Letter

Heterogeneity-induced synchronization in delay-coupled electronic oscillators

Nirmal Punetha [1,2,*] and Lucas Wetzel [1,3]
[1] *Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Straße 38, 01187 Dresden, Germany*
[2] *Amity University Haryana, Gurugram (Manesar), 122413 Haryana, India*
[3] *Center for Advancing Electronics (CFAED), Würzburger Straße 46, 01187 Dresden, Germany*

(Received 19 July 2022; accepted 4 October 2022; published 30 November 2022)

We study synchronization in networks of delay-coupled electronic oscillators, so-called phase-locked loops (PLLs). Using a phase-model description, we study the collective dynamics of mutually coupled PLLs and report the phenomenon of heterogeneity-induced synchronization. This phenomenon refers to the observation that heterogeneity in the system's parameters can induce synchronization by stabilizing the states which are unstable without such heterogeneity. In systems where component heterogeneity can be tuned and controlled, we show how the complex collective self-organized dynamics can be guided towards synchronized states with specific operational frequencies and phase relations. This is of importance for the technical applicability of self-organized dynamics. In electrical engineering, for example, where components can be strongly heterogeneous, our theoretical framework can inform the design process for networks of spatially distributed PLLs. The results presented here are also useful in understanding the collective dynamics in ensembles of phase oscillators with time-delayed interactions, inertia, and heterogeneity.

DOI: 10.1103/PhysRevE.106.L052201

The collective dynamics of ensembles of coupled-oscillators has been explored extensively due to their numerous applications [1–4]. In any realistic situation, oscillators in such ensembles and their connections are not perfectly identical either due to natural diversity, by design, or as a result of engineering limitations. Examples include system-level variations arising in electronic devices due to unavoidable irregularities in the production processes [5–7] or intrinsic biophysical diversity leading to heterogeneous cellular oscillators in a tissue [8–10]. Such heterogeneities are frequently encountered in practice and can have significant effects on the dynamics of a system. It is therefore crucial to ask how the collective dynamics of oscillator ensembles is affected by such heterogeneities.

In this work, we investigate the effects of heterogeneity on a system of mutually delay-coupled electronic oscillators, so-called phase-locked loops (PLLs). Arrays of such electronic oscillators are used in, e.g., mobile and radio communications, navigation systems, and antenna arrays. In such systems, efficient and concerted (synchronized) operations require an accurate and precise timing, usually realized by hierarchical entrainment of electronic clocks through a dedicated and precise reference oscillator. This hierarchical approach to synchronization is widely used in electronic devices, though it has its limitations, e.g., due to the accumulation of phase errors and increasingly difficult design constraints such as system size increases [11,12]. Studies have suggested self-organized synchronization based on mutual interactions in large spatially distributed electronic systems as an alternative to address these issues [13–19]. However, for both approaches, maintaining robust synchronization and having effective control over the resulting dynamics remain the key objectives for the calibration and performance optimization of such systems [19].

One of the practical difficulties in achieving these operational requirements in real systems arises due to their heterogeneous nature. Here we address how such heterogeneities affect the synchronization properties of such systems. Apart from component heterogeneity, the complexity introduced by time-delayed coupling, noise, and environmental effects (PVT variations) pose additional challenges to the reliable operation of these devices [20–23]. Time delay, for example, has significant effects on the collective dynamics, especially when the systems are operating at high frequencies and/or involved in long-range signal transmissions [24–27]. In fact, such systems are often designed to minimize these undesirable but also unavoidable factors, e.g., time delays and parameter heterogeneities, in order to reduce their effects on the collective dynamics [7,19,28]. Here we focus on heterogeneities in the system—especially those related to the processing, sensitivity, and signaling delays—which play vital role for the collective behavior of mutually coupled PLLs.

In this rapid communication, we demonstrate that if heterogeneity can be controlled, it can actually be helpful to achieve a required collective response in mutually delay-coupled oscillators. Specifically, we tune parameter heterogeneity to induce robust synchronization in such systems, and we report the occurrence of the phenomenon of heterogeneity-induced synchronization. Further, it is shown that by adjusting pa-

*punethanirmal@gmail.com

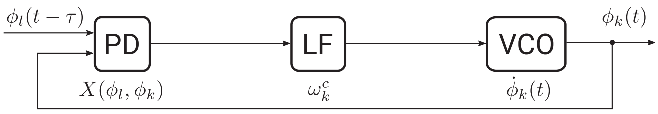

FIG. 1. Schematic of a phase-locked loop. The phase detector, the loop filter, and the voltage controlled oscillator are denoted by PD, LF, and VCO, respectively.

rameter heterogeneity, one can fulfill key requirements for the efficient operations of such devices, i.e., the ability to access a specific synchronized state with required operational frequency and phase relation. For this study, we consider a system of mutually delay-coupled PLLs [29,30]. These oscillators consist of three main components: a phase-detector (PD) that receives external input and internal feedback signals, a loop-filter (LF) to rectify (filter) these signals, and a voltage controlled oscillator (VCO) as an output unit; see Fig. 1. The dynamics of mutually delay-coupled PLLs is determined by the instantaneous VCO frequency,

$$\dot{\phi}_k(t) = \omega_k + \frac{K_k}{n_k}\sum_l c_{kl}\int_0^\infty du p_k(u) X(\phi_l,\,\phi_k),$$
$$\text{with}\quad X(\phi_l,\,\phi_k) = h\big(\phi_l(t-u-\tau_{kl}) - \phi_k\big(t-u-\tau_k^f\big)\big). \quad (1)$$

Here ω_k denotes the intrinsic frequency of the kth PLL, K_k represents its coupling strength, n_k the number of input signals, and c_{kl} reflects the connection topology. The convolution of $p_k(u)$ and $X(\phi_l,\,\phi_k)$ represents the filtering process, where $X(\phi_l,\,\phi_k)$ is the phase difference term from a phase detector signal and $p_k(u)$ the impulse response function of the loop filter. The LF is a buffered chain of a_k RC low-pass filters characterized by its scale parameter b_k, where the cutoff frequency is given by $\omega_k^c = 1/(a_k b_k)$. The filter's impulse response function $p_k(u)$ is given by the gamma distribution

$$p_k(u) = u^{(a_k-1)}\frac{e^{-u/b_k}}{b_k^{a_k}\,\Gamma(a_k)} \text{ with } \int_0^\infty du p_k(u) = 1. \quad (2)$$

The diffusive term $X(\phi_l,\,\phi_k) = h[\phi_l(t-u-\tau_{kl}) - \phi_k(t-u-\tau_k^f)]$, where τ_{kl} corresponds to the time delay in the signal transmitted from lth to kth PLL and τ_k^f the delay in the PLL's feedback signal. The coupling function $h(\cdot)$ depends on the nature of the signals—for digital PLLs, the PD is an XOR gate and $h(\cdot)$ is a triangular function $\mathbf{\Delta}(\cdot)$ [31], whereas in analog PLLs, multipliers are used as a PD and $h(\cdot)$ is a cosine function. This simple model allows us to study the effects of transmission and feedback time delays and parameter heterogeneity on the collective dynamics of phase-oscillator ensembles. It also incorporates inertial effects in the dynamics as larger integration times (large b_k, small ω_k^c) imply a more inert response of such systems [32]. Furthermore, note that the convolution can be interpreted as a distributed delay, with $p_k(u)$ being the delay distribution [26,32,33]. This phase model is independent of the details of the underlying circuitry and can be used to study the collective dynamics of ensembles of oscillators with time-delayed coupling (including discrete and distributed), inertia, and heterogeneity.

Our theoretical framework allows to precisely predict how the system parameters have to be tuned to achieve robust synchronization with specific synchronization frequency and phase configuration. In the following, we discuss the general case of a network of N oscillators, and then present the analytically tractable case of two delay-coupled PLLs. The conclusions drawn from the tractable case are helpful to understand the behavior of N-coupled oscillators. For heterogeneous systems with larger N, even for $N = 3$, it is not straightforward to obtain an explicit analytic form of the solutions. However, a simplified solvable set of equations can be obtained by inserting the general ansatz for phase-locked solutions, i.e., $\phi_k = \Omega t + \beta_k$, $k = 1, 2, \ldots, N$ into equations of motion (1), which gives

$$\Omega = \omega_1 + \frac{K_1}{n_1}S_1 = \omega_2 + \frac{K_2}{n_2}S_2 = \cdots = \omega_N + \frac{K_N}{n_N}S_N,$$
$$\text{where}\quad S_k = \sum_{l=1}^N c_{kl}\; h[-\Omega\tau_{kl} - \beta_{kl}]. \quad (3)$$

In this equation, it is assumed that the delay in the feedback signal is negligible, i.e., $\tau^f \to 0$. The phase differences between the kth and lth oscillator are represented by $\beta_{kl} = (\beta_k - \beta_l)$. Assuming $\beta_1 = 0$ as a reference angle, these N transcendental equations can be solved numerically to obtain synchronized solutions characterized by N values: $(\Omega, \beta_2, \ldots, \beta_N)$.

Using this procedure, we now obtain the analytic form of the solutions for $N = 2$ oscillators as follows. For this case, the equations of motion are given by

$$\dot{\phi}_{1,2}(t) = \omega_{1,2} + K_{1,2}\int_0^\infty du p_{1,2}(u)$$
$$\times\, h[\phi_{2,1}(t-u-\tau_{12,21}) - \phi_{1,2}(t-u)]. \quad (4)$$

This simple case can be used to illustrate the effects of heterogeneous parameters on the collective behavior of the systems with time-delayed coupling. The synchronized solutions of this system are characterized by a global frequency Ω and constant phase difference β, i.e., $\phi_1(t) = \Omega t$, $\phi_2(t) = \Omega t + \beta$. Therefore, from Eq. (4) we find

$$\Omega = \omega_1 + K_1 h(-\Omega\tau_{12} + \beta) = \omega_2 + K_2 h(-\Omega\tau_{21} - \beta). \quad (5)$$

When $h(\cdot) \equiv \mathbf{\Delta}(\cdot)$ (digital signals) [31], one can solve these transcendental equation numerically to find synchronized solutions. For analog signals, $h(\cdot)$ is a cosine function, and the analytical expressions for synchronized solutions can be obtained applying trigonometric identities. To utilize the analytic form of the solutions for the digital case, the triangular coupling function can be approximated by its first harmonic $\mathbf{\Delta}(\cdot) \approx -\cos(\cdot)$. Note that the coupling function approximation introduces only small quantitative deviations but well represents the qualitative behavior of such oscillators. For this case, the solutions Ω and β are given by the following interdependent transcendental equations:

$$\Omega = \bar{\omega} - \bar{K}\cos(\Omega\bar{\tau})\cos\left(\frac{\Omega\Delta\tau}{2} + \beta\right)$$
$$+ \frac{\Delta K}{2}\sin(\Omega\bar{\tau})\sin\left(\frac{\Omega\Delta\tau}{2} + \beta\right),$$

$$\beta = -\frac{\Omega\Delta\tau}{2} - \sin^{-1}\left(\frac{\Delta\omega}{H}\right) + \sin^{-1}\left[\frac{\Delta K\cos(\Omega\bar{\tau})}{H}\right], \quad (6)$$

where $H = \{[2\bar{K}\sin(\Omega\bar{\tau})]^2 + [\Delta K\cos(\Omega\bar{\tau})]^2\}^{1/2}$. The bar and delta notation of parameters represents respectively their average and difference. The transcendental nature of Eq. (5) and Eq. (6) confirms the presence of multiple synchronized solutions, which is a known property of time-delay coupled oscillator systems [34,35].

In order to study how robust these solutions are, we perform linear stability analysis by weakly perturbing the synchronized state, $\phi_k = \Omega t + \beta_k + \xi_k$; $k = 1, 2, \ldots, N$ and study the nature of the perturbation dynamics, i.e., $\dot{\xi}_k$. Assuming these perturbations evolve exponentially $\xi_k \sim \exp(\lambda t)$, the characteristic equation for the eigenvalues λ is given by

$$\det(\mathbf{G} - \zeta\cdot\mathbf{I}) = 0, \quad (7)$$

where $\zeta = 1$, $\mathbf{I}$ is identity matrix and the elements of matrix $\mathbf{G}$ are

$$G_{ij} = \left(\frac{\tilde{c}_{ij}\alpha_{ij}e^{-\lambda\tau_{ij}}}{\frac{\lambda}{\hat{p}_i(\lambda)} + f_i(\tilde{c},\alpha)}\right). \quad (8)$$

We denote $\tilde{c}_{ij} = c_{ij}/n_i$, $\alpha_{ij} = K_i\, h'[(-\Omega\tau_{ij} - \beta_{ij})]$, where $h'(\cdot)$ is the derivative of function $h(\cdot)$ with respect to its argument. $\hat{p}_k(\lambda) = \int_0^\infty du p_k(u)\exp(-\lambda u) = (1+\lambda b_k)^{-a_k}$ is the Laplace transform of the impulse response function, and $f_i(\tilde{c},\alpha) = \sum_{j=1}^{N}\tilde{c}_{ij}\,\alpha_{ij}$. The largest $\mathrm{Re}(\lambda)$ of the set of solutions λ determines the stability of such synchronized states, either unstable $[\mathrm{Re}(\lambda) > 0]$, marginal stable $[\mathrm{Re}(\lambda) = 0]$, or linearly stable $[\mathrm{Re}(\lambda) < 0]$.

Using this analysis, the behavior of synchronized solutions along with their stability can be predicted. To demonstrate our results, we consider a network of four PLLs ($N = 4$) connected in a ring topology. These digital PLLs have detuned intrinsic frequencies $\omega_k \in 2\pi \times \mathbf{N}(0.978, 0.04)$ [rad kHz] and coupling strengths $K_k \in 2\pi \times \mathbf{N}(0.40875, 0.0015)$ [rad kHz]. Here $\mathbf{N}$ represents a Gaussian distribution described by $\mathbf{N}$(mean, deviation). Cutoff frequencies of the RC filters (LF) [36] are initially equal and tuned to $\omega_k^c = 2\pi \times 0.05$ [rad kHz]. For this system, multiple synchronized states exist due to the delayed coupling, and we plot them as a function of the transmission delay in Fig. 2. The results from linear stability analysis [Eq. (7)] are used to distinguish stable [shown as solid-green (dark gray)] and unstable [shown as dashed-gray (light gray)] solutions. Note that the frequencies of the synchronized states and the associated phase difference are independent of LF parameters, but they do become unstable for large integration time of the filter. Such instabilities have not been observed in the absence of filtering processes [34,35]. We highlight the branch of the solution that has become unstable due to such an instability with dashed arrows in Fig. 2. We find that this branch can be stabilized. However, to our surprise, this occurs when parameter heterogeneity is increased in the system.

To demonstrate this behavior, which we termed heterogeneity-induced synchrony in our system of coupled digital PLLs, we consider heterogeneous LF cutoff frequencies for our system and analyze its effects on the stability using Eq. (7); see Fig. 3. We introduce heterogeneity

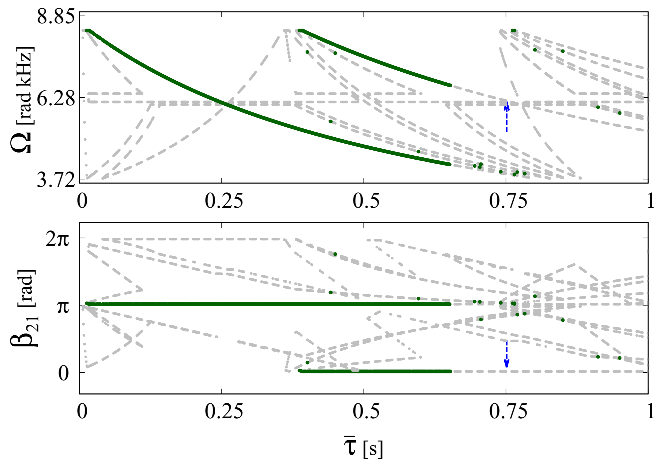

FIG. 2. Synchronized solutions characterized by their global frequency Ω and phase difference β as a function of mean time delay $\bar{\tau}$ for a system of four ($N = 4$) coupled detuned digital PLLs. Results are obtained for homogeneous transmission delays ($\Delta\tau = 0$) and homogeneous cutoff frequencies ($\Delta\omega^c = 0$, $\omega_k^c = 0.052\pi$ [rad kHz], $k = 1, 2, 3, 4$). Stable and unstable solutions are shown by solid green (dark gray) and dashed gray (light gray) curves [37], respectively. The arrows on one of the unstable solution branch points to an unstable synchronized state, which we consider for heterogeneity-induced stabilization (see Figs. 3 and 4).

in the LF cutoff frequencies by drawing their values from Gaussian distribution $\omega_k^c \in 2\pi \times \mathbf{N}(0.05, 0.018)$. Figure 3 shows that the unstable synchronized states observed for homogeneous cutoff frequencies can be stabilized by introducing heterogeneity in the systems. The stabilization of synchronized states that we predicted can also be verified by simulating the time evolution of the system's dynamics. In

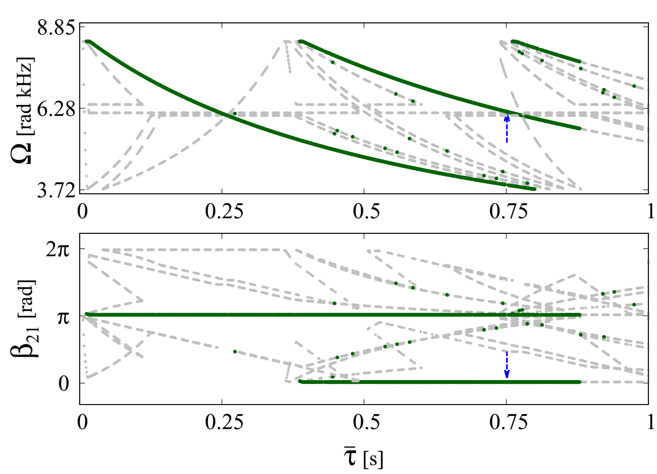

FIG. 3. Stabilization of synchronized solution branch using heterogeneous cutoff frequencies ω_k^c in four ($N = 4$) coupled DPLLs. Heterogeneous cutoff frequencies are $\omega_k^c \in 2\pi\mathbf{N}(0.05, 0.018)$ while the rest of the parameters are unchanged. The solid green (dark gray) and dashed lines (light gray) denote stable and unstable synchronized branches, respectively. We see that the stable branches get extended towards larger mean-delay values as a result of heterogeneity in cutoff frequencies (see Fig. 2 for comparison).

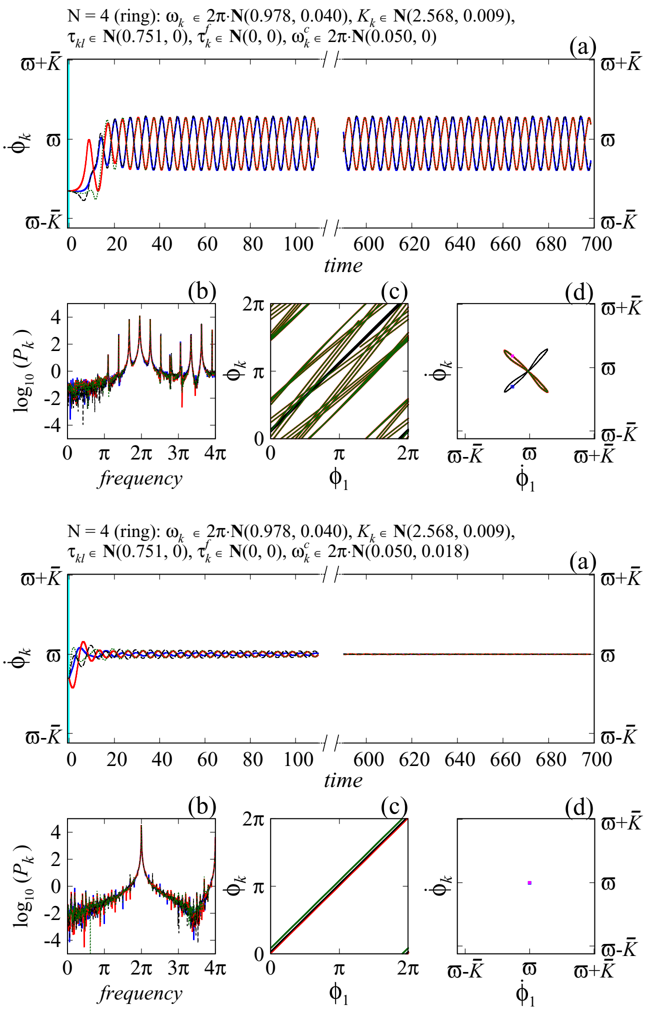

FIG. 4. Simulation results obtained with homogeneous (upper plot) and heterogeneous (lower plot) parameters for the system of four ($N = 4$) coupled DPLLs. For both cases, (a) shows the time variations of oscillator frequencies $\dot{\phi}_k$ ($k = 1, 2, 3, 4$). The frequency spectrums, phase-space ($\phi_1, \phi_{k=\{2,3,4\}}$) behaviors, and frequency-space ($\dot{\phi}_1, \dot{\phi}_{k=\{2,3,4\}}$) behaviors are shown in (b) (c), and (d), respectively. We see that unstable synchronized state (with time-dependent frequencies) in the homogeneous system can be stabilized in heterogeneous system with different cutoff frequencies. The transmission delays are fixed at $\bar{\tau} = 0.751$ ms pointed at by the arrow in Figs. 2 and 3.

Fig. 4 we examine the dynamics of both homogeneous (upper subfigure) and heterogeneous systems (lower subfigure) by evolving them computationally. For the homogeneous case, we observe that frequencies $\dot{\phi}_k$ do not settle to a constant value and show oscillations; see upper subfigure (a) and (d). This indicates desynchronized dynamics due to unstable synchronized states. Additionally, the frequency spectrum in (b) indicate the presence of multiple frequencies, therefore the phases plotted in (c) also do not show constant phase differences. However, by introducing heterogeneity through cutoff frequencies, we are able to stabilize synchronized dynamics (see lower subfigure). In this case, we can clearly see a single synchronized frequency (a), (b), and (d) and constant phase differences (c). We also have verified this

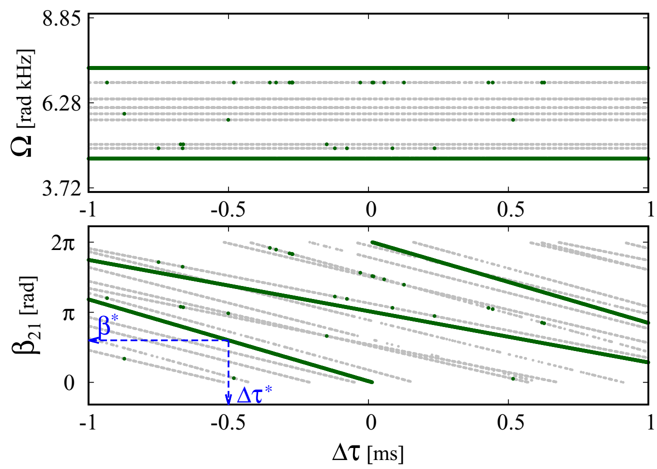

FIG. 5. The variation in the phase difference between first and second oscillator β_{21} is plotted as a function of delay heterogeneity $\Delta\tau$ for a system of $N = 4$ coupled digital PLLs. Delay heterogeneity is introduced in all the incoming and outgoing signals to or from first oscillator in the following manner: $\tau_{12} = \tau_{13} = \tau_{14} = (\bar{\tau} - \Delta\tau/2)$ and $\tau_{21} = \tau_{31} = \tau_{41} = (\bar{\tau} + \Delta\tau/2)$, while mean delay is kept fixed at $\bar{\tau} = 0.536$ ms. Stable and unstable analytic solutions are shown in solid green (dark gray) and dashed gray (light gray) curves, respectively. One can observe that the global frequency and its linear stability are unaffected by $\Delta\tau$, while phase difference β_{21} has linear dependence on $\Delta\tau$. The arrow points to an arbitrary phase difference value β^* which can be achieved by tuning delay heterogeneity to a pertinent value indicated by $\Delta\tau^*$.

stabilizing behavior through experiments for a two oscillator system—these results are shown in a related publication [32]. We found that as the cutoff frequencies are tuned [36] from identical ($\omega^c_{1,2} = (0.055, 0.055)2\pi$ [rad kHz]) to heterogeneous values ($\omega^c_{1,2} = (0.0148, 0.0957)2\pi$ [rad kHz]), a transition is observed from a desynchronized state with different frequencies and time-dependent phase difference to a stable synchronized state with common frequency and constant phase difference.

So far we have illustrated how component heterogeneities can be used to recover synchronization in a system of delay-coupled electronic oscillators. Another important objective in engineered or electronic systems is to control the phase relations between the oscillators in synchronized states. This is required for, e.g., "beamforming" and communication-related applications of such electronic oscillators [38–41]. In the following we discuss a method to control the phase dynamics through tunable heterogeneity in the transmission delays. This method is particularly effective for controlling phase relations as changes in the delay heterogeneity do not alter the frequency and stability of a synchronized state so long as the mean delay $\bar{\tau}$ is kept constant. We observe that β has a linear dependence on the delay heterogeneity $\Delta\tau$; see Eq. (6) and Fig. 5. Due to this linear dependence, the resolution with which the time delays (and hence $\Delta\tau$) can be changed determines how precisely β can be controlled. The result has been experimentally verified for a two-oscillator system (experimental details discussed in [32]) and shows how a synchronized state with a specific phase difference β can be achieved by appropriately tuning the delay heterogeneity.

In conclusion, we analyze the effects of heterogeneity on the synchronization of delay-coupled electronic oscillators, so-called phase-locked loops (PLLs). We demonstrate how tuning parameter heterogeneity can be used to control its collective response and achieve synchronized states with desired properties, i.e., specific operational frequencies and phase relations. Thus, we analytically predict the occurrence of a unique phenomenon: stabilization of synchronized states through parameter heterogeneity. In addition to the heterogeneity in LF cutoff frequencies and coupling strengths, similar stabilizing effects have been observed with feedback-delay heterogeneity as well [32]. This outcome is counterintuitive since heterogeneity, in general, is expected to create hindering effects on synchrony [7–9]. Recent studies have conjectured similar interesting scenarios where structural and parametric asymmetries can generate new symmetric (synchronized) states [42–45]. Here, however, symmetric states already exist in the homogeneous setup and can be stabilized by tuning parameter heterogeneities. In our system, the introduction of heterogeneity, for example, in the LF cutoff frequencies, reduces the destabilizing effects of inert dynamics induced by filtering. Further research is necessary to understand the connection between these seemingly similar behaviors resulting from asymmetries and heterogeneity. Another extension would be to experimentally study collective dynamics of large oscillator arrays, which is necessary for implementing these results in large-scale practical applications. Technological utilization of these results includes the synchronization layer design of distributed electronic systems such as mobile communication systems, navigation systems, and antenna arrays, with the potential to bring significant improvement to currently available state-of-the-art technology. In applications, the tuning of component heterogeneity can be restricted by technical limitations. Hence, the effectiveness of heterogeneity-induced synchronization and control depends on how well heterogeneous parameters can be controlled in the system. In PLLs for example, the time delays, coupling strengths, and component characteristics such as cutoff frequencies can be controlled and used for guiding the collective response as required by the application. Moreover, our analysis provides a general framework and sets the basis for studying collective dynamics of systems with parameter heterogeneity, inertia, and coupling delays. Thus, these results are also useful in understanding collective behavior of oscillator ensembles in other disciplines, e.g., the self-organization of biological oscillators [46,47], neural networks [48,49], social [50], and technological networks [17,19,51,52]. Implementations of these results to design real electronic devices are currently underway and have shown promising initial outcomes [53].

This work is partly supported by the German Research Foundation (DFG) within the Cluster of Excellence "Center for Advancing Electronics Dresden" and the Federal Ministry of Education and Research (BMBF) under Reference No. 03VP06431. N.P. would like to thank the Science and Engineering Research Board (SERB), India for the research grant Reference No. SRG/2022/002002.

[1] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University Press, Cambridge, 2001).
[2] S. H. Strogatz, *Sync: The Emerging Science of Spontaneous Order* (Hyperion, New York, 2003).
[3] A. T. Winfree, *The Geometry of Biological Time* (Springer, New York, 1980).
[4] Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence* (Springer-Verlag, Berlin, 1984).
[5] K. Verma, B. Kaushik, and R. Singh, Microelec. Int. **26**, 49 (2009).
[6] M. Alioto, G. Palumbo, and M. Pennisi, IEEE Trans. Very Large Scale Integrat. (VLSI) Syst. **18**, 697 (2010).
[7] M. Onabajo and J. Silva-Martinez, *Analog Circuit Design for Process Variation-Resilient Systems-on-a-Chip* (Springer Science+Business Media, New York, 2012).
[8] K. Padmanabhan and N. N. Urban, Nat. Neuroci. **13**, 1276 (2010).
[9] J. A. White, C. C. Chow, J. Ritt, C. Soto-Treviño, and N. Kopell, J. Comput. Neurosci. **5**, 5 (1998).
[10] S. D. Burton, G. B. Ermentrout, and N. N. Urban, J. Neurophysiol. **108**, 2115 (2012).
[11] R. Ho, K. Mai, and M. Horowitz, Proc. IEEE **89**, 490 (2001).
[12] E. Mensink, D. Schinkel, E. Klumperink, E. Van Tuijl, and B. Nauta, IEEE J. Solid-State Circuits **45**, 447 (2010).
[13] F. M. Orsatti, R. Carareto, and J. R. C. Piqueira, IET Circuits Devices Syst. **2**, 495 (2008).
[14] M. Javidan, E. Zianbetov, F. Anceau, D. Galayko, A. Korniienko, E. Colinet, G. Scorletti, J. M. Akre, and J. Juillard, *IEEE International Symposium on Circuits and Systems* (IEEE, New York, 2011), pp. 2589–2592.
[15] A. Pollakis, L. Wetzel, D. J. Jörg, W. Rave, G. Fettweis, and F. Jülicher, New J. Phys. **16**, 113009 (2014).
[16] L. Wetzel, D. J. Jörg, A. Pollakis, W. Rave, G. Fettweis, and F. Jülicher, PLoS One **12**, e0171590 (2017).
[17] E. Koskin, D. Galayko, and E. Blokhina, IEEE Access **6**, 18723 (2018).
[18] L. Minati, K. K. Tokgoz, M. Frasca, Y. Koike, J. Iannacci, N. Yoshimura, K. Masu, and H. Ito, IEEE Access **8**, 36536 (2020).
[19] W. C. Lindsey, F. Ghazvinian, W. C. Hagmann, and K. Dessouky, Proceedings of the IEEE **73**, 1445 (1985).
[20] M. A. Scarpato, Digital circuit performance estimation under PVT and aging effects, Ph.D. thesis, Université Grenoble Alpes (2017).
[21] A. Malkov, D. Vasiounin, and O. Semenov, Circuits Syst. **02**, 162 (2011).
[22] M. Maiti, S. K. Saw, A. J. Mondal, and A. Majumder, Ain Shams Eng. J. **11**, 265 (2020).
[23] S. Nejadhasan, F. Zaheri, E. Abiri, and M. R. Salehi, Int. J. RF Microwave Comput.-Aided Eng. **30**, 11 (2020).

[24] F. M. Atay, *Complex Time-Delay Systems* (Springer-Verlag, Berlin, 2010); M. Lakshmanan and D. V. Senthilkumar, *Dynamics of Nonlinear Time-Delay Systems* (Springer-Verlag, Berlin, 2011).
[25] J. M. Cushing, *Integrodifferential Equations and Delay Models in Population Dynamics*, Lecture Notes in Biomathematics Vol. 20 (Springer-Verlag, New York, 1977).
[26] N. MacDonald, *Time Lags in Biological Models*, Lecture Notes in Biomathematics Vol. 27 (Springer, Berlin, 1978).
[27] W. Just, A. Pelster, M. Schanz, and E. Schöll (Eds.), Phil. Trans. R. Soc. A. **368**, 303 (2010).
[28] M. Heibert, S. R. Madala, and H. Djahanshahi, Methods and apparatus for controlling mismatch in a voltage controlled oscillator array, U.S. Patent US 009356608B1 (2016).
[29] H. Meyr, L. Popken, and H. Mueller, IEEE Trans. Commun. **34**, 436 (1986).
[30] F. M. Gardner, *Phaselock Techniques* (John Wiley & Sons, Hoboken, NJ, 2005).
[31] $\mathbf{\Delta}$ is a piecewise linear and 2π-periodic function defined as $\mathbf{\Delta}(\phi) = \phi(2/\pi) - 1$, if $0 \leqslant \phi < \pi$, and $\mathbf{\Delta}(\phi) = \phi(-2/\pi) + 3$, if $\pi \leqslant \phi < 2\pi$.
[32] N. Punetha and L. Wetzel, Phys. Rev. E **106**, 054216 (2022).
[33] F. M. Atay, Phys. Rev. Lett. **91**, 094101 (2003).
[34] H. G. Schuster and P. Wagner, Prog. Theor. Phys. **81**, 939 (1989).
[35] M. K. S. Yeung and S. H. Strogatz, Phys. Rev. Lett. **82**, 648 (1999); M. G. Earl and S. H. Strogatz, Phys. Rev. E **67**, 036204 (2003).
[36] The cutoff frequency of the RC loop filter is given by $\omega^c = 1/RC$, which can be tuned by changing the resistance R using a potentiometer. The capacitance C is fixed at 22 nF in our setup.
[37] Few irregular stable points in the unstable solution curves are observed due to limitations of the root-finding numerical methods, which are based on iterative convergence of initial guess values. These numerical artifacts appear when this convergence fails and even after long iterations the algorithm does not reach to the appropriate solution.
[38] A. K. Agrawal and E. L. Holzman, IEEE Trans. Antennas Propag. **47**, 432 (1999).
[39] A. Hajimiri, H. Hashemi, A. Natarajan, X. Guan, and A. Komijani, Proc. IEEE **93**, 1637 (2005).
[40] D. Theodoropoulos, G. Kuzmanov, and G. Gaydadjiev, *IEEE 7th Symposium on Application Specific Processors* (IEEE, New York, 2009), pp. 80–87.
[41] S. Hu, G. Hong, and W. Jian, *IEEE International Conference on Signal Processing, Communication and Computing (ICSPCC 2013)* (IEEE, New York, 2013), pp. 1–5.
[42] T. Nishikawa and A. E. Motter, Phys. Rev. Lett. **117**, 114101 (2016).
[43] Y. Zhang, T. Nishikawa, and A. E. Motter, Phys. Rev. E **95**, 062215 (2017).
[44] J. D. Hart, Y. Zhang, R. Roy, and A. E. Motter, Phys. Rev. Lett. **122**, 058301 (2019).
[45] A. E. Motter and M. Timme, Annu. Rev. Condens. Matter Phys. **9**, 463 (2018).
[46] L. G. Morelli, S. Ares, L. Herrgen, C. Schröter, F. Jülicher, and A. C. Oates, Delayed coupling theory of vertebrate segmentation, HFSP **3**, 55 (2009).
[47] A. C. Oates, L. G. Morelli, and S. Ares, Development **139**, 625 (2012).
[48] F. C. Hoppensteadt and E. M. Izhikevich, Phys. Rev. Lett. **82**, 2983 (1999).
[49] V. V. Matrosov, M. A. Mishchenkoa, and V. D. Shalfeev, Eur. Phys. J.: Spec. Top. **222**, 2399 (2013).
[50] H. A. Tanaka, A. J. Lichtenberg, and S. Oishi, Phys. Rev. Lett. **78**, 2104 (1997).
[51] M. Rohden, A. Sorge, M. Timme, and D. Witthaut, Phys. Rev. Lett. **109**, 064101 (2012).
[52] F. Dörfler and F. Bullo, Automatica **50**, 1539 (2014).
[53] C. Hoyer, L. Wetzel, D. Prousalis, J. Wagner, F. Jülicher, and F. Ellinger, IEEE Trans. Circuits Syst. II: Express Briefs **69**, 3689 (2022).