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Interplay of self, epiphany, and positive actions in shaping individual careers

Rafael A. Bittencourt,[1,2,*] H. B. de B. Pereira,[1,3] M. A. Moret,[1,3] S. Galam,[4,5] and I. C. da Cunha Lima[1,2]

[1]*Centro Universitário SENAI-Cimatec, Salvador, 41650-010 Bahia, Brazil*
[2]*Instituto Nacional de Ciência e Tecnologia–Geofísica do Petróleo, Salvador, 40170-290 Bahia, Brazil*
[3]*Bahia State University, Salvador, 41180-045 Bahia, Brazil*
[4]*Centre for Political Research, 75007 Paris, France*
[5]*Centre National de la Recherche Scientifique, 75016 Paris, France*

(Received 1 May 2023; accepted 10 July 2023; published 29 August 2023)

In this work, we model an individual social career by a finite-size trajectory along a hexagonal lattice moving only forward. At each bifurcation, the individual makes a free-will choice to follow one or the other branch within an uncertain outcome. Considering that those choices are determined by an individual self built from endogenous characteristics, we assume they are random following a binomial distribution. As a result, the individual ascends or descends on the social scale via random progress through the series of bifurcations made at the encountered junctions. The related stochastic process is found to be diffusive. For different selves coming from different points on the social scale, progress does overlap. In addition, we include the possibility of continuous transition across the lattice caused external influences as an epiphany. The occurrence of a quantum leap resulting from an affirmative action opportunity is also included. We also treat the case of a social group being acted by a collective epiphany as with education. The results highlight the key effect of epiphanies and quantum leaps to promote upward mobility across social classes.

DOI: 10.1103/PhysRevE.108.024314

I. INTRODUCTION

In this paper, we address social mobility by modeling the course of an individual's career through the successive decisions made by the person concerned when professional opportunities arise throughout their career. The paths are embedded in a hexagonal lattice, with only forward movements. At each bifurcation, the individual choice to go one way or another is supposed to be determined by a self built from endogenous characteristics. We thus assume the choice to be a random process monitored by a binomial distribution. The series of choices made at all encountered bifurcations frame the person's career.

On this basis, we consider the occurrence of epiphanies, which are defined as influences of a person or an event on the individual assessment of the world, which modifies the probability of choosing either of the two branches at the bifurcation. The epiphany value can be a positive, meaning, for example, that an individual will now make better choices in its life, or negative bias, which can represent the rejection after the entrance in a specific social group.

In addition, we include the effect of affirmative action on an individual career. In our model, it is represented by a sudden change in the social scale produced by a vertical transition on the hexagonal lattice, without passing by intermediate nodes. The related individual is moved upward, bypassing the structure of the hexagonal lattice.

While we are using a hexagonal lattice to describe the stochastic process of the dynamics of the individual progress through different routes on the social career, other types of networks could, in principle, be used with an appropriate change of the model dynamics. The hexagonal lattice has the advantage of providing self-avoiding, once back-scattering is forbidden. The fact that junctions have their inward paths at left and their outward path to the right, while the reverse occurs for the bifurcations, prevent their creating backward progress when limited to left-to-right is represented by the hexagonal lattice. There is no need to provide an additional mechanism for self-avoiding routes. This property is relevant for our purpose. Usually, self-avoiding random walks must be treated by imposing restrictions to make the progress independent of the history. If this were included, then the stochastic process would become non-Markovian. Camilleri *et al.* [1] treat the case of one-dimension quantum walks. Another important property of the hexagonal lattice to represent the individual social career is the fact that the number of steps necessary to cross the web does not depend on the particular route to be followed.

We think the model is applied, adapting the external actions to the specific system, for situations involving decision trees (e.g., organizational flow, rules/knowledge base in fuzzy models [2], educational games [3], analysis of diagnoses in healthcare [4,5], financial fraud [6], etc. Our work subscribes to the field of Sociophysics [7–11].

The paper is organized as follows. In Sec. II we review the concepts of self and affirmative action policies. In Sec. III we review hexagonal lattices. In Sec. IV we identify the progress through the lattice as a diffusive process generated by a

*almeidabittencourt@gmail.com

binomial distribution and calculate the probability density function (PDF) of this Markovian stochastic process. We also obtain the influence of the number of choices taken by an element during its lifetime, and compare its progress with another element starting at a possible higher position on the social layer, who takes choices more frequently. In Sec. V we interfere in the progress by changing the choice probability at a position in the lifetime with an epiphany. The change is included in our model by applying a bias to the choice probability. While a bias may change through the social career, we consider it as a constant in our calculation. With a bias, the moments of the PDF change at that node, changing as well the overlap with symmetric progresses, starting at a higher level. We also explore vertical transitions to higher levels together or not with epiphanies and discuss how handling a bias on the choice probability and the height and node of the transition as control parameters contributes to an increase in the overlap region corresponding to the coexistence of elements starting from different points on the social layer. In Sec. VI we consider how a collective epiphany, called there education, acting in a way to invert the skewness of a group distribution of choice probability, leads the individual to progress in the most favorable zone allowed to their social career. In Sec. VII we discuss several aspects of our model and present our conclusions in Sec. VIII.

II. ABOUT SELF AND AFFIRMATIVE ACTION POLICIES

Self, a term widely used in social psychology, refers to an individual capacity for self-reflection, communication, and decision-making [12]. The self qualifies an individual's—a child or an adult—ability to think, communicate, and make decisions by himself [13]. The development of the self is shaped not only by genetic factors but also by environmental and social factors, such as family upbringing, education, and cultural background [12]. In the words of José Ortega y Gasset, emphasizing the inseparable relationship between the individual and the social environment, *"Yo soy yo y mi circunstancia"* (I am I and my circumstance) [14].

The concept of self has been the subject of much research in social psychology in recent years. Studies have explored various aspects of the self, including its structure, development, and impact on behavior [15]. One influential theory of self, proposed by William James, suggests that the self consists of two parts: the I, which is the subject of experience, and the me, which is the object of experience [16]. More recent research has focused on the dynamic nature of the self, emphasizing how the self evolves through social interaction and experience [17]. This dynamic view of the self has led to a growing interest in the study of how individuals make choices and navigate opportunities in their lives. The self, however, is not static, by the very interaction of the individual with the other members of the society, as it progresses in his education, or as it builds his culture. The concept of social career refers to the trajectory of an individual progression through various social roles and positions, influenced by factors such as education, religion, family background, and environment. The social career is a dynamic and complex process that shapes the development of an individual's self and decision-making abilities [18]. Social careers are also influenced by social structures and institutions [19]. The term "epiphany" describes sudden and profound realizations that can change an individual's perspective on themselves and the world around them. These changes may come from either a powerful influence of another individual, a religious change, a recovering from a severe illness, a sudden change of environment, etc. In social careers, epiphanies can significantly alter an individual trajectory and decision-making processes [20]. Recent research has shed light on the mechanisms underlying epiphanies and their impact on decision-making. For example, Neville and Cross [21] conducted a study on the effects of racial awakening epiphanies on individuals social careers in Bermuda, South Africa, and the United States. Their results showed epiphanies had a profound impact on participants' self-concept, leading to changes in their social behavior and interactions with others. A computational model developed by Chen and Krajbich [22] investigated the role of epiphanies in reinforcement learning, which is a fundamental mechanism for decision making. Their model found epiphanies can modulate learning by promoting exploration of alternative options and leading to more informed decision-making.

Recent research has focused on the impact of social structures and institutions on social careers. For example, research has shown that social networks and relationships can also have a significant impact on social careers, with individuals decisions often influenced by their social connections and the expectations of their social group [23].

Another question we deal with is the effect of enforcement of affirmative action policies. In the past few decades, affirmative action policies have been a subject of intense debate and research in various countries. These policies aim to provide equal opportunities to disadvantaged groups by establishing quotas in job markets, universities, and other areas [24]. However, the effect of these policies on individuals and society is complex and multifaceted. The change of an individual scenario by these policies may be for good or for bad, depending on how the individual is accepted in his new environment. This change depends itself on the individual self, once it may face rejections because of racial, religious, or job market disputes. Kent and Wade [25], studying affirmative action policies in Brazil, considered the high admixture of the Brazilian society, making it difficult to use the race category as a basis to enforcement of affirmative actions. They rather highlight how important are the political relations and political interest in the public engagement with genetic research and their social consequences.

Several recent studies have explored this topic from different angles, shedding light on various aspects of affirmative action policies. One area of research focuses on the impact of affirmative action on the performance and achievement of beneficiaries. For instance, a study by Riehl [26] found that race-based affirmative action in college admissions leads to a decline in academic performance among beneficiaries. Similarly, a study by Hoxby and Avery [27] found that affirmative action policies in college admissions do not significantly improve the representation of underrepresented minorities.

Another area of research explores the psychological effects of affirmative action policies on beneficiaries and nonbeneficiaries alike. A study by Coil [28] found that white Americans tend to overestimate the benefits of affirmative action policies to minorities, leading to resentment and negative attitudes towards these policies. However, a study by Steele and Aronson [29] found that affirmative action policies can reduce stereotype threat and improve the academic performance of minority students.

Moreover, recent research has also examined the legal and ethical implications of affirmative action policies. A study by Sander and Taylor [30] found that race-based affirmative action in law school admissions can harm the academic performance and career prospects of beneficiaries. Meanwhile, a study by Krysan, Cuddy, and Lewis [31] found that affirmative action policies can be perceived as unfair and lead to increased racial tensions in society. In the United States, however, besides Blacks, Latins have their portion in the concerns of affirmative action policies. Anderson *et al.* [32] were concerned with the concept of critical mass in establishing affirmative action policies in American universities. They built a mathematical model to forecast undergraduate student body racial/ethnic demographics in the form of a Markov chain that tracks students through application to graduation.

In conclusion, affirmative action policies remain a complex and contentious issue, with various studies producing conflicting results and interpretations. More research is needed to understand the effects of these policies on individuals and society better.

III. ABOUT HEXAGONAL LATTICES

Hexagonal lattices occur in nature, for instance, in honeycombs. The mechanical properties of a honeycomb are used in technology from simple applications, as in pillows, up to sophisticated applications, as structures of spacecrafts. In the laboratory, a perfect hexagonal lattice of a two-dimensional web of carbon atoms lying on a substrate was built two decades ago. This material was named graphene. Geim *et al.* [33] reported the discovery of that new material. The authors discovered that graphene is transparent, flexible and strong, and it conducts electricity, making it an attractive material for a number of electronic applications. Another important publication by Meyer *et al.* [34] reports they produced suspended graphene sheet showing that it is remarkably stable, elastic, and strong, and can withstand deformation and strain while retaining their structure. More than one million scientific reports were published since then.

The hexagonal lattice has been a topic of interest in various fields, including physics, chemistry, materials science and engineering. Ding *et al.* [35] addressed the challenge of deploying sensing nodes in an effective and efficient manner by proposing a novel static node center hexagonal deployment strategy for hybrid crowd sensing. Their study showed that this strategy can improve the accuracy and efficiency of data collection, as compared to other commonly used deployment methods. By using computer simulations, the authors showed that their proposed strategy can lead to a more even distribution of sensing nodes and better coverage of the sensing area.

In their study on mathematical modeling of complex contagion on clustered networks, O'Sullivan *et al.* [36] demonstrated the effectiveness of hexagonal lattices in simulating the spread of infectious diseases within communities. They developed a model that accounts for clustering effects and also showed, via simulation, that their complex contagion model could produce similar spreading behavior between a hexagonal lattice and comparable random network as the mentioned analytic results for the clique-type networks. Also found that the hexagonal lattice produces faster spreading.

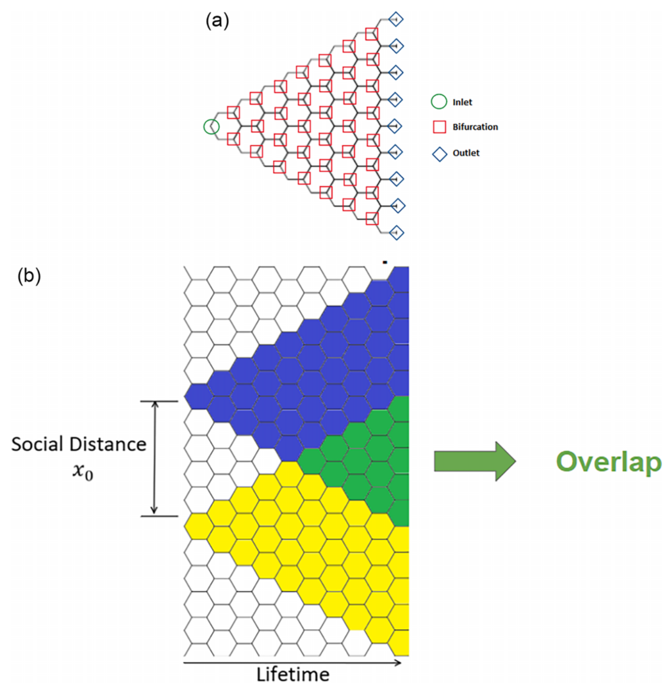

FIG. 1. The hexagonal lattice is a web of alternating junctions and bifurcations. (a) The square sites (nodes) are used to represent opportunities for the individual to make a choice. (b) The yellow (lower) triangle is the region accessible to an individual, taken as reference, while the blue (upper) triangle is that region for an individual starting at a higher social position; the green (superimposed) triangle is the allowed overlap nodes for both individuals.

IV. RANDOM PROGRESS IN A HEXAGONAL LATTICE

Our simulations use the network shown in Fig. 1(a). There, a green circle represents an inlet to the web. It is taken as the first bifurcation. Every branch leads to other bifurcations represented by red squares, which leave and meet again at junctions. Progress continues forward across the web, with paths limited to a triangular region. The routes end at outlets represented by blue diamonds. Figure 1(b) shows how the social distance (x_0) between the entrance of two individuals in the web limits the zone of overlapping of two triangles representing the common access zones in their social career. The access zones do not depend on the probability of making choices, but on the starting positions. However, the probability distribution for occupying different levels in the vertical side of the triangle depends on the individual probabilities of choice. Our calculations were performed using the Python programming language making use of external libraries: Numpy, Math, and Scipy for calculations and creation of arrays and matrices. Matplotlib for creating the graphs shown in this work.

The random progress through the hexagonal lattice obeys a binomial probability distribution. At each bifurcation, the

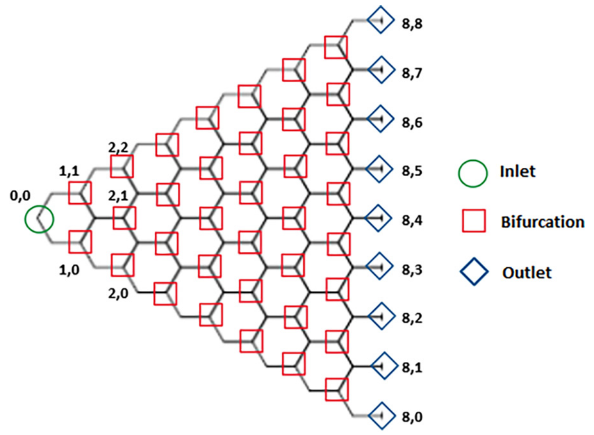

FIG. 2. A column index n labels the square site in the web, which is equal to the number of steps the individual took to reach that point. The index m labels the height in that column, which represents how high on the social scale lies that site.

individual has a probability p of taking the upper branch (considered the good choice, since it leads to a higher node), and the probability $1 - p$ of taking the lower branch. Figure 2 shows how we identify each square-type site. The index n labels the column; it starts at the inlet with $n = 0$, and it ends at $n = N$, the number of choice opportunities presented to the individual during his lifetime and the last column of the lattice. The index m labels the position on the social scale accessed by the individual choices. For each column, m ranges from the lowest viable position, $m = 0$, up to the highest one, $m = n$. The probability of reaching the mth level after n choices is

$$P_{n,m} = \frac{n!}{m!(n-m)!} p^m (1-p)^{n-m}, \quad 0 < m \leqslant n. \tag{1}$$

The expected value of m after n bifurcations is $\bar{m} = np$, and the first four central moments of the binomial distribution are

$$\begin{aligned} \mu_1 &= np, \\ \mu_2 &= np(1-p), \\ \mu_3 &= np(1-p)(1-2p), \\ \mu_4 &= np(1-p)[1 + (3n-6)p(1-p)]. \end{aligned} \tag{2}$$

When $p = 0.5$, we can fit a Gaussian to the PDF for continuous variables t, m:

$$P(m;t) = \frac{1}{\sqrt{2\pi}\sigma(t)} \exp\left[-\frac{1}{2}\left(\frac{m-\bar{m}}{\sigma(t)}\right)^2\right], \tag{3}$$

where t represents the time in units of steps n, and $\bar{m}$ coincides with the starting point on the social scale. The dispersion (the second moment of the distribution) depends on t as $\sigma^2 \propto t$ for any p, a signature of a diffusive stochastic process.

Next, we consider a bias α applied at $n = 0$, $p + \alpha$. We calculated the mean value with a positive bias for the binomial distribution together with its width. Figures 3(a) and 3(b) are aid to eyes for the dependence of the increments on the expected value $\Delta\bar{m}^{(1)} = n\alpha$, and on the variance $\Delta\mu_2^{(1)} = -n\alpha^2$

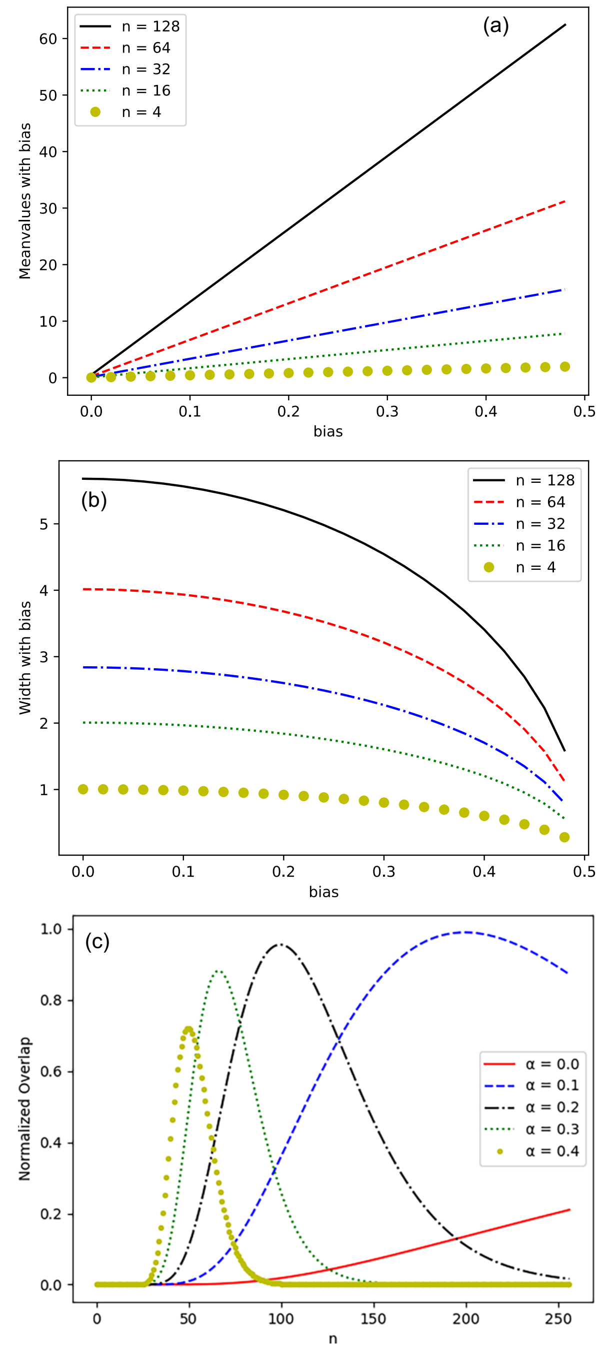

FIG. 3. (a) Expectation value of access on the social column ($\bar{m}$) as a function of bias for different columns. (b) Width of the PDF distribution as a function of the bias. (c) Overlap between two PDFs, one biased, other unbiased but with $x_0 = 20$ across the web; a maximum appears as the bias increases, moving to the left of the web. The maximum appears when the lower PDF bypass that with a positive x_0.

with α. Figure 3(c) shows the normalized overlap,

$$\mathcal{O}(\alpha, x_0; n) = \frac{\sum_m P_{n,m}^{(1)}(p+\alpha, 0) P_{n,m}^{(2)}(p, x_0)}{\sum_m \left[P_{n,m}^{(1)}(p+\alpha, 0)\right]^2}, \tag{4}$$

between two binomial distributions according to Eq. (1). $P_{n,m}^{(1)}(p+\alpha, 0)$ is the probability biased by a value α, while

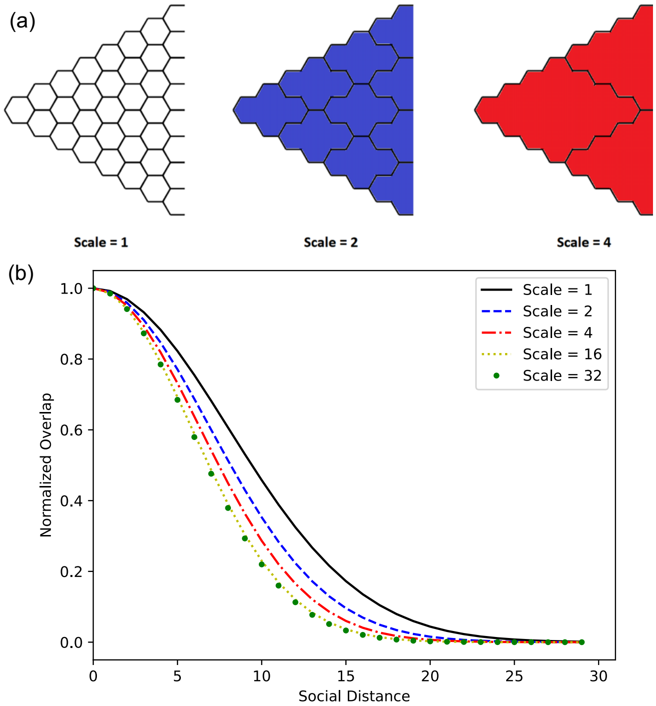

FIG. 4. (a) Visualization of the effect of decimation: the complete lattice with its hexagonal cell (scale 1), decimation of one every two steps (scale 2), and one every three steps (scale 3); we see the unit cell approaching the limit of a diamond. (b) Overlap between two PDFs as a function of the social distance for different scales.

$P^{(2)}_{n,m}(p, x_0)$ is unbiased with a positive social distance of $x_0 = 20$. Both have $p = 0.5$ and start at $n = 0$.

We extended the range of n although it sounds large in a lifetime, but it is necessary to get a clear visibility of the curves. Then, we show that, for a small value of α (0.1), the overlap increases monotonically throughout the lattice. As soon as α reaches 0.2, we see a maximum in the overlap. This maximum shows that the expected value $\bar{m}^{(1)}$ crossed $\bar{m}^{(2)}$ before reaching the end of the range of n. As α increases still more, the maximum of the overlap moves to an earlier time (smaller values of n), and the overlap decreases because of the decrease in the width of the biased distribution.

Besides the probability of going up or down along the paths, the number of opportunities an individual is exposed during his lifetime also influences his social career. Two individuals lying in the same position on the social layer, having the same conditions determining their choice probability, may decide to be exposed more or less frequently to choices in their lifetimes. It depends on how each of them is exposed to risks. There are many factors leading to this situation. Challenges occur naturally in one's life. But we can always accommodate to a comfortable, less risky position, avoiding such challenges. We consider here this possibility. We define a scale of risk given by the number of passes-overs, i.e., we decimate a certain number of bifurcations on the web. Figure 4(a) shows at left the exposure to every single opportunity, where the unit lattice cell is hexagonal. At the center we decimated one every two bifurcations, $n = 1, 3, \ldots$. We see that the scaled unit cell is no more a hexagon. At right, which is a still less risky case, we decimated three every four decisions; the unit cell is a jagged edge diamond (rhombus), the edges becoming smoother as the avoiding of risk scale increases. Notice that the progress is one step up or down at each time unit for all scales. This is what the unit cell means. In consequence, the diffusion has the dispersion scaled down by the risk avoiding parameter. Figure 4(b) shows the overlap of two PDFs, the reference one without decimation at a social distance x_0, and others scaled by 1, 2, 4, 16, and 32. We calculated all the PDFs for $p = 0.5$ and $N = 128$. As expected, the overlap decreases more rapidly with the social distance as the risk-avoiding increases.

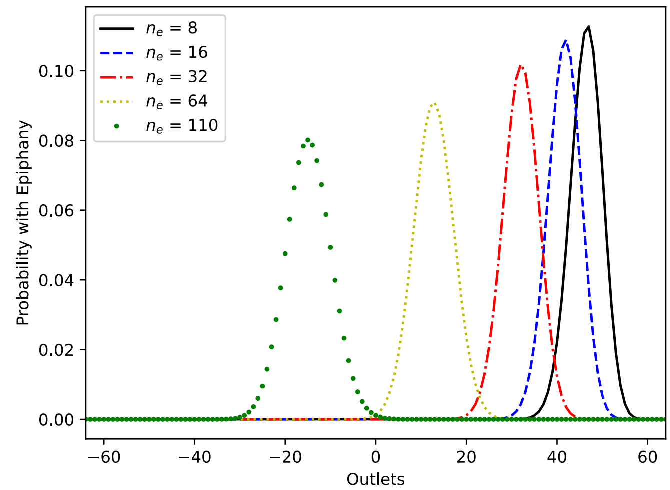

FIG. 5. PDF at the outlet, $n = 128$, for epiphany $\alpha = 0.6$ applied on $p = 0.3$ at different time steps $n_e = 8$, 12, 32, 64, and 110.

V. THE EPIPHANY AND THE QUANTUM LEAP

An epiphany, represented by a bias in our model, changes the way the individual makes his choices, but it does not change the position of the individual on the social scale at the time the bias is applied. Affirmative actions provide the latter, giving a transition to higher levels on the social scale without passing by intermediate levels, and with no change in the way the individual makes his choices. We assumed that the cohabitation in these higher layers will model the individual self. Next, we treat both processes.

A. Epiphany

We assume that at a point on his life, n_e, a bias α (can be positive or negative despite the name epiphany) changes the way the individual makes choices (for better or worse), from p_0 to $p_0 + \alpha$. In Fig. 5 we calculated the PDF for $p = 0.3$ up to $n = n_e$, and a sudden change to $p = 0.9$ from that point on up to $n = 128$. The x axis represents the index m of the outcome after 128 exposures to challenges. The curves show how the outcomes are distributed on the social scale. We see that for $n_e = 8$, where the bias occurs early in the individual life (this is supposed to occur around the period when the individual is still a child), we have a huge response, bringing the adult much higher on the social scale with a small uncertainty in this good outcome. The epiphany becomes less effective as it

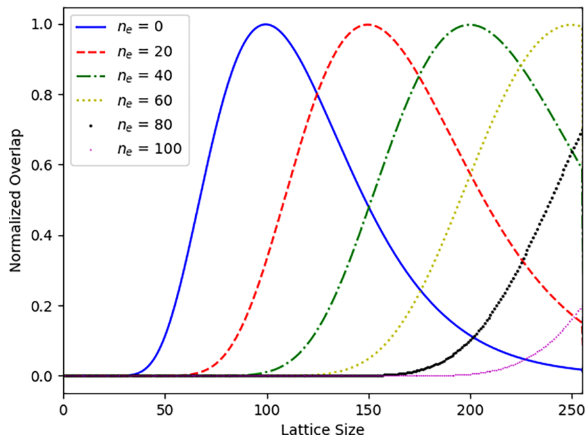

FIG. 6. Overlap of two PDFs, one under epiphany of $\alpha = 0.5$ at $n = n_e$, another with a social distance $x_0 = 20$. We performed the calculation for $N = 256$, and $n = 0, 1, \cdots N$

is applied later in his life. Applied on an adult, it is almost irrelevant, as shown by the black line peaked at $\bar{m} = -15$. In all cases calculated, we started with $x_0 = 0$. These results show how the present model gives a picture of the effect of an epiphany on the children with less favorable backgrounds. It is important to emphasize the continuous upward progress in the social scale of the individual due to the positive change of his self, mathematically represented by a change from $p = 0.3$ to $p = 0.9$ (application of a positive bias) in the beginning of life: a jump of 50 levels.

We calculated the overlap of two binomial distributions, one describing the social progress subjected to an epiphany on a less favored individual labeled by his starting position ($\bar{m} = 0$) on the social scale and bad choice probability when facing challenges (p_1), and another regular one following the progress given by a binomial distribution with $p_2 = 0.5$ and starting position $\bar{m} = x_0$. In the first case, the progress follows Eq. (1) up to $n = n_e$ with $p = p_1$. So, at $n = n_e$ we have $m = m_e$, with $m_e = 0, 1, 2 \dots n_e$.

$$P^{(1e)}_{nm} = \begin{cases} P_{n,m}(p_1) & 0 < n < n_e, \quad m = 0, 1, 2, \cdots n, \\ P_{n_e,me}(p_1) & \quad\quad n = n_e, \quad me = 0, 1, 2, \cdots n_e. \end{cases} \tag{5}$$

From that point on, it follows the binomial distribution with $n - n_e$ steps and centered at each accessible value of m_e:

$$P^{(1e)}_{n,m} = \sum_{me} P_{n_e,me}(p_1) P_{n-n_e,m-me}(p_1 + \alpha). \tag{6}$$

The second process follows Eq. (1) for every step:

$$P^{(2)}_{n,m} = P_{n,m+x_0}(p_2). \tag{7}$$

The overlap becomes

$$\mathcal{O}_n = \frac{\sum_m P^{(1e)}_{n,m} P^{(2)}_{n,m}}{\sqrt{\sum_m \left(P^{(1e)}_{n,m}\right)^2 \sum_m \left(P^{(2)}_{n,m}\right)^2}}. \tag{8}$$

Figure 6 shows the result of Eq. (8) for a bias $\alpha = 0.5$ given by an epiphany at several steps of the individual progress through the lattice. We assumed $p_1 = 0.2$, $p_2 = 0.5$, and $x_0 = 20$. We extended the lattice side up to $n = 256$ to capture all the features of the overlap as the epiphany occurs at a wide range of possibilities. The extended range shows that the curves are displaced later in time as the epiphany occurs in the individual life. Considering that in real life the number of successive choices is really important for the social progress in less frequent, the overlap may never occur for a later epiphany. This result illustrates how significant is the bias being applied earlier in the individual life is.

B. Quantum leap

The vertical transition because of an affirmative action cannot be achieved by following any path within the lattice. It changes the social position by $\Delta m = 1, 2, \dots$ at a time n_l in the lifetime. Transitions in the hexagonal lattice are allowed either with $\Delta m = 0$ or $\Delta m = 1$ after a bifurcation, with $\Delta n = 1$. We call these vertical transitions a quantum leap, corresponding to a quantum $\Delta m = 1, 2, \dots$ with $\Delta n = 0$. The term quantum leap used here has no direct relation with quantum mechanics but it shares the concept of a jump from a node to another through no existing path, like electrons tunneling across a potential barrier [37]. In the present case, an affirmative action makes an individual tunnel across a social barrier. A quantum leap occurs without a change in the individual self. This sudden change of social environment may, and will, change the probability of choice when facing a follow up choice at a next bifurcation.

First, we assume a quantum leap to occur without an epiphany. In that case the effect of the leap applies to the n_l nodes, $m_l = 0, 1 \dots, n_l$ with probability $P_{n_l,m_l}(p)$. From those nodes on we consider a leap Δm. We use the super index (1) to identify this individual:

$$P^{(1)}_{nm} = \begin{cases} P_{n,m}(p_1) & 0 < n < n_l, \quad m = 0, 1, 2, \cdots n, \\ P_{n_l,m_l}(p_1) & \quad\quad n = n_l, \quad m_l = 0, 1, 2, \cdots n_l. \end{cases} \tag{9}$$

From the node where the quantum leap occurs, we have

$$P^{(1h)}_{n,m} = \sum_{m'} P^{(1)}_{n,m'}(p_1) P_{n-n_l,m-m'+\Delta m}(p_1),$$
$$P^{(2)}_{n,m} = P_{n,m+x_0}(p_2). \tag{10}$$

The normalized overlap becomes

$$\mathcal{O}_n = \frac{\sum_m P^{(1h)}_{n,m} P^{(2)}_{n,m}}{\sqrt{\sum_m \left(P^{(1h)}_{n,m}\right)^2 \sum_m \left(P^{(2)}_{n,m}\right)^2}}. \tag{11}$$

Figure 7 shows the PDF overlap of two individuals coming from social nodes separated by $x_0 = 20$ after a quantum leap of $\Delta m = 30$, with $p_1 = 0.2$, and $p_2 = 0.5$. The leap occurs at $n_l = 10$, 30, 50, 70, and 90. Initially, there is no overlap, and for $n_l = 90$ it raises to small value decreasing after that according to the dashed black line. As the leap is applied sooner and sooner, we see that a jump to a guiding line describing a decay which is common to all cases. The conclusion to be taken is that a leap to higher nodes is unsustainable by itself, decaying according to a common law, although an earlier leap be able to coexist for a longer period with the individual starting at a higher node.

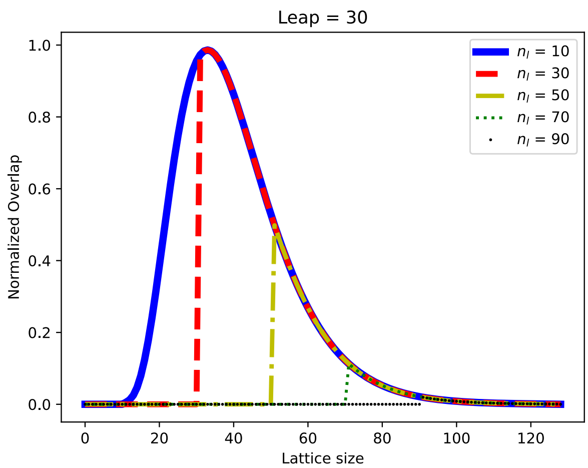

FIG. 7. Overlap of two PDFs, one starting at $x_0 = 0$ with $p_1 = 0.2$, and the other starting at $x_0 = 20$ with $p_2 = 0.5$ when a leap $\Delta m = 30$ occurs at $n_l = 10$, 30, 50, 70, and 90. The calculation is for $N = 128$

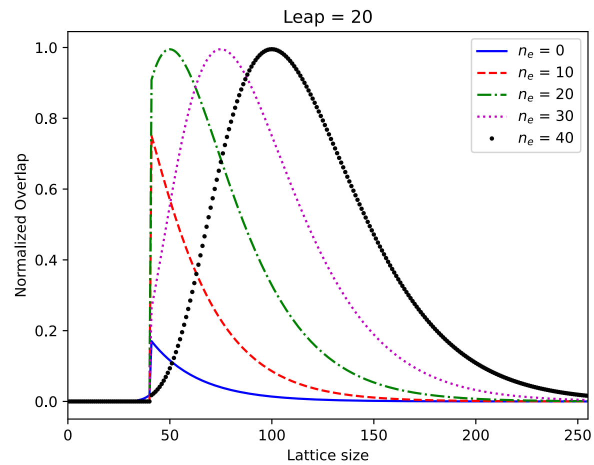

FIG. 8. Overlap of two PDFs, one with a social distance $x_0 = 20$ and $p = 0.5$ and another with $p = 0.2$ to which is applied an epiphany of $\alpha = 0.5$ at $n_e = 0, 10, \cdots 40$ and a quantum leap of $\Delta m = 20$ at $n_l = 40$. We performed the calculation for $N = 256$.

C. Quantum leap associated with epiphany

We now consider an epiphany applied before a quantum leap. The epiphany in the way we used until now is something to occur very much unlikely in the individual life, and from the point of view of a social group, it has negligible effect. However, we may as well consider epiphany as a collective initiative, for instance, improving the education level of a limited social group, all of them with the same p and being given the same bias.

We repeat here the same case as above, but now with an epiphany by applying a bias α at a time $n_e < n_l$. Then,

$$P^{(1eh)}_{n,m} = \sum_{m''} P^{(1e)}_{n,m''} P_{n-nh,m-m''+\Delta m}(p_1 + \alpha),$$
$$P^{(2)}_{n,m} = P_{n,m+x_0}(p_2), \tag{12}$$

and

$$\mathcal{O}_n = \frac{\sum_m P^{(1h)}_{n,m} P^{(2)}_{n,m}}{\sqrt{\sum_m \left(P^{(1h)}_{n,m}\right)^2 \sum_m \left(P^{(2)}_{n,m}\right)^2}}. \tag{13}$$

As we have already seen, the allowed nodes do not depend on p, so there is no overlap before $n = 40$. However, the median of the PDF increases with the bias, while the width decreases. So, just after the crossing of the two triangles starts, the overlap increases because of the raising of the lower distribution median. As soon as this median cross that of the initial upper distribution, the two distributions get apart from each other and the overlap decreases. And it decreases as fast as earlier the epiphany is applied because of the shrinkage of the distribution.

Figure 8 shows the overlap of two PDF,one for an individual with $p = 0.5$ starting his career at a higher level in the social scale $x_0 = 20$, another with $p = 0.2$ at $x_0 = 0$. To better observe the consequences of the epiphany before leaping, we extended once again the range on N up to 256, a rather artificial number of relevant opportunities for a lifetime.

A leap $\Delta m = 20$ at $n_l = 40$ applies to individual together with an epiphany given by a bias $\alpha = 0.5$ at several times before the leap. In the last case, $n_e = 40$, when epiphany and leap coincides, the two actions together are not the most effective, indeed the overlap is relevant late in the individual lifetime (assuming it is still alive) reaching the value 1.0 at $n = 100$, when its maximum cross that of the reference distribution and starts decaying. For $n = 30$ and $n = 20$ the behavior of the overlap is similar, while the maximum gets being earlier, showing the combination epiphany and leap to be more effective the earlier the former occurs. However, for the last two cases, $n_e = 10$ and $n_e = 0$, the action is so effective that not even a significant maximum of the overlap occurs. The median of the probability distribution of the individual starting at the lower node grows so fast after epiphany that after leaping it has already bypassed that above and the overlap always decays. These results show how important is that the epiphany occurs early in the lifetime and, for the individual receiving the epiphany, how effective is the quantum leap.

D. Epiphany, quantum leap, and social rejection

Next, we treat the case in which two individuals separated by a social distance $x_0 = 30$ are lead to cohabit after a quantum leap. We consider three different values of p_1: 0.3, 0.4, and 0.5, and the choice probability $p_2 = 0.5$. The quantum leap is $\Delta m = 30$, at $n_l = 40$. We consider three cases: (a) once the individual 1 leaps to the upper node cohabiting with individual 2, a bias of 0.2 is applied; (b) no bias; (c) because of rejection, the individual self suffers a negative bias of -0.2. The three cases are represented by Figs. 9(a), 9(b) and 9(c), respectively. We expect that in case (a), when a bias $\alpha = 0.2$ is applied on $p_1 = 0.3$, before leaping there will be zero overlapping; just after leaping $p_1 = p_2$ and the overlapping tends to one; when the bias is applied to $p_1 = 0.4$, it becomes 0.1 higher than p_2, the median cross that of the second individual, and the overlap decreases as time goes by; the process occurs still more sudden for $p_1 = 0.6$. Figure 9(a) represents the case where successful individuals coming from lower social layers are accepted and stimulated by the new environment. Figure 9(b) represents the leap without epiphany

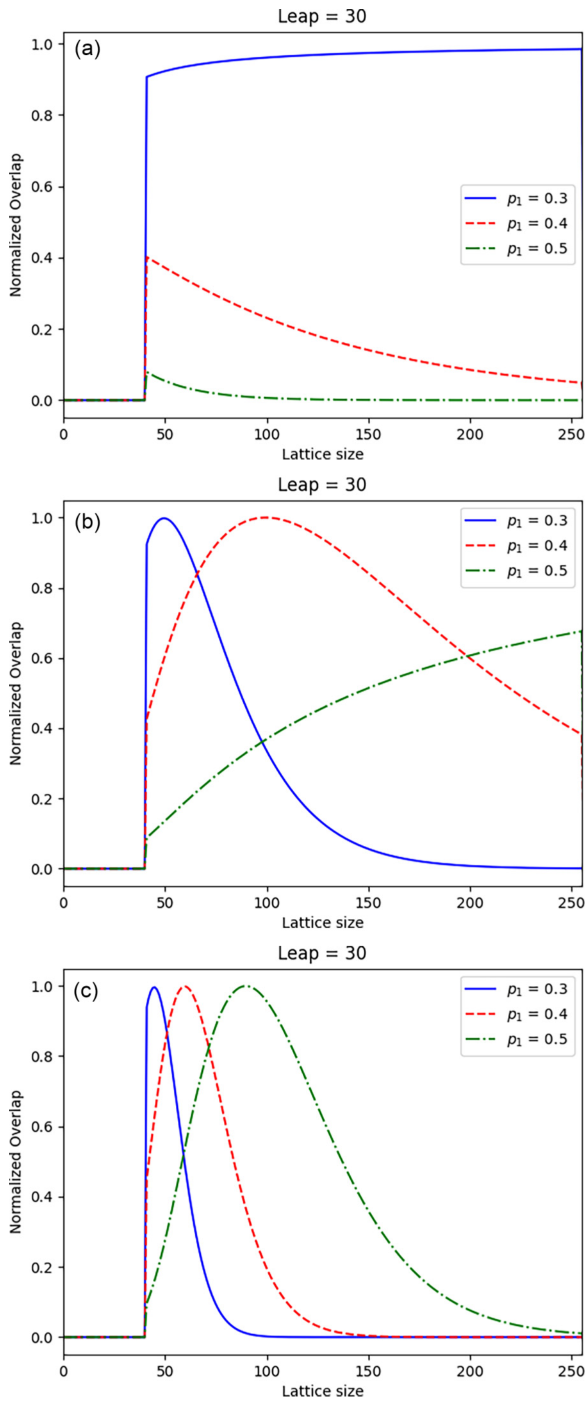

FIG. 9. Two individuals separated by a social distance $x_0 = 30$ are lead to cohabit after a quantum leap. We consider three different values of p_1: 0.3, 0.4, and 0.5. The choice probability $p_2 = 0.5$. The quantum leap is $\Delta m = 30$, at $n_l = 40$. We consider three cases: (a) once the individual 1 leaps to the upper node cohabiting with individual 2, a bias of 0.2 is applied; (b) no bias; (c) because of rejection, the individual self suffers a negative bias of -0.2.

and is included here just for reference. Figure 9(c) is typical of affirmative actions with a rejection by the new environment; we observe here what occurs, for instance, when individuals migrate from a critical area into a society that rejects them and change their self by not allowing them to adjust to their new reality.

VI. EDUCATED GUESS AND AFFIRMATIVE ACTION POLICIES

Up to now, we treated epiphany and quantum leap acting on individuals. Next, we consider a group of individuals lying on a tiny node interval between x and $x + dx$ of the social scale. Independently of their selves determining p, they all move through the web in a diffusive process inside a triangle as discussed in Sec. IV, and described by Fig. 1. As a group, we assume they have different selves determining reactions facing opportunities. Inside the group, there are people better qualified than others to make choices, so the distribution of p must be characterized by a width σ. It is also characterized by a median $\bar{p}$. The skewness, related to the third moment of the p distribution, gives the symmetry or asymmetry of the distribution, and being positive or negative, shows to where, in the scale of p points a tail of the distribution. The skewness tells how people in that group make good or bad choices. A tail to right means that most people are poorly qualified for making choices, and vice versa.

We are interested in the social dynamics of a lower laying group of the society, and we assume that, for diverse reasons, the p-PDF has a tail in higher values of p, making $\bar{p} < 0.5$. We may not submit every single member to an epiphany. In fact, epiphany is something very rare in the individual lifetime. So, we seek to improve the access to the upper part of the triangle by moving the tail of the distribution to the lower value of p. We can name this collective epiphany by education. People which p lie in the upper part of the distribution, in fact, make educated choices. Of course, in our simulations, we may change at will the way p will be distributed. The education is not free to be manipulated as the epiphany, so we will implement the education just by inverting the p-PDF.

Our next calculation aim to determine how the raising on the social scale is affected by applying a bias on the distribution, resulting in a positive change in the skewness distribution. Let $g(p)$ be the PDF for the p distribution of the specific group entering the web at $n = 0, m = 0$. Then, the progress through the web is given by the probability distribution:

$$P_{n,m} = \frac{n!}{m!(n-m)!} \int_0^1 g(p) p^m (1-p)^{n-m} dp, \quad 0 < m \leqslant n. \tag{14}$$

We stress once more that the occupation area of the web will be a triangle independent of the choice we make for $g(p)$. The p-PDF is bounded in the interval $0 \leqslant p \leqslant 1$. Our choice is the Beta distribution for $g(p)$, a family of continuous distribution bounded in that interval, and defined by two parameters, α and β:

$$B(p; \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}, \tag{15}$$

where $\Gamma(z)$ is the γ function. The most likely value of p, the mode, is

$$\bar{p} = \frac{\alpha - 1}{\alpha + \beta - 2}. \tag{16}$$

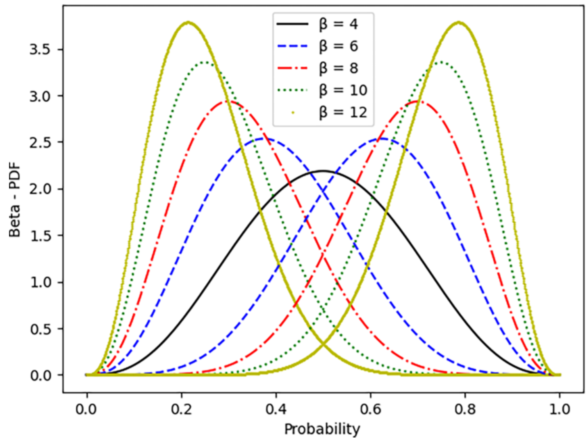

FIG. 10. Original (at left) and modified (at right) Beta-PDF according to Eqs. (15) and (19) for $\alpha = 4$. The symmetric distribution occurs when $\alpha = \beta$.

The width of the distribution, σ, is

$$\sigma = \frac{\sqrt{\alpha\beta}}{(\alpha+\beta)\sqrt{\alpha+\beta+1}}. \tag{17}$$

The skewness is

$$\gamma = \frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}}. \tag{18}$$

We consider a positive affirmative action as that where education (collective epiphany) acts on the social group inverting an existing upright tail of $P_{n,m}$ by inverting the skewness signal of the p-PDF. For this purpose, we define a modified Beta-PDF:

$$B^{\mathrm{mod}}(p;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(1-p)^{\alpha-1}p^{\beta-1}. \tag{19}$$

Figure 10 shows the modified Beta probability density function for $\alpha = 4$ and $\beta = 4, 6, 8, 10, 12$. Notice that the modified Beta-PDF is obtained by interchanging α and β:

$$B^{\mathrm{mod}}(p;\alpha,\beta) = B(p;\beta,\alpha). \tag{20}$$

However, once these parameters have no physical reality, we just take as a target to invert the distribution by education, knowing that it will result in giving people in the group access to upper zones of the allowed triangle in the web. Collective leap, then, completes the inclusive process by affirmative action policies.

If we establish a border x as a lower limit we intend part of the group to cross, as much as possible, then we may use β as a control parameter, like the bias in the epiphany. Then, the entire group is raised to upper nodes of the society because of a collective effort (education). Let us establish, for instance, that starting from a symmetric $g(p)$ leading the group to occupy the allowed triangular region of access either going down or up with equally like possibility, we want to use education to provide the progress mostly inside the higher 25% portion of the triangle. Initially,with $B(p;\alpha,\beta)$ the probability

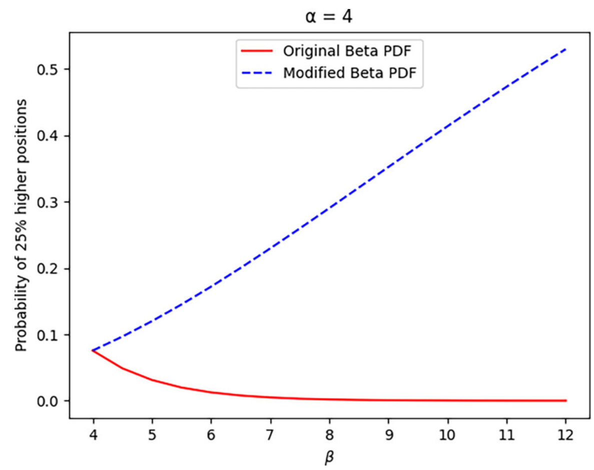

FIG. 11. Probability of occupying the 25% upper part of the access triangle in the social progress: the red (solid) line is an aid to eyes for a $g(p)$ given by a Beta distribution with $\alpha = 4$ and increasing β, what means making the distribution more and more asymmetric; the blue (dashed) line is for the modified Beta distribution when the skewness is inverted.

of occupying those nodes is

$$\mathcal{S}(n) = \sum_{m=0.75n}^{n} \frac{n!}{m!(n-m)!}\int_0^1 B(p;\alpha,\beta)p^m(1-p)^{n-m}dp. \tag{21}$$

However, after education with $B^{\mathrm{mod}}(p;\alpha,\beta)$, we have

$$\mathcal{S}(n) = \sum_{m=0.75n}^{n} \frac{n!}{m!(n-m)!}\int_0^1 B^{\mathrm{mod}}(p;\alpha,\beta)p^m(1-p)^{n-m}dp. \tag{22}$$

Figure 11 shows how the probability increases when the collective epiphany (education) acts inverting the skewness of the original p distribution. It is easy to imagine the effect of this education on the social progress of that group when affirmative policies act on them imposing collective quantum leaps (without changing the modified p distribution).

VII. DISCUSSION

Our model is based on the concept that social careers are made of successive choices, which we represent by the route the individual takes through the web of junctions and bifurcations. It considers that individuals coming out of different nodes on the social scale may always meet at a node, the coexisting probability at that node depending on time, initial social distance and choices probabilities. This is allowed by the junctions on the web. A succession of junctions and bifurcations builds our model to produce a hexagonal lattice. All together, our model has the following characteristics:

(i) It allows the progress head-on simulating the progress from the past to the future.

(ii) It contains junctions and bifurcations. The latter represents a choice to be done in one's life. The former allows for individuals following different routes in their lives to meet at some restricted nodes, so coexisting in the same social scale position.

(iii) It opens the possibility of jumping over choices for those conservative individuals which are used to avoid risky situations in their lives.

(iv) It blocks the vertical jump across different levels in the web, what represents very well the social barriers.

(v) It assumes an individual choice probability to determine the routes it follows in his social career.

(vi) It determines the range of the social scale allowed to an individual who does not receive any external influence, like epiphanies or quantum leaps.

(vii) It delivers the range of overlap of two individual's careers who started at different nodes on the social scale. That range depends exclusively on the starting points. However, the coexistence probability at each node depends on their choice probability.

(viii) The paths across the web follow a binomial distribution throughout the nodes, what permits the calculation of allowed regions of coexistence.

VIII. CONCLUSION

Our simulations show the different roles of epiphany and quantum leap. Aiming for social equity, epiphany and quantum leap must act together. At an early stage of the individual it must be submitted to an epiphany (for instance, a better education compatible with his potential, social help to his family, etc.), up to reach a strength to support a quantum leap (for instance, enter an affirmative action program). Our model illustrates the combined effect of these two agents. A change of the choice probability by an epiphany has the effects of pushing up the median and decreasing the width of this distribution of the node occupation distribution. A posterior leap to an elevated position keeps the time evolution of the distribution and allows even for decreasing the overlap with a mediocre distribution starting at higher position. This occurs when the two medians cross over each other, leading to the less favorable individual to have the possibility of occupying much higher nodes with time.

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