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Wealth dynamics in a market with information asymmetries

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Financial markets are usually investigated through time series such as prices and volumes that describe the behavior of the system as a whole. Such observables emerge from microeconomic interactions between market participants. Agent-based models have been utilized to shed light on this process. The model's ability to produce statistics frequently found in empirical data is evidence of some correspondence with real markets. Here, an agent-based market model is proposed. Different trader profiles with short- and long-term motivations are considered, and limitations on the agents' skills to manipulate information are inserted into the model. According to their profile and limitations, agents are rational. A differential equation approximation is employed to find the value to which the price converges and the timescale of this process. The relationship between agents' attributes and the evolution of their wealth is explored in different scenarios. Agent's average wealth was not significantly affected by information processing accuracy, but the standard deviation was. The increased risk is the main consequence of low accuracy. The model yielded price series with multifractal behavior and heavy-tailed return distributions, which are nontrivial statistics frequently observed in empirical series.

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I. INTRODUCTION

Financial markets are complex systems [1] composed of a plurality of agents that interact through the exchange of information and asset negotiation. The result of such interactions, which occur at the microeconomic level, can be observed from indicators like prices and volumes that describe the dynamics of the market as a whole. Often, these series display the emergence of stylized facts typical of financial systems [2,3]. Non-Gaussian distributions of returns have been reported: the heavy tails made explicit in inverse cumulative distributions originate from values on different scales [4–9]. The Hurst exponent is frequently employed to show fractal behavior in financial series and its generalized version reveals multifractal scaling through the singularity spectrum [10–17].

Experimental methods have been utilized to shed light on the manner economic agents tend to operate in different situations [18]. Investigating search engine query data also provided insights into their behavior [19]. However, economic agents' actions remain essentially unpredictable at the individual level. The decision-making process under uncertainty was proven to move away from the strict rationality that underlies many economic models [20]. In addition to behavioral issues, deviations from market efficiency are associated with a variety of factors, such as political uncertainties [21], financial crises [22], central bank management [23,24], and information flow [25,26].

Although they inevitably involve great simplification, market models are often employed to improve the understanding of how the relationships between agents yield the phenomena observed at the macroscale. Random elements representing external influences on the market are often combined with endogenous dynamics. In the past few decades, agent-based models have played a prominent role [27–42]. Market models based on differential equations were also explored [43–46]. Differential equation-based models have been proposed to analyze the outcomes of laboratory markets [47,48]. To investigate whether the elements that integrate a model are likely to correspond to real markets, one can verify if the model is capable of generating stylized facts usually found in financial series. The process of adjusting parameters for this purpose can be a source of information about the interactions between agents.

In the present contribution, we propose an agent-based model for a market where traders have short-term and long-term motivations. The degree of influence of such motivations on each agent can vary, which allows the modeling of different profiles of traders. Agents are rational, in the sense of making the best use of information, according to their limitations and profile. Such information concerns price assessment according to fundamental analysis, as well as the prediction of the price of the next time step. Through errors in the evaluation of these prices, limitations of the agent's ability to deal with information are inserted into the model. In the next section, we describe the model in detail. In Sec. III, we study price dynamics through a differential equation approximation. Such approximation discloses the value towards which the price tends, according to the agents' evaluations, as well as the timescale of this process. Four different simulations are explored in Sec. IV. In the first one, three agent profiles are considered, without introducing evaluation errors. In the following two simulations, only one profile is assumed; uncertainties in the valuation of the fundamental price are inserted in one and errors in forecasting the price for the next step are modeled in the other. By using the model in a more

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advanced version, the fourth simulation regards three different agent profiles, including valuation inaccuracies. In all cases, the relationship between the agents' characteristics and the variation in their wealth is investigated. It is also shown that the model can generate price series with multifractal scaling and heavy-tailed return distributions. The conclusions are summarized in Sec. V.

II. MODEL

Let us consider a market where n agents trade a single asset at discrete time steps t. The variable $X(a,t)$ describes the agent's behavior: when $X(a,t) \geqslant 0$ $[X(a,t) \leqslant 0]$, the agent a demands (supplies) $X(a,t)$ assets. The asset is bought or sold at the price $P(t)$. Each agent knows the action of all agents $[X(a,t)]$ and the price at which the trade is carried out $[P(t)]$.

The agent sets its position by comparing the price at t with (a) a reference price $P_r(a)$ related to valuation from fundamental analysis and (b) the estimate for the price in the next time step, $P_s(a)$. Note that such prices may vary from agent to agent, which allows us to model traders with different information handling skills. The agent's conduct is driven by

$$X(a,t) = c_r(a)[P_r(a) - P(t)] + c_s(a)[P_s(a) - P(t)], \quad (1)$$

where $c_r(a)$ and $c_s(a)$ are non-negative parameters defining the portion of the agent's behavior targeting the long term and the short term, respectively. Different investor profiles can be modeled this way. The price that agent a estimates for the next step is

$$P_s(a) = P(t+1)[1 + \Delta(a)]. \quad (2)$$

Agents are able to forecast the next price approximately; a non-null $\Delta(a)$ models limitations in the ability to make such a prediction. The variations, from agent to agent, in the reference price $P_r(a)$ are, in turn, associated with imperfections in the fundamental analysis valuation; the accurate fundamental price corresponds to the mean of $P_r(a)$ computed over all agents. To model external inputs in the market, the values of $P_r(a)$ can change during the simulation.

The price evolution is defined as

$$P(t+1) = P(t) + c_p S(t), \quad (3)$$

where c_p is a positive parameter that relates the variation in price and excess demand

$$S(t) = \sum_{a=1}^{n} X(a,t). \quad (4)$$

The amounts of assets $A(a,t)$ and money $M(a,t)$ owned by the agents are updated assuming that their entire supply or demand has been fulfilled. The wealth of one agent is the sum of its money and the value of its assets. In what follows, we call *wealth variation* at time t the difference between the wealth at time t and the initial wealth.

III. DIFFERENTIAL EQUATION APPROXIMATION

From Eqs. (1), (3), and (4), we find, after some algebra,

$$P(t+1) = \frac{1 - nc_p(\overline{c_s} + \overline{c_r})}{1 - nc_p(\overline{c_s} + \overline{c_s\Delta})} P(t) + \frac{nc_p\overline{c_r P_r}}{1 - nc_p(\overline{c_s} + \overline{c_s\Delta})}, \quad (5)$$

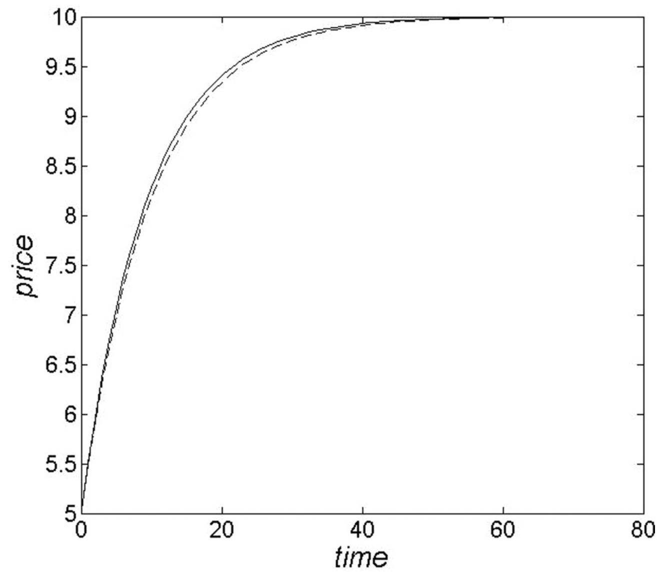

FIG. 1. Differential equation approximation. Comparison of price evolution computed by the agent-based model (solid line) with its differential equation approximation (dashed line) for $n = 101$ and $c_p = 0.001$. The parameters $c_r(a) = 1$ and $c_s(a) = 0.01$ are the same for all agents. Values of $P_r(a)$ are distributed from 9 to 11 with step 0.02; here, they do not vary in time. Values of $\Delta(a)$ are distributed between -0.1 and 0.1 with step 0.002. The curves show good agreement.

where

$$\overline{c_s} = \frac{1}{n}\sum_{a=1}^{n} c_s(a), \quad \overline{c_r} = \frac{1}{n}\sum_{a=1}^{n} c_r(a),$$
$$\overline{c_s\Delta} = \frac{1}{n}\sum_{a=1}^{n} c_s(a)\Delta(a), \quad \overline{c_r P_r} = \frac{1}{n}\sum_{a=1}^{n} c_r(a)P_r(a). \quad (6)$$

Equation (5) allows us to write

$$P(t+1) - P(t) = \gamma_1[\gamma_2 - P(t)], \quad (7)$$

where

$$\gamma_1 = \frac{(\overline{c_r} - \overline{c_s\Delta})nc_p}{1 - nc_p(\overline{c_s} + \overline{c_s\Delta})}, \quad \gamma_2 = \frac{\overline{c_r P_r}}{\overline{c_r} - \overline{c_s\Delta}}. \quad (8)$$

To get some insight into price dynamics, the differential equation

$$\frac{d}{dt}P(t) = \gamma_1[\gamma_2 - P(t)] \quad (9)$$

can be used as an approximation of Eq. (7). The solution reads

$$P(t) = \gamma_2 + [P(0) - \gamma_2]e^{-\gamma_1 t}. \quad (10)$$

Note that the differential equation parameters are functions of the agent-based model parameters, which allows a fair comparison between the results from both sources.

Figure 1 compares this solution with the price evolution from the agent-based model, showing good agreement. In what follows we choose $nc_p(\overline{c_s} + \overline{c_s\Delta}) < 1$, which is related to the low responsiveness of (a) the price to excess demand and (b) the agent's behavior to the estimated next price. This guarantees a positive denominator in the expression of γ_1.

If we also set a relatively high fundamentalist influence on agents' decisions ($\overline{c_r} > \overline{c_s \Delta}$), positive γ_1 and γ_2 are assured. Under such conditions, the equilibrium solution $P(t) = \gamma_2$ is positive and stable. If all agents accurately estimate the price at the next time step $[\Delta(a) = 0]$ and perform the same fundamentalist valuation $[P_r(a) = P_r$ for $a = 1, 2, \ldots, n]$, the price tends asymptotically to the reference price. The reciprocal of γ_1 defines a timescale for the price stabilization process. The increase in the parameter $\overline{c_r}$ is a factor that leads to the decrease in this time. In all simulations described here, we chose agent-based model parameters that lead to positive and stable price equilibrium solutions and price stabilization timescales that did not exceed the mean time between reference price changes.

IV. SIMULATIONS

We explore the model in different configurations referring to distinct investor profiles and information manipulation skills. In all the simulations, we assume agents with satisfactory competence to assess the fundamental price and predict the price of the next time step. Thus the values of $P_r(a)$ and $\Delta(a)$ were chosen so that the maximum errors in such estimates are 10% of the accurate value, i.e., one order of magnitude smaller.

Wealth distribution is examined. We do not set initial values for the amount of money and assets owned by agents. We focus attention on the difference between the agent's wealth at each time step and its initial wealth. Unlimited resources are assumed: as a consequence, the agent is always able to operate, regardless of the losses it has suffered during the simulation.

We look into the model's ability to produce price series with nontrivial statistics typical of empirical financial data, focusing on multifractal scaling and heavy tails in return distributions. Inverse cumulative distribution functions of normalized returns were built to examine the heavy tails. The normalization corresponds to subtracting the mean from each return and dividing the result by the standard deviation. Multifractal scaling was investigated through generalized Hurst exponents. The Hurst exponent H is a tool to analyze long-term memory [49]. Values of H above (below) 0.5 are found in persistent (antipersistent) series; if $H = 0.5$, there is no trend. Fractal behavior is associated to the computation of H. Multifractal scaling is made explicit through the singularity spectrum [50], which is attained from generalized Hurst exponents $H(q)$ (q is a parameter that assumes the value $q = 2$ for the canonical Hurst exponent). Here, the singularity spectra were performed with q ranging from -3 to 3.

In all scenarios described below, we found heavy-tailed return distributions. This stems from the tendency of the dynamics to seek a price that depends on the agent's reference prices, demonstrated by the differential equation approximation. If we choose the model parameters so that the timescale for such convergence is small (see Sec. III), large returns arise after each change in reference prices. These returns occur in many timescales and populate the heavy tails. Multifractal scaling is also present for the simulations explored here. We found significant singularity spectra widths for all original series and a relevant width decrease for each corresponding

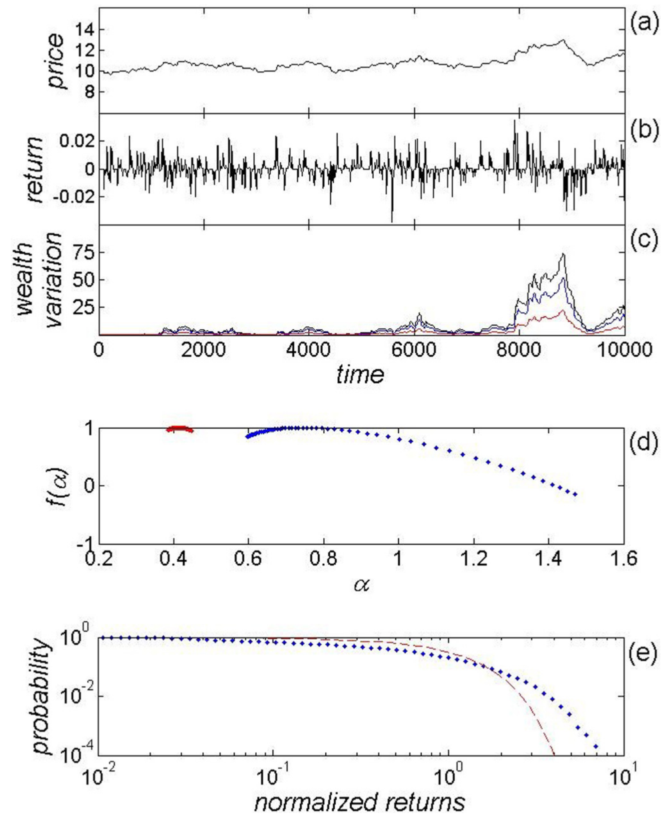

FIG. 2. Three homogeneous sets. A total of 303 agents were grouped into three sets of 101 elements, within which the agents' parameters do not vary. In set 1, agents are driven only by the reference price $[c_r(a) = 5$ and $c_s(a) = 0]$ and, in set 2, only by the next step price $[c_r(a) = 0$ and $c_s(a) = 1]$; in set 3, both prices influence the agents $[c_r(a) = 5$ and $c_s(a) = 1]$. The initial price is $P(0) = 10$. The reference price is the same for all agents affected by it $[P_r(a) = P_r]$. It starts at $P_r = 10$ and at each time step can change with a probability of 0.04. The logarithm of such variation follows a Gaussian distribution: the new value is computed by multiplying the current one by $\exp(\eta/100)$, where η is a random variable with standard normal distribution. Agents guided by the price of the next step predict it accurately $[\Delta(a) = 0]$. The parameter that relates excess demand to price variation is $c_p = 3 \times 10^{-4}$. The price dynamics is given in (a) and the corresponding returns in (b). In (c), we present the evolution of wealth variation for a typical agent of set 1 (blue line), set 2 (red line), and set 3 (black line). Singularity spectra are displayed in (d): blue dots correspond to the original series and red dots to the shuffled ones. The dots in (e) represent the inverse cumulative distribution of normalized returns; for comparison, the dashed line gives the Gaussian profile.

shuffled series. Such behavior, which evinces nonlinear correlations in the original series, may also be associated with price convergence to the stable value after changes in reference prices. During this process, clustering of high returns is observed, which leads to those correlations. Such stylized facts are often described in the examination of empirical financial data. They are related to augmented variations in traders' wealth and therefore increased risk.

In Fig. 2, three groups of agents are assumed: one is driven only by the reference price $[c_r(a) \neq 0$ and $c_s(a) = 0]$; another is ruled just by the estimated value for the next time step

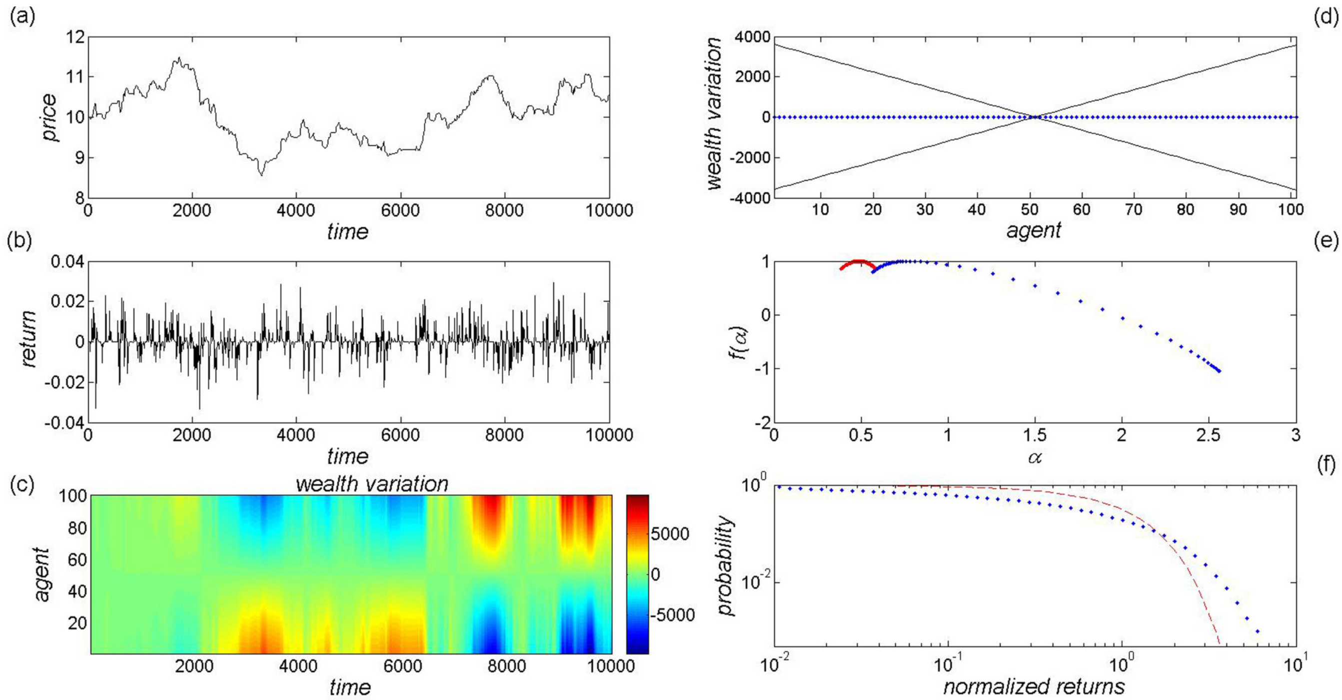

FIG. 3. Uncertainty in the fundamental analysis. Market dynamics for $n = 101$ agents driven by both fundamental analysis $[P_r(a)]$ and short-term expectations $[P_s(a)]$. The simulation was performed with $c_r(a) = 1$ and $c_s(a) = 0.01$ for all agents, and $c_p = 0.001$. The initial price is $P(0) = 10$. The reference prices take values equally spaced between $0.9P_{rc}$ (for the agent with index $a = 1$) and $1, 1P_{rc}$ (for the agent with index $a = 101$), according to $P_r(a) = P_{rc}[0.9 + (a-1)0.002]$, where P_{rc} is a central price that begins at $P_{rc} = 10$ and can change at each time step with probability 0.04. The logarithm of such variation follows a Gaussian distribution: the new value is computed by multiplying the current one by $\exp(\eta/100)$, where η is a random variable with standard normal distribution. The agents accurately predict the next step price $[\Delta(a) = 0]$. Price evolution is given in (a) and the corresponding returns in (b). In (c), we present the wealth variation in time for each agent. In (d), dots represent the mean of the wealth variation computed over the whole simulation period; the upper (lower) line displays the mean plus (minus) the standard deviation. Singularity spectra are displayed in (e): blue dots correspond to the original series and red dots to the shuffled ones. The dots in (f) represent the inverse cumulative distribution of normalized returns; for comparison, the dashed line gives the Gaussian profile.

$[c_r(a) = 0$ and $c_s(a) \neq 0]$; the third takes both factors into account $[c_r(a) \neq 0$ and $c_s(a) \neq 0]$. The agents influenced by the next step price forecast it accurately $[\Delta(a) = 0]$. The reference price is the same for all agents affected by it $[P_r(a) = P_r]$. When not null, the parameters $c_r(a) = c_r$ and $c_s(a) = c_s$ do not change with the agent. Therefore, the behavior of agents does not vary within each group. In Figs. 2(a) and 2(b), price and return dynamics are shown; the wealth evolution for one agent of each group is found in Fig. 2(c): it is easy to see the relationship between wealth and price movements. Much of this relationship concerns the valuation and devaluation of assets owned by agents. The agents that employed the reference price (fundamentalist assessment) for their decision earned more than the ones focused only on the next step. The agents that utilized both pieces of information were the most successful. The fundamental assessment and the forecast for the next step are evaluations of the future price referring to different timescales. Operating based on the former tends to produce gains that go along with the approximation to the reference price, which occurs in several time steps; trading according to the latter aims at gains concerning the movement of each step. The absence of errors in such estimations makes them more relevant to the growth of agents' wealth. Although the fundamentalist evaluation was more effective for wealth accumulation, the variation of the model parameters can change this outcome. On the other hand, it is reasonable to expect that agents employing all available information tend to have the best performance, which occurred in our exploration. Figure 2(d) displays the singularity spectrum. The width found for the original price series is a signature of multifractal behavior; the relevant reduction in width for the shuffled series indicates that it comes mostly from nonlinear correlations. In Fig. 2(e), we show the normalized returns inverse cumulative distribution function. The heavy tail becomes explicit from the comparison with the Gaussian profile.

Figure 3 shows the results of a simulation where all the agents are influenced by both the reference price and the price of the next time step $[c_r(a) \neq 0$ and $c_s(a) \neq 0]$. All agents evaluate the next price accurately $[\Delta(a) = 0]$, but they differ concerning the fundamental reference price. The values of $P_r(a)$ are symmetrically distributed around a central price which randomly changes from time to time. Price and return series are displayed in Figs. 3(a) and 3(b), respectively. Figure 3(c) shows the wealth variation dynamics of each agent. The reference price of the agent in the middle of the heatmap (agent 51) is the central price. Moving towards the top of the chart (from agent 52 to 101), the reference price increases; moving towards the bottom (from agent 50 to 1), the reference

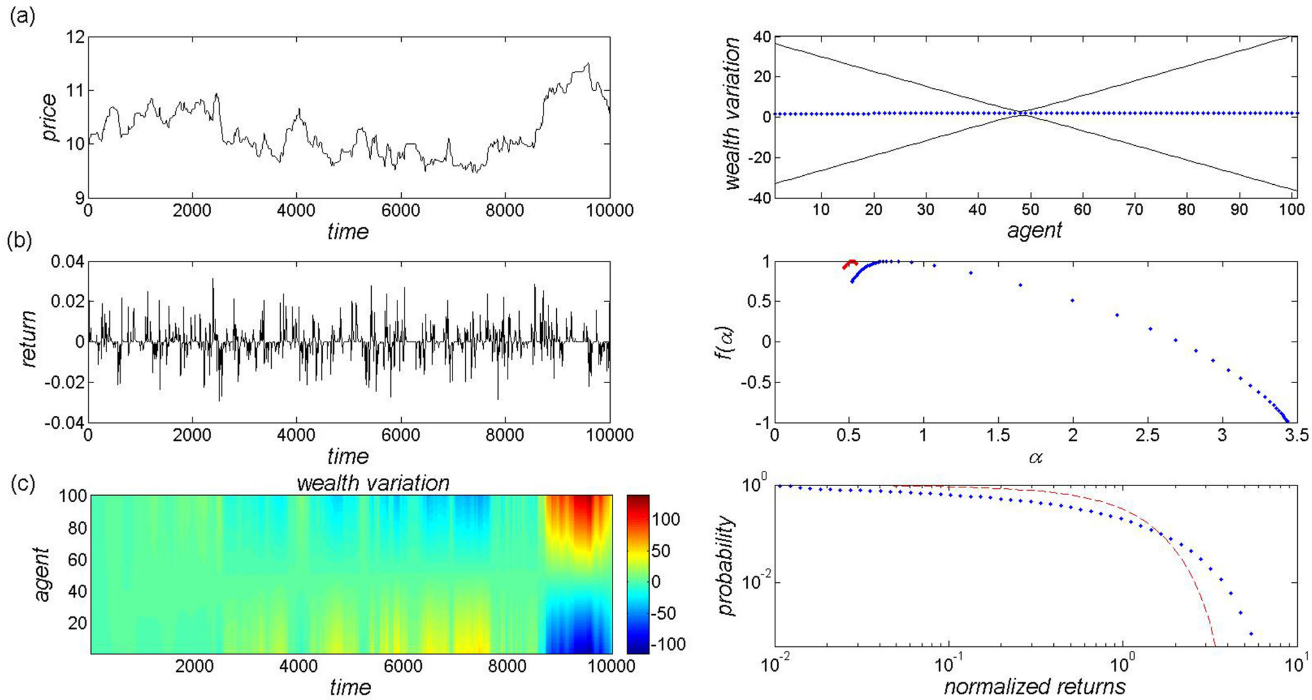

FIG. 4. Uncertainty in the short-term forecast. Market dynamics for $n = 101$ agents driven by both fundamental analysis $[P_r(a)]$ and short-term expectations $[P_s(a)]$. The simulation was performed with $c_r(a) = 1$ and $c_s(a) = 0.01$ for all agents and $c_p = 0.001$. The initial price is $P(0) = 10$. The reference price is the same for all agents $[P_r(a) = P_r]$. It starts at $P_r = 10$, and at each time step can change with a probability of 0.04. The logarithm of such variation follows a Gaussian distribution: the new value is computed by multiplying the current one by $\exp(\eta/100)$, where η is a random variable with standard normal distribution. The next time step price prediction is imperfect: $\Delta(a)$ varies from -0.1 (for the agent with index $a = 1$) to 0.1 (for the agent with index $a = 101$), according to $\Delta(a) = -0, 1 + 0, 002(a - 1)$. Price evolution is given in (a) and the corresponding returns in (b). In (c), we present the wealth variation in time for each agent. In (d), dots represent the mean of the wealth variation computed over the whole simulation period; the upper (lower) line displays the mean plus (minus) the standard deviation. Singularity spectra are displayed in (e): blue dots correspond to the original series and red dots to the shuffled ones. The dots in (f) represent the inverse cumulative distribution of normalized returns; for comparison, the dashed line gives the Gaussian profile.

price decreases. By the comparison of Figs. 3(a) and 3(c), one can see that price rising (falling) favors wealth accumulation for the agents with higher (lower) reference prices; the magnitude of the wealth variation increases when we move away from the center. Agents near the center experience a much lesser variation. This is confirmed by Fig. 3(d), which exhibits the mean and standard deviation of the wealth variation computed over the whole simulation period. While the average values are close for all agents, the standard deviation, which may be associated with risk, grows substantially for agents whose reference prices are further away from the price center. To understand this result, consider, for example, that the reference price has increased. Agents who estimate the fundamental price above the accurate value maintain a momentary advantage, as they have already been operating according to a higher reference price. Agents with valuations below the accurate value have a symmetrical disadvantage. When the reference price decreases, the advantage is reversed. As this process is random, sometimes one group of agents is favored and sometimes the other group benefits. The singularity spectra displayed for the original and shuffled price series in Fig. 3(e) indicate multifractal behavior originating mainly from nonlinear correlations. The inverse cumulative distribution function of the normalized returns presents a heavy tail, as can be seen in Fig. 3(f).

In the simulation presented in Fig. 4, both the reference price and the price of the next time step affect the agent's behavior $[c_r(a) \neq 0$ and $c_s(a) \neq 0]$. The reference price, which changes randomly from time to time, is the same for all agents $[P_r(a) = P_r]$. The evaluation of the next price, in turn, varies with the agent: $\Delta(a)$ goes from -0.1 (for agent 1) to 0.1 (for agent 101) through equally spaced steps. In this scenario, agent 51 gets the higher prediction accuracy $[\Delta(51) = 0]$. Figures 4(a) and 4(b) show the price and return series, respectively. Figure 4(c) exposes the wealth variation of the agents. The relation between wealth variation and price movements can be seen by comparing Figs. 4(a) and 4(c): the rapid price increase (decrease) favors the agents that predict, for the next price, a value above (below) the accurate one. A smaller wealth variation is found for the agents near the center of the heatmap, which are the ones with higher forecast skills. This can also be seen in Fig. 4(d): although the average variation of wealth is almost the same for all agents, the standard deviation increases when moving away from the central agent. Forecast accuracy is mainly related to reducing risk rather than increasing gain. The situation here is analogous to the previous case: variations in reference prices are at the origin of momentary advantages or disadvantages for agents with inaccurate evaluations. The multifractality in the price series mostly originating from nonlinear correlations is evidenced in

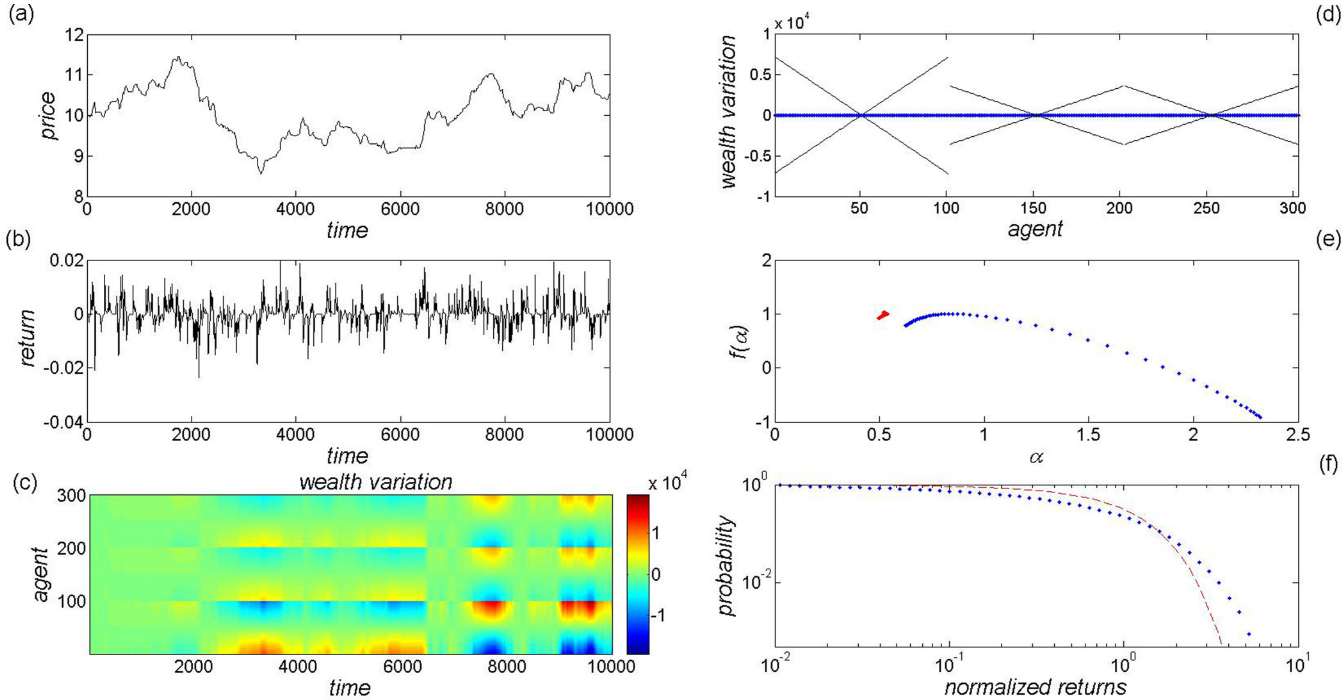

FIG. 5. Uncertainty in both fundamental analysis and short-term forecast. A total of 303 agents were grouped into three sets of 101 elements. Set 1 consists of agents from $a = 1$ to $a = 101$, which consider both $P_r(a)$ and $P_s(a)$ [$c_r(a) = c_s(a) = 1$]. In this set, the reference prices take values equally spaced between $0.9P_{rc}$ (for the agent with index $a = 1$) and $1, 1P_{rc}$ (for the agent with index $a = 101$), according to $P_r(a) = P_{rc}[0.9 + (a - 1)0.002]$, where P_{rc} is a central price that begins at $P_{rc} = 10$ and can change at each time step with probability 0.04. The logarithm of such variation follows a Gaussian distribution: the new value is computed by multiplying the current one by $\exp(\eta/100)$, where η is a random variable with standard normal distribution. The next time step price prediction is imperfect: $\Delta(a)$ varies from -0.1 (for the agent with index $a = 1$) to 0.1 (for the agent with index $a = 101$), according to $\Delta(a) = -0, 1 + 0, 002(a - 1)$. Agents from $a = 102$ to $a = 202$ integrate set 2, where agents are only influenced by $P_r(a)$ [$c_r(a) = 1$ and $c_s(a) = 0$]. The values of $P_r(a)$ in this group are the same as in set 1: $P_r(a + 101) = P_r(a)$, for $a = 1, 2, \ldots, 101$. Set 3 is composed of agents from $a = 203$ to $a = 303$; they are driven just by $P_s(a)$ [$c_r(a) = 0$ and $c_s(a) = 1$]. The values of $\Delta(a)$ are distributed similarly to those of set 1: $\Delta(a + 202) = \Delta(a)$, for $a = 1, 2, \ldots, 101$. Price begins at $P(0) = 10$; its evolution is governed by excess demand according to $c_p = 3 \times 10^{-4}$. Price evolution is given in (a) and the corresponding returns in (b). In (c), we present the wealth variation in time for each agent. In (d), dots represent the mean of the wealth variation computed over the whole simulation period; the upper (lower) line displays the mean plus (minus) the standard deviation. Singularity spectra are displayed in (e): blue dots correspond to the original series and red dots to the shuffled ones. The dots in (f) represent the inverse cumulative distribution of normalized returns; for comparison, the dashed line gives the Gaussian profile.

Fig. 4(e). The heavy tail of the returns distribution is made explicit in Fig. 4(f).

Figure 5 presents the results of a simulation involving the two types of uncertainty—the one concerning the reference prices, $P_r(a)$, and the one concerning the forecasted prices for the next time step, $P_s(a)$. We consider a universe of 303 agents divided into three sets. Agents with indexes from $a = 1$ to $a = 101$ (set 1) take into account both $P_r(a)$ and $P_s(a)$ [$c_r(a) \neq 0$ and $c_s(a) \neq 0$]. In this group, the values of $P_r(a)$ are distributed symmetrically around $P_r(51)$, taken as a central reference price which varies from time to time at random. As we move away from $a = 51$, the prices $P_r(a)$ move away from $P_r(51)$ at fixed steps. Similarly, the $\Delta(a)$ are symmetrically distributed around $\Delta(51) = 0$, departing from this central value taking negative values for a between 1 and 50 and positive values for a between 52 and 101. From $a = 102$ to $a = 202$ (set 2), the agents are only influenced by $P_r(a)$ [$c_r(a) \neq 0$ and $c_s(a) = 0$]. The values of $P_r(a)$ in this group are the same as in the previous one: $P_r(a + 101) = P_r(a)$, for $a = 1, 2, \ldots, 101$, centered in $P_r(152) = P_r(51)$. Agents from $a = 203$ to $a = 303$ (set 3) are, in turn, directed only by $P_s(a)$ [$c_r(a) = 0$ and $c_s(a) \neq 0$]. The values of $\Delta(a)$ are distributed around $\Delta(253) = 0$ similarly to those of set 1: $\Delta(a + 202) = \Delta(a)$, for $a = 1, 2, \ldots, 101$. Figures 5(a) and 5(b) present the dynamics of prices and returns. Figure 5(c) shows the evolution of wealth variation for the agents of the three sets. Increases (decreases) in the price are associated with positive variations in the wealth of agents with higher (lower) both reference and next-step prices. Such variations are greater for the agents that use both pieces of information (set 1). Figure 5(d) shows that the averages of wealth variation (calculated over the simulation period) fluctuate little between agents. The standard deviations, in turn, increase in each set as we move away from its central agent. This increase is more pronounced in set 1. Standard deviation is associated with risk. When we depart from the central element of each set, valuation inaccuracies increase, which leads to greater risks. Such results can be understood the same way as in previous cases. When reference prices change, agents with inaccuracies pointing in the direction of that change are momentarily benefited and those with errors in the opposite direction are momentarily harmed. This increases the range of variations in wealth, but since changes in both directions occur with equal probability, average wealth does not vary significantly within each set. This effect is stronger in set 1, where short- and long-term motivations drive the agents to buy and sell larger asset amounts. On average, we find greater earnings for the agents of set 1: taking the mean of the wealth variation over all time steps and all agents, we get 4.2429 for set 1, 3.9858 for set 2, and 0.2571 for set 3. Such averages are not noticeable in Fig. 5(d) because the scale of the standard deviations, which determine the figure scale, is much larger. Nevertheless, they indicate greater gains for the agents that utilize $P_r(a)$ and $P_s(a)$, followed by those that employ only $P_r(a)$. The agents influenced just by $P_s(a)$ present the lower average profits. Although such gains are irrelevant when compared to the risks (standard deviations), these results are in line with those regarding Fig. 2(c). Despite the presence of errors, the agents' estimations are relevant for operation, as the biggest mistakes are 10% of the accurate values. Thus it is expected that agents who use both pieces of information tend to perform better than the other, as we found. The advantage of agents motivated by long-term expectations over the ones guided by the short term is not inevitable. This depends on the parameters chosen for the model. Nonlinear correlations in the price series are evidenced by the singularity spectra in Fig. 5(e). Heavy tails in the distribution of returns are made explicit in Fig. 5(f).

V. CONCLUSION

The model studied here involves agents motivated by two types of information: the price predicted for the next time step and the price derived from a fundamental valuation. Such estimates proved to be relevant to the agents' earnings: simulations indicate that the agents that employ both pieces of information have, on average, the best performance with regard to the variation of their wealth. Different information manipulation skills were modeled by introducing errors in the evaluation of both prices. The average wealth of agents, calculated over time, is not significantly influenced by such errors, but the standard deviation is. Thus the major consequence of lower accuracy is increased risk. The simulations also yielded price series with multifractal behavior and heavy-tailed return distributions, which are nontrivial statistics usually found in financial data.

Among the studied scenarios, the last one, which comprises the agents with the most varied motivations and skills, seems to be the most plausible. Even there, immense simplification is evident. Adding new ingredients to the model is a way to increase its likelihood. Some possibilities along these lines are considering other kinds of agents, such as momentum traders, or the simultaneous negotiation of different types of assets. In actual markets, agents own limited resources, often having to stop trading after heavy losses. Introducing such limitations probably leads to different wealth dynamics than those displayed here. Model developments like these make its exploration more complex, given the increase in the number of parameters.

In general, we start by considering agents that interact in a credible way to compose a market model. Its validation can be performed by comparing its outcomes with what is expected or observed in actual cases. Some parallels involve qualitative attributes. In this context, the results that indicate that agents who use more information earn more and that increased evaluation errors lead to greater risk are in line with reasonable expectations. From a quantitative perspective, statistics synthesized by the model are compared with stylized facts typical of financial markets. This was carried out through the analysis of return distributions and multifractal scaling. The agreement observed is an indication that the model components keep some correspondence with processes that effectively take place in actual markets.

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