Comment on "Renormalization group approach to connect discrete- and continuous-time descriptions of Gaussian processes"

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F. Ferretti *et al.* [Phys. Rev. E **105**, 044133 (2022)] show that time discretization of linear Gaussian continuous-time stochastic processes are either first-order Markov processes or non-Markovian ones. Specializing to ARMA(2,1) processes, they propose a general redundantly parametrized form for a stochastic differential equation giving rise to this dynamics as well as a candidate nonredundant parametrization. However, the latter does not give rise to the full range of possible dynamics allowed by the former. I propose an alternative nonredundant parametrization which does.

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The authors in Ref. [1] compute ARMA coefficients corresponding to the time discretization of a partially observed continuous-time linear process given by Eqs. (B2) and (B3), performed in Appendix B. As noted by the authors, the parameters of the partially observed process are not uniquely determined by the observed process. In the main text, the possibility of choosing $\lambda = 0$ and $\sigma_{xv}^2 = 0$ is put forth [Eqs. (19) and (20)]. We may consider the possible values of the parameters u, z, s, and b [Eqs. (B9), (B12), and (B13)], in general [Eqs. (B2) and (B3)] versus under these choices [Eqs. (19) and (20)]. In general, from nonpositivity of the eigenvalues of the matrix $\mathbf{A}$, an allowable combination of parameters must satisfy $\mathrm{tr}(\mathbf{A}) \leqslant 0$ (sum of eigenvalues) and $\det(\mathbf{A}) \geqslant 0$ (product of eigenvalues). Therefore, $u = \mathrm{tr}(\mathbf{A}) \leqslant 0$ and $z = u^2/2 - \det(\mathbf{A}) \leqslant u^2/2$. Clearly, we have $s \leqslant 0$. For b, we use the inequality $(dv + \eta\, dx)^2 \geqslant 0$, which implies $\sigma_{vv}^2 + 2\eta\sigma_{xv}^2 + \eta^2\sigma_{xx}^2 \geqslant 0$, to deduce $b \geqslant s(z/3 + u^2/2)$. These inequalities are the tightest possible for the general dynamics given by Eqs. (B2) and (B3). However, for a process given by Eqs. (19) and (20), the parameter b additionally satisfies $b \geqslant s(z/3 + u^2/3)$. Thus, it does not recover in full generality the possibilities for the (observed) dynamics. If one wants to take $\lambda = 0$, then to recover full generality, the possibility of negative σ_{xv}^2 needs to be included in order to allow $\sigma_{vv}^2 + 2\eta\sigma_{xv}^2 + \eta^2\sigma_{xx}^2$ to fall below $\eta^2\sigma_{xx}^2$. Alternatively, rather than taking $\lambda = 0$, instead taking $\eta = 0$ and $\sigma_{xv}^2 = 0$ recovers the full range of possible values of u, z, s, and b without any redundant parameters.

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[1] F. Ferretti, V. Chardès, T. Mora, A. M. Walczak, and I. Giardina, Renormalization group approach to connect discrete- and continuous-time descriptions of Gaussian processes, Phys. Rev. E **105**, 044133 (2022).

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