal{R}$, and will be defined below, whereas $\mathcal{W}_{\rm F} = \frac{1}{2}[K_1(\nabla\cdot\hat{\mathbf{n}})^2 + K_3(\hat{\mathbf{n}}\times\nabla\times\hat{\mathbf{n}})^2]$ denotes the elastic energy

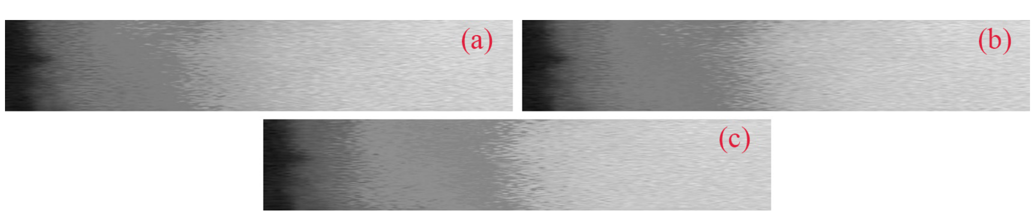

FIG. 3. Same as in Fig. 2, but at times (a) $t = 0.30$ ms, (b) $t = 0.40$ ms, and (c) $t = 0.50$ ms, respectively.

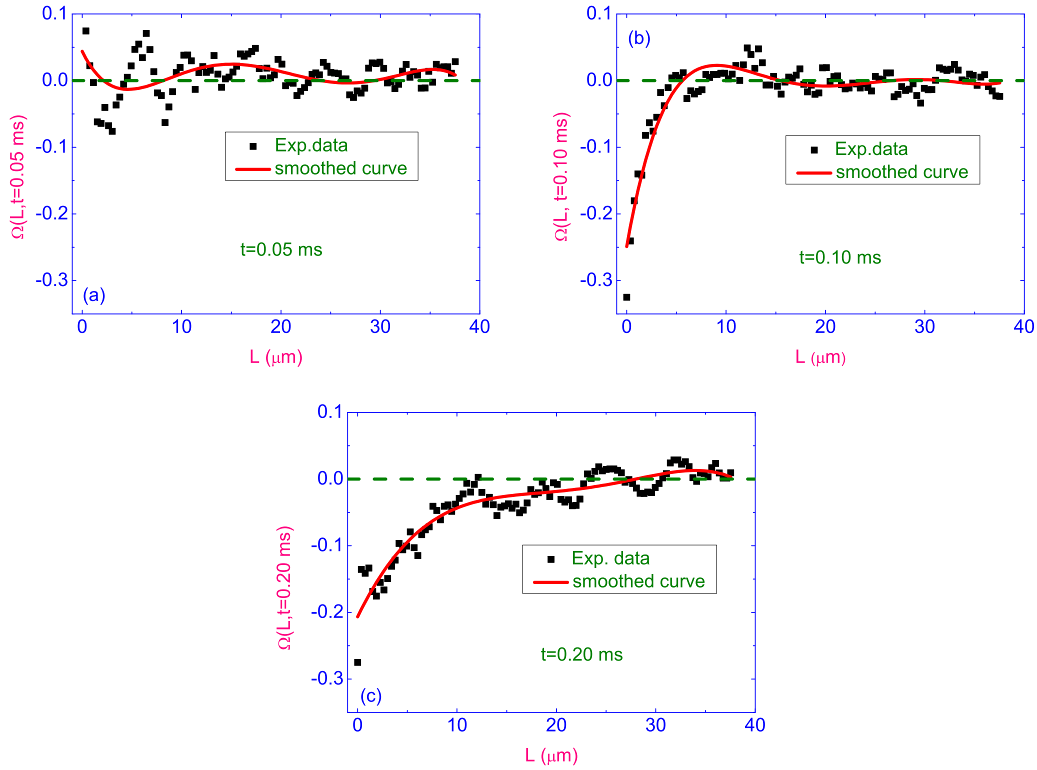

FIG. 4. The normalized intensity of $\Omega(L, t)$ depending on the coordinate L at different times: (a) $t = 0.05$ ms, (b) $t = 0.10$ ms, and (c) $t = 0.20$ ms, respectively. Solid lines show the smoothing by a five-order polynomial.

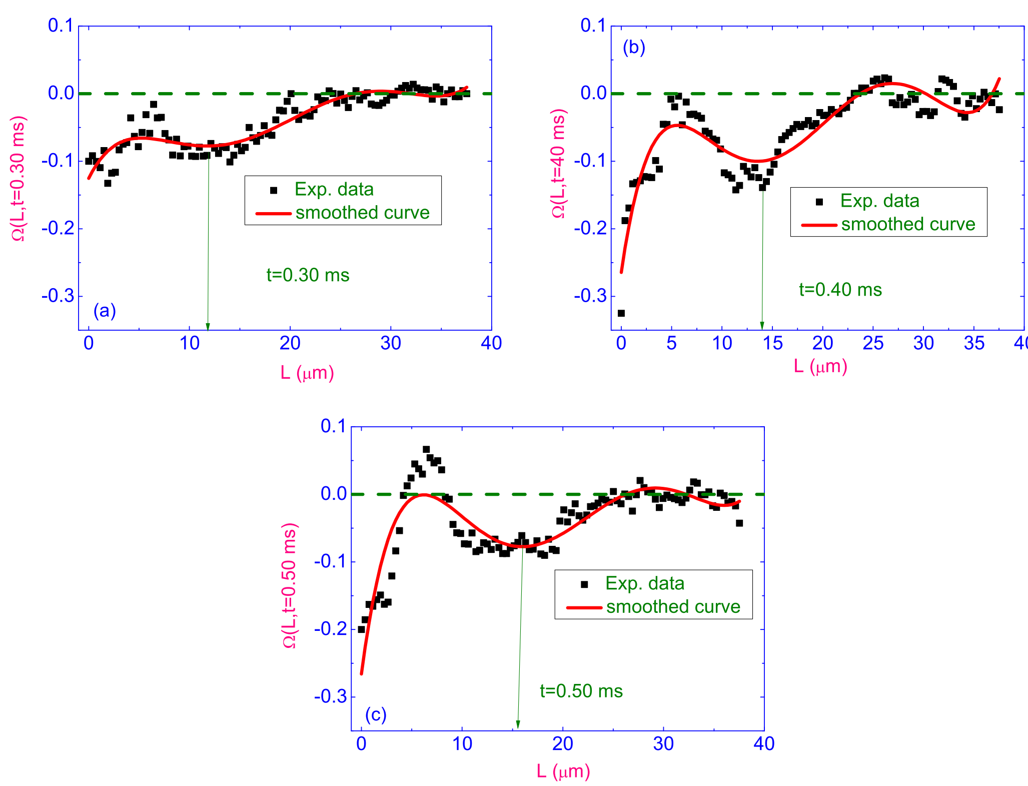

FIG. 5. Same as in Fig. 4, but at different times: (a) $t = 0.30$ ms, (b) $t = 0.40$ ms, and (c) $t = 0.50$ ms, respectively.

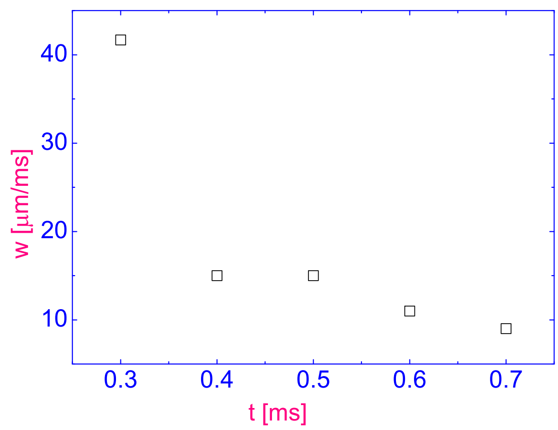

FIG. 6. Time dependence of the velocity of the movement of the narrow area w (traveling kinklike wave) under the effect of the voltage applied across the HALC cell.

density, K_1 and K_3 are splay and bend elastic coefficients, $\psi_{\rm el} = -\frac{1}{2}\epsilon_0\epsilon_a(\hat{\mathbf{n}}\cdot\mathbf{E})^2$ is the electric energy density, and $\hat{\mathbf{n}}_{,t} = \frac{d\hat{\mathbf{n}}}{dt}$ is the material derivative of the director $\hat{\mathbf{n}}$, respectively.

The application of the voltage across the nematic film results in a variation of $E(z)$ through the channel, which is obtained from [12]

$$\frac{\partial}{\partial z}\left[\left(\frac{\epsilon_\perp}{\epsilon_a} + \sin^2\theta(z,\tau)\right)\overline{E}(z)\right] = 0,$$

$$1 = \int_0^d \overline{E}(z)dz, \qquad (3)$$

where $\overline{E}(z) = \frac{E(z)}{E}$, $E = \frac{U}{d}$, and U is the voltage applied across the channel, $\epsilon_a = \epsilon_\parallel - \epsilon_\perp$, and $\epsilon_\parallel$ and $\epsilon_\perp$ are the dielectric constants parallel and perpendicular to the director, respectively. Note that the overbars in the electric field $\overline{E}$ will be eliminated in the following equations.

The hydrodynamic equations describing the reorientation of the LC phase in our case can be derived from the torque balance equation [7,12] $\mathbf{T}_{\rm el} + \mathbf{T}_{\rm elast} + \mathbf{T}_{\rm vis} = 0$, which has the form

$$\left[\frac{\delta\psi_{\rm el}}{\delta\hat{\mathbf{n}}} + \frac{\delta\mathcal{W}_{\rm F}}{\delta\hat{\mathbf{n}}} + \frac{\delta\mathcal{R}^{\rm vis}}{\delta\hat{\mathbf{n}}_{,\rm t}}\right]\times\hat{\mathbf{n}} = 0. \qquad (4)$$

The linear momentum equation for the velocity field $\mathbf{v}$ can be written as [7,12]

$$\rho\frac{d\mathbf{v}}{dt} = \boldsymbol{\nabla}\cdot\sigma, \qquad (5)$$

where $\sigma = \sigma^{\rm elast} + \sigma^{\rm vis} - \mathcal{P}\mathcal{I}$ is the full stress tensor (ST), and $\sigma^{\rm elast} = -\frac{\partial\mathcal{W}_{\rm F}}{\partial\boldsymbol{\nabla}\hat{\mathbf{n}}}\cdot(\boldsymbol{\nabla}\hat{\mathbf{n}})^{\rm T}$ and $\sigma^{\rm vis} = \frac{\delta\mathcal{R}^{\rm vis}}{\delta\boldsymbol{\nabla}\mathbf{V}}$ are the ST components corresponding to the elastic and viscous forces, respectively. Here $\mathcal{P}$ is the hydrostatic pressure in the HALC system and $\mathcal{I}$ is the unit tensor, respectively.

Taking into account the microsize of the HALC channel, one can assume the mass density ρ to be constant over the LC volume, and thus we can deal with an incompressible fluid. The incompressibility condition $\boldsymbol{\nabla}\cdot\mathbf{v} = 0$ assumes that only one nonzero component of the vector $\mathbf{v}$ exists, viz., $\mathbf{v}(z,t) = u(z,t)\hat{\mathbf{j}}$.

To be able to observe the evolution of the director field $\hat{\mathbf{n}}$ [or the angle $\theta(z,t)$] to its stationary orientation $\hat{\mathbf{n}}_{\rm st}(z)$, and excitation of the velocity field $\mathbf{v}(z,t)$ caused by the external electric field, we consider the dimensionless analog of the torque and linear momentum balance equations.

The dimensionless torque balance equation describing the reorientation of the LC phase can be written as [7,12]

$$\theta_{,\tau} = \mathcal{A}(\theta)u_{,z} + \Delta\left[[\mathcal{G}(\theta)\theta_{,z}]_{,z}]_{,z} - \frac{1}{2}\mathcal{G}_{,\theta}(\theta)\theta_{,z}^2\right] - \frac{E^2(z)}{2}\sin 2\theta, \qquad (6)$$

where $\mathcal{A}(\theta) = \frac{1}{2}(1-\gamma_{21}\cos 2\theta)$, $\mathcal{G}(\theta) = \sin^2\theta + K_{31}\cos^2\theta$, $\mathcal{G}_{,\theta}(\theta)$ is the derivative of $\mathcal{G}(\theta)$ with respect to θ, $\theta_{,z} = \partial\theta(z,\tau)/\partial z$, $\Delta = (\frac{E_{\rm th}}{\pi E})^2$, $\gamma_{21} = \gamma_2/\gamma_1$, γ_1 and γ_2 are the rotational viscosity coefficients, $K_{31} = K_3/K_1$, K_1 and K_3 are the splay and bend elastic constants of the LC phase, $\tau = (\epsilon_0\epsilon_a E^2/\gamma_1)t$ is the dimensionless time, $\overline{z} = z/d$ is the dimensionless distance measured from the lower electrode, and $\overline{u} = (\frac{\gamma_1}{d\epsilon_0\epsilon_a E^2})u$ is the dimensionless velocity. Notice that the overbars in the space variable z and velocity u have been eliminated. In the case of incompressible fluid the dimensionless Navier-Stokes equation reduces to [7,12]

$$\frac{1}{\Delta}\delta_1 u_{,\tau}(z,\tau) = [\overline{h}(\theta)u_{,z} - \overline{\mathcal{A}}(\theta)\theta_{,\tau}]_{,z}. \qquad (7)$$

Here $\mathcal{R}(z,\tau) = \frac{\gamma_1 d^4}{K_1^2}\mathcal{R}(z,t)$ is the full dimensionless Rayleigh dissipation function, where $\mathcal{R}(z,t) = \mathcal{R}^{\rm vis}$, and $\mathcal{R}^{\rm vis} = \frac{1}{2}h(\theta)u_{,z}^2 - \overline{\mathcal{A}}(\theta)\theta_{,t}u_{,z} + \frac{1}{2}\gamma_1\theta_{,t}^2$ is the viscous contribution. Here $h(\theta) = \alpha_1\sin^2\theta\cos^2\theta - \overline{\mathcal{A}}(\theta) + \frac{1}{2}\alpha_4 + g(\theta)$, $\overline{\mathcal{A}}(\theta) = \gamma_1\mathcal{A}(\theta)$, $g(\theta) = \frac{1}{2}(\alpha_6\sin^2\theta + \alpha_5\cos^2\theta)$, $u_{,z} = \partial u(z,t)/\partial z$, $\theta_{,z} = \partial\theta(z,t)/\partial z$, and $\delta_1 = \rho K_1/\gamma_1^2$ is a parameter of the system. Here α_i $(i = 1,\ldots,6)$ are six Leslie coefficients. The stress tensor component σ_{zx} is given by [12] $\sigma_{zx}(\tau) = \frac{\delta\mathcal{R}(\tau)}{\delta u_{,z}} = \overline{h}(\theta)u_{,z} - \mathcal{A}(\theta)\theta_{,\tau}$, where $\overline{h}(\theta) = h(\theta)/\gamma_1$.

In order to elucidate the role of the electric field $\mathbf{E}$ in the process of reorientation in the microsized HALC channel, we consider a regime when the director $\hat{\mathbf{n}}$ is strongly anchored to both electrodes, parallel to the upper and lower bounding surfaces, where

$$\theta(z)_{z=0} = \frac{\pi}{2}, \quad \theta(z)_{z=1} = \frac{\pi}{2}, \qquad (8)$$

and its initial orientation is perturbed parallel to the interface, with

$$\theta(z,\tau=0) = \frac{\pi}{2}, \qquad (9)$$

and then allowed to relax to its stationary distribution $\theta_{\rm st}(z)$ across the microsized HALC channel.

The velocity on these surfaces has to satisfy the no-slip boundary condition

$$u(z)_{z=0} = 0, \quad u(z)_{z=1} = 0. \qquad (10)$$

Now the reorientation of the director in the microsized HALC channel confined between two electrodes, when the relaxation regime is governed by the viscous, electric, and elastic forces, and with accounting the flow, can be obtained by solving the system of the nonlinear partial differential equations (3), (6), and (7), with the appropriate boundary

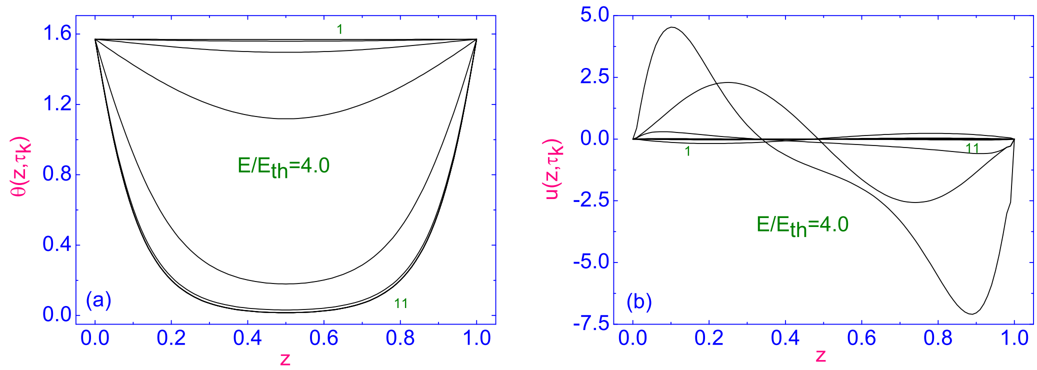

FIG. 7. (a) Plot of evolution of the angle $\theta(z, \tau_k)$ to its stationary distribution across the HALC microfluidic channel, under the effect of the electric field $E/E_{th} = 4.0$, at different times $\tau_k = \Delta\tau(k-1)$ $(k = 1, \dots, 11)$, respectively. (b) The same as in (a), but for the evolution of the dimensionless velocity $u(z, \tau_k)$ to its stationary distribution across the HALC channel.

conditions both for the angle $\theta(z, \tau)$ [Eqs. (8)] and the velocity $u(z, \tau)$ [Eqs. (10)], as well as with the initial condition [Eqs. (10)].

IV. NUMERICAL AND EXPERIMENTAL RESULTS FOR RELAXATION MODES IN THE HALC CHANNEL

Accounting the electric field $\mathbf{E} = E\hat{\mathbf{k}}$ directed across the homogeneously aligned nematic channel leads to reorientation of the director field $\hat{\mathbf{n}}$, being initially directed parallel to the electrodes, to be parallel to $\mathbf{E}$. Thus, in initially homogeneously aligned LC volume, the hybrid-aligned microsized domain may arise, with nonzero gradient of the director field $\nabla\hat{\mathbf{n}}$.

Case $E > E_{th}$

In the case when the electric field is equal to $E/E_{th} = 4.0$, the evolution of the director field $\hat{\mathbf{n}}$ to its stationary orientation $\hat{\mathbf{n}}_{st}$ in the microsized HALC channel, which is described by the angle $\theta(z, \tau_k)$, at different times $\tau_k = \Delta\tau(k-1)$ $(k = 1, \dots, 11)$, is shown in Fig. 7(a). Here $\Delta\tau = 0.05$ and $\tau_{11}(E/E_{th} = 4.0) = 0.5$ ($\sim$2.25 ms). In the calculations, by means of the numerical relaxation method [16], the relaxation criterion $\epsilon = |[\theta_{(m+1)}(z, \tau) - \theta_{(m)}(z, \tau)]/\theta_{(m)}(z, \tau)|$ was chosen to be equal to 10^{-4}, and the numerical procedure was then carried out until a prescribed accuracy was achieved. Here m is the iteration number.

The evolution of the dimensionless velocity field $u(z, \tau) = (\frac{\gamma_1}{d\epsilon_0\epsilon_a E^2})v_x(z, \tau)$ to its stationary distribution $u(z, \tau_{11})$ across the microsized HALC channel, at different times τ_k $(k = 1, \dots, 11)$, under the effect of the electric field $E/E_{th} = 4.0$, is shown in Fig. 7(b).

The highest velocity value $|u_{max}(E/E_{th} = 4.0)| \sim 7.3$ ($\sim 3.24 \times 10^{-3}$ m/s) is reached near the upper ($z = 0.8$) electrode.

Our calculations have shown that the dependence of the maximum value of the absolute equilibrium velocity $|u_{max}(E/E_{th})|$ on the value of electric field E/E_{th} is characterized by the monotonic increase of $|u_{max}(E/E_{th})|$ up to a maximum value 1.66 at $E/E_{th} \sim 2.0$, whereas the further increase of the value of E/E_{th} leads to the decrease in $|u_{max}(E/E_{th})|$ (see Fig. 8). Such behavior of $|u_{max}(E/E_{th})|$ vs E/E_{th} can be explained by the rapid growth of the coefficient $1/\Delta = (\frac{\pi E}{E_{th}})^2$, in the left-hand part of Eq. (7), with the growth of E/E_{th}. In this case, the contribution of electric forces prevails over the contributions of viscous, elastic, and anchoring forces. In this case, any flow of the LC phase stops in the microsized HALC channel, since under the influence of strong external electric field $\mathbf{E}$ the dipoles of molecules forming the LC phase are oriented along this field.

It should be noted that the microscale observation of the velocity field $u(E/E_{th})$ excited by the electric field $E/E_{th} \in [1, 20]$ can be performed by tracking a single tracer of a nanoparticle [17]. Now we will consider the case $E \gg E_{th}$; since $U_{th} \sim 1$ V, this is much less than the value of 250 V applied across the HALC cell.

Case $E \gg E_{th}$

In the case when the electric field E is much greater than E_{th}, the evolution of the velocity field $u(z, \tau)$ in the microsized HALC channel is described by the reduced dimensionless Navier-Stokes equation (7), which can be written as

$$\lim_{E\to\infty} \frac{1}{\Delta}\delta_2 u_{,\tau}(z, \tau) \to \infty, \tag{11}$$

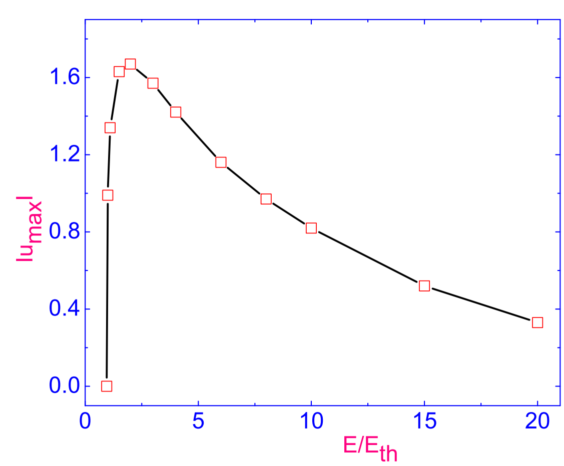

FIG. 8. Dependence of the maximum value of the absolute stationary velocity $|u_{max}(E/E_{th})|$ on the value of electric field E/E_{th}.

where $1/\Delta = (\frac{\pi E}{E_{\text{th}}})^2$. In this case when $\lim_{E\to\infty} \frac{1}{\Delta} \to \infty$, one has, with accounting the no-slip boundary condition [see Eqs. (10)], that $u(z,\tau) = 0$. As a result, in the case when $E \to \infty$, the dimensionless torque balance equation (6) is reduced to equation

$$\theta_{,\tau}(z,\tau) = -\frac{E^2(z)}{2}\sin 2\theta(z,\tau). \tag{12}$$

By substitutions $\overline{\theta} = 2\theta$ and $\beta = E^2\tau$, the last equation takes the form

$$\overline{\theta}_{,\beta}(z,\beta) = -\sin\overline{\theta}. \tag{13}$$

There is exact solution of Eq. (13):

$$\overline{\theta}(z,\beta) = \tan^{-1}[\sinh^{-1}(w\beta - z + z_0)], \tag{14}$$

where z_0 is a constant, and w is the solitary wave velocity along the axis z. Indeed, $\partial_\beta\overline{\theta}(z,\beta) = -\cosh^{-1}(w\beta - z + z_0)$, and taking into account the relation $\tan\overline{\theta}(z,\beta) = \sinh^{-1}(w\beta - z + z_0) = \mathcal{A}$, one has that $\sin\overline{\theta}(z,\beta) = \mathcal{A}/\sqrt{1+\mathcal{A}^2} = \cosh^{-1}(w\beta - z + z_0)$. Finally, one can rewrite Eq. (12) in the following way: $\partial_\beta\overline{\theta}(z,\beta) = -\cosh^{-1}(w\beta - z + z_0) = -\sin\overline{\theta}(z,\beta)$. Solution (14) describes the solitary kink $\overline{\theta}(z,\beta)$ which is spreading along the z axis with the velocity

$$w(z,t) = \frac{E^2\overline{E}(z)d\epsilon_a\epsilon_0}{\gamma_1} = \frac{1}{\Delta}\frac{K_1}{d\gamma_1}\overline{E}(z), \tag{15}$$

where $\overline{E}(z) = E(z)/E$, $E = U/d$, and U is the voltage applied across the LC channel. Physically, this means that in the case of $E \gg E_{\text{th}}$, directed across the HALC channel, the director field $\hat{\mathbf{n}}$ has initially been disturbed, for instance, at the bottom of the LC channel, with the condition $\theta(z_0 = 0, \tau = 0) = \frac{\pi}{2}$, and that disturbance must propagate in the form of the traveling kinklike wave along the z axis with the velocity w.

V. DISCUSSIONS AND CONCLUSION

In order to compare the experimentally obtained results for the velocity w of movement of the narrow area (traveling kinklike wave) under the effect of the voltage applied to the HALC cell with the theoretically predicted results, we used a simple model in which the total gap of the cell d can be represented as $d = L_\parallel + L_\perp$, where $L_\parallel$ is the distance at which the LC molecules have already reoriented in the direction of the electric field $\mathbf{E}$, and $L_\perp$ is the distance, at which the LC molecules have not yet reoriented.

Using the two-layer model of series-connected dielectrics (Fig. 9), it is possible to write simple expressions for the total voltage drop on the LC cell, as well as for the distribution of the field strength $E > E_{\text{th}}$.

$$U = E_\parallel L_\parallel + E_\perp L_\perp,$$
$$E_\parallel\epsilon_\parallel = E_\perp\epsilon_\perp. \tag{16}$$

Taking into account that at the initial moments of time the director's reorientation occurs in a small area near the wall, it

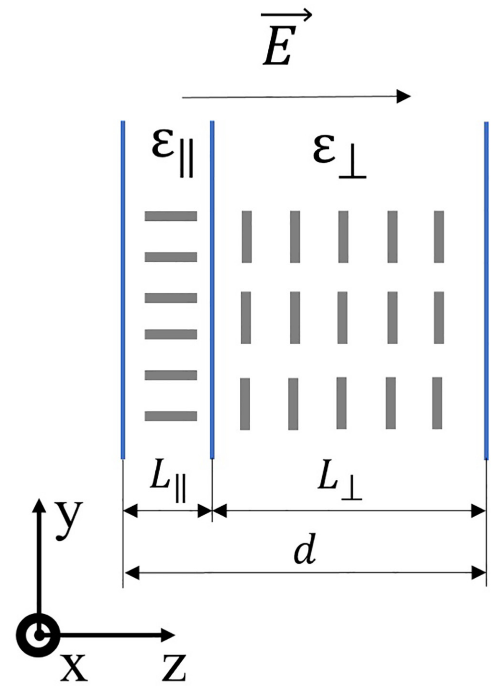

FIG. 9. Model of series-connected dielectrics.

can be assumed that $L_\parallel \gg L_\perp$ and, consequently,

$$E = E_\perp \approx \frac{U}{d},$$
$$E(z) = E_\parallel \approx \frac{\epsilon_\perp}{\epsilon_\parallel}E. \tag{17}$$

For this purpose, we will consider a sample of 5CB, at the temperature 299.5 K and density 10^3 kg/m^3, confined between two bounding electrodes at the distance of $d = 270$ μm. The measured value of the elastic constant K_1 of this compound is equal to 10.0 pN [18], the calculated value of the dielectric anisotropy is equal to $\epsilon_a = 11.5$ [19], while the experimental value of the rotational viscosity coefficient γ_1 is equal to 0.077 Pa s [20], respectively. Using the parameters of the 5CB liquid crystal material, the size between the electrodes and inserting Eqs. (16) and (17) into Eq. (15), we calculated the value of the wave velocity w, equal to 0.056 m/s. The maximum experimentally obtained value of w at the initial moments of time is equal to 0.042 m/s. The experimentally obtained value of the velocity of the traveling kinklike wave is close to the value calculated theoretically within the framework of the model used above. Physically, this means that in the case of $E \gg E_{\text{th}}$, directed across the HALC channel, distortion of the director field $\hat{\mathbf{n}}$, under the effect of the externally applied electric field must propagate in the form of the kinklike wave across the LC channel.

Finally, we can conclude that both by experimentally—using the setup including a polarizing microscope, a HALC cell, and a high-speed camera capable of capturing grayscale video at 20 000 frames per second—and theoretically—based upon the nonlinear extension of the classical Ericksen-Leslie theory—the excitation and evolution of the running kinklike wave has been visualized. Having obtained the evolution of the normalized light intensity, which was recorded by this high-speed camera, the process of excitation and evolution of the traveling wave in the HALC was visualized. Theoretically, it was shown that in the case when the electric field $E \gg E_{\mathrm{th}}$, the flow of liquid crystal material completely stops and a novel mechanism for converting of the electric field arises in the form of the electrically driven distorting traveling kinklike wave which can be excited in the LC channel.

This once again shows that the macroscopic description of the nature of the hydrodynamic flow of an anisotropic liquid subtly senses the microscopic structure of the LC material.

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