Implications of nonzero photon mass on plasma equilibria

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The Beltrami state in a single-species (electron or ion) ideal plasma in the context of massive electromagnetism has been explored. The inclusion of photon mass, essentially treating the massive photon field as a mobile fluid in the ideal plasma vortical dynamics, has resulted in a triple curl Beltrami state of the magnetic vector potential $\vec{A}$. A variational principle is constructed which shows this state can also be obtained by constrained minimization of the system's energy with appropriate helicity invariants. Such a state is endowed with three different length scales; one of which is the system length, and the other two are species' skin depth and photon Compton wavelength, respectively. An analytical solution of this state in the cylindrical geometry is presented, which is the linear combination of three single Beltrami states. Possible observational signatures of this state in astrophysical and laboratory settings are also discussed.

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I. INTRODUCTION

The standard theory of electrodynamics is based on the premise that the photon is massless [1–3]. It follows from the requirement of gauge invariance of the Lagrangian of the electromagnetic field. Like the cosmological constant and neutrino masses, once assumed to be precisely zero till empirical data proved otherwise, it is reasonable to assume that a photon is likely to have a tiny mass, but nonzero [4,5]. The continued interest in massive photons can also be attributed to dark matter research where massive dark photons are proposed to be force carriers that can kinetically mix with the standard model photon [6,7].

To account for photon mass, the Lagrangian for the electromagnetic field can be modified, first considered by Proca, to include a mass term as follows [1]:

$$\mathcal{L}_{\text{proca}} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} + \frac{1}{8\pi\lambda_p^2}A_\mu A^\mu - \frac{1}{c}J_\mu A^\mu, \quad (1)$$

which results in the Proca equation of motion

$$\partial^\nu F_{\mu\nu} + \frac{1}{\lambda_p^2}A_\mu = \frac{4\pi}{c}J_\mu \quad (2)$$

while satisfying the same homogeneous equations $\partial_\mu \mathcal{F}^{\mu\nu} = 0$ in the Maxwell theory. Here, $F_{\mu\nu}$ ($\mathcal{F}^{\mu\nu}$) is the electromagnetic field tensor (dual tensor), A_μ is the four potential, J_μ is the four current, and c is the speed of light. The parameter λ_p is the Compton wavelength of a photon ($\lambda_p = h/m_p c$) where m_p and h are the photon rest mass and Planck's constant, respectively. Some of the immediate consequences of nonzero photon mass are wavelength dependence of the speed of light, deviations from the exactness of Coulomb's and Ampere's law, longitudinal polarization of electromagnetic waves, and Yukawa-like dependence of magnetic field created by a magnetic dipole [2,3,8–10].

Laboratory investigations to measure photon mass have reached their limits because of the extraordinary precision required of the experiments [11]. It was realized in the early 1960s that even a tiny photon mass might affect astrophysical phenomena occurring at length scale $L > \lambda_p$ [12,13]. Therefore, direct astrophysical observations such as measuring the dispersion of photon velocity from cosmic gamma-ray bursts, solar wind data, and measurement of Jupiter's magnetic field have improved the accuracy of photon mass limit over the last few decades [14–20]. The currently accepted upper bound for photon mass $m_p < 10^{-49}$ g for which the Compton wavelength is $\lambda_p > 1$ *au* [8,18,19,21,22].

While exploring the astrophysical phenomena occurring on length scales comparable to the Compton wavelength of photons in galaxies and galaxy clusters, we generally encounter large-scale ordered and random magnetic fields. The large-scale ordered magnetic field is probably sustained by the ambient plasma [13,23–25]. Since plasma exhibits the tendency to self-organize resulting in coherent magnetic and flow configurations, it is important to analyze plasma equilibria to improve the estimates of photon mass limit and better understand the electromagnetic stress on galactic rotation curves [26]. One possible equilibria is the single Beltrami state in Magnetohydrodynamics (MHD) where plasma relaxes into a minimum energy state while keeping the global magnetic helicity intact. By assuming the pressure to be negligible, a single Beltrami state satisfies the relation $\vec{\nabla} \times \vec{B} = \lambda\vec{B}$, where $\vec{B}$ and λ are the magnetic field and proportionality constant, respectively [27,28]. This type of equilibria is also known as the force-free equilibrium because the force $\vec{J} \times \vec{B} = 0$, where J is the electric current. Because MHD is inherently scale-free, the simultaneous presence of both small- ($l < \lambda_p$) and large-scale ($L > \lambda_p$) fields makes it highly unlikely that the force-free equilibria can exist [18].

In this article, we study a wider class of large-scale equilibria in plasmas by incorporating the inertia of plasma species in the dynamics. These types of equilibria were first explored in the context of Hall MHD—a two-fluid system with ions and massless electrons [29,30]. Some of the fundamental attributes of the equilibria (also called Beltrami states) are the alignment of plasma flow $\vec{v}$ with a composite quantity known as Generalized vorticity

$$\vec{\Omega} = \vec{B} + \frac{mc}{q}\vec{\nabla} \times \vec{v}, \tag{3}$$

and conservation of Generalized helicity. Here, m and q stand for the mass and charge of the plasma species. These minimum energy states are characterized by the number of independent single Beltrami states required to construct them. These multi-Beltrami states have been investigated in plasma confinement, solar prominences, blackhole accretion disk, etc. [29,31–34].

This paper will explore the characteristics of the ordered large-scale magnetic field associated with the magnetic vector potential. This paper aims to investigate the possible existence of Beltrami states in galactic length scales and how they differ due to nonzero photon mass. Unlike the Beltrami state for zero photon mass, these states will primarily be of the magnetic vector potential $\vec{A}$ because it is a dynamical quantity in massive electromagnetism. Since multi-Beltrami equilibria are endowed with multiple scale lengths, this study will provide a possible resolution to some well-known analytical difficulties in the context of massive electromagnetism, such as photo mass limit in the presence of both small- and large-scale fields, galactic rotation curves, etc.

First, we construct the vortical dynamics of plasmas in massive electromagnetism. Next, we show how Beltrami states can be derived from the variational principle. Finally, we provide the analytical solution of the Beltrami state and discuss the features of the state.

II. PLASMA DYNAMICS IN MASSIVE ELECTROMAGNETISM

We present the dynamics of an ideal, incompressible plasma in massive electromagnetism in this section. The momentum equation obeyed by each species (labeled i) in a multispecies plasma can be written as [34]

$$\rho_i\left(\frac{\partial}{\partial t} + \vec{v}_i\cdot\vec{\nabla}\right)\vec{v}_i = \rho_i\vec{g} + q_i n_i(\vec{E} + \vec{v}_i \times \vec{B}) - \vec{\nabla}p, \tag{4}$$

where ρ_i, $\vec{E}$, and p represent the plasma density, electric field, and plasma pressure, respectively.

Now, by using (a) vector identity $(\vec{v}_i\cdot\vec{\nabla})\vec{v}_i = \vec{\nabla}(v_i^2/2) - \vec{v}_i \times (\vec{\nabla}\times\vec{v}_i)$ and (b) $\vec{E} = -\vec{\nabla}\phi - 1/c\partial_t\vec{A}$, where $\vec{A}$ and ϕ are vector and electrostatic potentials, respectively. Then, we can rewrite Eq. (4) as

$$\frac{\partial\vec{P}_i}{\partial t} - (\vec{v}_i \times \vec{\Omega}_i) = -\frac{\vec{\nabla}p}{\rho_i} - \vec{\nabla}\psi_i, \tag{5}$$

where all the potentials are incorporated into a single potential $\psi_i = c/q_i(m_i v_i^2/2 + m_i\Phi_g + q_i\phi)$. Here, Φ_g is the gravitational potential and related to the gravitational field as $\vec{g} = -\vec{\nabla}\Phi_g$.

It should be noted here that nonzero photon mass makes the magnetic vector potential and electrostatic potential observable quantities having a direct dynamical effect on the plasma motion and also satisfy the Lorentz condition

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0. \tag{6}$$

For a barotropic fluid, the equation of state yields a relationship between enthalpy H_i and pressure: $\vec{\nabla}H_i = \rho_i^{-1}\vec{\nabla}p$. Using this relationship, the new generalized potential can be written as $\tilde{\psi}_i = c/q_i(m_i v_i^2/2 + m_i\Phi_g + q_i\phi + m_i H_i)$.

Now, taking the curl of Eq. (4) we obtain the source-free vorticity evolution equation:

$$\frac{\partial\vec{\Omega}_i}{\partial t} = \vec{\nabla}\times(\vec{v}_i \times \vec{\Omega}_i), \tag{7}$$

in terms of the generalized vorticity $\vec{\Omega}_i = \vec{\nabla}\times\vec{P}_i = \vec{B} + m_i c/q_i\ (\vec{\nabla}\times\vec{v}_i)$. It should be noted that $\tilde{\psi}$ includes all potential terms which do not play a direct role in the evolution of vorticity. The dynamics of magnetic and flow vorticity are now governed by Eq. (7), and all species evolving independently are connected through Ampere's law modified by the photon mass [23]

$$\vec{\nabla}\times\vec{B} + \frac{\vec{A}}{\lambda_p^2} = \frac{4\pi}{c}\vec{J} = \frac{4\pi}{c}\sum_i n_i q_i \vec{v}_i \tag{8}$$

with the assumption that the displacement current is negligible in the nonrelativistic dynamics.

A. Constant of motion and Variational principle

We can explore the possible constants of motion of the vortex dynamics presented in the previous section. Using Eqs. (5) and (7), it is straightforward to show that the vortex dynamics leads to two constants of motion, i.e., helicity [35]

$$\begin{aligned} h_i &= \frac{1}{8\pi}\int \vec{P}_i\cdot\vec{\Omega}_i\, d^3x \\ &= \frac{1}{8\pi}\int\left(\vec{A} + \frac{m_i c}{q_i}\,\vec{v}_i\right)\cdot\left(\vec{B} + \frac{m_i c}{q_i}\,\vec{\nabla}\times\vec{v}_i\right)d^3x \end{aligned} \tag{9}$$

and energy

$$E = \int\left(\frac{1}{2}\rho_i v_i^2 + \frac{B^2}{8\pi} + \frac{A^2}{8\pi\lambda_p^2}\right)d^3x. \tag{10}$$

It should be noted here that the total volumetric magnetic energy density in Proca electrodynamics retains a similar feature to Maxwellian electrodynamics. However, the departure from the Maxwellian feature is noticeable in total volumetric magnetic pressure in Proca electrodynamics because the massive photon contribution to this quantity is negative. This can considerably alter the plasma dynamics as the negative Proca pressure will pull plasmas toward a stronger magnetic field.

A variational principle is usually constructed by assuming that energy is more fragile in the presence of dissipation than generalized helicity. As a result, we choose to extremize energy as a target functional subject to the helicity acting as a constraint, i.e., [36,37]

$$\delta Q = \delta\left(E - \sum_i \mu_i^{-1}h_i\right) = 0, \tag{11}$$

where μ_i is the Lagrange multiplier and $\vec{A}$ and $\vec{v}$ are treated as the independent variables. Performing the variation in

Eq. (11), we obtain

$$\delta Q = \frac{m_i c}{q_i}\sum_i\left(\frac{\vec{\Omega}_i}{\mu_i} - \frac{4\pi n_i q_i}{c}\vec{v}_i\right)\cdot\delta\vec{v}_i + \frac{1}{4\pi}\left(\vec{\nabla}\times\vec{B} + \frac{\vec{A}}{\lambda_p^2} - \sum_i\frac{\vec{\Omega}_i}{\mu_i}\right)\cdot\delta\vec{A} = 0. \quad (12)$$

By equating the variation of δv to zero independently, we obtain the Beltrami condition

$$\vec{\Omega}_i = \mu_i\frac{4\pi}{c}\sum_i n_i q_i\vec{v}_i = \mu_i\frac{4\pi}{c}\vec{J}. \quad (13)$$

By setting the coefficient of $\delta\vec{A}$ to zero and using Eq. (13), we obtain a relation equivalent to the time-independent Ampere's law.

Furthermore, we notice that Eq. (13) is also a solution of time-independent vorticity equation $\vec{v}_i\times\vec{\Omega}_i = 0$ when all the gradient forces are constrained to zero, i.e., $\vec{\nabla}\tilde{\psi}_i = 0$, or by integrating

$$\frac{m_i v_i^2}{2} + m_i\Phi_g + q_i\phi + m_i H_i = constant. \quad (14)$$

Equation (14) relates the enthalpy (pressure) to the flow velocity and other potentials without reference to the magnetic field or vector potential explicitly, and the only coupling to the magnetic field and vector potential is through the flow velocity. This is known as Bernoulli's condition. Then, the Beltrami condition, along with Ampere's law, constitutes the minimum energy states of the N-species plasma.

Next, to understand the physical meaning of the Lagrange multiplier μ_i, we substitute Eq. (13) into Eq. (9) and obtain

$$h_i = \frac{\mu_i}{2c}\int\left(\vec{A} + \frac{m_i c}{q_i}\vec{v}_i\right)\cdot n_i q_i\vec{v}_i\, d^3x. \quad (15)$$

It is straightforward to show that Eq. (15) yields the following:

$$\sum_i\frac{h_i}{\mu_i} = \sum_i\int\left(\frac{1}{2}\rho_i v_i^2 + \frac{B^2}{8\pi} + \frac{A^2}{8\pi\lambda_p^2}\right)d^3x = E, \quad (16)$$

which also satisfies Eq. (11). It should be noticed that the amount of energy in the multi-Beltrami states is not an independent invariant but a quantity fixed by the Lagrange multipliers and helicities of individual species [36]. For single Beltrami equilibria in MHD, the relationship in Eq. (16) can be expressed as $\mu = h/E$, the ratio of the helicity to the total energy.

III. BELTRAMI STATE

We consider a nonrelativistic system of single-species plasma, either of dynamic electrons or ions (to be labeled "d"), in a background stationary (nondynamic) electron-ion bulk plasma. The dynamic fluid carries all the current in the system while the presence of bulk plasma ensures charge neutrality [35]. In this case, the system is fully defined by a single ($i = 1$) generalized helicity h and the corresponding Beltrami condition for the dynamic single fluid.

The equilibrium fields can then be calculated by combining Eqs. (13) and (8) into the following equation:

$$\vec{\nabla}\times\vec{\nabla}\times\vec{\nabla}\times\vec{A} - k_1\vec{\nabla}\times\vec{\nabla}\times\vec{A} + k_2\vec{\nabla}\times\vec{A} - k_3\vec{A} = 0, \quad (17)$$

where we have normalized all the gradients to the system length scale L and defined the following

$$k_1 = \frac{\mu L}{\lambda_d^2},\quad k_2 = \frac{L^2}{\lambda_d^2}\left(1 + \frac{\lambda_d^2}{\lambda_p^2}\right),\quad k_3 = \frac{\mu L^3}{\lambda_p^2\lambda_d^2} \quad (18)$$

with dynamic species' skin depth $\lambda_d = c/\omega_{pd}$ and plasma frequency $\omega_{pd} = \sqrt{4\pi n_d q_d^2/m_d}$. For the sake of simplicity, the dynamic plasma density is assumed to be constant here.

Examining Eq. (17), we notice that the system relaxes into a triple curl Beltrami state and has a general solution

$$\vec{A} = \sum_i^3 C_i\vec{G}_i, \quad (19)$$

where $\vec{G}_i$, known as Beltrami fields, are the solutions of

$$\vec{\nabla}\times\vec{G}_i = \alpha_i\vec{G}_i \quad (20)$$

with the eigenfunctions, and C_i's are the constants that can be determined from the boundary conditions. The eigenvalues of the curl operator (α_i) are solutions of the cubic equation [38,39]

$$\alpha^3 - k_1\alpha^2 + k_2\alpha - k_3 = 0 \quad (21)$$

which can be real or complex, and the following relations between Beltrami parameters and eigenstates are satisfied:

$$k_1 = \alpha_1 + \alpha_2 + \alpha_3, \quad (22)$$

$$k_2 = \alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_1\alpha_3, \quad (23)$$

$$k_3 = \alpha_1\alpha_2\alpha_3. \quad (24)$$

The three roots for Eq. (21) are

$$\alpha_1 = s - \tau + \frac{k_1}{3}, \quad (25)$$

$$\alpha_2 = -\frac{1}{2}(s-\tau) - i\frac{\sqrt{3}}{2}(s+\tau) + \frac{k_1}{3}, \quad (26)$$

$$\alpha_3 = -\frac{1}{2}(s-\tau) + i\frac{\sqrt{3}}{2}(s+\tau) + \frac{k_1}{3}, \quad (27)$$

where

$$s = \left(-\frac{q}{2} - \sqrt{D}\right)^{1/3}, \quad (28)$$

$$\tau = \left(\frac{q}{2} + \sqrt{D}\right)^{1/3}, \quad (29)$$

$$p = k_2 - \frac{k_1^2}{3}, \quad (30)$$

$$q = \frac{k_1 k_2}{3} - \frac{2k_1^3}{27} - k_3, \quad (31)$$

$$D = \frac{q^2}{4} + \frac{p^3}{27}. \quad (32)$$

The discriminant of Eq. (21) is given by

$$\Delta = k_1^2k_2^2 + 18k_1k_2k_3 - 4k_2^3 - 4k_1^3k_3 - 27k_3^2. \quad (33)$$

When $\Delta < 0$, all three roots of Eq. (21) are real; when $\Delta > 0$, then one of the roots is real while the other two are complex conjugate pairs. If $\Delta = 0$, all the roots are real, and at least two are equal.

Based on the preceding analysis, one can surmise that the magnetic field should also be a triple curl Beltrami state in this system. Using the relation $\vec{B} = \vec{\nabla} \times \vec{A}$, we obtain the following expression for the magnetic field:

$$\vec{B} = \sum_{i}^{3} C_i \alpha_i \vec{G}_i. \tag{34}$$

The inclusion of nonzero photon mass in plasma dynamics has fundamentally altered the vector potential and magnetic field structures. The fact that a triple curl Beltrami state emerges in this system is due to the inclusion of inertia of dynamic plasma species and photons. The construction of vortical dynamics treats the massive photon field, flow field, and magnetic field on an equal footing where the photon can be viewed as a mobile fluid in the system. It should be emphasized here that a photon can acquire an effective mass in Maxwellian electrodynamics as it propagates through the plasma [40,41]. The formalism presented here is strictly based on the assumption that a photon has a nonzero rest mass and its dynamics follow from the Proca Lagrangian.

Also, we notice that the scale parameters (α) characterize the size of the system's structure because they are dimensionally equal to the inverse of length. One of the three structures corresponds to system size L, and the other two will be related to the Compton length and species skin depth, respectively. In the limit, $\lambda_p \to \infty$ ($m_p = 0$), Eq. (17) results in the double curl Beltrami equation associated with dynamic single-species fluid.

IV. ANALYTICAL SOLUTION

In a cylindrical geometry, the solution is given by [39]

$$A_\theta = C_1 J_1(\alpha_1 r) + C_2 J_1(\alpha_2 r) + C_3 J_1(\alpha_3 r), \tag{35}$$

$$A_z = C_1 J_0(\alpha_1 r) + C_2 J_0(\alpha_2 r) + C_3 J_0(\alpha_3 r), \tag{36}$$

where J_0 and J_1 are the Bessel functions of the zeroth and first order. If we take $|A_z|_{r=0} = a_1$, $B_z = |(\vec{\nabla} \times \vec{A})_z|_{r=0} = a_2$, and $B_\theta = |(\vec{\nabla} \times \vec{A})_\theta|_{r=L} = a_3$, where L is the length of the system, we obtain

$$a_1 = C_1 + C_2 + C_3, \tag{37}$$

$$a_2 = \alpha_1 C_1 + \alpha_2 C_2 + \alpha_3 C_3, \tag{38}$$

$$a_3 = \alpha_1 C_1 J_1(\alpha_1 L) + \alpha_2 C_2 J_1(\alpha_2 L) + \alpha_3 C_3 J_1(\alpha_3 L). \tag{39}$$

Solving the equations, we obtain

$$C_1 = \frac{\beta_1}{Q}, \quad C_2 = \frac{\beta_2}{Q}, \quad C_3 = \frac{\beta_3}{Q}, \tag{40}$$

where

$$\begin{aligned} \beta_1 &= a_3(\alpha_3 - \alpha_2) + \alpha_2 J_1(\alpha_2 L)(a_2 - \alpha_3 a_1), \\ &\quad + \alpha_3 J_1(\alpha_3 L)(\alpha_2 a_1 - a_2) \end{aligned} \tag{41}$$

$$\begin{aligned} \beta_2 &= a_3(\alpha_1 - \alpha_3) + \alpha_3 J_1(\alpha_3 L)(a_2 - \alpha_1 a_1), \\ &\quad + \alpha_1 J_1(\alpha_1 L)(\alpha_3 a_1 - a_2) \end{aligned} \tag{42}$$

$$\begin{aligned} \beta_3 &= a_3(\alpha_2 - \alpha_1) + \alpha_1 J_1(\alpha_1 L)(a_2 - \alpha_2 a_1), \\ &\quad + \alpha_2 J_1(\alpha_2 L)(\alpha_1 a_1 - a_2) \end{aligned} \tag{43}$$

$$\begin{aligned} Q &= \alpha_1 J_1(\alpha_1 L)(\alpha_3 - \alpha_2) + \alpha_2 J_1(\alpha_2 L)(\alpha_1 - \alpha_3) \\ &\quad + \alpha_3 J_1(\alpha_3 L)(\alpha_2 - \alpha_1). \end{aligned} \tag{44}$$

Then, the corresponding magnetic fields are

$$B_\theta = C_1 \alpha_1 J_1(\alpha_1 r) + C_2 \alpha_2 J_1(\alpha_2 r) + C_3 \alpha_3 J_1(\alpha_3 r), \tag{45}$$

$$B_z = C_1 \alpha_1 J_0(\alpha_1 r) + C_2 \alpha_2 J_0(\alpha_2 r) + C_3 \alpha_3 J_0(\alpha_3 r). \tag{46}$$

V. CONCLUSION

In light of the preceding discussion and analysis, we make the following observations about the electrodynamical signatures of plasmas with nonzero photon mass

(i) Inclusion of both nonzero photon and species inertia in a single fluid plasma in Maxwell-Proca electrodynamics yields a multiscale equilibrium state where generalized helicity emerges as the fundamental determinant of magnetic and flow configurations in plasmas [35,36].

(ii) Similar to massless electrodynamics, the termination of this equilibria can result in eruptive events leading to the transfer of magnetic energy (now modified due to nonzero photon mass) into flow energy or vice versa [42,43].

(iii) When $\mu = 0$, one obtains the plasma "superconducting" solution $\Omega = 0$ where the magnetic flux is expelled from the interior of the plasma [35,44]. This state is electrodynamically equivalent to the Meissner effect displayed by classical superconductors. Compared to the traditional electrodynamics, we notice in Eq. (17) that the London skin depth is modified by the Compton wavelength of the photon. Since zero helicity is the major determinant of this state, one can prepare a system with vanishingly small helicity to measure any changes to the traditional London skin depth in a laboratory.

(iv) The inclusion of inertia of plasma species has played a significant role in plasma self-organization and created field structures of different length scales. One can compute the electromagnetic stress on the plasma element and the corresponding change in flow profiles due to magnetic fields in these different length scales. For large enough photon mass (or small enough λ_P), the deviations from the pure MHD flow structures could help set in setting a refined upper bound on the photon mass [19].

(v) The ordered magnetic field generated at small scales in this system can lead to large-scale flow through the reverse dynamo mechanism [45]. Any observational signatures related to this can also serve as an essential basis for refining the photon mass upper bound estimates.

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