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Letter

Boosted excitation of the fifth cyclotron harmonic based on frequency multiplication in conventional gyrotrons

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We discovered a specific property of the eigenmodes of a cylindrical waveguide, due to which efficient mode excitation at ultrahigh ($s = 4n + 1$, n is integer) multiples of the gyrofrequency can be provided by a polyhelical weakly relativistic electron beam, standardly used for gyrotron operation. In the proof-of-principle experiment with a V-band gyrotron driven by a 25-keV, 2-A beam, about 100 mW radiation power at the fifth cyclotron harmonic (0.22 THz) has been detected in the cw regime. Based on the gyrotron theory, we demonstrate in simulations that, using parameters of existing gyrotrons, this method can be applied for producing cw radiation at a power level of up to several watts in the terahertz gap.

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Multiple applications, including spectroscopy with high resolution, remote sensing, high-speed wireless communications, biomedicine, security, and many others [1,2], require cw, compact, and powerful radiation sources operating in the terahertz gap, i.e., at frequencies from 0.3 to 10 THz, to be developed. This problem still remains pretty much unresolved, as THz frequencies are "too low" for quantum devices and "too high" for classical electronics associated with radiation of rectilinear electron beams in slow-wave structures. Lack of powerful radiation sources is especially critical in the 0.5–3 THz band [2–4].

Meanwhile, one of the most natural sources of high-frequency radiation is an ensemble of electrons gyrating in a magnetic field H. Each electron with an axial velocity of $v_{||} = \beta_{||}c$ and transverse velocity of $v_\perp = \beta_\perp c$ rotates with a gyrofrequency of $\omega_H = eH/m\gamma c$, where $\gamma = (1-\beta_{||}^2-\beta_\perp^2)^{-1/2}$ is the relativistic mass factor and radiates at the frequencies equal to Doppler-shifted multiples of ω_H:

$$\omega_s = \frac{s\omega_H}{1-\beta_{||}\cos\psi}, \tag{1}$$

where s an integer number of a cyclotron harmonic (CH), and ψ is the angle of wave propagation with respect to the magnetic field.

Devices that exploit radiation of gyrating electrons are numerous. Making use of strongly relativistic beams with $\gamma >> 1$, synchrotrons [5] generate a wide CH spectrum at very high frequencies. However, these are cumbersome plants which cannot be used in many of the demanding applications. Alternately, weakly relativistic ($\gamma \approx 1$) gyrating beams are exploited in cyclotron resonance masers (CRMs) [6,7], which may be rather compact and, simultaneously, provide significantly higher power of high-frequency radiation, as compared to the devices with rectilinear beams.

Among CRMs, gyrotron [7–15] is arguably the most efficient, versatile, and stable source of high-power radiation at 0.1–1 THz [11,12]. Gyrotron's main feature is the operation at the close-to-cutoff waveguide mode, which is characterized by transverse propagation with $\cos\psi \approx 0$ ($k_{||} \to 0$). This allows for minimization of the influence of the electron's velocity spread on wave-beam interaction, making all the electrons radiate at the same frequency [see Eq. (1)]. Accordingly, the guiding magnetic field H should be chosen so as to provide the resonance between the mode at a frequency close to critical ω_c and the radiation frequency ω_s, which is determined by the gyrofrequency ω_H or its sth harmonic:

$$\omega_s \approx \omega_c \approx s\omega_H. \tag{2}$$

As follows from Eq. (2), the critical issue for cw THz generation by fundamental-harmonic ($s = 1$) gyrotrons is the requirement of very strong permanent magnetic fields. Accordingly, considerable effort is directed towards high-harmonic generation, which leads to proportional reduction of the required magnetic field. However, harmonic generation encounters a number of difficulties in the standard schemes of gyrotrons powered by weakly relativistic electron beams, for which radiation efficiency rapidly decreases with a harmonic number. Thus, only the second and the third CH generation are typically achievable.

In this Letter we propose a method of boosted excitation of ultrahigh ($s = 4n + 1$) cyclotron harmonics. The method is based on a peculiarity of the cylindrical waveguide mode spectrum that follows from an obscure property of the Bessel derivative roots. We present the theoretical concept, the simulations, and the results of the proof-of-principle experiment in which 0.22-THz, 80–100-mW radiation at the fifth cyclotron

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harmonic has been detected in the cw regime. The feasibility of the proposed method for providing powerful radiation in the terahertz gap is discussed at the conclusion.

For gyrotrons, two concepts of harmonic operation are known. The most efficient way of obtaining the high-frequency radiation is by selective excitation of a mode resonant to the electron beam at one of the high cyclotron harmonics so the azimuthal bunching occurs only at this harmonic. However, the transition to higher-order harmonics is hindered by the problem of mode competition [7] as the wavelength gets smaller. As a result, at frequencies above 0.1 THz, conventional gyrotrons with tubular polyhelical electron beams provide operation at the second cyclotron harmonic only [13,14]. Higher-frequency (0.4 THz) cw generation at the third harmonic [15] was achieved in the so-called large-orbit gyrotrons (LOGs) with axis-encircling electron beams.

An alternate way of producing high-harmonic radiation in a gyrotron is by using the effect of frequency multiplication [16–22], when the high-frequency (HF) radiation is excited as a consequence of a nonlinear interaction at the low-frequency (LF) cyclotron resonance. As a result, the electron beam is bunched at the frequency of the LF wave and simultaneously, at all its multiples. Thus, unlike the case of selective excitation, the HF harmonic output is not directly excited but coexists with LF radiation, thus evading the mode competition.

For efficient frequency multiplication in a conventional first-harmonic gyrotron, the electron beam should be resonant to two cutoff $TE_{m,p}$ modes characterized by (m_1, p_1) and (m_s, p_s) indexes at the first and sth cyclotron harmonics [Fig. 1(a)]. Thus the cutoff frequencies of these modes $\omega_c^{HF} = \nu_{m_s,p_s}/R$ and $\omega_c^{LF} = \nu_{m_s,p_s}/R$ should be divisible with the coefficient s (here R is the waveguide radius, φ is the azimuthal angle, $\nu_{m,p}$ is the pth root of the equation $J'_m(x) = 0$, $J_m(x), J'_m(x)$ are the mth-order Bessel function and its first derivative, and m and p are azimuthal and radial mode's indexes). In addition, as follows from the gyrotron theory, the electron current bunched at the first harmonic in the mode with field of $\sim e^{-im_1\varphi+i\omega t}$ would be orthogonal to all modes except for those proportional to $e^{s(-im_1\varphi+i\omega t)}$. As a result, the following selection rules can be formulated [16]:

$$\nu_{m_s,p_s} \approx s\nu_{m_1,p_1}, \quad m_s = sm_1. \tag{3}$$

Conditions (3) would have been easy to exactly satisfy if the eigenspectrum of cutoff frequencies was equidistant, which is not the case of the cylindrical waveguide. Thus the efficiency of nonlinear conversion (i.e., the ratio of powers at the sth and at the fundamental harmonics) is rather low and, moreover, rapidly reduces with the harmonic number. For example, recent experiments with a sub-THz gyrotron [21] demonstrated the conversion ratio of 10^{-4} and 10^{-6} at the second and third multiples, respectively. In order to increase the harmonic power, different arrangements have been proposed, including sectioning the interaction space [18,19], using planar waveguides [22], or introducing the selective corrugations [20]. Nevertheless, even in the improved schemes, the excitation of harmonics above the fourth CH was not considered promising.

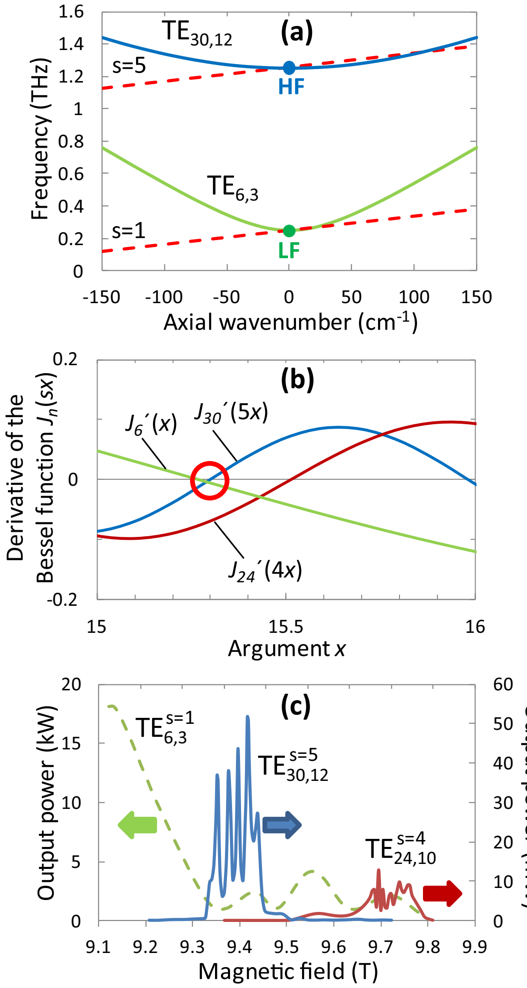

FIG. 1. (a) Dispersion diagrams of the proper pair of waveguide modes (solid curves) and electron cyclotron characteristics $\omega - k_{||}v_{||} = s\omega_H$ (dashed lines) for the case of frequency multiplication with a factor $s = 5$. (b) Intersections of the derivatives of Bessel functions $J'_6(x)$ and $J'_{30}(5x)$ demonstrating the accuracy of fulfillment of conditions (4) for the case of $s = 5$, $m_1 = 6$, $p_1 = 3$. For comparison, intersections with $J'_{24}(4x)$ are shown, which demonstrates significantly worse accuracy for $s = 4$. (c) Results of simulations for excitation of the fifth cyclotron harmonic with the operating frequency of 1.25 THz in accordance with Fig. 1(a).

However, among the Bessel derivative roots there is a subset that satisfies the conditions (3) with fairly high accuracy. Indeed, the Debye asymptotic valid at $m >> 1$ [23] yields the following asymptotic property of the roots of the Bessel function derivative:

$$\nu_{m,p} \sin\Psi = m\Psi + \pi\left(p - \frac{3}{4}\right) + O\left(\nu_{m,p}^{-1}\right), \tag{4}$$

where $\sin\Psi = \sqrt{1 - m^2/\nu_{m,p}^2}$. According to (4), at

$$p_s = sp_1 - 3(s-1)/4, \tag{5}$$

conditions (3) are asymptotically satisfied as $\nu_{sm_1,p_s} = s\nu_{m_1,p_1} + O(\nu_{m_1,p_1}^{-1})$. Since the index p is an integer, $s-1$ must be a multiple of four, i.e., $s = 5, 9, 13\ldots$. Note that the property (4) follows rather obviously from the McMahon asymptotic [24], valid at $p >> m$, but its validity is much broader. Figure 1(b) demonstrates that the accuracy of fulfillment of conditions (3) for the case of $s = 5$, $m_1 = 6$, $p_1 = 3$ is $(\nu_{sm_1,p_s} - s\nu_{m_1,p_1})/s\nu_{m_1,p_1} \approx 0.2\%$.

Thus Eqs. (3) and (5) provide a simple recipe for boosting the efficiency of frequency multiplication with excitation at the cyclotron harmonic with $s = 4n + 1$ (n is an integer): (a) one should choose the magnetic field so that it is close to the resonance with the $\mathrm{TE}_{m,p}$ mode on the first CH; it would simultaneously be resonant to the $\mathrm{TE}_{sm,sp-3(s-1)/4}$ at the sth CH; (b) the radius R_b of a thin tubular electron beam in the operating waveguide should be chosen in such a way that provides efficient interaction with both HF and LF modes, as determined by the radial dependence of the acting electric field given by $J_{m-s}(\nu_{m,p}R_b/R)$. Note that as most of the equalities in Eqs. (3)–(5) are approximate, the described system needs some tuning in order to be optimized. At the same time, the advantage of the proposed method is that LF and HF mode profile (coupling coefficients) overlapping would always take place, as the radius of the mode caustic is determined as $R_c \approx Rm/\nu_{m,p}$. Since the azimuthal indexes and Bessel derivative roots are proportional according to Eq. (3), the caustic radiuses are close, so the conditions of both modes coupling with the beam is easily satisfied.

The proposed approach was verified in the proof-of-principle experiment based on the setup of the fundamental-harmonic 45-GHz, 20-kW gyrotron [25] driven by a 25-keV, 2-A beam. The experiment was aimed at generating the fifth cyclotron harmonic in the regime of frequency multiplication. Since the gyrotron operated at the $\mathrm{TE}_{6,3}$ mode, harmonic excitation of $\mathrm{TE}_{30,12}$ at the frequency of 225 GHz was expected. This required a slight change in the injection radius of the electron beam (by the use of a cathode solenoid) in order to provide maximum coupling with the fifth harmonic mode.

The principal scheme for detecting the HF signal is presented in Fig. 2. The output radiation coming from the gyrotron was converted into a quasioptical wave beam with simultaneous LF and HF content. Since the excited modes have close caustics, the quasioptical mode converter designed for the LF mode would efficiently output the HF mode as well. According to simulations, the diffraction losses at a frequency of 225 GHz do not exceed 5%. Then radiation fell on the parabolic mirror with a small hole of 0.8 mm diameter. The mirror has high reflecting properties for the LF radiation, as the cutoff frequency of the hole exceeds the first-harmonic frequency. At the same time, the hole provides partial transparency of the HF radiation, intercepting about 5×10^{-5} of its total power (transmission coefficient of -43 dB). Behind the hole, a rectangular waveguide brought radiation to the mixer of the Keysight N9010A spectrum analyzer, which detected the maximum signal of -92 dBm [Fig. 3(a)]. Taking into account the calibration of the analyzer (0.3-mW radiation power corresponds to -70 dBm), the transmission coefficient to the waveguide, and additional losses in the waveguide of about -3 …-4 dB, the power of the gyrotron output signal at the frequency of 225 GHz was estimated to be of $\sim 80 \pm 20$ mW. This value is in a good agreement with simulations [Fig. 3(c)] within the frame of a standard nonstationary self-consistent

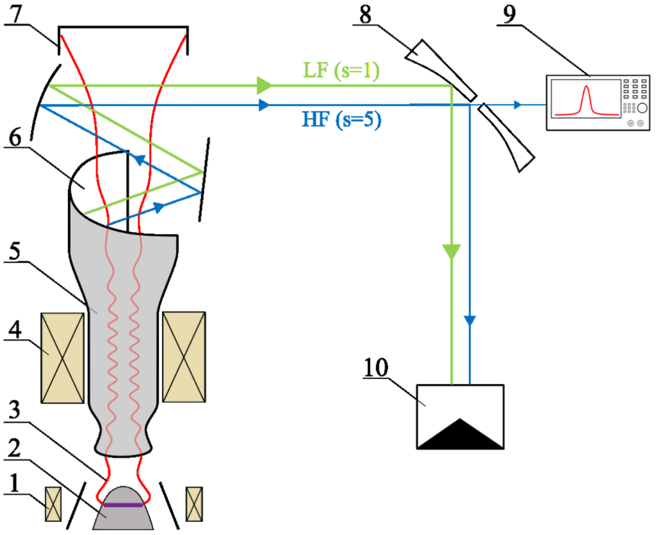

FIG. 2. Block diagram of the experiment on the fifth harmonic frequency multiplication on the basis of a V-band gyrotron setup: 1 – control cathode solenoid, 2 – cathode, 3 – tubular polyhelical electron beam, 4 – solenoid of the guiding magnetic field, 5 – operating waveguide, 6 – quasioptical converter of output radiation, 7 – collector, 8 – parabolic mirror with a diagnostic hole, 9 – spectrum analyzer, and 10 – calorimeter load.

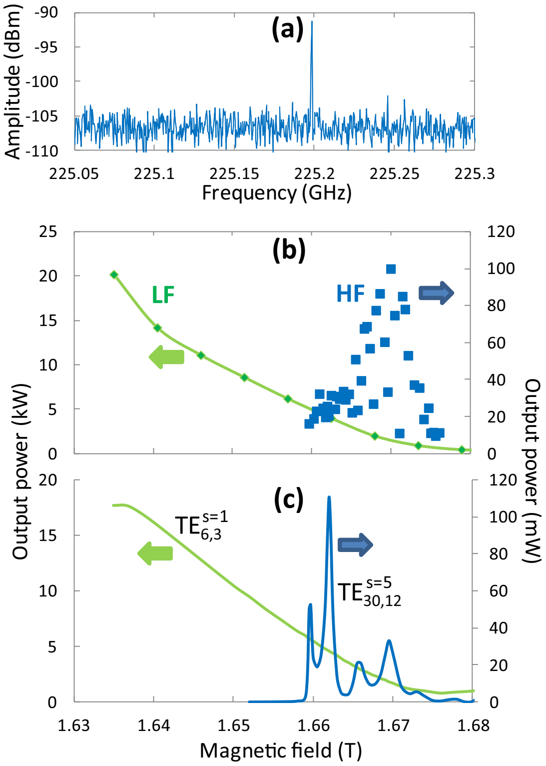

FIG. 3. Results of the experiment on observation of the fifth harmonic frequency multiplication based on standard setup of the fundamental-harmonic, 45-GHz gyrotron driven by weakly relativistic 25-keV, 2-A beam of gyrating electrons. (a) Spectrum analyzer readings confirming the excitation of the fifth cyclotron harmonic. (b) Experimental dependences of the output power on the guiding magnetic field strength for LF and HF radiation. (c) Results of simulations with experimental parameters.

model of electron-wave interaction described in [21] (cf. [26] and references within). According to simulations, the optimal conditions for the excitation at the fifth harmonics were fulfilled at magnetic fields of 1.66–1.67 T [Fig. 3(b)], where the power at the fundamental resonance reduced down to 5 kW. Thus the conversion ratio at the fifth multiple was of 2×10^{-5}, which is an order of magnitude higher than for the case of the third CH multiplication obtained experimentally in [21].

Thus the discovered property of the roots of the Bessel function derivative allows us to propose a method for generating powerful high-frequency radiation based on the effect of frequency multiplication of gyrating electron beams in a cylindrical waveguide with a nonequidistant spectrum of eigenmodes. We demonstrate experimentally that proper choice of parameters provides boosted excitation of the fifth cyclotron harmonic in standard gyrotron setup. For practical applications, LF and HF radiation components can be further separated using frequency-selecting filters.

The experiment performed is in good agreement with theoretical predictions, including the adjustment of the beam radius and the magnitude of the guiding magnetic field. This makes it possible to evaluate the capabilities of the proposed method for generating the terahertz radiation with using the model [21]. As an example of a THz source, we consider here the fifth harmonic excitation in a gyrotron with an operating frequency of 250 GHz, keeping the modes and the beam parameters as indicated above. Such parameters are very close to those used in the gyrotron driver designed in [27]. Results of simulations [Fig. 1(c)] clearly demonstrate the boosted excitation of the fifth cyclotron harmonic $s = 5$ in comparison with $s = 4$, while, in accordance with existing concepts, the radiation power should reduce with increase of the harmonic number. Estimates promise a power of several dozen milliwatts at a frequency of 1.25 THz, which cannot be reached by any other state-of-the-art devices excluding bulky accelerator-based sources like free-electron lasers. Since the fundamental-harmonic gyrotron equipped by cryomagnets can operate at frequencies up to 0.4 THz, powerful generation at the 5th cyclotron harmonic can be provided even in the most “uncultivated” regions of the terahertz gap of 0.8–2 THz. Radiation sources with indicated parameters are in high demand for applications, for instance, for molecular gas spectroscopy, including studies of unsaturable resonance transitions [28,29].

Note, in conclusion, that the efficiency at the fifth harmonic can in principle be increased by an order of magnitude by means of operation at higher transverse modes, when the accuracy of condition (4) increases. Such modes are employed in MW gyrotrons, which would also increase the power in the fifth harmonic component (up to several watts in the terahertz band, according to estimations). However, this scheme may be limited by mode competition at the fundamental harmonic. At the same time, it is possible to suppress undesirable competitors due to the locking of a gyrotron by an external signal (see, for example, [30]).

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