Letter

Janssen effect in columns of fire ants

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We study fire-ant columns, an active version of passive granular columns, and find that, despite the inherent activity of the ants and their natural tendency to rearrange, the ants develop force-chain structures that help support the weight of the column. Hence, the apparent mass at the bottom of the column saturates with added mass in a Janssen-like fashion, reminiscent of what is seen in passive-grain columns in wide containers. Activity-induced rearrangements within the column, however, lead to changes in the force-chain structure that slightly reduce the supportive nature of the force-chains over time and to fluctuations in the pressure at the bottom of the column that scale like the law of large numbers. We capture the experimental results in simulations that include not only friction with the walls, but also a fluctuating force that introduces activity and that effectively affects the force-chain structure of the ant collective.

Consider a vertical cylindrical column with cross-section diameter D filled with a simple fluid, such as water. The pressure at the base of the column, P, increases linearly with the height of the free surface of the fluid, h [1]: $P = \rho g h$, where g is the acceleration of gravity, and ρ is the fluid density. Parametrizing the pressure with an apparent mass that would be detected by a scale at the base of the column, $m_{app} = P\pi D^2/(4g)$, and the height with the mass added to the container, $m_{add} = \rho h \pi D^2/4$, results in the intuitive result that $m_{app} = m_{add}$ for any m_{add}. In contrast, Janssen famously found that, for granular matter, m_{app} linearly grew with m_{add} initially, but that it eventually saturated for sufficiently large m_{add} [2–4]. To rationalize this behavior, Janssen modeled the grains as a continuum elastic solid interacting with the walls via friction, and applied the condition of mechanical equilibrium to an infinitesimal horizontal slice a distance z from the free surface, as illustrated in Fig. 1(a); the force balance includes friction, gravity, and the force due to the pressure difference above and below the slice. Rearrangement of this force balance allows rewriting the equilibrium condition in terms of m_{app} [5]:

$$dm_{app}(z) = \rho \frac{\pi D^2}{4} dz - \frac{4\mu_s k}{D} m_{app} dz, \qquad (1)$$

which after separation and integration, yields $m_{app} = m_{sat}[1 - \exp(-m_{add}/m_{sat})]$, where $m_{sat} = \rho \pi D^3/(16\mu_s k)$ is the satu-

rated apparent mass corresponding to $m_{add} \to \infty$. In these expressions, μ_s is the static friction coefficient, and k is a proportionality constant relating vertical and horizontal stresses.

The success of Janssen's model is that it qualitatively captures the behavior of m_{app} in granular colums and the importance of friction in screening the base of the column from the effects of further added mass. In practice, however, the details of how grains are placed in the column can lead to quantitatively different results in measurements of m_{app}, as the loading protocol determines the structure of the force chains that form within the column. These frictional force chains, which are roughly linear strings of adjacent particles compressed together in such a manner that they help support the load stresses [6–9], terminate on the walls, providing the normal force required for friction to play a significant role [10]. In narrow columns, however, an overshoot in m_{app}, where $m_{app} > m_{add}$, has been observed for an intermediate range in m_{add}. The behavior is attributed to rearrangements of the grains that resulted in inverted force chains that compress rather than support the granular column [11].

In this Letter, we use Janssen's experiment to probe whether collectives of fire ants, an active version of the grains Janssen used in his experiment, support themselves via force chains, and examine if and how activity alters the classic results for passive grains.

The anticipated answer is not obvious. Fire ants, *Solenopsis invicta*, naturally form rafts [12–14] and towers [15,16] to survive floods or to forage for food. The ants within these
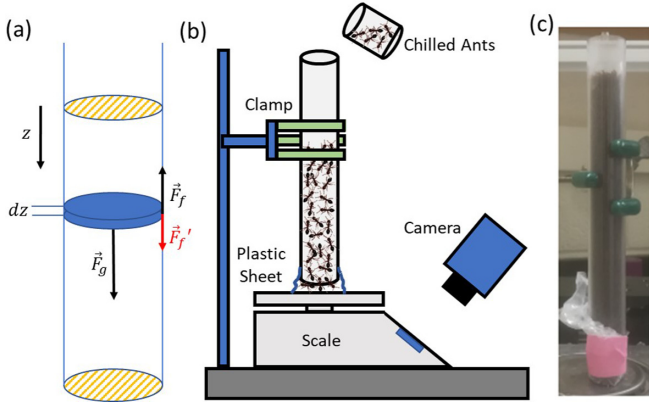
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FIG. 1. (a) One of the horizontal slices used to obtain m_{app} in Janssen's model; $\vec{F}_f$ is a supportive friction force, $\vec{F}_g$ is the gravitational force, and $\vec{F}'_f$ is the random force used in our simulations. (b) Schematic of the experimental set up. (c) Image of a fully loaded cylinder at the end of an incremental experiment.

FIG. 2. Apparent mass m_{app} vs added mass m_{add} increases. Differently shaded points are results from trials with ants from different colonies.

structures form linkages with their neighbors that are sustained through frictional forces, forming a network that can support the structure against compression or extension. This is reminiscent of the way passive grains support themselves against compression via force-chain networks. Since, in addition, the average size of the ants, $l = (3.4 \pm 0.7)$ mm, is in the granular range, one may perhaps expect a behavior comparable to that obtained with conventional grains. However, the network that supports the ants in towers and rafts evolves in time because the ants continually make and break connections. This behavior allows dense ant collectives to flow in response to an applied shear stress [17,18]. For high stresses, the ants exhibit shear thinning, but they also exhibit flow regimes at low stresses reminiscent of the behavior of simple fluids [12,19,20]. This behavior distinguishes ant collectives from granular materials, which are known to flow only above a certain threshold stress that depends on the elastic properties of the grains, their density, and on whether they are already flowing [21–23]. Hence, whether fire-ant columns, and active granular columns in general, continuously support their weight by forming force chains is not known.

We clamp an open cylinder with an inner diameter $D = (22.2 \pm 0.1)$ mm above a balance, as shown in the schematic in Fig. 1(b). To ensure that the ants remain contained in the cylinder, we secure a polyethylene sheet over the outside of the base of the cylinder with enough slack that an area of the sheet roughly equal to the cross-section of the cylinder can rest on the scale without tension. We coat the inside of the cylinder and sheet with talcum powder, which absorbs the oil on the ants' feet and eliminates their ability to walk along the walls of the cylinder. Without this layer of talcum powder, the ants would not be confined to a column and they would escape out of the top of the cylinder. We add ants to the cylinder in approximately $1g$ increments from the top, mimicking the loading procedure with grains in Ref. [11]. In particular, we add the increments every two minutes and record m_{app} at a rate of 3.75 fps with a CCD camera. To facilitate manipulation, we chill each increment of ants for two minutes immediately prior to its addition to the cylinder
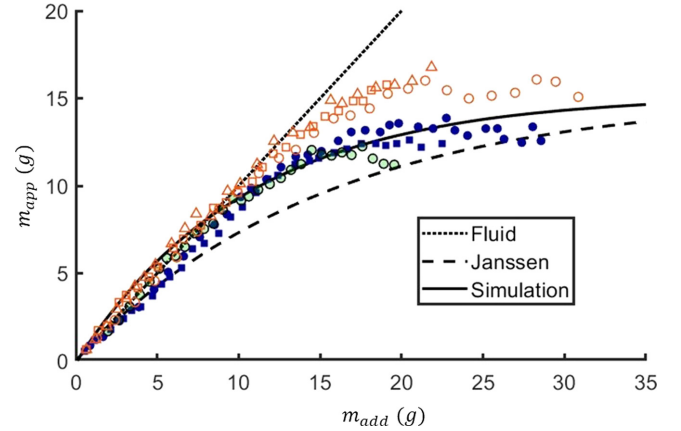
to temporarily reduce their motion. These increments of ants appear to warm up and become fully active within $15s$ after they are added to the room-temperature column [5]. We find that this set-up results in columns of ants with a defined free surface and in good contact between the ants and the scale through the plastic sheet, as shown in Fig. 1(c).

Figure 2 shows our measurements of m_{app} at 110 s after each incremental ant addition, for various trials. For comparison, the dotted line represents the expected result if the ants displayed simple fluid behavior, $m_{app} = m_{add}$, and the dashed line shows Janssen's model with $m_{sat} = 15.2$ g. Points with the same shading represent measurements made in experiments that used ants from the same colony. Aside from the reproducibility of the results in experiments that used a single colony, which validates our experimental methods, the most significant feature of these plots is that m_{app} always saturates, indicating the presence of supportive force chains in the dense ant collectives and a strong deviation from fluid behavior. Evidently, even though these force chains must be continually broken and reformed as the ants rearrange in the column and crawl past each other, the ants never relax enough to behave like a fluid. Using that previous studies with spontaneously-formed ant-towers have shown that a load of 90 mg on an ant at their base prevents the ant from walking [15], and considering the average mass density in our columns, $\rho = 0.22$ g/cm^3 [5], together with the average ant mass, $m_a \approx 0.8$ mg, we estimate that this situation corresponds to $m_{app} \approx 15$ g, which is where saturation is seen to begin in our experiments.

Remarkably, we do not observe a strong overshoot, $m_{app} > m_{add}$ for intermediate m_{add}, as one would find if we had used passive grains with a similar ratio of length-scales between the particles and the cylinder containing them; this indicates that secondary rearrangements near the free surface of the column, which would cause a strong overshoot, are absent in fire-ant columns. We believe this is due to the ability of the ants to cling tightly to each other, in effect drastically increasing the inter-particle friction. Increased friction between the particles changes the available avenues for energy dissipation and makes surface rearrangements much less likely, as has been shown in computer simulations of spherical particles
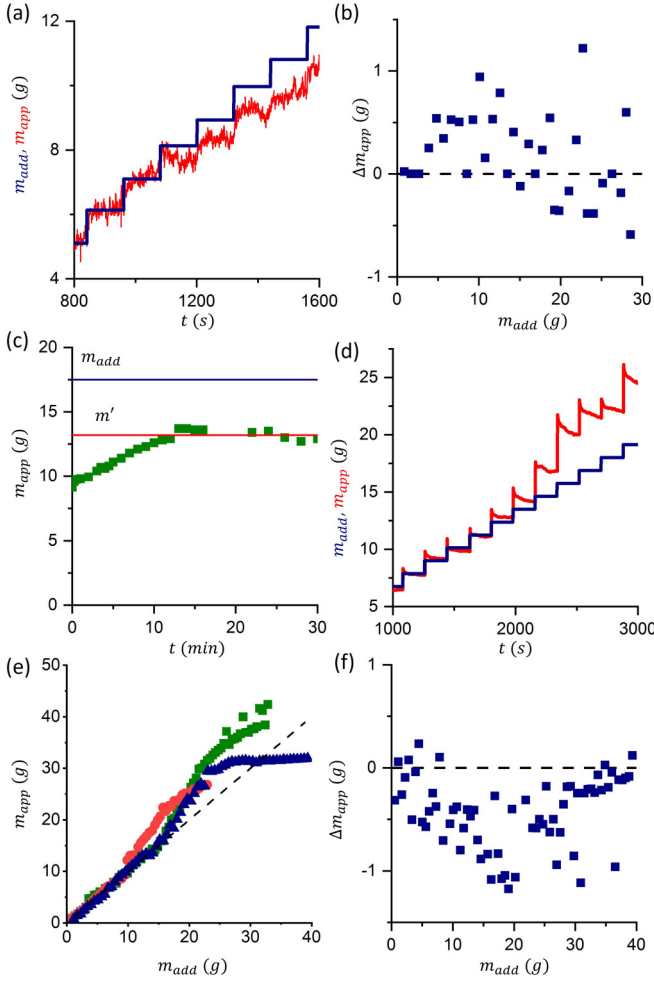
FIG. 3. (a) Added (stepping line) and apparent (fluctuating line) masses of the ants during a portion of a representative experiment in the region where departure from fluid-like behavior begins. (b) The change in the apparent mass over the first 60 s after a new addition of mass. (c) Time dependence of the apparent mass in an experiment in which 17.5 g of ants were added at once at $t = 0$. The horizontal line labeled m' shows the apparent mass measured in the incremental experiment for the same m_{add}. The upper horizontal line corresponds to m_{add}. (d) Added (stepping line) and apparent (fluctuating line) masses of passive spherical grains in a representative experiment. (e) m_{app} vs m_{add} for the grains in various experimental realizations. Note the strong overshoot in m_{app}. The dashed line represents fluid behavior. (f) Change in the apparent mass of the passive spheres over the first 60 s after a new addition of mass.

[11]. It may also be that the interconnections between the ants formed by their limbs serve to absorb enough of the impact that secondary rearrangements are suppressed. Whatever the cause, fire-ant columns feature supportive force chains, but lack the compressive force chains that would result in a strong $m_{app} > m_{add}$ regime, for some intermediate range of m_{add} if the particles in the column were not active.

Reflective of the active nature of the fire ants, we find that our measurements of m_{app} are time dependent, as demonstrated in Fig. 3(a), which shows m_{add} (stepping line) and m_{app} (fluctuating line) as a function of time in a representative experiment near the moments when the behavior of the fire-ant

column begins to depart from fluid-like behavior. Notice that, after the addition of an increment, the apparent mass often increases slightly over time before the next increment is added to the cylinder. This behavior is easily captured by measuring the change in m_{app} over the 60 s immediately following an increment, Δm_{app}. Though the fluctuations in m_{app} make this measurement noisy, we find that most often $\Delta m_{app} > 0$, as shown for a representative experiment in Fig. 3(b). The internal rearrangements that occur within the column of ants after the addition of each increment consistently lead to larger stresses at the bottom of the column. We find similar behavior if instead of using increments of added mass, we add a single large mass of ants to the column. For example, Fig. 3(c) shows the time dependence of m_{app} when $m_{add} = 17.5$ g is added to the column all at once at $t = 0$. Over the course of the first 15 min, m_{app} increases by about 30% and then plateaus at a value consistent with the apparent mass measured in the incremental experiments. For reference, the horizontal line labeled m' in Fig. 3(c) shows the apparent mass 110 s after a $\sim 1g$ increment of ants was added to bring m_{add} to 17.5 g in an experiment with incremental addition of ants performed about 2 hours before with the same colony. Hence, the fire-ant rearrangements leading to $\Delta m_{app} > 0$ appear robust and independent of the loading protocol.

To see whether this relaxation of the frictional force-chain structure is unique to active columns, we perform experiments with columns of passive grains contained inside the cylinder used for the fire-ant experiments. Instead of ants, we use plastic beads with a diameter of 6 mm, which is comparable to l, and determine the time dependence of the apparent mass in incremental experiments in which the grains are added at 2 min intervals [5]; see Fig. 3(d). Notice that in these passive experiments, $m_{app} > m_{add}$ when m_{app} first departs from fluid-like behavior, as shown in Fig. 3(e), which is consistent with the overshoot previously observed for passive grains in narrow cylinders. However, each time an increment of beads is added to the cylinder, the apparent mass sharply increases before slowly decreasing, in contrast to the behavior of the fire-ant columns. Clearly, $\Delta m_{app} < 0$ for passive grains, as shown in Fig. 3(f).

The apparent mass relaxation in passive granular columns must result from small rearrangements of the grains within the column that serve to strengthen the force chain structure and allow more of the mass of the column to be supported by the cylinder walls via friction. Notice that these rearrangements are also present for values of m_{add} that correspond to saturated measurements of m_{app}, so these rearrangements do not necessarily serve to drive m_{app} towards its hydrostatic value. In contrast, the behavior of the ants, which must also result from rearrangements within the column, indicates that over time and up to a point, the number or the strength of the supportive forcechains decreases in time. This suggests that fire-ants inside the column yield when they are subjected to shear stress. While the ants can temporarily support shear stresses because of the links they make with each other, modeling work on ant rheology suggests that rearrangements of the connections between ants occur faster in areas under higher shear stress [18]. This would result in an initially strong force-chain network that subsequently weakens so that individual ants support less shear stress. Our results suggest that this only happens up to
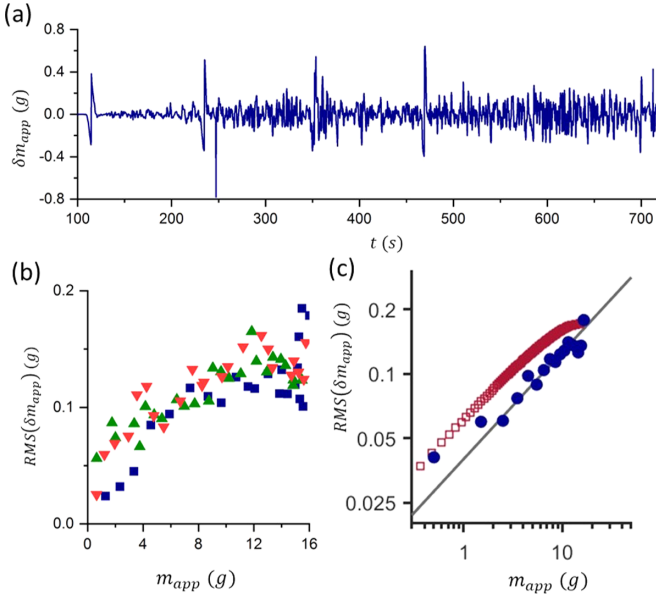
FIG. 4. (a) Fluctuations in the apparent mass as a function of time in a representative experiment. (b) Root-mean-square (RMS) fluctuations as a function of apparent mass for three experiments using ants from the same colony. (c) Binned average of the experimental RMS fluctuations of m_{app} (closed circles) and simulated RMS fluctuations of m_{app} (open squares) as a function of m_{app}. The black line corresponds to $\sim m_{app}^{1/2}$.

a point, since ultimately the observed Janssen-like saturation indicates that the force-chain structure of the ant network helps support the weight of the column.

On top of the slow timedependence of m_{app}, it is also clear from Fig. 3(a) that our measurements of m_{app} in the ant experiments features prominent fluctuations on a much faster timescale than the trend we measured with Δm_{app}. We highlight these fast fluctuations by subtracting a moving average of m_{app} from itself: $\delta m_{app} = m_{app} - \langle m_{app} \rangle_{13.3s}$, where the brackets represent the running average over a 13.3 s window, which is 50 frames at the frame rate we collect our data and corresponds to a timescale in which the autocorrelation function of m_{app} falls below 0.1. Figure 4(a) shows δm_{app} as a function of time for the first 1000 s of an experiment. At early times, additions of mass are clear as successive negative and positive spikes that are an artifact of our averaging window. Disregarding these, we find that the fluctuations grow stronger over time as more mass is added to the column, which we do at 120s timeintervals.

We quantify the strength of these fluctuations by plotting the root mean square fluctuations, $\sqrt{\langle(\delta m_{app})^2\rangle}$, as a function of m_{app}, choosing our averaging window to span from 10 s after an increment of ants is added up to 10 s before the next increment to avoid including the artificial spikes in δm_{app} mentioned above. We find that the fluctuations in apparent mass become more pronounced with increased apparent mass, as shown in Fig. 4(b), where we show results for three trials conducted using one ant colony. To improve our statistics, we bin the measurements from these three experiments by m_{app} into 2 g bins and average the results in each bin to

find that $\sqrt{\langle(\delta m_{app})^2\rangle} \propto \sqrt{m_{app}}$, as shown in Fig. 4(c). Here the solid circular points represent the binned measurements and the solid line is $\sqrt{\langle(\delta m_{app})^2\rangle} \propto \sqrt{m_{app}}$. This result is reminiscent of the law of large numbers expected for sums of random, independent, and identically distributed variables [24]. In addition, because $m_{app} \neq m_{add}$, $\sqrt{\langle \delta m_{app}^2 \rangle} \not\propto \sqrt{m_{add}}$, so the effects of rearrangements of the ants far from the base of the column are completely screened from the scale.

To understand the nature of the forces that lead to the fluctuations in m_{app}, we build a discrete 1D simulation, based on the PDE for m_{app} (Eq. 1). We calculate the apparent mass in discrete steps of height Δh, starting at the free surface, and include discrete versions of the two right-hand-side terms in Eq. 1:

$$m_{app}(i+1) = m_{app}(i) + \rho \frac{\pi D^2}{4} \Delta h$$
$$- \frac{4\mu_s k}{D} \Delta h \, m_{app}(i) + \frac{f_f'}{g} \Delta h \qquad (2)$$

To account for activity, we also include a third term, f_f'/g, that represents a random force of friction per unit length at the wall. This term is randomly drawn from a uniform distribution and represents the change in the frictional forces from the ants rearranging their connections, and thus the orientation and magnitude of the forces in the force chains that contact the wall of the column within the slice. The added mass is simply calculated as $m_{add} = \rho \frac{\pi D^2}{4} \Delta h \cdot i$. We then perform 10^5 independent simulations, and calculate $\langle m_{app} \rangle$ and $\sqrt{\langle(\delta m_{app})^2\rangle}$ for each m_{add}. We find that if the friction force F_f' is centered at 0 and $|F_f'| < \rho g \frac{\pi D^2}{4} \Delta h$, $\langle m_{app} \rangle$ lies on top of the curve predicted by Janssen's model for a given m_{sat}. However, there is no a priori reason for F_f' to be centered at 0, as the ants might bias the resultant frictional force towards a net positive or negative value of F_f'. Note that $\langle F_f' \rangle < 0$ would imply that the ants use their activity to effectively exert a downward frictional force that will allow increasing the supportive nature of the force-chain structure, while $\langle F_f' \rangle > 0$ would imply their inherent dynamics results in an increase in the apparent mass measured at the bottom of the column. Consistent with our result that $\Delta m_{app} > 0$, we find that the latter scenario most closely replicates our experimental results. Using $\mu_s \approx 0.6$, as measured using the inclined plane method [5], and the average density of the ants in the column, we find that our experiments are well fit if F_f' is a uniformly distributed random variable with $0 \leqslant F_f' \leqslant 0.64N$, and $k \approx 0.1$, in a simulation with $\Delta h = 1$ mm [5]. The results of the simulation are shown with a solid line in Fig. 2 and with the open square points in Fig. 4(c). Note that k is smaller than the typical value often obtained when fitting data for passive grains using Janssen's model; this indicates that the proportionality between vertical and horizontal stresses is weaker for ants, perhaps reflective of their inability to effectively support shear, and consistent with postive values of Δm_{app} and F_f'. Our result confirms that the force-chain structure in ant columns rearranges to increase their apparent mass on the scale. Furthermore, these rearrangements are independent from each other and well captured by a random force at the walls. Notably, if we include

the fluctuations in our simulation through a random normal force, essentially by randomizing the product $\mu_s k$, we are unable to reproduce the correct scaling of the fluctuations [5]; this confirms that the most important effect of activity is to randomize the force at the wall in the region of the column where frictional effects are most important.

We have shown that the force chains in active networks of ants can support a column against compression over long times, even as they constantly rearrange due to their inherently active nature. Interestingly, the dense ant collective relaxes each time there is mass added to the column to reduce the shear stress sustained by the ants. Eventually, however, the ant collective is supported by the force chain structure force chain structure. Activity manifests in fluctuations in the apparent mass, which are well described by attributing a random com-

ponent to the frictional forces between the ants and the walls. The random component in the frictional forces may be due to rearrangements within the bulk of the column that change the structure of the internal force chains. Taken together, our results highlight the interplay between the granular and active character of the ants, a fact likely at play in ant towers that are formed by the ants to create shelter from the elements in the absence of a nest [15].

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