Letter

Role of initial conditions in one-dimensional diffusive systems: Compressibility, hyperuniformity, and long-term memory

Tirthankar Banerjee,[1,*] Robert L. Jack,[1,2] and Michael E. Cates[1]

[1]DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom

[2]Yusuf Hamied Department of Chemistry, University of Cambridge, Lensfield Road, Cambridge CB2 1EW, United Kingdom

(Received 23 June 2022; accepted 4 November 2022; published 7 December 2022)

We analyze the long-lasting effects of initial conditions on dynamical fluctuations in one-dimensional diffusive systems. We consider the mean-squared displacement of tracers in homogeneous systems with single-file diffusion, and current fluctuations for noninteracting diffusive particles. In each case we show analytically that the long-term memory of initial conditions is mediated by a single static quantity: a generalized compressibility that quantifies the density fluctuations of the initial state. We thereby identify a universality class of hyperuniform initial states whose dynamical variances coincide with the quenched cases studied previously, alongside a continuous family of other classes among which equilibrated (or annealed) initial conditions are but one member. We verify our predictions through extensive Monte Carlo simulations.

DOI: 10.1103/PhysRevE.106.L062101

In single-file diffusion, particles move in a single lane with no possibility of overtaking [1–11]. Modern technology has made this process increasingly relevant in experimental [12–16], industrial [11,17,18], biological [19], and biomedical [20] settings. Of fundamental interest is the mean-squared displacement (MSD) of a tagged particle (tracer) within a collection of identical particles executing single-file diffusion. This MSD grows with time as $\sqrt{t}$, in contrast to normal diffusion, where it grows as t.

Remarkably, theoretical studies [8–10,21–24] have shown that MSDs can depend on the initialization of the system, prior to measurement. For example, in one-dimension (1D) one might either initialize point particles uniformly at random, or prepare an initial state with equispaced particles. The MSD grows as $\sqrt{t}$ in both cases, but the prefactors are different [8]. This is an *everlasting* dependence on the initial state, affecting the asymptotic behavior at large times. Similar everlasting effects of the initialization protocol are also observed in other 1D systems (not necessarily single file), for example when measuring the variance of particle currents [25,26]. In both cases, one may additionally choose to perform either a quenched or annealed average over initial states [9,25]: this choice also has an everlasting effect on the resulting behavior. These results establish that 1D particle systems can retain long-term memory of their initialization: this is a form of nonergodicity, which may occur (in general) by many different mechanisms [25,27–35].

We show here that these systems' long-term memory can be explained by a unified framework, greatly generalizing previous results known for specific cases [8,9,25,26,36–38]. Physically, we note that the large-scale density fluctuations of the initial condition relax very slowly: this causes long-term memory. Remarkably, the role of these fluctuations can be quantified by a single static quantity, the Fano factor [39] or generalized compressibility:

$$\alpha_{\rm ic} = \lim_{\ell\to\infty} \frac{\mathrm{Var}[n(\ell)]}{\overline{n}(\ell)}. \tag{1}$$

Here $n(\ell)$ is the number of particles initially found within a distance ℓ of the origin and $\overline{n}(\ell)$ is its average; $\alpha_{\rm ic}$ encapsulates the effects of initial density fluctuations on systems' asymptotic (large-time) dynamical behavior. Notably, $\alpha_{\rm ic}$ in Eq. (1) is a static property of the initial state that is experimentally accessible (e.g., via microscopy) without knowing details of particle interactions [40–42]. Via Eq. (2) below, it directly controls a dynamical quantity that is similarly accessible [15], namely the variance of particle displacements up to time T.

We now summarize our main results, before presenting their derivation. First, consider single-file diffusion of a homogeneous interacting particle system under conditions where macroscopic fluctuation theory (MFT) is applicable [9,43–49]. MFT is a hydrodynamic theory that captures numerous microscopic processes [43] including the simple exclusion process, point Brownian particles, zero range processes, and random average processes [22,50]. The system is initialized in a macroscopically homogeneous state with mean density $\bar{\rho}$ and density fluctuations determining $\alpha_{\rm ic}$ via Eq. (1). Identifying a single tracer particle starting from the origin, we will show that the variance of its position $X(T)$ satisfies

$$\mathrm{Var}[X(T)] \simeq \Delta X^2_{\rm noise}(T,\bar{\rho}) + \alpha_{\rm ic}\Delta X^2_{\rm dens}(T,\bar{\rho}), \tag{2}$$

where $\simeq$ indicates asymptotic equality for large T. Expressions for $\Delta X^2_{\rm noise}$ and $\Delta X^2_{\rm dens}$ are given in Eqs. (24) and (25), respectively. Both are proportional to $\sqrt{T}$, and depend on $\bar{\rho}$ and on the transport coefficients; diffusivity $D(\bar{\rho})$ and mobility $\sigma(\bar{\rho})$ encoded in MFT [43,44]. The variance in Eq. (2) includes the stochastic motion of the particles in the system, and any random aspects of the initialization process; it

*tb698@cam.ac.uk

corresponds to an annealed variance in the terminology of Ref. [25].

Several previous results are special cases of Eq. (2). If the initial condition is the thermal equilibrium ensemble of the system's own dynamics, then $\alpha_{\rm ic}$ is the thermal compressibility factor [51,52], and we recover a result of Refs. [9,36]. If in contrast one chooses initial conditions that are hyperuniform (HU), then $\alpha_{\rm ic} = 0$ by definition [53] and Eq. (2) correctly predicts a different variance, $\Delta X^2_{\rm noise}$. This was known for the equispaced initial condition of Ref. [8], which now emerges as representative of a much larger, HU universality class. Another special case was obtained in Ref. [9] by considering a quenched average over initial conditions: in our framework, this also represents $\alpha_{\rm ic} = 0$ (see Ref. [54]). Moving beyond these special cases, we emphasize that Eq. (2) reveals a continuous spectrum of classes with variances Var$[X(T)]$ parameterized by $\alpha_{\rm ic}$, which is tunable via the initialization protocol.

To elucidate the physics behind results (1) and (2), we also analyze below a diffusive system of *noninteracting* particles. After initializing a homogeneous system at density $\bar\rho$, we remove all particles to the right of an arbitrary origin. The remaining particles have dynamics that at large times is Brownian with diffusivity D. Let $Q(T)$ be the integrated flux of particles through the origin, up to time T. We show that

$$\mathrm{Var}[Q(T)] \simeq \Delta Q^2_{\rm noise}(T,\bar\rho) + \alpha_{\rm ic}\Delta Q^2_{\rm dens}(T,\bar\rho). \tag{3}$$

Here $\Delta Q^2_{\rm noise}$ and $\Delta Q^2_{\rm dens}$, given explicitly in Eqs. (13) and (12), respectively, both grow as $\sqrt{T}$ for large times, and $\alpha_{\rm ic}$ again obeys Eq. (1).

The similarity between Eqs. (2) and (3) is striking: both variances depend on initialization solely via the mean density $\bar\rho$ and the Fano factor $\alpha_{\rm ic}$. Moreover, they share a physical origin: everlasting memory of the initial condition can only survive through modes whose lifetime is unbounded. These are the (conserved) mean density, and the large-scale ($\ell \to \infty$) density fluctuations, quantified by $\alpha_{\rm ic}$.

We next derive the above results, first obtaining Eq. (3) by direct computation. To derive Eq. (2) is more complicated, but the same physical principles are at work. Indeed, while Eq. (2) is more relevant for applications involving single-file diffusion, Eq. (3) provides a simpler illustration of the underlying physics.

Current fluctuations for noninteracting particles. Consider noninteracting particles with initial positions $\mathbf{y} = (y_1, y_2, \dots)$ lying to the left of the origin ($y_i < 0$). At time t, define $\chi_i(t) = 1$ if particle i has position $x_i(t) > 0$, and $\chi_i(t) = 0$ otherwise. Also define the propagator $G(x, y, t)$ as the probability density that a particle is at position x, given that it was at position y a time t earlier. For given initial conditions $\mathbf{y}$, we have $\langle\chi_i(t)\rangle_{\mathbf{y}} = U(-y_i, t)$, where [26]

$$U(z,t) = \int_0^\infty dx\, G(x,-z,t). \tag{4}$$

The notation $\langle\dots\rangle_{\mathbf{y}}$ represents an average over the stochastic particle dynamics, for a given initial condition $\mathbf{y}$. The mean integrated flux through the origin follows by summing over all particles: $\langle Q(t)\rangle_{\mathbf{y}} = \sum_i U(-y_i, t)$. Defining the empirical density of the initial condition $\hat\rho(x|\mathbf{y}) = \sum_i \delta(x - y_i)$, we write

$$\langle Q(t)\rangle_{\mathbf{y}} = \int_0^\infty dz\, \hat\rho(-z|\mathbf{y}) U(z,t). \tag{5}$$

(Here and below, positive z lies to the *left* of the origin.)

For independent particles, and since $\chi_i \in \{0, 1\}$, the quantity

$$\langle Q(t)^2\rangle_{\mathbf{y}} - \langle Q(t)\rangle^2_{\mathbf{y}} = \int_0^\infty dz\, \hat\rho(-z|\mathbf{y}) U(z,t)[1 - U(z,t)] \tag{6}$$

measures how much $Q(t)$ fluctuates between trajectories, for a fixed initial condition $\mathbf{y}$. For many initialization protocols, the initial condition is itself stochastic, so the next step is to average over $\mathbf{y}$, (denoted as $\overline{(\dots)}$). We define the variance of the flux to include both sources of randomness [54,55]:

$$\mathrm{Var}[Q(t)] = \Delta Q^2_{\rm noise}(t,\bar\rho) + \Delta Q^2_{\rm ic}(t,\bar\rho), \tag{7}$$

where

$$\begin{aligned}\Delta Q^2_{\rm noise} &= \overline{\langle Q(t)^2\rangle_{\mathbf{y}}} - \overline{\langle Q(t)\rangle^2_{\mathbf{y}}},\\ \Delta Q^2_{\rm ic}(t) &= \overline{\langle Q(t)\rangle^2_{\mathbf{y}}} - \overline{\langle Q(t)\rangle_{\mathbf{y}}}^2.\end{aligned} \tag{8}$$

In the disordered-systems terminology of Ref. [25], Var$[Q(t)]$ and $\Delta Q^2_{\rm noise}$ are, respectively, "quenched" and "annealed" variances [32,33,56]. Physically, $\Delta Q^2_{\rm noise}$ measures how much Q fluctuates between trajectories with the same initial condition, while $\Delta Q^2_{\rm ic}$ depends additionally on the fluctuations of the initial condition, which are never forgotten. Using Eq. (5) yields

$$\Delta Q^2_{\rm ic}(t) = \int_0^\infty\int_0^\infty dz\,dz'\, U(z,t)U(z',t)C_2(z,z'), \tag{9}$$

where $C_2(z,z') = \overline{\hat\rho(-z|\mathbf{y})\hat\rho(-z'|\mathbf{y})} - \overline{\hat\rho(-z|\mathbf{y})}\,\overline{\hat\rho(-z'|\mathbf{y})}$. Hence, the initial fluctuations enter the variance through the one- and two-point density correlations only.

So far, this analysis is general. We now specialize to the case where, for large t, the propagator G is diffusive, such that

$$U(z,t) \simeq \frac{1}{2}\mathrm{erfc}\left[\frac{z}{\sqrt{4Dt}}\right] \quad \text{as } t \to \infty. \tag{10}$$

This assumption covers passive diffusers, and many kinds of active particle whose late-time motion is also diffusive. Second, we consider initial conditions found by taking an infinite, translationally invariant system and erasing all particles to the right of the origin. This means that $\overline{\hat\rho(-z|\mathbf{y})} = \bar\rho\Theta(z)$ and $C_2(z,z') = \bar\rho\Theta(z)\Theta(z')C(z-z')$ where $\Theta(z)$ is the Heaviside function and $C(z-z')$ the two-point correlator before erasure. These assumptions can be relaxed, but are sufficient here.

For large times T, (9, 10) yield

$$\begin{aligned}\Delta Q^2_{\rm ic}(T) \simeq \frac{\bar\rho\sqrt{DT}}{4\pi}\int_0^\infty dy\int_0^\infty dy'\int_{-\infty}^\infty dp\, e^{ip(y'-y)}\\ \times S\left(\frac{p}{\sqrt{4DT}}\right)\mathrm{erfc}(y)\,\mathrm{erfc}(y'),\end{aligned} \tag{11}$$

where $S(q) = \int_{-\infty}^\infty dz\, C(z)e^{-iqz}$ is the structure factor. The fluctuations of the initial condition enter this expression solely through $\alpha_{\rm ic} = \lim_{q\to 0} S(q)$, which is equivalent to Eq. (1)

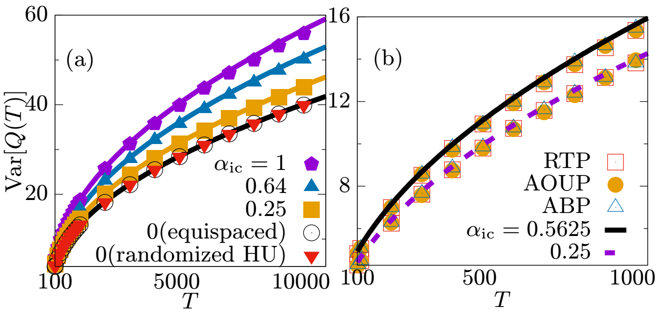

FIG. 1. (a) The variance Var$[Q(T)]$ of noninteracting Brownian particles with $D = 1$ and $\bar{\rho} = 1$, at different values of α_{ic}. The MC simulation results (points) match the theoretical prediction (3) (solid lines). For HU initial states ($\alpha_{ic} = 0$), two different initial set-ups are shown: equispaced initial positions with or without additional random displacements. (b) The variance Var$[Q(T)]$ for noninteracting active particles at long times. Points show simulation results; lines are the prediction (3).

[52,54,57]. Replacing the structure factor by its limit, the integrals in Eq. (11) yield $\Delta Q^2_{ic}(T,\bar{\rho}) = \alpha_{ic}\Delta Q^2_{dens}(T,\bar{\rho})$ with

$$\Delta Q^2_{dens}(T,\bar{\rho}) \simeq (\sqrt{2}-1)\sqrt{\frac{\bar{\rho}^2 DT}{2\pi}}. \quad (12)$$

Similarly using Eq. (6), ΔQ^2_{noise} is

$$\Delta Q^2_{noise}(T,\bar{\rho}) \simeq \sqrt{\frac{\bar{\rho}^2 DT}{2\pi}}. \quad (13)$$

Combining Eqs. (7),(12), and (13) gives the promised result, Eq. (3).

These results confirm that the current variance has an everlasting dependence on the fluctuations of the initial state, through α_{ic}. The case $\alpha_{ic} = 0$ arises if the initialization has no randomness at all (e.g., equispaced particles) or is hyperuniform; the variance in these cases is simply ΔQ^2_{noise}, referred to as the quenched variance in Refs. [25,26] (see Ref. [54]). On the other hand, if the initial condition has $C(z) = \delta(z)$ (as holds for an equilibrated ideal gas), then $\alpha_{ic} = 1$. This coincides with the annealed variance computed in Refs. [25,26], which is larger than the quenched variance by a factor $\sqrt{2}$. These previously-studied cases now emerge as two specific choices within an infinite family of classes of initial condition, parameterized by α_{ic} which may take any non-negative value.

To understand the physical mechanism, note that together, Eqs. (5) and (10) imply that particles starting within $\sqrt{4DT}$ from the origin have passed it with probability $1/2$ after time T, while particles starting much further away are unlikely to have done so. Hence, the average flux is controlled by the number of particles within $\sqrt{4DT}$ of the origin. For large T the variance of this number is determined by α_{ic} via Eq. (1), thereby controlling $\Delta Q^2_{ic}(T,\bar{\rho})$. The everlasting effect of the initial conditions stems from the longest-wavelength density fluctuations that determine α_{ic}, whose unbounded relaxation times are the source of long-term memory.

Figure 1(a) shows Var$[Q(T)]$ for point Brownian particles in 1D, obtained from Monte Carlo (MC) simulations for various α_{ic}. In the initial state for these numerics, particles are placed at random, with spacings constrained to exceed some constant r_0: this yields $\alpha_{ic} = (1 - r_0\bar{\rho})^2$, providing a family of initialization protocols spanning α_{ic} between 0 (equal spacing $r_0 = 1/\bar{\rho}$) and unity (ideal gas, $r_0 = 0$) [54]. All results match Eqs. (3),(12), and (13). To check that all dependence on the initial conditions comes from α_{ic}, we also simulated a different HU initial ensemble where equispaced particles receive independent random displacements of fixed size [54]: the long-time behavior matches that for equispaced initial particles.

Figure 1(b) shows results for three popular models of active particles [58–65]: active Ornstein-Uhlenbeck particles (AOUPs), active Brownian particles (ABPs), and run-and-tumble particles (RTPs), see Ref. [54] for details. These systems all satisfy Eq. (10) with D their late-time diffusivity. They precisely obey our predictions (3),(12), and (13).

Notably, Eq. (3) also applies in higher dimensions, to fluctuations of the flux passing through a planar boundary: it suffices that the particles' normal distances from the boundary are independent, Markovian, and obey Eq. (10). Moreover, the current in a system of hard Brownian point particles undergoing single-file motion has the same statistics as in the noninteracting case [66]. Hence Eq. (3) also applies in that single-file system, which we address next.

Single-file tracer motion: We now consider an infinite 1D system of diffusive, hardcore particles, whose initial condition is homogeneous, with mean density $\bar{\rho}$ and two-point correlation $C(z)$. A single tracer particle is identified, and its displacement between times 0 and T is denoted by $X(T)$, whose variance obeys Eq. (2), as we now show. The physical mechanism for this result is the same as that leading to Eq. (3), although the computation is more involved. The method follows previous work [9]; we outline it here, with details in Ref. [54].

We first assume that the hydrodynamic density field ρ obeys the MFT equation [43]

$$\partial_t \rho(x,t) = \partial_x[D(\rho)\partial_x\rho(x,t) + \sqrt{\sigma(\rho)}\eta(x,t)], \quad (14)$$

where $\sigma(\rho)$ and $D(\rho)$ are the mobility and diffusivity and $\eta(x,t)$ is Gaussian spatiotemporal white noise. Examples of such MFT systems include hard Brownian particles [9], and the symmetric simple exclusion process [49].

The moment generating function (MGF) of the tracer position is $\langle e^{\lambda X(T)}\rangle$, which can be expressed as a path integral in the Martin-Siggia-Rose formalism [9]:

$$\langle e^{\lambda X(T)}\rangle = \int \mathscr{D}[\rho(x,t),\hat{\rho}(x,t)]\, e^{-\mathcal{S}[\rho(x,t),\hat{\rho}(x,t)]}, \quad (15)$$

where the average $\langle\ldots\rangle$ now includes both the random initial condition and the stochastic dynamics of the density; $\hat{\rho}(x,t)$ is a response field, and the action is

$$\mathcal{S}[\rho,\hat{\rho}] = -\lambda X(T) + F[\rho] + \int_0^T dt \int_{-\infty}^{\infty} dx\, \mathcal{L}(\rho,\hat{\rho}). \quad (16)$$

Here $F[\rho]$ is the log-probability of the initial condition, and

$$\mathcal{L}(\rho,\hat{\rho}) = \hat{\rho}\partial_t\rho - \frac{\sigma(\rho)}{2}(\partial_x\hat{\rho})^2 + D(\rho)(\partial_x\rho)(\partial_x\hat{\rho}). \quad (17)$$

For single-file motion, $X(T)$ is fully determined by the dynamics of the density: at time T, all particles between the tracer and the origin must have had initial positions $y_i < 0$. This implies [9,54]

$$\int_0^{X(T)} dx\, \rho(x,T) = \int_0^{\infty} dx\, [\rho(x,T) - \rho(x,0)]\,. \qquad (18)$$

On hydrodynamic time scales, noise becomes weak and Eq. (15) can be evaluated by a saddle-point method. Physically, this involves the computation of an instanton that generates a large tracer displacement $X(T)$, whose size is determined by the parameter λ. To find the variance of $X(T)$, the computation is required to $O(\lambda^2)$. At this level, the instanton dynamics is $\rho(x,t) \approx \bar{\rho} + \lambda q_1(x,t)$ and $\hat{\rho}(x,t) \approx \lambda p_1(x,t)$, where p_1, q_1 are canonically conjugate fields; terms at higher order in λ can be neglected.

Within MFT, the log-probability of the initial state is determined by a function $g_{\rm ic}$, as $F[\rho] = \int_{-\infty}^{\infty} dx\, g_{\rm ic}(\rho(x,0))$, specifying the probability of local density fluctuations [43,54]. We emphasize that ρ is the hydrodynamic density: there may be density correlations on the scale of the interparticle spacing, but $g_{\rm ic}$ is still a local function of ρ. Since the density fluctuations are of order λ, it is consistent to approximate [43,54]

$$g_{\rm ic}(\rho) \approx [\rho(x,0) - \bar{\rho}]^2/(2\alpha_{\rm ic}\bar{\rho}), \qquad (19)$$

where $\alpha_{\rm ic} = 1/(\bar{\rho} g''_{\rm ic}(\bar{\rho}))$ is the Fano factor defined in Eq. (1).

Two special cases are relevant: first, if the initial condition has no fluctuations then $\rho(x,0) = \bar{\rho}$ exactly and $\alpha_{\rm ic} \to 0$. The corresponding result for Var$[X(T)]$ coincides with the quenched case addressed in Ref. [9]. Second, if the initial condition is a thermally equilibrated steady state of Eq. (14) then a fluctuation-dissipation theorem requires $g''_{\rm ic}(\bar{\rho}) = 2D(\bar{\rho})/\sigma(\bar{\rho})$ [43]. In this case, $g_{\rm ic}$ follows from the model's free energy, and Var$[X(T)]$ coincides with the annealed variance of Ref. [9]. Hence (as for the noninteracting particles considered above) our formalism incorporates these two special cases, but extends them to arbitrary initial protocols with no assumption of thermal equilibration.

The instanton dynamics is obtained by extremizing the action, leading to

$$\begin{aligned} \partial_t q_1(x,t) &= \partial_x[D(\bar{\rho})\partial_x q_1(x,t) - \sigma(\bar{\rho})\,\partial_x p_1(x,t)] \\ \partial_t p_1(x,t) &= -D(\bar{\rho})\partial_{xx} p_1(x,t)\,, \end{aligned} \qquad (20)$$

with boundary conditions [54]

$$q_1(x,0) = \bar{\rho}\,\alpha_{\rm ic}[p_1(x,0) - p_1(x,T)], \qquad (21)$$

$$p_1(x,T) = \Theta(x)/\bar{\rho}. \qquad (22)$$

The equations for p_1 are closed and exactly solvable, following Ref. [9]. Hence Eq. (21) sets the initial condition for the instanton, which is the fluctuation of the initial condition $\rho(x,0)$ associated with the prescribed fluctuation of $X(T)$:

$$q_1(x,0) = \alpha_{\rm ic}\left[\frac{1}{2}{\rm erfc}\left(\frac{-x}{\sqrt{4D(\bar{\rho})T}}\right) - \Theta(x)\right]. \qquad (23)$$

This result is shown in Fig. 2(a). In physical terms, if the initial density is large on the left of the tracer, then it tends to

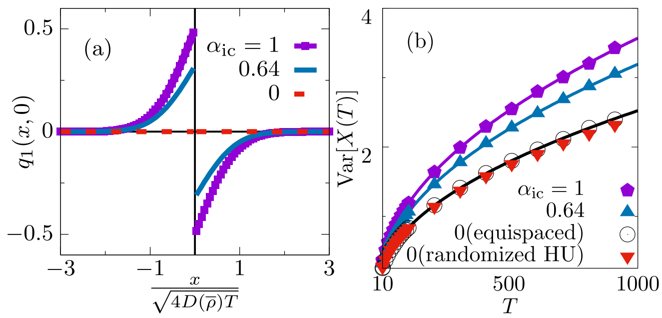

FIG. 2. (a) The nontypical initial condition $q_1(x,0)$ that appears when considering fluctuations of $X(T)$ for various $\alpha_{\rm ic}$. Large currents correspond to an excess of particles to the left of the origin, biasing the tracer to the right. (b) The mean-squared tracer displacement Var$[X(T)]$ in a system of point Brownian particles at density $\bar{\rho} = 10$. Numerical results are shown as points; solid lines show the theoretical prediction (26). We show results for two different HU initial conditions, with initialization protocols the same as Fig. 1.

move to the right, and vice versa. The size of the fluctuation is set by $\alpha_{\rm ic}$ (vanishing for HU initial conditions, which lack these large-scale density fluctuations by definition); the associated length scale is $\sqrt{4D(\bar{\rho})T}$. Like the current fluctuations considered earlier, this shows that fluctuations of tracer position are strongly coupled to the number of particles within this distance on either side of the origin.

Finally, the variance of the tracer position is set by the second derivative of the MGF. We thereby obtain Eq. (2) with

$$\Delta X^2_{\rm noise}(T) \simeq \frac{1}{\bar{\rho}}\sqrt{\frac{\sigma(\bar{\rho})^2 T}{2\pi\bar{\rho}^2 D(\bar{\rho})}}\,, \qquad (24)$$

$$\Delta X^2_{\rm dens}(T) \simeq \frac{\sqrt{2}-1}{\bar{\rho}}\sqrt{\frac{2D(\bar{\rho})T}{\pi}}. \qquad (25)$$

Once again, $\alpha_{\rm ic} = 0$ corresponds to the quenched result given in Ref. [9], while their annealed result is recovered for the thermally equilibrated value of $\alpha_{\rm ic}$.

For the specific case of hardcore Brownian particles, one also has $\sigma(\bar{\rho}) = 2D\bar{\rho}$ and $D(\bar{\rho}) = D$. Hence,

$${\rm Var}[X(T)] \simeq \frac{1}{\bar{\rho}}\sqrt{\frac{2DT}{\pi}}[1 + \alpha_{\rm ic}(\sqrt{2}-1)]. \qquad (26)$$

This result is verified numerically in Fig. 2(b), for three different values of $\alpha_{\rm ic}$ with $D = 1$, $\bar{\rho} = 10$. For the HU case ($\alpha_{\rm ic} = 0$), we also show that two different initial preparations yield the same result, similarly to Fig. 1.

To summarize, we have considered two different situations where the initial preparation of a 1D diffusive system has long-lasting effects on its fluctuating dynamics. We showed that this dependence is captured by the single parameter $\alpha_{\rm ic}$, which quantifies large-scale density fluctuations in the initial state. The resulting framework generalizes previous results to a vastly wider range of initialization protocols. The coupling of $\alpha_{\rm ic}$ to the long-time dynamics occurs because $X(T)$ and $Q(T)$ are both correlated with the number of particles that were initially within a distance $\sqrt{4DT}$ of the origin: for large T, the fluctuations in this number are set by $\alpha_{\rm ic}$. We suspect that similar effects arise in several other systems [10,21,67] where initialization protocols have everlasting effects (see also Ref. [68]).

These results may be important for quantitative experiments on single-file diffusion, such as NMR measurements inside molecular sieves [17,18]. Equation (26) predicts that the molecular transport ratio $\mathrm{Var}[X(T)]/\sqrt{T}$ has an everlasting dependence on α_{ic}, which itself depends on the (typically nonequilibrium) conditions under which the molecules are loaded into the sieve [17,18]. Without controlling for this dependence, measurements of this ratio may not yield reproducible results. Our predictions could also be tested in colloidal systems, following Ref. [15]. We also note that our results on single-file motion could lead to interesting consequences for related harmonization studies on bead-spring systems [37].

We have emphasized the special role of HU initial states, whose long-wavelength density fluctuations have vanishing amplitude. All such states lie within a single universality class that also contains quenched systems [9,25,26]. While many previous works focused on their creation [69–71], our work reveals instead a dynamical *consequence* of HU states (see also Refs. [72,73]).

While we have analyzed the variances of dynamical quantities, their higher moments (and large deviations) presumably depend on higher-order statistics of the initial state, suggesting rich new possibilities for future research. Further open questions arise for single-file systems not described by MFT [66]; for driven 1D systems [22–24,74–77]; and in higher dimensions [78–80].

We thank T. Agranov, C. Scalliet, J. Pausch, and J.-F. Derivaux for helpful discussions. This work was funded in part by the European Research Council under the Horizon 2020 Programme, ERC Grant Agreement No. 740269. MEC is funded by the Royal Society.

[1] A. L. Hodgkin and R. D. Keynes, J. Physiol. **128**, 61 (1955).
[2] T. E. Harris, J. Appl. Probab. **2**, 323 (1965).
[3] P. M. Richards, Phys. Rev. B **16**, 1393 (1977).
[4] S. Alexander and P. Pincus, Phys. Rev. B **18**, 2011 (1978).
[5] R. Arratia, Ann. Probab. **11**, 362 (1983).
[6] C. Hegde, S. Sabhapandit, and A. Dhar, Phys. Rev. Lett. **113**, 120601 (2014).
[7] Q-H. Wei, C. Bechinger, and P. Leiderer, Science **287**, 625 (2000).
[8] N. Leibovich and E. Barkai, Phys. Rev. E **88**, 032107 (2013).
[9] P. L. Krapivsky, K. Mallick, and T. Sadhu, Phys. Rev. Lett. **113**, 078101 (2014).
[10] J. Cividini and A. Kundu, J. Stat. Mech. (2017) 083203.
[11] A. Taloni, O. Flomenbom, R. Castaneda-Priego, and F. Marchesoni, Soft Matter **13**, 1096 (2017).
[12] V. Gupta, S. S. Nivarthi, A. V. McCormick, and H. Ted Davis, Chem. Phys. Lett. **247**, 596 (1995).
[13] V. Kukla *et al.*, Science **272**, 702 (1996).
[14] A. Das *et al.*, ACS Nano **4**, 1687 (2010).
[15] C. Lutz, M. Kollmann, and C. Bechinger, Phys. Rev. Lett. **93**, 026001 (2004).
[16] J. C. Rasaiah, S. Garde, and G. Hummer, Annu. Rev. Phys. Chem. **59**, 713 (2008).
[17] L. Reikert, Adv. Catal. **21**, 281 (1970).
[18] J. Kärger, M. Petzold, H. Pfeifer, S. Ernst, and J. Weitkamp, J. Catal. **136**, 283 (1992).
[19] M. Wanunu, Phys. Life Rev. **9**, 125 (2012).
[20] S. Y. Yang *et al.*, ACS Nano **4**, 3817 (2010).
[21] J. Rana and T. Sadhu, arxiv:2203.01609.
[22] R. Rajesh and S. N. Majumdar, Phys. Rev. E **64**, 036103 (2001).
[23] S. N. Majumdar and M. Barma, Phys. Rev. B **44**, 5306 (1991).
[24] H. van Beijeren, J. Stat. Phys. **63**, 47 (1991).
[25] B. Derrida and A. Gerschenfeld, J. Stat. Phys. **137**, 978 (2009).
[26] T. Banerjee, S. N. Majumdar, A. Rosso, and G. Schéhr, Phys. Rev. E **101**, 052101 (2020).
[27] J. P. Bouchaud, J. Phys. I (France) **2**, 1705 (1992).
[28] G. Bel and E. Barkai, Phys. Rev. Lett. **94**, 240602 (2005).
[29] A. G. Cherstvy, A. V. Chechkin, and R. Metzler, New J. Phys. **15**, 083039 (2013).
[30] G. M. Schütz and S. Trimper, Phys. Rev. E **70**, 045101(R) (2004).
[31] A. A. Budini, Phys. Rev. E **94**, 022108 (2016).
[32] R. G. Palmer, Adv. Phys. **31**, 669 (1982).
[33] N. Goldenfeld, *Lectures on Phase transitions and the Renormalization Group* (Frontiers in Physics-85) (CRC Press, Boca Raton, FL, 2018.
[34] E. Fermi, J. Pasta, and S. Ulam, Los Alamos Report LA-1940 (1955).
[35] G. P. Berman and F. M. Izrailev, Chaos **15**, 015104 (2005).
[36] M. Kollmann, Phys. Rev. Lett. **90**, 180602 (2003).
[37] L. Lizana, T. Ambjörnsson, A. Taloni, E. Barkai, and M. A. Lomholt, Phys. Rev. E **81**, 051118 (2010).
[38] A. Grabsch, A. Poncet, P. Rizkallah, P. Illien, and O. Bénichou, Sci. Adv. **8**, eabm5043 (2022).
[39] U. Fano, Phys. Rev. **72**, 26 (1947).
[40] R. Dreyfus, Y. Xu, T. Still, L. A. Hough, A. G. Yodh, and S. Torquato, Phys. Rev. E **91**, 012302 (2015).
[41] Ü. Seleme Nizam *et al.*, J. Phys.: Condens. Matter **33**, 304002 (2021).
[42] S. F. Knowles, N. E. Weckman, V. J. Y. Lim, D. J. Bonthuis, U. F. Keyser, and A. L. Thorneywork, Phys. Rev. Lett. **127**, 137801 (2021).
[43] L. Bertini, A. De Sole, D. Gabrielli, G. Jona-Lasinio, and C. Landim, Rev. Mod. Phys. **87**, 593 (2015).
[44] B. Derrida, J. Stat. Mech. (2011) P01030; (2007) P07023.
[45] T. Bodineau and B. Derrida, Phys. Rev. Lett. **92**, 180601 (2004).
[46] L. Bertini, A. De Sole, D. Gabrielli, G. Jona-Lasinio, and C. Landim, Phys. Rev. Lett. **87**, 040601 (2001).
[47] G. Jona-Lasinio, J. Stat. Mech. (2014) P02004.
[48] T. Imamura, K. Mallick, and T. Sasamoto, Phys. Rev. Lett. **118**, 160601 (2017).
[49] K. Mallick, H. Moriya, and T. Sasamoto, Phys. Rev. Lett. **129**, 040601 (2022).

[50] A. Poncet, A. Grabsch, P. Illien, and O. Bénichou, Phys. Rev. Lett. **127**, 220601 (2021).
[51] P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, Cambridge, 2013).
[52] J. S. Bell, Phys. Rev. **129**, 1896 (1963).
[53] S. Torquato, Phys. Rep. **745**, 1 (2018).
[54] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.106.L062101 for further technical details.
[55] L. Berthier and R. L. Jack, Phys. Rev. E **76**, 041509 (2007).
[56] K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986).
[57] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, Course on Theoretical Physics, Vol. 5 (Pergamon Press, Oxford, 1970).
[58] I. Santra, U. Basu, and S. Sabhapandit, J. Phys. A **55**, 385002 (2022).
[59] M. Bothe and G. Pruessner, Phys. Rev. E **103**, 062105 (2021).
[60] D. Martin, J. O'Byrne, M. E. Cates, É. Fodor, C. Nardini, J. Tailleur, and F. van Wijland, Phys. Rev. E **103**, 032607 (2021).
[61] C. Bechinger, R. Di Leonardo, H. Löwen, C. Reichhardt, G. Volpe, and G. Volpe, Rev. Mod. Phys. **88**, 045006 (2016).
[62] U. Basu, S. N. Majumdar, A. Rosso, and G. Schehr, Phys. Rev. E **98**, 062121 (2018).
[63] K. Malakar *et al.*, J. Stat. Mech. (2018) 043215.
[64] T. Demaerel and C. Maes, Phys. Rev. E **97**, 032604 (2018).
[65] M. E. Cates and J. Tailleur, Europhys. Lett. **101**, 20010 (2013).
[66] T. Banerjee, R. L. Jack, and M. E. Cates, J. Stat. Mech. (2022) 013209.
[67] P. L. Krapivsky and B. Meerson, Phys. Rev. E **86**, 031106 (2012).
[68] R. Dandekar and K. Mallick, J. Phys. A Math. Theor., **55**, 435001 (2022).
[69] E. Tjhung and L. Berthier, Phys. Rev. Lett. **114**, 148301 (2015).
[70] D. Hexner and D. Levine, Phys. Rev. Lett. **114**, 110602 (2015).
[71] S. Torquato, G. Zhang, and F. H. Stillinger, Phys. Rev. X **5**, 021020 (2015).
[72] S. B. Lee, J. Stat. Mech. (2019) 053201.
[73] S. Kwon and J. M. Kim, Phys. Rev. E **96**, 012146 (2017).
[74] E. Barkai and R. Silbey, Phys. Rev. Lett. **102**, 050602 (2009).
[75] A. De Masi and P. A. Ferrari, J. Stat. Phys. **38**, 603 (1985).
[76] E. Mallmin, R. A. Blythe, and M. R. Evans, J. Stat. Mech. (2021) 013209.
[77] A. Poncet, O. Bénichou, and P. Illien, Phys. Rev. E **103**, L040103 (2021).
[78] T.-H. Liu and C. C. Chang, Nanoscale **7**, 10648 (2015).
[79] M. M. R. Williams, Math. Proc. Camb. Philos. Soc. **84**, 549 (1978).
[80] R. Villavicencio-Sanchez, R. J. Harris, and H. Touchette, Europhys. Lett. **105**, 30009 (2014).