Opinion formation models with extreme switches and disorder: Critical behavior and dynamics

Kathakali Biswas [1,2] and Parongama Sen [2]
[1] Department of Physics, Victoria Institution (College), 78B Acharya Prafulla Chandra Road, Kolkata 700009, India
[2] Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata 700009, India

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In a three-state kinetic exchange opinion formation model, the effect of extreme switches was considered in a recent paper. In the present work, we study the same model with disorder. Here disorder implies that negative interactions may occur with a probability p. In the absence of extreme switches, the known critical point is at $p_c = 1/4$ in the mean-field model. With a nonzero value of q that denotes the probability of such switches, the critical point is found to occur at $p = \frac{1-q}{4}$ where the order parameter vanishes with a universal value of the exponent $\beta = 1/2$. Stability analysis of initially ordered states near the phase boundary reveals the exponential growth (decay) of the order parameter in the ordered (disordered) phase with a timescale diverging with exponent 1. The fully ordered state also relaxes exponentially to its equilibrium value with a similar behavior of the associated timescale. Exactly at the critical points, the order parameter shows a power-law decay with time with exponent $1/2$. Although the critical behavior remains mean-field-like, the system behaves more like a two-state model as $q \to 1$. At $q = 1$ the model behaves like a binary voter model with random flipping occurring with probability p.

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I. INTRODUCTION

To address the problem of opinion formation in a society [1–3], several models with three opinion states have been considered recently [4–20]. Typically these opinions are taken as ± 1 and 0, where ± 1 may represent extreme ideologies. In a recent paper [17], using a mean-field kinetic exchange model, the present authors studied the effect of extreme switches of opinion, which is not usually considered in such models. Several interesting results were obtained; in particular, for the maximum probability of such a switch, the model was shown to effectively reduce to a mean-field voter model beyond a transient time. In this paper we extend the previous work by including negative interaction between the agents which acts as a disorder. Such negative interactions have been incorporated in three-state kinetic exchange models previously [10–14] and several properties have been studied in different dimensions. However, the effect of extreme switches and negative interaction both occurring together has not been studied earlier. Since these two features can occur simultaneously in reality, the dynamics of a model incorporating both is worth studying. In the absence of the extreme switches the critical point as well as the critical behavior is known [10–12]. The interest is primarily to see how the critical behavior is affected in the presence of the extreme switches.

In the present two-parameter model, representing the probabilities of negative interaction and extreme switches, in addition to obtaining the phase boundary and behavior of the order parameter, we have studied the dynamical behavior close to the fixed point. The relaxation of the order parameter from a fully ordered state is also studied at and away from criticality. The static critical behavior as well as the dynamical behavior are found to be similar to the mean-field model without extreme switches. However, we find that the nature of the phases in terms of the densities of the three types of opinions is quite different. Especially, the case with maximum extreme switches in the presence of the negative interaction leads to an interesting mapping to a disordered binary model. As a starting point, the mean-field model has been studied where the majority of the results can be obtained analytically. We derive the time derivatives of the three densities of population in terms of the transition rates which are then either solved analytically or numerically. A small-scale simulation is also made particularly to study the finite size scaling behavior of the order parameter.

In Sec. II, the model is described. Results are presented in Sec. III and some further analyses are made in the last section which also includes the concluding remarks.

II. THE MODEL

We have considered a kinetic exchange model for opinion formation with three opinion values $0, \pm 1$. Such states may represent the support for two candidates or parties and a neutral opinion [17,21,22] or three different ideologies where ± 1 represent radically different ones. The opinion of an individual is updated by taking into account her present opinion and an interaction with a randomly chosen individual in the fully connected model. The time evolution of the opinion of the ith individual opinion denoted by $o_i(t)$, when she interacts with the kth individual, chosen randomly, is given by

$$o_i(t+1) = o_i(t) + \mu o_k(t), \tag{1}$$

where μ is interpreted as an interaction parameter, chosen randomly. The opinions are bounded in the sense $|o_i| \leqslant 1$ at all times and therefore o_i is taken as 1 (-1) if it is more (less) than 1 (-1). There is no self-interaction so $i \neq k$ in general. The values of the interaction parameter are taken

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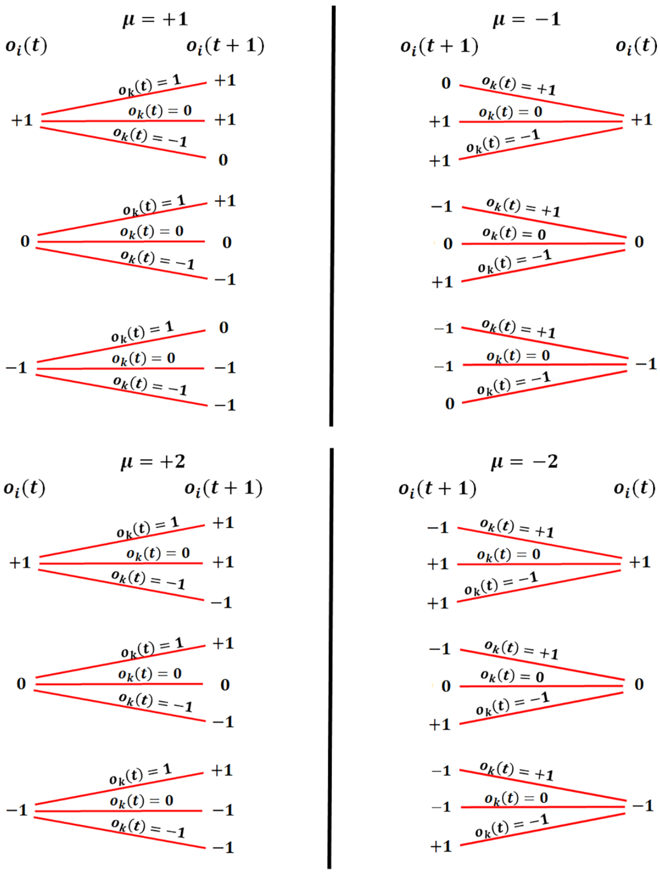

FIG. 1. Updated opinions of the ith individual for $\mu = \pm 1$ and ± 2 after interacting with the randomly chosen kth individual following Eq. (1) are shown.

to be discrete: $\mu = \pm 1$ and ± 2. Here we take $|\mu| = 1$ and 2 with probability $1 - q$ and q, respectively, and negative interactions, i.e., a negative value of μ occurs with probability p.

The updated opinion of an individual after interaction with another agent, with all possible values of opinion for both agents, are shown schematically in Fig. 1 for the four values of $\mu = \pm 1$ and ± 2.

III. RESULTS

The mean-field results are known for the limits $q = 0$, any p and also for $p = 0$, any q. For $q = 0$, the system undergoes a order-disorder phase transition at $p = 1/4$. For $p = 0$ on the other hand, the fate of the system starting from initially ordered configurations showed that it reached consensus for $q \neq 1$ while for $q = 1$, there is a quasiconservation leading to a partially ordered state. Initially disordered states flow to a q-dependent frozen fixed point which is disordered for all q [17]. These results are ensemble averaged and valid in the thermodynamic limit.

A. Rate equations

The densities of the three populations with opinion $0, \pm 1$, are denoted by $f_0, f_{\pm 1}$, with $f_0 + f_{+1} + f_{-1} = 1$. The ensemble averaged order parameter is $\langle O \rangle = f_{+1} - f_{-1}$ with $-1 \leqslant \langle O \rangle \leqslant 1$. A nonzero value of the order parameter indicates an ordered state while $\langle O \rangle = 0$ would correspond to a disordered state.

To set up the rate equations for the f_i's, we need to treat the time variable as continuous. Let the opinion change from i to j ($i, j = 0, \pm 1$) in time Δt with the transition rate given by $w_{i \to j}$. Then we have the following set of w_{ij}'s:

$$
\begin{aligned}
w_{+1\to+1} &= f_0 f_{+1} + f_{+1}^2(1-p) + f_{+1} f_{-1} p, \\
w_{0\to+1} &= (1-p) f_0 f_{+1} + p f_0 f_{-1}, \\
w_{-1\to+1} &= q(1-p) f_{-1} f_{+1} + p q f_{-1}^2, \\
w_{+1\to 0} &= (1-q)(1-p) f_{-1} f_{+1} + p(1-q) f_{+1}^2, \\
w_{0\to 0} &= f_0^2, \\
w_{-1\to 0} &= (1-q)(1-p) f_{-1} f_{+1} + (1-q) p f_{-1}^2, \\
w_{+1\to-1} &= q(1-p) f_{+1} f_{-1} + q p f_{+1}^2, \\
w_{0\to-1} &= (1-p) f_0 f_{-1} + p f_{+1} f_0, \\
w_{-1\to-1} &= f_0 f_{-1} + (1-p) f_{-1}^2 + p f_{-1} f_{+1}.
\end{aligned}
$$

In general, we have $f_i(t+\Delta t) = f_i(t) + \sum_j w_{j\to i}\Delta t - \sum_j w_{i\to j}\Delta t$ such that taking $\Delta t \to 0$, we get

$$\frac{df_{+1}}{dt} = qpf_{-1}^2 + f_0((1-p)f_{+1} + pf_{-1}) - (1-q)(1-p)f_{-1}f_{+1} - pf_{+1}^2, \tag{2}$$

$$\frac{df_{-1}}{dt} = qpf_{+1}^2 + f_0((1-p)f_{-1} + pf_{+1}) - (1-q)(1-p)f_{-1}f_{+1} - pf_{-1}^2. \tag{3}$$

B. Steady states and critical behavior

Solving Eqs. (2) and (3), it is possible to obtain the time evolution of the order parameter $\langle O \rangle$ which satisfies

$$\frac{d\langle O \rangle}{dt} = [-qp - p + f_0(1 - p + qp)]\langle O \rangle. \tag{4}$$

To reach a steady state the right-hand side of the above equation should be zero at $t \to \infty$. It is obvious that any initially disordered configuration will remain disordered, i.e., the order parameter will be zero always which can be achieved whenever $f_{+1} = f_{-1}$.

It was already observed in [17] that the $q = 1$ case is unique. Here also, it should be discussed separately. Precisely, for an ordered state to exist, $f_0 = 2p$ when $q = 1$. However, we note that f_0 is expected to vanish very fast as there is no flux to the zero state from other states for $q = 1$. Assuming f_0 vanishes within a transient time, one can show directly from the dynamical equations for the individual densities that for an ordered state to exist, p can only take the zero value when $q = 1$.

Taking the origin of time as that when f_0 becomes zero and for $q = 1$, one can rewrite Eqs. (2) and (3) as

$$\frac{df_{+1}}{dt} = pf_{-1} - pf_{+1}, \tag{5}$$

and

$$\frac{df_{-1}}{dt} = pf_{+1} - pf_{-1}. \tag{6}$$

As $f_{+1} + f_{-1} = 1$, one can easily obtain the solutions,

$$f_{\pm}(t) = \frac{1 - [1 - 2f_{\pm}(0)]e^{-2pt}}{2}, \tag{7}$$

where $f_{\pm 1}(0)$ are the values of $f_{\pm 1}$ when f_0 reaches 0. These equations are valid with the origin of time shifted but it does not matter as we are interested in the $t \to \infty$ results. When $p = 0$ we get the result that $f_{\pm}(t \to \infty) \to f_{\pm}(0)$. This will then result in an ordered state (but not a consensus state in general). Here it is assumed that the initial state is ordered; for initial disordered states, $\langle O(t) \rangle = 0$ for all times as already mentioned. For any nonzero value of p the system will reach an equilibrium state only with $\langle O \rangle = 0$ as $f_{\pm 1} \to 1/2$ according to Eq. (7). Thus for the $q = 1$ point, we see that any $p \neq 0$ makes the system disordered.

For values of $q \neq 1$, Eq. (4) indicates that in the steady state (at $t \to \infty$), the system may reach an ordered state with $\langle O \rangle \neq 0$ when $[-qp - p + f_0(1 - p + qp)] = 0$, i.e., $f_0 = \frac{qp+p}{qp-p+1}$ (this puts a restriction $p \leqslant 0.5$ as $f_0 \leqslant 1$, hence no ordered state is possible if $p \geqslant 0.5$). Now at the steady state if we also demand that the individual densities attain a fixed point, i.e., $\frac{df_{+1}}{dt} = 0$, etc., then from Eq. (2), putting $f_0 = \frac{qp+p}{qp-p+1}$, we get

$$f_{+1} = \frac{-q - 2p + 2pq + 1 \pm \sqrt{(1 - q)(1 - q - 4p)}}{-2(p + q - 2pq + pq^2 - 1)}. \tag{8}$$

Note that the above is valid for $q \neq 1$.

Therefore the expression of $\langle O \rangle$ in the steady state will be

$$\langle O \rangle = \frac{-q - 2p + 2pq + 1 \pm \sqrt{(1 - q)(1 - q - 4p)}}{-(p + q - 2pq + pq^2 - 1)} + \frac{2p - 1}{pq - p + 1}. \tag{9}$$

As $\langle O \rangle$ is real, a nonzero solution for $\langle O \rangle$ implies $(1 - q - 4p)$ should be greater than or equal to zero. This provides a more stringent bound for the ordered phase given by $\frac{1-q}{4} \geqslant p$.

On the other hand, when the steady state (at $t \to \infty$) is disordered, $f_{+1} = f_{-1}$, which when put in Eq. (2), one gets

$$f_{+1} = f_{-1} = \frac{1}{3 - q}; \quad f_0 = \frac{1 - q}{3 - q}. \tag{10}$$

Interestingly, the above values are independent of p.

Phase boundary.

At the critical point between ordered-disordered phase transition the fraction f_0 at the steady state for both the phases should be equal which gives

$$f_0 = \frac{1 - q}{3 - q} = \frac{qp + p}{qp - p + 1}.$$

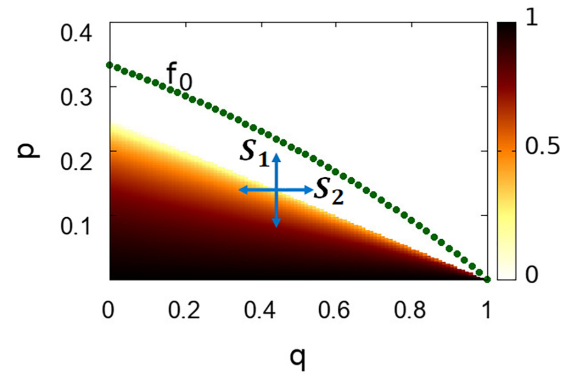

FIG. 2. The phase diagram in the $p - q$ phase space is shown with the phase boundary given by $p = \frac{1-q}{4}$. The color code corresponds to the value of the order parameter. The dashed line in the disordered phase represents $f_0 = \frac{1-q}{3-q}$.

Hence one gets an equation of a straight line,

$$p = \frac{1 - q}{4}, \tag{11}$$

which is the phase boundary in the $p - q$ phase space shown in Fig. 2. The value of f_0, a function of q only in the disordered phase, is also shown. Note that the order-disorder boundary obtained at $p = 0$ for $q = 1$ can be obtained as an analytical continuation from the above equation. However, all results discussed henceforth are for $q \neq 1$ in general.

Behavior of $\langle O \rangle$ close to a critical point.

Each point on the phase boundary is a critical point. We analyze the behavior of the order parameter close to a critical point along two different routes S_1 and S_2 as indicated in Fig. 2. For path S_1, we take $x = p_c - p$ and $q = q_c$ to get from Eq. (9),

$$\langle O \rangle = \frac{2\left(4x - 4q_c x - q_c^2 + 1 \pm 4\sqrt{(1 - q_c)x}\right)}{\left(4x - q_c - 8q_c x + 4q_c^2 x - 3q_c^2 + q_c^3 + 3\right)} - \frac{\frac{q_c}{2} + 2x + \frac{1}{2}}{\frac{q_c}{4} + x - q_c\left(\frac{q_c}{4} + x - \frac{1}{4}\right) + \frac{3}{4}}. \tag{12}$$

As $x \to 0$ behavior of $\langle O \rangle \to \sqrt{x}$, i.e., the critical exponent β is 0.5 along the path S_1.

Similarly, for path S_2, rewriting Eq. (9) taking $y = q_c - q$ and $p = p_c$ we get

$$\langle O \rangle = \frac{2p_c y - y - 4p_c + 8p_c^2 \pm \sqrt{y(4p_c + y)}}{16p_c^3 + 8p_c^2 y + p_c y^2 - 4p_c - y} - \frac{2p_c - 1}{p_c + p_c(4p_c + y - 1) - 1}. \tag{13}$$

Thus $\langle O \rangle \to \sqrt{y}$ as $y \to 0$ showing that the value of the exponent $\beta = 1/2$ does not depend on the path. In fact, the full variation of $\langle O \rangle$ can easily be seen to depend on $[x + 4y]^{1/2}$ as the leading order term if we allow both p and q to vary about the critical point as before, i.e., $p = p_c - x$ and $q = q_c - y$ with the restriction that $x + 4y \geqslant 0$ for a general direction. Note that for S_1 and S_2, both $x, y \geqslant 0$.

We have also numerically solved the time evolution equations to obtain the values of $\langle O(t \to \infty) \rangle$ along paths S_1

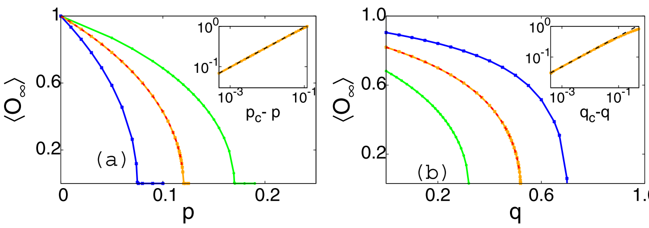

FIG. 3. $\langle O_\infty \rangle$ calculated numerically by solving Eqs. (2) and (3) for different sets of values of q_c and p_c along paths S_1 and S_2, respectively, on the phase boundary (see Fig. 2). In (a) the curves are for the path S_1 with fixed critical values of q; $q_c = 0.70, 0.52, 0.32$ from left to right; in (b) the curves are for the path S_2 with fixed critical values of p; $p_c = 0.17, 0.12, 0.075$ from left to right. Analytical values of $\langle O \rangle$ from Eq. (9) are also plotted with dashed lines for $q_c = 0.52$ in (a) and $p_c = 0.12$ in (b). Insets of (a) and (b) show the scaling behavior of $\langle O \rangle$, indicating $\beta = 0.5$.

and S_2 for a particular point on the phase boundary to find that indeed the results are compatible with $\beta = 0.5$ shown in Fig. 3.

C. Stability analysis

In the disordered phase, we obtained a fixed point characterized by $f_{+1} = f_{-1} = \frac{1}{3-q}$. As there are three variables, in principle it is possible that in the disordered state, the values of f_{+1} and f_{-1} still evolve keeping $f_{+1} = f_{-1}$. However, for the above values there can be no further change and hence we call this the frozen fixed point [17].

Stability of this frozen fixed point can be studied by introducing a deviation δ about it. Taking $f_{+1} = x^* + \delta_1$ and $f_{-1} = x^* + \delta_2$, where $x^* = \frac{1}{3-q}$, a stability analysis leads to $\delta_i(t) = \delta_i^0 \exp[\gamma t]$ for both $i = 1, 2$. Here δ_i^0 is the initial deviation considered about the fixed point at $t = 0$ and

$$\gamma = \frac{1 - q - 4p}{3 - q}. \tag{14}$$

As $\langle O \rangle = f_{+1} - f_{-1} = \delta_1 - \delta_2$, we get

$$\langle O \rangle = \left(\delta_1^0 - \delta_2^0\right) \exp[\gamma t]. \tag{15}$$

The above equation shows that for an initially ordered state, there will be a growth or decay of the order parameter according to the sign of γ which changes at $1 - q - 4p = 0$. This is consistent with the phase boundary obtained at $1 - q = 4p$ and as expected one ends up in a disordered state even after starting from an ordered state (when $\delta_1 \neq \delta_2$) in the disordered phase. Once again, the time-dependent equations are solved numerically by taking initial states close to the frozen fixed point and the results agree with the above as shown in Fig. 4.

Equation (15) shows that there is a timescale $\tau = \gamma^{-1}$ associated with the dynamics of the growth or decay. Since $\tau \propto (1 - q - 4p)^{-1}$, it diverges with an exponent 1 close to the critical point. The diverging timescale indicates critical slowing down known to be present in continuous phase transitions. The exponent with which τ diverges is related to the critical dynamical exponent z; this is to be discussed further in the next section.

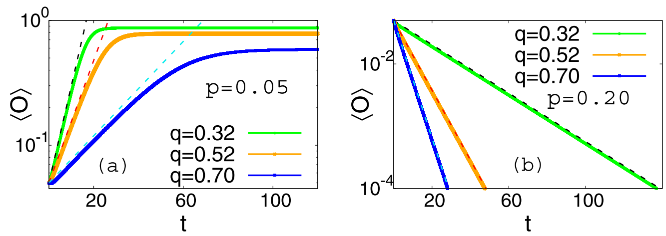

FIG. 4. The time evolution of $\langle O \rangle$ generated numerically from Eqs. (2) and (3) for $\frac{1-q}{4} > p$ (ordered phase) and $\frac{1-q}{4} < p$ (disordered phase) are shown in (a) and (b), respectively, with the initial configuration $f_{+1} = x^* + \delta_1$ and $f_{-1} = x^* + \delta_2$ (see text), where we have taken $\delta_1 = 0.07$, $\delta_2 = 0.02$. The dotted line in each graph represents the fitted graph following Eq. (15).

D. Relaxation from a perfectly ordered state

While for initial states with small order, the order parameter will either decay or grow depending on whether one is in the disordered or ordered phase, for a fully ordered initial state the order parameter will decrease in time in both phases. We study the relaxation behavior by numerically solving the rate equations taking initial condition $f_{+1} = 1$.

In the disordered phase the decay of the order parameter is expected to follow Eq. (15) at late times as it comes closer to the frozen fixed point, indicating again the presence of a timescale $\propto |\gamma|^{-1}$. In the ordered phase, the order parameter will initially decay and then attain a nonzero saturation value. We find that both behaviors are captured by a single equation,

$$(\langle O \rangle - \langle O_\infty \rangle) \propto \exp[-t/\tau_R], \tag{16}$$

where $\langle O_\infty \rangle$ is the ensemble averaged equilibrium value of the ordered parameter at $t \to \infty$. We have plotted the data for some particular points above and below the phase boundary in Fig. 5, and the timescales τ_R extracted from the slopes of the log-linear graphs are shown in Figs. 6(a) and 6(b). The results show that τ and τ_R have identical scaling behavior in the disordered phase as argued above, while in the ordered phase also, τ_R diverges close to the critical point with the same exponent 1 [see Figs. 6(c) and 6(d)].

Lastly, we plot $\langle O \rangle$ as a function of time at several points exactly on the phase boundary in Fig. 7 to get a power-law decay with an exponent close to 0.5. This discussion of course excludes the $q = 1$ point which is unique, one can easily check

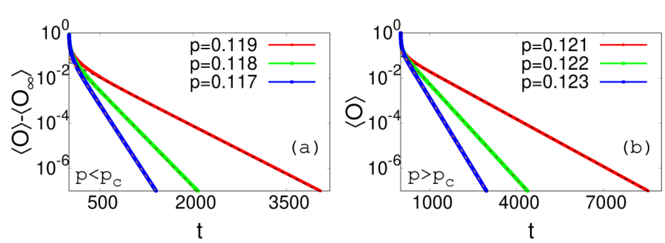

FIG. 5. Time evolution of $\langle O \rangle$ calculated by numerically solving Eqs. (2) and (3) with a fixed value of $q = 0.52$ and different values of p (a) below and (b) above the corresponding p_c when the initial condition is $(f_+, f_0, f_{-1}) = (1, 0, 0)$. The dotted lines show the fitted curve with Eq. (16).

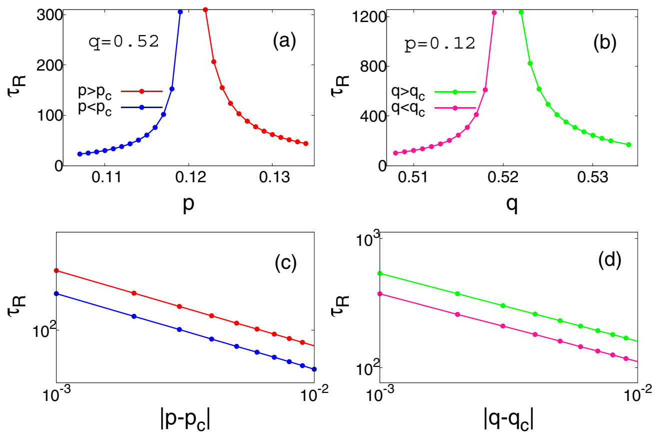

FIG. 6. The timescale τ_R calculated from the slopes of the log-linear plots in Fig. 5 using Eq. (16) is shown (a) against p for a constant value of q and (b) against q for a constant value of p. In (c) and (d), these values of τ_R are plotted against $|p - p_c|$ and $|q - q_c|$, respectively, both above and below the critical values to show a power-law divergence with the associated exponent very close to unity. The color codes for the upper and lower panels are the same.

that with the critical value $p = 0$ here, no evolution of the initially fully ordered state is possible.

IV. SUMMARY AND DISCUSSIONS

In this paper, we have studied the case of extreme switches in opinion in a three-state kinetic exchange model where the interactions may be both positive as well as negative. The two parameters characterizing the probabilities of the extreme switches and negative interactions are q and p, respectively. Our main findings are the following.

(1) The presence of a phase boundary given by $p = \frac{1-q}{4}$.

(2) β, the exponent associated with the order parameter, is universal with the value 1/2.

(3) The phase boundary can also be obtained using stability analysis. Additionally one gets the time evolution of the partially ordered state showing exponential growth or decay. The associated timescale τ diverges with an exponent -1.

(4) The overall behavior is mean-field-like for $q \neq 1$.

(5) Relaxation behavior of the fully ordered state shows the expected exponential decay of the order parameter with time in the disordered phase; for the ordered phase, it relaxes exponentially to a saturation value. The relaxation timescales τ_R and τ have identical scaling behavior. Exactly on the phase boundary, the order parameter shows a power-law decay.

(6) The $q = 1$ point has a special significance.

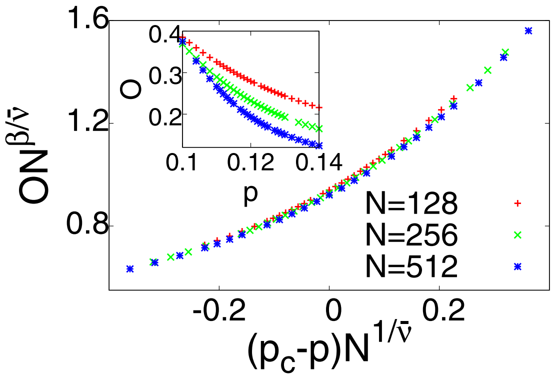

FIG. 8. Simulation results. Data collapse of the scaled order parameter $ON^{\beta/\bar{\nu}}$, for $q = 0.52$ when plotted against $(p_c - p)N^{1/\bar{\nu}}$, where $p_c = 0.12$, is obtained using the values $\bar{\nu} = 2.0$ and $\beta = 0.5$. Inset shows the raw data.

While the first three issues have already been discussed in detail in the previous section, we focus on the last three points here. It had been already known that in the mean-field three-state kinetic exchange model without extreme switches, the value of the exponent $\beta = 1/2$. Also, assuming the mean-field model has an effective dimension d, the exponent $\bar{\nu} = \nu d = 2$ was obtained previously where ν is the correlation length exponent [10]. We have found β to be equal to 1/2 at any point on the phase boundary in the two-parameter model for $q \neq 1$. To get $\bar{\nu}$, we conduct small-scale simulations about a particular point on the phase boundary. Indeed the scaled order parameter curves collapse when plotted against $|\epsilon|N^{1/\bar{\nu}}$, where ϵ denotes the deviation from the critical point, with $\beta = 1/2$ and $\bar{\nu} = 2$. The raw data and the collapse are shown in Fig. 8. Hence the static critical behavior is unaffected by the parameter $q \neq 1$.

The critical slowing down phenomena is observed with a timescale diverging as $|\epsilon|^{-1}$, again independent of q. Since in a continuous phase transition, the timescale diverges as ξ^z where ξ is the correlation length $\propto |\epsilon|^{-\nu}$ and z the dynamic critical exponent, one gets $\tau \propto |\epsilon|^{-\nu z}$. Hence our results indicate $\nu z = 1$.

Next one can consider the relaxation of the order parameter exactly at criticality. The power-law behavior $t^{-1/2}$ can be shown to be compatible with the theory of dynamic critical phenomena. In general the dynamical behavior of the order parameter is given by $O(t) \propto t^{-y} f(t/\xi^z)$ with $O(t) \propto t^{-y}$ exactly at the critical point. Equilibrium behavior indicates $y = \beta/\nu z$. Therefore, using the values $\beta = 1/2$ and $\nu z = 1$, one gets $y = 1/2$, which is the value obtained here. We also remark that the values of the static and dynamic exponents

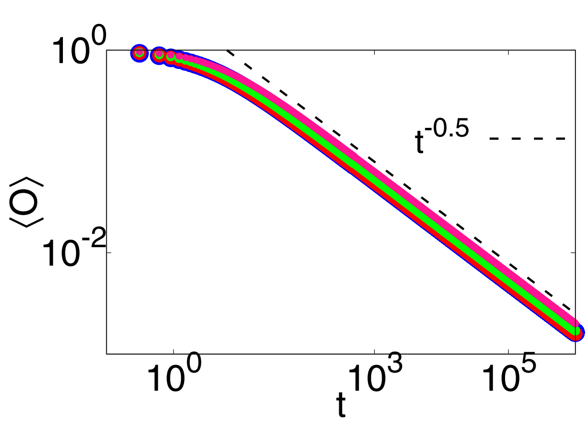

FIG. 7. Time variation of $\langle O \rangle$ calculated by numerically solving Eqs. (2) and (3) exactly at different critical points on the phase boundary indicates $O(t \to \infty) \propto t^{-0.5}$.

obtained here coincide with those of the mean-field Ising model ($\beta = 1/2$, $\nu = 1/2$, $z = 2$) if one uses $d = 4$, the upper critical dimension of the Ising model.

Even though the critical behavior is unchanged with a nonzero value of q, we note that the fixed point values are independent of p in the disordered phase [Eq. (10)]. Also, the variation of f_0 in the disordered phase (see Fig. 2) suggests that as q is increased, the model tends to become a binary one. The interpretation is, with increasing probability of extreme switches, the system in the disordered phase tends to be polarized as it becomes more difficult to retain a neutral opinion. Thus the nature of polarization is affected by the presence of the extreme switches. This can correspond to cases where there are two major candidates represented by opinions ± 1 and the society is strongly polarized. All other candidates can be clubbed to the opinion zero. People switch their opinions from one strong candidate to the other and also from the weaker candidates to one of the stronger ones with considerably higher probability, depleting the votes for the less significant candidates, a situation known to occur in reality. On the other hand, without the extreme switches, the disordered state has a fixed point where all the densities become equal. In fact, the polarization can be quantified as $f_{\pm 1} - f_0 = q/(3-q)$ which is zero for $q = 0$ and takes a maximum value $1/2$ at $q = 1$.

For $q = 1$, we get two equations [Eqs. (5) and (6)] after a transient time when f_0 becomes zero. These can be easily identified as the equations governing the dynamics of a two-state binary model with random flipping probability p. Obviously, it becomes disordered at any value of p. Previously it was noted that for $p = 0$, the model is identical to the mean-field voter model for $q = 1$; with $p \neq 0$ we thus obtain a mapping to a model with random flipping.

Hence the main conclusion from the present study is that the extreme switches act as additional noise for the model considered in [10] thereby lowering the critical values p_c without changing the critical behavior. The point $q = 1$ has a special interpretation. The role of the two kinds of disorder are, however, different; the system becomes disordered for a finite value of p (for $q = 0$) but remains ordered up to the extreme value of q (for $p = 0$) [17]. It is, therefore, not surprising that the critical behavior is dominated by p while q acts as an irrelevant variable. However, the nature of the disordered phase is dictated by q alone.

The results obtained in the present paper are based on the mean-field dynamical equations. Of course, if we consider the system on lattices with nearest-neighbor coupling, there will be quantitative changes. As a future study, it will be interesting to investigate how the extreme switches affect qualitatively the criticality and dynamics in finite dimensions.

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