# Initial phase and frequency modulations of pumping a playground swing

Chiaki Hirata [*] and Shun'ichi Kitahara
*Faculty of Education and Humanities, Jumonji University, Sugasawa, Niiza, Saitama 352-8510, Japan*

Yuji Yamamoto
*Research Center of Health, Physical Fitness and Sports, Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464-8601, Japan*

Kazutoshi Gohara
*Division of Applied Physics, Graduate School of Engineering, Hokkaido University, Sapporo 060-8628, Japan*

Michael J. Richardson
*School of Psychological Sciences, Macquarie University, Sydney 2109, Australia*
*and Center for Elite Performance, Expertise and Training, Macquarie University, Sydney 2109, Australia*

(Received 1 November 2022; accepted 17 February 2023; published 10 April 2023)

The playground swing is a dynamic, coupled oscillator system consisting of the swing as an object and a human as the swinger. Here, we propose a model for capturing the effect of the initial phase of natural upper body motion on the continuous pumping of a swing and validate this model from the motion data of ten participants pumping swings of three different swing chain lengths. Our model predicts that the swing pumps the most if the phase of maximum lean back, which we call the initial phase, occurs when the swing is at a vertical (midpoint) position and moving forward when the amplitude is small. As the amplitude grows, the optimal initial phase gradually shifts towards an earlier phase of the cycle, the back extreme of the swing's trajectory. As predicted by our model, all participants shifted the initial phase of their upper body movements earlier as swing amplitude increased. This indicated that swingers adjust both the frequency and initial phase of their upper body movements to successfully pump a playground swing.

DOI: 10.1103/PhysRevE.107.044203

## I. INTRODUCTION

The pumping of a playground swing can be defined as a coupled oscillator system consisting of an object and a human, the swing and the swinger. This understanding and the commonality of swinging as a playground activity have resulted in more than a half century of research on the dynamics of playground swings (from the 1970s [1–4] to the 2020s [5–8]). In a seated position, the way to add energy to the playground swing is by rotating the center of mass around the end of the chain of the swing, corresponding to a driven oscillation [9]. Accordingly, because the motion of the upper body rotates the center of mass, one can simply focus on the phase between the upper body and the swing to understand the dynamics of pumping a playground swing.

To build our model of the dynamics of pumping the playground swing, we combined and expanded upon the key aspects of two types of models that have been proposed to capture how the upper body moves to pump a swing. The first or classic approach [10] defines the upper body movement of a swinger as $\cos \omega t$, where $\omega$ is the frequency of movement oscillations, such that the upper body of the swinger moves in a smooth and natural way [see Fig. 1(a)]. One issue with this model is that it assumes the swinger periodically moves the upper body at a fixed frequency [hereafter referred to as the *fixed-frequency model* (FFM)]. When the amplitude of the swing grows [see blue curve in Fig. 1(a) bottom], the period of the swing extends, which results in the fixed frequency of the swinger's upper body movement moving outside the swing's basin of frequency entrainment. Once this happens, the swinger can no longer pump the swing effectively [6]. Thus, by assuming swingers' natural upper body movement occurs at a fixed frequency, the FFM predicts that effective swing pumping only occurs up to a certain amplitude (i.e., $\approx 40^\circ$).

The second and more recent approach assumes that the swinger detects when the velocity of the swing equals zero and abruptly moves the upper body at this zero-velocity point (e.g., [6,11]). As illustrated in Fig. 1(b), this model, hereafter referred to as the *square-wave model* (SWM), enables the frequency of the upper body movement to adjust to that of the swing, such that effective swing pumping has no amplitude limit. One issue with this model is that the movement of the upper body is defined in a less natural way, because this model assumes that a swinger makes square-wave-like changes to the angle of the upper body when the seat of the swing reaches the highest extremes, in front and in the back (phase $= 0\pi$, $1.0\pi$).

The aim of the current paper was to propose a model of playground swing pumping that better captures human swing behavior, by combining the smooth upper body movement

*chirata@jumonji-u.ac.jp

 

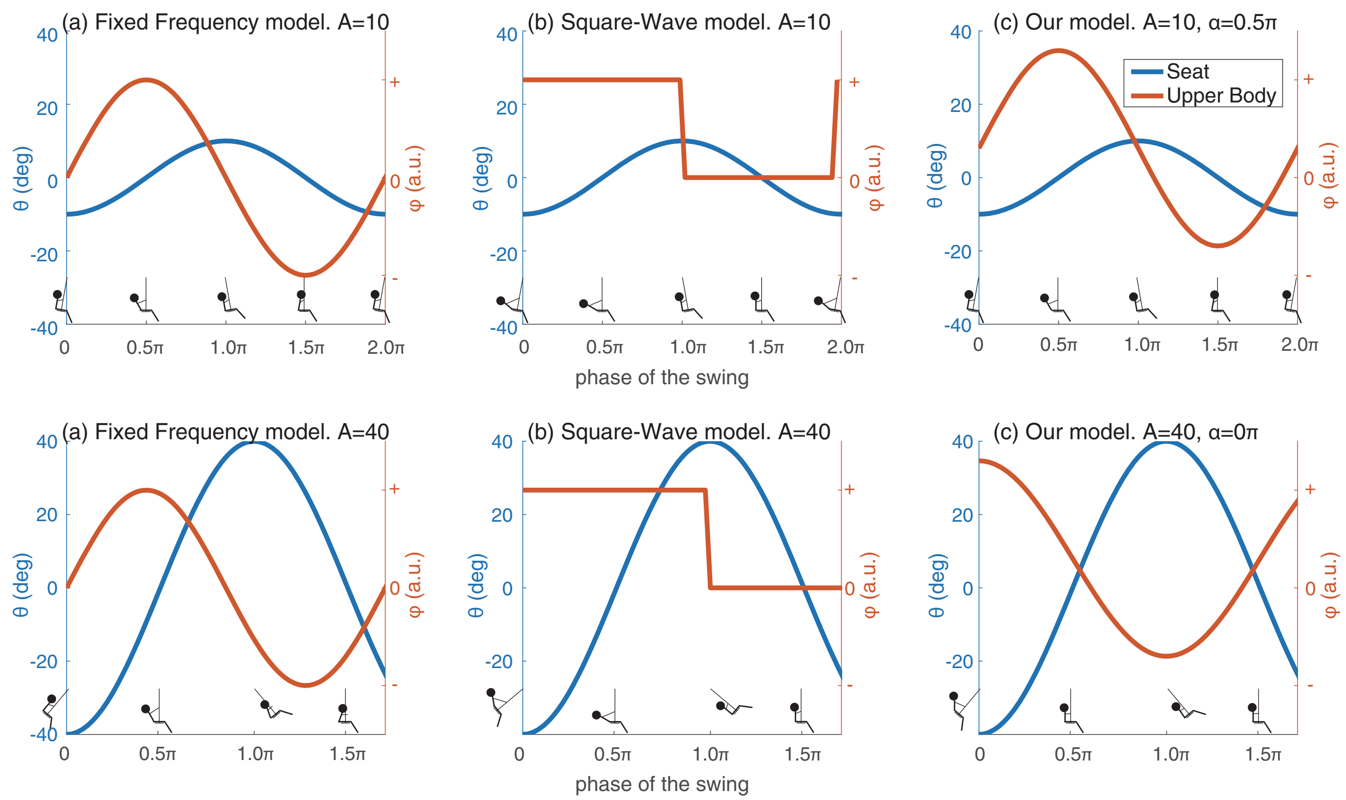


FIG. 1. Schematic drawing of different ways of modeling capturing the phase between the seat of the swing (blue) and the upper body (red). (a) Fixed frequency model. (b) Square-wave model. (c) Model proposed here. Top row: $A$ (amplitude of the swing) $= 10^{\circ}$. Bottom row: $A = 40^{\circ}$. The unit for the upper body angle (ordinate axis on the right of each panel) is arbitrary. Note that in the bottom row a whole cycle of the swing to the phase of $2.0\pi$ is not shown because the period of the swing is extended with the gain of the amplitude.

characteristics of the FFM with the frequency adaption characteristic of the SWM. As shown in Fig. 1(c), the model proposed here assumes that the swinger smoothly moves the upper body to modulate the movement to the state of the swing. This is achieved by including a term for both the initial phase $\alpha$ and frequency $\omega$ of the upper body movement. Here, we defined the initial phase $\alpha$ of upper body movement as the interval between the onsets of a cycle of the swing and upper body, where the cycle of the swing is defined to start from the back extreme of swing movement and the onset of the cycle of upper body movement is defined as the point at which a swinger's leaning backwards movement reaches its maximum. As illustrated in Fig. 1(c) top, when the amplitude of the upper body reaches its maximum when the swing is at its vertical position, the initial phase of the upper body $\alpha$ equals $0.5\pi$. When the initial phase $\alpha$ is $0\pi (= 2.0\pi)$, the maximum leaning back of the swinger occurs at the back extreme of the swing [see Fig. 1(c) bottom]. As can also be discerned from an inspection of Fig. 1(c) bottom, the upper body of the swinger remains in sync with the swing even when the period of the swing increases. In short, this model predicts that swing pumping requires that a swinger not only modify the frequency of upper body movement $\omega$, but also the initial phase $\alpha$ of the upper body movement to keep this coupling system effective.

To validate the proposed model and the role of initial movement phase in pumping a playground swing, we recorded the motion data of ten participants pumping a playground swing. An in-laboratory swing was constructed, such that lengths of the chains from the top bar of the swing to the swing seat could be manipulated. The chain length of a swing is one of the key parameters that determines the resonant frequency of the swing. The motion data of participants swinging for three different chain lengths were therefore examined to verify the reliability of our model and the role of the initial phase on pumping a swing.

## II. MODEL

### A. Model formulation

As illustrated in Fig. 2(a), our model assumes the swing is a uniform rigid body, a chain and seat, with mass $M_0$ and length $L$. Consistent with Klimina and Formalskii [5], we also assume two degrees of freedom in the hip and the knee joints of the swinger, with the weight and length of torso, legs, and lower legs defined as $m_{1-3}$ and $l_{1-3}$, respectively. The angles of the seat, the upper body, and the lower leg are denoted as $\theta$, $\phi$, and $\psi$, with the origin for $\theta$ (i.e., $\theta = 0$) corresponding to the vertical position of the swing. The angles of $\phi$ and $\psi$ are defined with respect to $\theta$, such that $\phi = 0^{\circ} = \theta$ and $\psi = 0^{\circ} = \theta$. All angular variables are positive for counterclockwise motion with the swinger assumed to be seated facing the + side (forward) of $\theta$. The ranges of motion of the hip and the knee joint are defined as $\bar{\phi} \pm \phi_0$ and $\bar{\psi} \pm \psi_0$, respectively, where $\bar{\phi}$ and $\bar{\psi}$ are the centers of the range [see Figs. 2(b) and 2(c)]. This manipulation captures the asymmetry of human

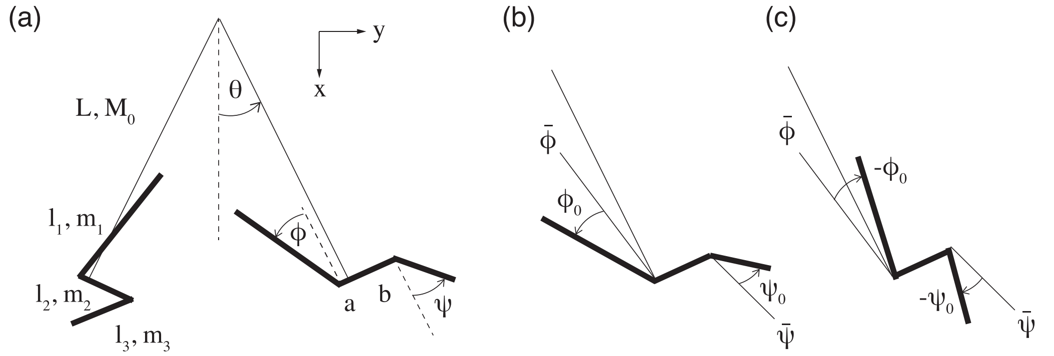


FIG. 2. Rigid body model with two degrees of freedom sitting on a swing. (a) Definitions of parameters. (b, c) Positive (b) and negative (c) range of motion of the hip and the knee joint.

leaning movements on a swing, whereby the leaning back motion of human upper body movement is larger compared to the leaning forward.

To obtain the sitting position of the human $a$ and $b$ shown in Fig. 2(a), we assume that the upper body and lower leg are perpendicular to the seat in the stationary state of the swing and that the moment around the seat is zero, such that $a = (m_2/2 + m_3)l_2/M$, $b = (m_1 + m_2/2)l_2/M$, where $M = m_1 + m_2 + m_3$. The Lagrangian $\mathcal{L}$ for the model is therefore

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{2}(I + I')\dot{\theta}^2 + Ng\cos\theta - \frac{m_1}{2}l_1 g\cos(\theta+\phi) + \frac{m_3}{2}l_3 g\cos(\theta+\psi) \\ & + \frac{m_1}{2}l_1\{(2\dot{\theta}\dot{\phi} + \dot{\phi}^2)l_1/3 + (\dot{\theta}^2 + \dot{\theta}\dot{\phi})(-L\cos\phi + a\sin\phi)\} \\ & + \frac{m_3}{2}l_3\{(2\dot{\theta}\dot{\psi} + \dot{\psi}^2)l_3/3 + (\dot{\theta}^2 + \dot{\theta}\dot{\psi})(L\cos\psi + b\sin\psi)\},\end{aligned} \tag{1}$$

where $I = (M_0/3 + M)L^2$, $I' = (m_1 l_1^2 + m_2 l_2^2 + m_3 l_3^2)/3 + (m_1 m_3 - m_2^2/4)l_2^2/M$, and $N = (M_0/2 + M)L$. This Lagrangian is an extension of the model proposed by Glendinning [6]. If the length and the weight of the upper legs are ignored, $m_2 = 0$, $l_2 = 0$, and $a = b = 0$; the mass of the chain is equal to zero, $M_0 = 0$; and the angles of the hip joints and the knee joints are equal, $\phi = \psi$, then Eq. (1) is equivalent to Glendinning's [6] model.

The Euler-Lagrange equation for the seat angle $\theta$ from Eq. (1) is

$$\begin{aligned}& (I + I')\ddot{\theta} + \{m_1 l_1(-L\cos\phi + a\sin\phi) + m_3 l_3(L\cos\psi + b\sin\psi)\}\ddot{\theta} \\ & = -NLg\sin\theta + \frac{m_1}{2}l_1 g\sin(\theta+\phi) - \frac{m_3}{2}l_3 g\sin(\theta+\psi) \\ & \quad - \frac{m_1}{2}l_1\left(\frac{2}{3}l_1 - L\cos\phi + a\sin\phi\right)\ddot{\phi} - \frac{m_3}{2}l_3\left(\frac{2}{3}l_3 + L\cos\psi + b\sin\psi\right)\ddot{\psi} \\ & \quad - \frac{m_1}{2}l_1(L\sin\phi + a\cos\phi)(2\dot{\theta}\dot{\phi} + \dot{\phi}^2) - \frac{m_3}{2}l_3(-L\sin\psi + b\cos\psi)(2\dot{\theta}\dot{\psi} + \dot{\psi}^2) - k\,\mathrm{sign}(\dot{\theta})(\dot{\theta}^2 L^3 + k').\end{aligned} \tag{2}$$

The term $k\mathrm{sign}(\dot{\theta})(\dot{\theta}^2 L^3 + k')$ appearing on the right-hand side of Eq. (2) embodies both velocity-dependent resistance and dry friction. In the context of pumping dynamics, friction is often neglected or represented as a simple linear air resistance proportional to velocity. Studies examining this resistance have primarily focused on contact resistance at the pillar of the swing [12,13]. In contrast, one can feel the airflow through hair and on the face, particularly with a large swing amplitude when pumping a swing. Our calculation indicates that the velocity of the swing can reach over 20 km/h when the chain length is 2.01 m. These observations have led us to incorporate significant air resistance in our model, which is characterized by a resistance proportional to the square of velocity. The coefficients $k = 0.4$ and $k' = 7/3$ were determined using the Runge-Kutta method, with $\phi_0 = 30^\circ$ in the range of $\theta < 100^\circ$.

The position $a$ of the swinger in Fig. 2(a) may shift during pumping and may vary among swingers and thus have a consequential impact on both Eqs. (1) and (2). We investigated the effect of position $a$ in the range of $\pm 0.1$ m, with the height of the swinger regarded as $L_1 + L_2 + L_3 = 0.792 + 0.393 + 0.395 = 1.58$ m and weight of $m_1 + m_2 + m_3 = 31.3 + 12.3 + 6.4 = 50$ kg. The height and weight of this typical swinger were derived from the mean values for Japanese females aged 20–24 according to the Japanese Ministry of Education, Culture, Sports, Science, and Technology. The ratio for three segments was sourced from Plagenhoef *et al.* [14]. As a result, the moment of inertia $I + I'$ shifts by 0.34%, the balanced position from rest varies within 2.9°, and the shift in the moment caused by gravity $N$ is 0.22%. Therefore, we considered these values small enough to regard the parameter $a$ as a constant.

The resonance between the swing and the swinger results in a gain in swing amplitude $\theta$. If we regard the swing as a single pendulum, the period of the swing will be longer as $\theta$ increases [6]. Under the condition that $\dot{\phi} = \dot{\psi} = \ddot{\phi} = \ddot{\psi} = k = 0$ in Eq. (2), the period $T_n$ of the $n$th cycle of the swing can be estimated using the complete elliptic function of the first kind $K$ as follows:

$$T_n = \frac{2\pi}{\omega_n} = 8\pi K\left[\sin\left(\frac{A_{n-1}}{2}\right)\right]\sqrt{\frac{I + I' + I_p}{g\sqrt{C_s^2 + C_c^2}}},$$
$$I_p = m_1 l_1(-L\cos\phi + a\sin\phi) + m_3 l_3(L\cos\psi + b\sin\psi),$$
$$C_s = N - \frac{m_1}{2}l_1\cos\phi + \frac{m_3}{2}l_3\cos\psi,$$
$$C_c = -\frac{m_1}{2}l_1\sin\phi + \frac{m_3}{2}l_3\sin\psi, \tag{3}$$

where $A_{n-1}$ denotes the amplitude of the swing at the back extreme of the cycle $n-1$, while $A_0 = 0$. Again, the onset of a swing cycle $n$ corresponds to when the seat is at the back extreme ($\theta < 0, \dot{\theta} = 0$).

Given the period obtained from Eq. (3), the motion of the upper body $\phi$ and the lower leg $\psi$ are then defined as sinusoidal functions of the form

$$\phi(t) = \phi_0\cos(\omega_n t + \alpha_n) + \bar{\phi}, \tag{4}$$

$$\psi(t) = \psi_0\cos(\omega_n t + \alpha_n) + \bar{\psi}, \tag{5}$$

where $\omega_n = 2\pi/T_n$ such that the frequencies of the upper body and the lower leg movements are defined with respect to the frequency of the swing $\omega t$, as in Case and Swanson [10]. Here, the time index $t$ is set to zero at the back extreme of the swing to start a new cycle of the swing. It is almost infeasible to predict the period precisely when the length of the pendulum is in a state of constant fluctuation. To overcome this difficulty, we compared periods between the maximum and the minimum of the pendulum length. Assuming that the pendulum length is defined as the distance between the pivot point of the swing and the center of mass of the swinger, it reaches its maximum when the swinger reclines horizontally and will be a minimum when the swinger sits with an upright posture. Here, we estimated the length of the pendulum for a typical swinger with a height of 1.58 m and a weight of 50 kg. For the swing with a short chain length 1.61 m, the pendulum length varied within $1.4669 \pm 0.0419$ m, and the size of the range 0.0419 m is about 2.86% of the median length of 1.4669 m. For the medium chain of 1.81 m, the range of the chain length was $1.6494 \pm 0.0395$ m, equating to 2.39%, and for the long chain of 2.01 m the range of the chain length was $1.8322 \pm 0.0375$ m, corresponding to 2.04%. We regarded these fluctuations as small enough to define the distance between the pivot of the swing and the estimated median of the center of mass of the swinger as the pendulum length. We put this pendulum length into Eq. (3) and obtained oscillatory period $T_n$. This estimate enabled us to successfully derive a numerical solution for Eq. (2) using the Runge-Kutta method, which demonstrates that the oscillation of the swing constantly increases its amplitude.

At the onset of the cycle of the swing $t = 0$, $\phi(t)$ and $\psi(t)$ are determined by Eqs. (4) and (5), with the term $\alpha$ as the initial phase of the upper body and the lower leg (updated at the onset of each cycle). Note that the initial phase $\alpha$ is the interval between the onset of a cycle of a swing ($t = 0$) and the time of maximum leaning back $\phi_0$ in that cycle. Thus, if the angle $\phi$ reaches its maximum at the onset of the swing cycle $t = 0$, the initial phase will be $\alpha = 0\pi$ as shown in Fig. 1(c) bottom. If the angle $\phi$ reaches its maximum when the swing is at the vertical ($\theta = 0, \dot{\theta} > 0$), the initial phase will be $\alpha = 0.5\pi$ as shown in Fig. 1(c) top.

For the initial phase $\alpha$, it is difficult to derive equations that define its relationship with other parameters, such as the amplitude and the period. Instead, we explore the relationship between the initial phase and the gain of amplitude per cycle by examining the contributions of different model parameters (e.g., $\alpha, \phi_0, \psi_0, \bar{\phi}$, and $\bar{\psi}$), to the gain of the amplitude of the swing by substituting $\phi$ and $\psi$ of Eqs. (4) and (5) into Eq. (2). To estimate the gain $\Delta$ of the amplitude $A$ per cycle, we assume that $\theta$ in Eq. (2) is expressed as $\theta(t) = -A\sin\omega t$, and then expand the trigonometric functions for $\theta, \phi$, and $\psi$ to fourth order terms. We then multiply Eq. (2) by $\dot{\theta}$ and integrate the energy of one cycle to estimate $\Delta$. Importantly, we do not put the initial phase $\alpha$ in the equation that determines the swing amplitude $\theta$ as in Case and Swanson [10]. Instead, the initial phase $\alpha$ is defined in Eqs. (4) and (5), which determine the motion of the swinger, $\phi$ and $\psi$. Accordingly, the gain of the amplitude per cycle can be expressed as

$$\Delta = \frac{A_n}{2Ng\sin A_n}\left\{c_\phi m_1 l_1\phi_0\pi\sin\alpha_n - c_\psi m_3 l_3\psi_0\pi\sin\alpha_n - k\left(\frac{8}{3}A_n^2\omega_n^2 L^3 + 4k'\right)\right\}$$

$$\text{where}\quad c_\phi = \left(1 + \frac{1}{4}A_n\phi_0\cos\alpha_n - \frac{1}{8}A_n^2 - \frac{1}{8}\phi_0^2 - \frac{1}{2}\bar{\phi}^2\right)g - \left(L - \frac{2}{3}l_1 - \frac{3}{8}L\phi_0^2 - \frac{L}{2}\bar{\phi}^2\right)\omega_n^2$$
$$+ \left(\frac{1}{2}(1-\bar{\phi})\phi_0 A_n\cos\alpha_n - \frac{1}{4}(1-\bar{\phi}/2)\phi_0^2\right)L\omega_n^2,$$
$$c_\psi = \left(1 - \frac{1}{4}A_n\psi_0\cos\alpha_n - \frac{1}{8}A_n^2 - \frac{1}{8}\psi_0^2 - \frac{1}{2}\bar{\psi}^2\right)g - \left(L + \frac{2}{3}l_3 - \frac{3}{8}L\psi_0^2 - \frac{L}{2}\bar{\phi}^2\right)\omega_n^2$$
$$+ \left(\frac{1}{2}(1-\bar{\psi})\psi_0 A_n\cos\alpha_n - \frac{1}{4}(1-\bar{\psi}/2)\psi_0^2\right)L\omega_n^2. \tag{6}$$

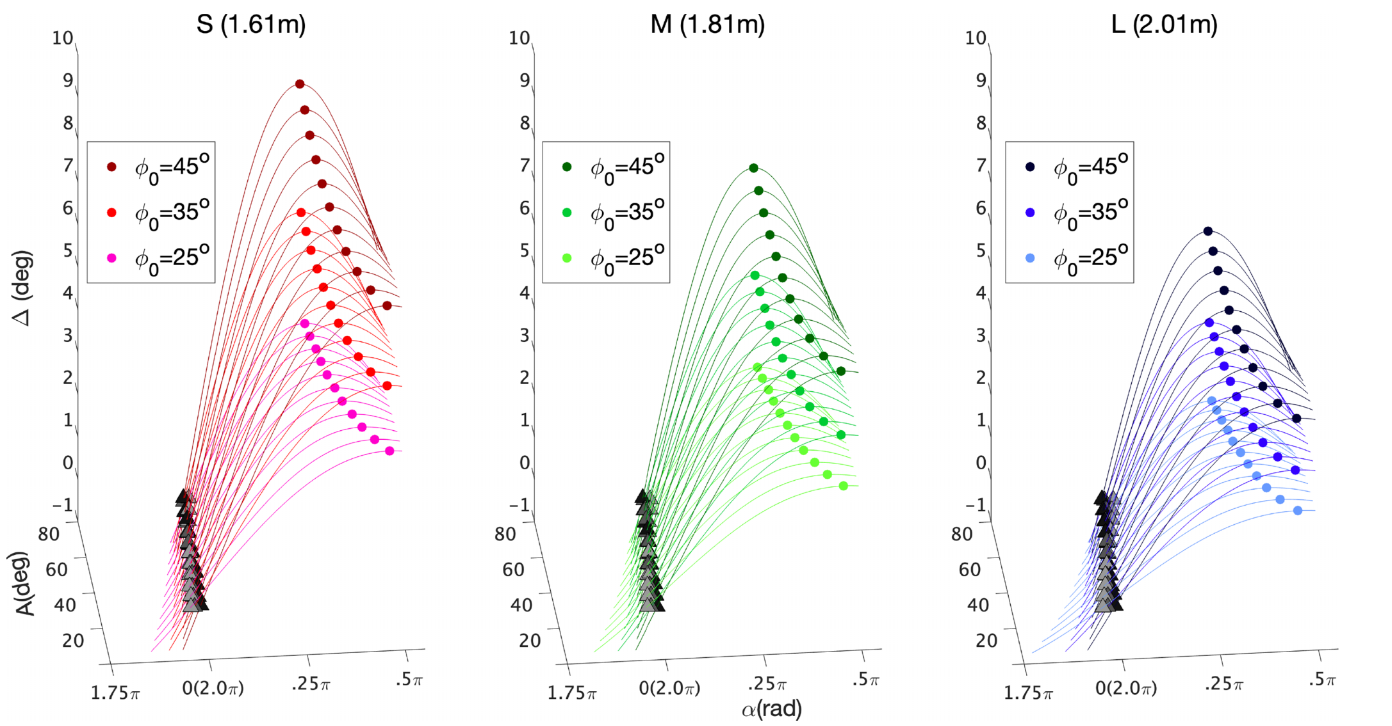


FIG. 3. Initial phase $\alpha$, amplitude of the swing $A$, and gain of the amplitude per cycle $\Delta$ (left, short chain; middle, medium chain; right, long chain). The bright, medium, and dark colors indicate small ($\phi_0 = 25^\circ$), medium ($\phi_0 = 35^\circ$), and large ($\phi_0 = 45^\circ$) amplitude of the upper body, respectively. Colored dots indicate the initial phase $\alpha$ to obtain the maximum $\Delta$ for a given amplitude $A$. Triangles in dark colors are initial phases for zero $\Delta$ values.

It is important to note that the term on the leftmost side in the wave brackets, $c_\phi m_1 l_1 \phi_0 \pi \sin\alpha_n$, illustrates that $\Delta$ increases with $\sin\alpha_n$. If the initial phase $\alpha_n$ is close to $0.5\pi$, the maximum gain will be achieved, and if the term $\sin\alpha_n$ is small, the gain of the amplitude will be limited. In Eq. (6), one of the coefficients of $\sin\alpha_n$ and $c_\phi$, depends on $\cos\alpha_n$ so that $\Delta$ would not reach its maximum at $\alpha_n = 0.5\pi$. If we assume the typical swinger of height 1.58 m and weight 50 kg, our numerical simulation showed that $c_\phi$ stays greater than zero when the amplitude of swing $\theta$ is less than 90°. Therefore the term of $\Delta$ in Eq. (6) will reach its maximum with $\alpha_n = 0.5\pi$ in the earlier stages of pumping while $\theta$ is small, and as the amplitude of the swing increases, $c_\phi$ also increases to a significant extent and the phase needed to obtain maximum $\Delta$ will gradually shift from $0.5\pi$ toward $0\pi$. Given that the mass of the lower leg $\psi_0$ is significantly smaller than that of the upper body [14], we will disregard the effect of the lower leg from this point on.

### B. Initial phase and amplitude gain of swing

Figure 3 illustrates the amount of gain per cycle, $\Delta$, predicted by Eq. (6) as a function of swing amplitude $A$, initial phase $\alpha$, and chain length. Three slopes for each panel differ in the amplitude of the swinger's maximum leaning back (bright color, $\phi_0 = 25^\circ$; medium color, $\phi_0 = 35^\circ$; dark color, $\phi_0 = 45^\circ$). These values of $\phi_0$ are determined as the mean $\pm$ 1 standard deviation (SD) of $\phi_0$ obtained from the subsequent motion analysis of ten participants [mean = 35.75, SD = 10.81; see Fig. 6(c)]. The colored dots indicate the $\alpha$ for the maximum $\Delta$ in a given swing amplitude $A$, and gray triangles are those where $\Delta = 0$. From an inspection of Fig. 3, it is clear that for the proposed model the gain of swing amplitude $\Delta$ increases in proportion to the amplitude of the upper body, $\phi_0$. Moreover, given the same amplitude $A$ and initial phase $\alpha$, the swing with a shorter chain is more likely to amplify. The initial phase $\alpha$ for the maximum gain $\Delta$ (colored dots) gradually shift from $0.5\pi$ as $A$ increases. Furthermore, if the initial phase $\alpha \approx 0$, that is, if maximum leaning back $\phi_0$ occurs near the back extreme of the swing, the swing will not amplify or decay.

In summary, our proposed model is an extension of the previous models presented in the literature, with the following key additions. First, a frictional force $k$ was introduced in Eq. (2), so that the gain of the amplitude of the swing did not explode along with the pump, and it remained within a realistic human movement range (see Fig. 3). Second, the initial phase of upper body movement was also introduced onto the equation of motion [Eq. (4)], with the resultant model defining how the initial phase $\alpha$ affects the amplitude gain of the swing. In what follows, we examined whether the corresponding predictions (also illustrated in Fig. 3) are also observed in the motion data of human swingers.

## III. MOTION ANALYSIS

### A. Method

#### 1. Participants

Ten female college students (mean age 20.3 years) participated in this study. All participants stated that they had

FIG. 4. A participant pumping the swing in the lab.

previously learned to effectively pump a playground swing (usually in their childhood). They were not specifically trained for the purpose of this study and did not practice pumping the swing in the laboratory prior to recordings. Participation in the experiment was entirely voluntary and all procedures and methods employed were approved by the Ethics Committee of Jumonji University, which adheres to the principles outlined in the Declaration of Helsinki.

#### *2. Experimental setup*

The swing with the horizontal pillar of 2.29 m high was set up in a room with a ceiling height of 3.00 m (see Fig. 4). Three lengths of the chains to hold the seat to the pillar were employed: 1.61 m as short, 1.81 m as medium, and 2.01 m as long. The lengths were measured from the pillar to the surface of the seat. These chains consist of four segments, with three knots for participants to easily grip the chain.

A motion capture system consisting of four cameras (OQUS 300, Qualisys, Sweden) was used to measure the position of optical markers attached to the swing and to the participants in two-dimensional coordinates ($x$ and $y$ in Fig. 2) at a frequency of 100 Hz. Three markers were attached to the swing, one at the top pillar, and one at both the front and the back of the seat of the swing. The average position of two markers attached to the seat defined the position of the intersection of the chain and the seat. Seven markers attached to the participants: (1) top of the head, (2) right acromion, (3) right greater trochanter, (4) right knee, (5) right ankle, (6) right elbow, and (7) right wrist.

#### *3. Task*

Participants were asked to pump the swing from rest, while seated until the amplitude increased more than 40° without kicking the floor. When the experimenter judged that the swing amplitude was large enough, the experimenter announced that the participant could stop. Thus, the trial duration and the maximum amplitude of the swing differed among trials (average trial duration of ≈60 s). Each participant performed one trial for each chain length, and the order of the trials was randomized among participants.

### B. Examination of model fit

Figure 5 shows the amplitude gain per cycle $\Delta$, predicted from the model versus the gain obtained from participant trials. To predict the amplitude gain, we put values of $A, \omega, \alpha,$ and $\phi$ of each cycle obtained from motion data and the chain length into Eq. (6). All swing cycles of all ten participants are plotted on each panel, consisting of 210, 226, and 260 cycles. The $R^2$ values of the linear fit between predicted and measured $\Delta$ were 0.57, 0.41, and 0.30 for short, medium, and long chains, respectively. The observed $\Delta$ values are constantly larger than predicted ones, especially in shorter chains, and this would lead to a better fit of the linear regression in shorter chains. Our model predicted a smaller amount of gain per cycle $\Delta$ than the actual gain, especially for shorter chains, due to the conservative setting of the friction term $k = 0.4$ in Eq. (2). Given that friction increases proportionally with the square of the seat velocity, this parameter setting leads to the prediction of a smaller gain, particularly in shorter chains where the seat swings more rapidly. Modifying the parameters

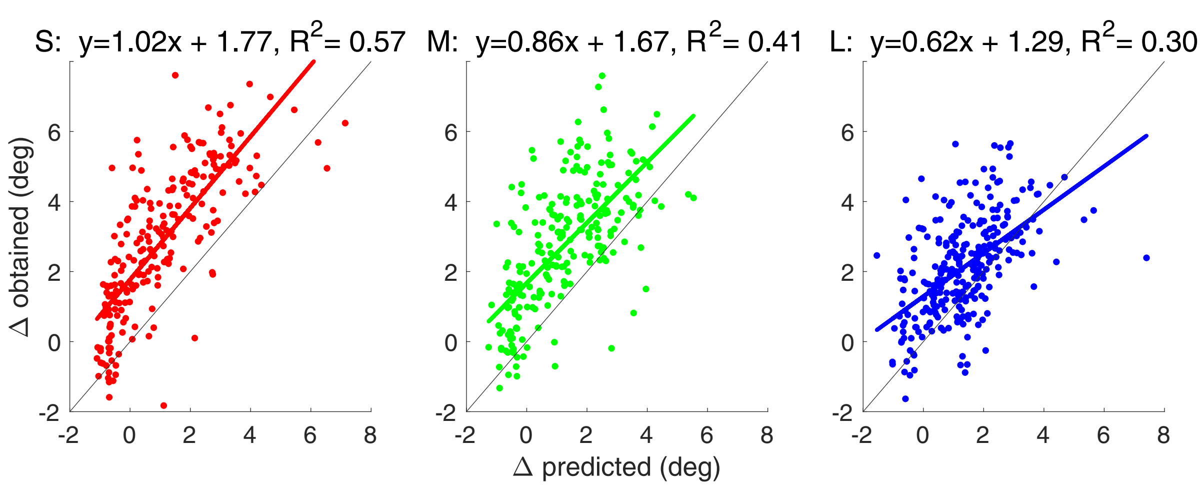


FIG. 5. Predicted and obtained gain of seat amplitude $\Delta$ per cycle (from left to right: short, medium, and long chain).

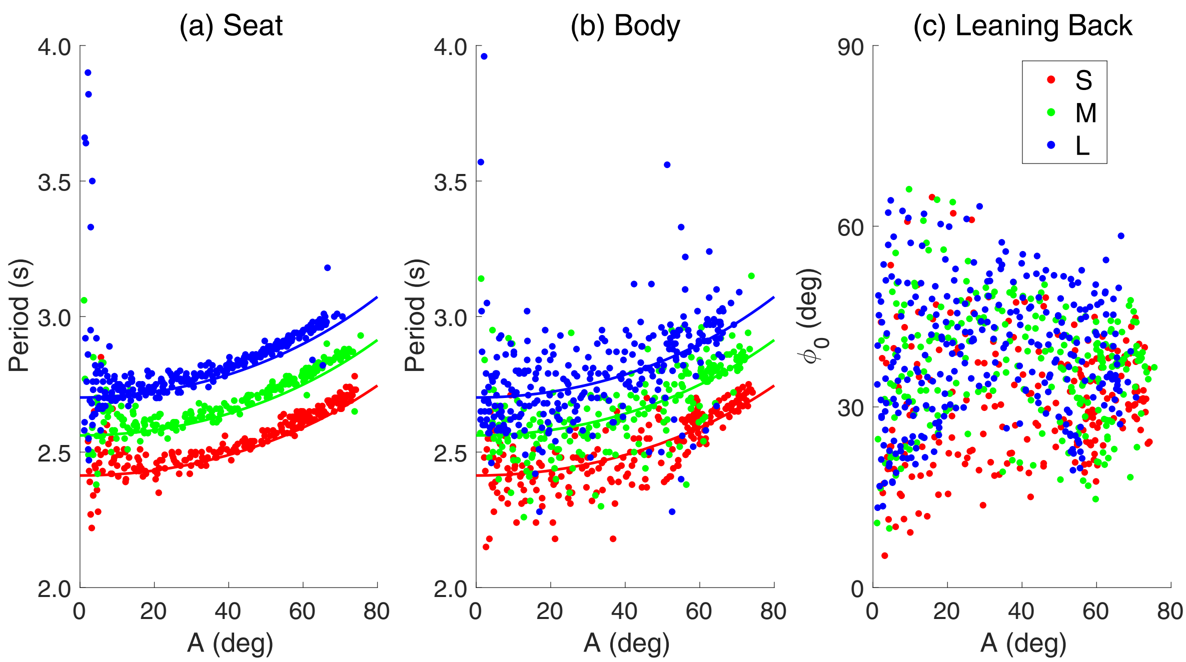


FIG. 6. Period of the (a) seat, (b) upper body, and (c) amplitude of maximum lean back as a function of the swing amplitude $A$. Red, green, and blue indicate short, middle, and long chains, respectively. The solid lines in panels (a) and (b) indicate the period of the swing predicted from Eq. (3).

for the friction term of Eq. (2) for each chain length and/or reconsidering the model of the resistance force itself would be necessary for a better fit in the future.

### C. Period of seat and upper body, and amplitude of lean back

Figure 6(a) shows the period of the seat, the interval between the onset of the seat cycle $n$ and $n + 1$, as a function of the amplitude of the seat at the back extreme ($\theta < 0, \dot{\theta} = 0$). As in Fig. 5, the data of ten participants overlap for all chain lengths, indicating the stability (and similitude) of the observed behavioral pattern across individuals (i.e., the observed pattern of behavior is individually independent). Colored curves indicate the theoretical period obtained from Eq. (3). As expected, the period of the swing converged to the predicted curve for all three chains, with increases in chain length and amplitude leading to a longer period of seat oscillation. The swing continued to amplify over 40° where the swing period extended rapidly. This indicates that the swingers did in fact adjust the frequency of the upper body movement along with the pump. For the long chain length, for example, when the amplitude of the swing $A$ increased from 0° to 70°, the period increased from 2701.4 to 2977.3 ms (a difference of 275.9 ms, or approximately 10% of the cycle).

Figure 6(b) shows the period of the upper body motion. Although the deviation around the curve is much larger than that of the seat, the upper body of the human swingers still followed the curve predicted by the model. The reason for the increased variance was likely due to the noise inherent to human movements [15], and/or because participants intermittently controlled both frequency and the initial phase of the upper body movement.

Contrary to the periods of the seat and the upper body, the amplitude of maximum leaning back, $\phi_0$, does not show clear relations with the length of the chain, or with the amplitude of the swing [Fig. 6(c)]. As Fig. 3 indicates, the amount of gain $\Delta$ increases with the size of leaning back $\phi_0$. However, even when the gain of the swing amplitude is necessary, the amplitude of leaning back stayed rather invariant. Therefore, to pump the swing, participants preferred to control temporal aspects of their movement (i.e., period), rather than the spatial aspects of upper body motion (i.e., $\phi_0$).

### D. Upper body cycle trajectories

Figure 7 shows the trajectory of the upper body angle $\phi$ as a function of the phase of the seat. The three columns from the left show the profile of the first five participants and the right half shows that of the remaining five. The shapes of the profile of the upper body movement are roughly sinusoidal as assumed by Case and Swanson [10] and assuming that there were no sudden jumps of upper body motion at either extremes or any fixed motion in between (see [6,11,16]). In addition, the variations in the trajectory of the upper body movement, caused by individual differences (vertical comparisons of panels), were greater than that of the chain length (horizontal comparison). This indicated that each participant moved their upper body with their own unique way (i.e., each participant has a slightly unique movement profile) within a resonance range with the swing.

The colored thick curves in Fig. 7 are the trajectories of the upper body in a cycle when the amplitude of the seat $A$ was the closest to 10°, as this is when a large gain of the amplitude was required. As expected, when the amplitude of the swing $A$ was small, the maximum lean back $\phi_0$ of participants occurred around the initial phase $\alpha \approx 0.5\pi$, such that the swing

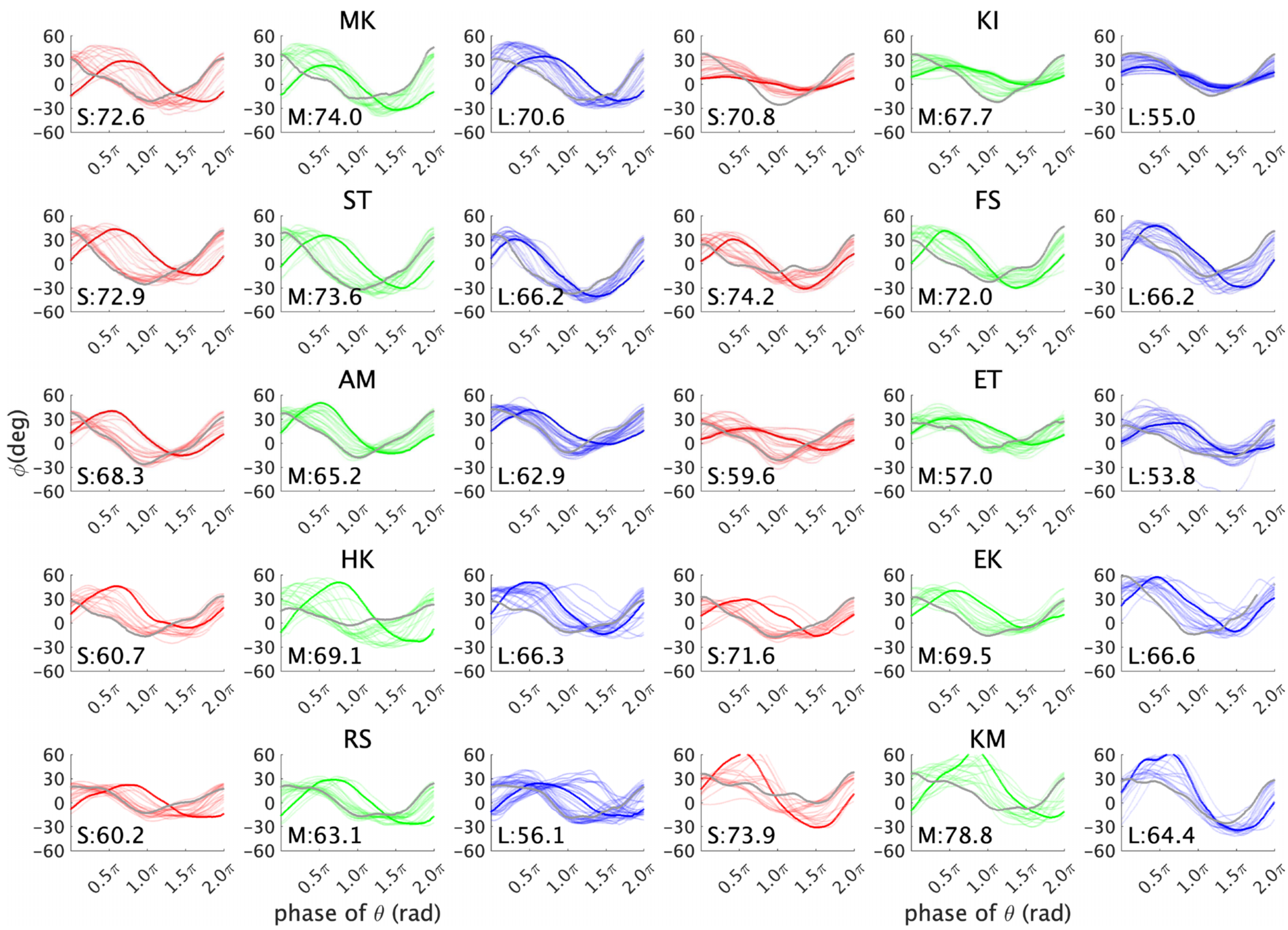


FIG. 7. Trajectories of upper body motion as a function of the phase of the seat. The colored thick curves are from the cycle of $A \approx 10^{\circ}$ when the pump is required. The gray curves are from the cycle of maximum $A$ of the trial where the gain of the amplitude was no longer necessary. The curves in faint colors are from cycles in between. The initials of the corresponding participant, length of chain, and maximum $A$ of the trial are shown.

continued to amplify in succeeding cycles (i.e., curves in faint colors), then the amplitude of the swing eventually exceeded $40^{\circ}$. The gray curve shows the trajectory of the cycle when the maximum amplitude of the trial is achieved and, thus, a gain in amplitude is no longer necessary. Comparing the gray curve to the colored thick line within each panel, the size of peaks and the valleys of the curves are similar in most cases. The most salient difference can be found in the horizontal (i.e., temporal) position of the curves, or the initial phase. This difference is detailed in the next section.

### E. Shift of initial phase during pumping

The initial phase of the upper body $\alpha$ as a function of $A$ is shown in Fig. 8, which captures the timing of maximum upper body lean back along with the pump of the swing. The three trials for each chain length for one participant are presented in each panel, with their initials shown above each panel column. Importantly, if it is critical for participants to control not only the frequency $\omega$ but also the initial phase $\alpha$ in order to pump the swing, the initial phase will be $\alpha = 0.5\pi$ at the beginning of the trial, and will exhibit a gradual decrease as amplitude increases, as Fig. 3 predicts. Furthermore, if the gain of the amplitude is no longer necessary, the initial phase will drop down to $\alpha = 0\pi$. In Fig. 8, the initial phase that maximizes the gain $\Delta$, obtained from Eq. (6), is shown in colored solid lines (if three lines are not seen in a panel, lines overlap), and the initial phase that maintains the swing amplitude is shown as gray lines. For Fig. 8, actual data of $A$, $\phi$, and $\alpha$ from trials were used to calculate theoretical values. As expected, at the beginning of a trial with small amplitude $A$, the initial phase of each participant is around $0.5\pi$, indicating that participants leaned back the most when the swing was at the vertical position while moving forward. Furthermore, the initial phase shifts gradually as the theoretical curve predicts with the pump of the swing. Finally, when the pump is no longer necessary, the initial phase converges into the gray line near $0\pi$ showing that the maximum leaning back occurs at the back extreme of the swing.

Although most of the trials start with the initial phase around $\alpha \approx 0.5\pi$ and end up with $\alpha \approx 0\pi$, the dynamics of the convergence to $\alpha = 0\pi$ varied among participants. It is important to note that there was only about 70 cm of space between the beam of the swing and the ceiling of the room.

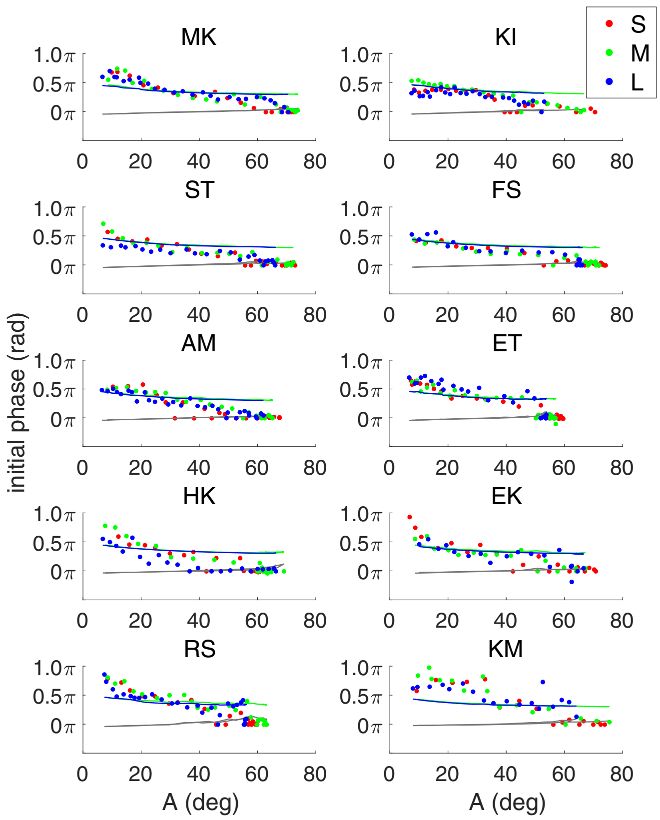


FIG. 8. Initial phase $\alpha$ as a function of amplitude $A$. Colored dots indicate the initial phase and amplitude for each cycle in colors of red, green, and blue for chain lengths of S, M, and L, respectively. Lines with color show theoretical values of initial phase to achieve maximum gain $\Delta$ with colors the same as dots. Gray lines indicate theoretical values of making $\Delta = 0$.

Thus, when the amplitude of the seat increased, participants faced a risk of hitting their head or leg on the ceiling. Therefore, participants appeared to switch their movement goals from pumping the swing to minimizing the amplitude gain, or maintaining its amplitude. Thus, in all trials, the initial phase converged to $0\pi$ in the last few cycles of a trial.

## IV. DISCUSSION

Here we proposed and validated a model of playground swing pumping, in which a swinger is predicted to modulate the amplitude of the swing via both the initial phase $\alpha$ and frequency $\omega$ of upper body movement. Consistent with these predictions, actual motion data of human swingers revealed that the smooth upper body movements of the swingers could amplify the movements of a swing over 40°, by adapting both the frequency and initial phase of their upper body movement.

From the model and the motion data, two important findings were observed in relation to the modulation of the initial phase of upper body movement. First, the initial phase gradually shifted from $0.5\pi$ toward $0\pi$ as the amplitude of the swing increased, which was consistent with the maximum gain predicted by the proposed model (see Fig. 3). Second, the initial phase changed according to the goal of the movement. That is, when pumping the swing was necessary, initial phase remained around $0.5\pi$. However, once the goal switched to maintaining the amplitude of the swing, the initial phase of upper body movements converged to $0\pi$.

In addition to modulating upper body frequency and initial phase, the amount of swing amplification could also be achieved by controlling the magnitude of swingers' leaning back $\phi_0$ movements, as illustrated in Fig. 3. The larger upper body angle $\phi_0$ could lead to the larger gain of $\Delta$ when required, but participants did not prefer to control this spatial variable (i.e., via changes in the magnitude or degree of leaning back $\phi_0$). Instead, it appeared that all participants primarily focused on controlling the temporal aspects of their movement to modulate swing amplitude, i.e., the frequency $\omega$ and the initial phase $\alpha$ of upper body movements. Indeed, participants tended to exhibit an invariant amplitude of leaning back across swing amplitudes, constantly leaning back maximally. Therefore, the dynamics of pumping the playground swing can be regarded as the system where swingers fix spatial degrees of freedom, while modulating (controlling) temporal degrees of freedom (see [16]).

Despite the effectiveness of the proposed model, it remains unclear how human swingers continuously modulated the upper body frequency and the initial phase from cycle to cycle. As illustrated in Fig. 3, it is rather simple to find the optimal period and initial phase for one cycle. However, the actual pumping of the playground swing is a sequence of consecutive cycles, and each cycle is tightly and continuously bound to the next. Thus, modifying $\alpha$ in a cycle $n$ may also affect $\omega$ in the next cycle $n + 1$. Moreover, the shift of the initial phase shown in Fig. 8 requires fine control. When the swing amplifies from 0° to 75° in the long chain condition, for example, the extension of the period of the swing will be less than 275 ms. The initial phase shifts from $0.5\pi$ to $0\pi$, that is, a quarter of the period, therefore the shift of $\alpha$ will be about 70 ms. If it takes ten cycles to amplify the swing to 75°, the amount of the shift will be about 7 ms per cycle, which is an extremely short duration to perceive and control for humans.

One possibility is that human swingers are able to achieve such tight control via attunement to centrifugal forces, which specify the required shift in the initial phase of the movement. For example, when the amplitude of the swing increases, the centrifugal forces on the upper body also increase. When the seat is moving backwards with high velocity, the larger inertial forces would push the upper body of the swinger backwards. As a result, the upper body may lean back earlier regardless of the intention of the swinger, which may cause a gradual shift of $\alpha$ and the optimal covariation of $\alpha$ with $\omega$. One way to test this hypothesis would be to examine the phase shift of swingers in a virtual reality environment where centrifugal forces have no effect. The expectation would be that in the virtual reality environment, there would be no shift of the initial phase $\alpha$ along with the pumping of the amplitude of the swing. This would limit the size of gain $\Delta$ and slow down the process of swing pumping, thereby preventing the swing from amplifying more than 40°.

In conclusion, the current paper demonstrates the importance of frequency and initial phase modulations in pumping a playground swing, and in contrast to previous work provides a more ecologically valid model and understanding of the dynamics of playground swinging.

## ACKNOWLEDGMENTS

We thank K. Kadota for his enriching contributions to our conversations. This work was supported by Japan Society for the Promotion of Science KAKENHI Grant No. 19K11470.

[1] P. L. Tea and H. Falk, Pumping on a swing, Am. J. Phys. **36**, 1165 (1968).
[2] B. F. Gore, The child's swing, Am. J. Phys. **38**, 378 (1970).
[3] J. A. Burns, More on pumping a swing, Am. J. Phys. **38**, 920 (1970).
[4] S. M. Curry, How children swing, Am. J. Phys. **44**, 924 (1976).
[5] L. A. Klimina and A. M. Formalskii, Three-link mechanism as a model of a person on a swing, J. Comput. Syst. Sci. Int. **59**, 728 (2020).
[6] P. Glendinning, Adaptive resonance and pumping a swing, Eur. J. Phys. **41**, 025006 (2020).
[7] S. G. Nikolov, V. M. Vassilev, and D. T. Zaharieva, Analysis of swing oscillatory motion, *Advanced Computing in Industrial Mathematics*, Studies in Computational Intelligence (Springer, New York, 2021), pp. 313–323.
[8] S. Koshkin and V. Jovanovic, Swinging a playground swing: Torque controls for inducing sustained oscillations, IEEE Control Syst. Mag. **42**, 18 (2022).
[9] A. A. Post, C. E. Peper, and P. J. Beek, Effects of visual information and task constraints on intersegmental coordination in playground swinging, J. Mot. Behav. **35**, 64 (2003).
[10] W. B. Case and M. A. Swanson, The pumping of a swing from the seated position, Am. J. Phys. **58**, 463 (1990).
[11] S. Wirkus, R. Rand, and A. Ruina, How to pump a swing, College Math. J. **29**, 266 (1998).
[12] E. K. Lavrovskii and A. M. Formal'skii, Optimal control of the pumping and damping of a swing, J. Appl. Math. Mech. **57**, 311 (1993).
[13] E. I. Butikov, Spring pendulum with dry and viscous damping, Commun. Nonlinear Sci. Numer. Simul. **20**, 298 (2015).
[14] S. Plagenhoef, F. G. Evans, and T. Abdelnour, Anatomical data for analyzing human motion, Res. Q. Exerc. Sport. **54**, 169 (1983).
[15] J. B. Dingwell and L. C. Marin, Kinematic variability and local dynamic stability of upper body motions when walking at different speeds, J. Biomech. **39**, 444 (2006).
[16] S. O. Linge, An assessment of swinger techniques for the playground swing oscillatory motion, Comput. Methods Biomech. Biomed. Engin. **15**, 1103 (2012).