Electrically driven torsional distortions in twisted nematic films

Izabela Šliwa*
Department of Operations Research and Mathematical Economics, Poznan University of Economics and Business, Al. Niepodleglosci 10, 61-875 Poznan, Poland

Pavel V. Maslennikov†
Institute of Living Systems, Immanuel Kant Baltic Federal University, Str. Universitetskaya 2, Kaliningrad 236040, Russia

Alex V. Zakharov‡
Saint Petersburg Institute for Machine Sciences, Russian Academy of Sciences, Saint Petersburg 199178, Russia

(Received 19 September 2022; accepted 9 November 2022; published 23 November 2022)

The purpose of this article is to describe the physical mechanism responsible for the appearance of both traveling and nontraveling distortions in a microsized homogeneously aligned nematic (HAN) film under the effect of a large electric field. Numerical studies have been carried out to describe both the traveling and nontraveling dynamic reorientation of the director's field in a thin, in a few tens of micrometers, the HAN film under the effect of a large electric field $\mathbf{E}$ ($\sim$1.0 V/μm). It is shown that in response to the electric field $\mathbf{E}$ applied parallel to the bounding surfaces, the torques acting on the director $\hat{\mathbf{n}}$ may excite the traveling distortion wave propagating normally to both boundaries, whose resemblance to a kinklike wave increases with increasing applied electric field $\mathbf{E}$. Calculations show that in the HAN film the physical mechanism that is responsible for the electric-field-induced distortion of the director field $\hat{\mathbf{n}}$ in the form of traveling wave provides a much faster relaxation regime than in the case of the nontraveling mode.

DOI: 10.1103/PhysRevE.106.054703

I. INTRODUCTION

This paper is motivated by a large number of studies of liquid crystals (LCs), via both experimental and theoretical techniques, aimed at showing that deformations of these LC materials, under the effect of a strong electric field, are characterized by several features. The field-induced effects, specifically, whether the applied voltage induces a homogeneous state or a structured one, are essential for both research and technology. Some researchers have studied the processes of excitation of traveling kinklike distortions in initially twisted nematic (TN) systems under the effect of a strong electric field [1,2]. In contrast, other researchers excluded the possibility of the formation of traveling distortions in initially homogeneous systems and considered the further evolution of these systems as with periodic domains or without structural changes [3–10].

Whatever the actual purpose, the condition for the initiation of excitation of traveling distortions under the influence of strong external fields is important information for this LC system. It is necessary to understand the conditions that define the boundary between the excitation of traveling distortions in the LC samples and the state when the exciting of this traveling distortions is impossible. Knowing this, it is possible to predict the further behavior of this LC system under the influence of a strong electric field.

Among all LC compounds, nematic LCs are currently the most popular LC material used not only in information technology [11], but also in various LC sensors and LC actuators [12]. The widely used flat-panel TN displays consist of a LC film sandwiched between two glasses of plastic surfaces on the scale of the order of micrometers across which a voltage may be applied independently to each pixel of the LC display (LCD). This applied electric field may alter the molecular configuration of the LC layer and thus alter the optical characteristics of the LCD [11].

For instance, the texture of TNs is obtained by orienting a drop of bulk LC material between two plates oriented perpendicular to each other with convenient treating. In turn, in the case of a homogeneously aligned nematic (HAN) drop placed between two plates oriented parallel to each other, the TN textures can be obtained by applying an electric field $\mathbf{E}$ directed orthogonally to a uniformly aligned LC material. It is shown [13] that there is a threshold value of the electric field $E_{\rm th}$, beyond which a distortion of the homogeneous texture of the director field $\hat{\mathbf{n}}$ occurs between the bounding surfaces. This form for threshold field $E_{\rm th}$ is based on assumptions that the director remains strongly, for instance, homogeneously, anchored at these bounding surfaces and the director $\hat{\mathbf{n}}$ is uniformly aligned across the nematic sample for $E < E_{\rm th}$. On the other hand, with an increase in the electric field $E > E_{\rm th}$, several relaxation regimes of the director field arise in the

*izabela.sliwa@ue.poznan.pl
†pashamaslennikov@mail.ru
‡Author to whom correspondence should be addressed: alexandre.zakharov@yahoo.ca

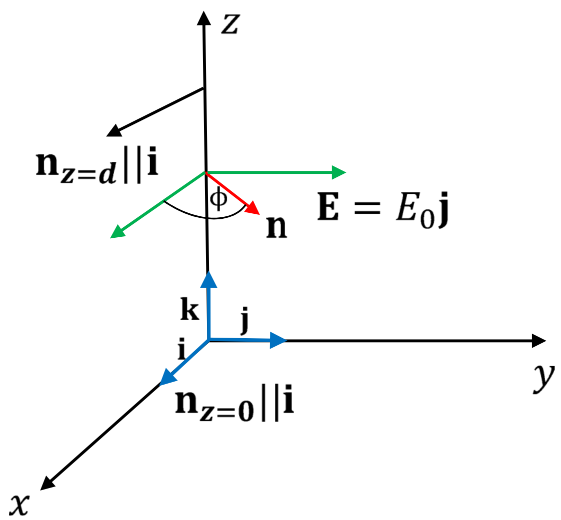

FIG. 1. Coordinate system used for theoretical analysis.

microsized volume of the TN film. These regimes, in which the director $\hat{\mathbf{n}}$ rotates in the plane parallel to both bounding surfaces, are created by torques acting on the unit volume of the LC phase.

The application of an electric field $\mathbf{E}$, directed along the y axis, to the nematic microvolume, initially homogeneously aligned in the x-y plane (see Fig. 1), can lead to two types of responses. First, the structure does not change in the microvolume of the HAN drop and the further evolution of this LC system as a whole proceeds without structural changes. Second, growing structured kinklike domains can be excited in the microvolume of the HAN drop.

In this regard, a quite natural question arises: Is it possible to form large distortions, in the form of a running kinklike wave, in the microsized TN or HAN layers under the effect of a strong electric field, several times higher than the threshold field [13] $E_{\rm th}$? The answer to this question was given, for the case of the TN volume, in the framework of the classical Ericksen-Leslie approach [14,15], which implies maintaining of the balance of torques and impulses acting per unit volume of the LC phase. It was shown that the torques acting on the director may excite in the TN film the distortion traveling wave spreading along the normal to the bounding surfaces, whose resemblance to a kinklike wave increases with an increasing of the applied electric field $\mathbf{E}$ [1,2]. This value of the critical field $E_{\rm cr}$ is based on the assumption that the director $\hat{\mathbf{n}}$ remains strongly anchored, in this TN case, at the two horizontal bounding surfaces, when the director on the upper bounding surface is inclined at a right angle to the director on the lower surface being in the x-y plane (see Fig. 1).

In turn, our main goal is to investigate how the HAN film is converted to TN film under the effect of the external electric field $\mathbf{E} = E_0\hat{\mathbf{j}}$ directed along the unit vector $\hat{\mathbf{j}}$, while the director on both bounding surfaces is aligned parallel to the unit vector $\hat{\mathbf{i}}$ (see Fig. 1). It will be shown that in this case, the torques acting on the director $\hat{\mathbf{n}}$ can excite the distortion of the HAN volume as a whole over the entire region or can excite the traveling distortion wave spreading normally to both boundaries. Thus, this mode of distortion of the director field $\hat{\mathbf{n}}$ under the effect of the strong electric field will be investigated within the framework of a hydrodynamic model that takes into account the balance of forces, torques, and linear momenta acting per unit volume of the HAN system.

The layout of this article is as follows. In the next section we will give the theoretical background for describing the physical mechanism responsible for the field-induced distortions of the director field in microsized HAN volumes. The numerical description of possible distortions of the director field in thin HAN films under the effect of the large electric field directed parallel to the bounding surfaces is given in Sec. III. A summary and our conclusions are given in Sec. IV.

II. BASIC HYDRODYNAMIC EQUATIONS AND THEIR SOLUTION

We are primarily interested in describing the physical mechanism responsible for the excitation of torsional distortions in the microsized HAN volume under the effect of a large electric field directed parallel to the bounding horizontal surfaces. We will consider the response of the nematic volume composed of cyanobiphenyl molecules without impurities confined in the microsized HAN volume delimited by two horizontal and two vertical surfaces at mutual distances of L and d ($L \gg d$) on a scale of the order of tens of micrometers. This problem will be treated within the framework of the Ericksen-Leslie theory [14,15], supplemented by the charge balance equation, and the geometry of this nematic system can be considered as two dimensional, since the director $\hat{\mathbf{n}} = (n_x, n_y, 0) = \cos\Phi\hat{\mathbf{i}} + \sin\Phi\hat{\mathbf{j}}$ belongs to the XY plane.

In the case of the microsized HAN system (see Fig. 1), the direction of director $\hat{\mathbf{n}}$ on both bounding surfaces is aligned parallel to the unit vector $\hat{\mathbf{i}}$ ($\hat{\mathbf{n}}_{z=0} \parallel \hat{\mathbf{i}}$ and $\hat{\mathbf{n}}_{z=d} \parallel \hat{\mathbf{i}}$), while the electric field $\mathbf{E} = E_0\hat{\mathbf{j}}$ is directed parallel to the unit vector $\hat{\mathbf{j}}$. Here d is the thickness of the nematic volume and z is the distance count away from the lower bounding surface. Thus, we consider the response of the HAN microvolume under the effect of the externally applied electric field $\mathbf{E} = E_0\hat{\mathbf{j}}$ directed parallel to the y axis when the initial orientation of the director $\hat{\mathbf{n}}(t = 0)$ is disturbed orthogonally to $\mathbf{E}$ and then relaxes to its stationary orientation $\hat{\mathbf{n}}_{\rm st}$ parallel to $\mathbf{E}$.

Our previous analysis of the influence of the large electric field on the reorientation of the director's field, which correctly describes the dynamic distortion of $\hat{\mathbf{n}}$, is based on the fact that the influence of the flow $\mathbf{v}$ can be neglected [1,2,9,10]. The dynamics of the director's distortion in the microsized HAN volume sandwiched between two parallel bounding surfaces and under the effect of the electric field $\mathbf{E}$ can be derived from the balance of elastic, electric, and viscous torques and linear momentum acting on the unit LC volume. Further assuming that the microsized HAN volume is limited by long horizontal surfaces significantly exceeding their thickness ($L \gg d$) allows us to consider all physical quantities as depending only on the coordinate z and time t. In this case the torque balance equation [1,2] $\mathbf{T}_{\rm el} + \mathbf{T}_{\rm elast} + \mathbf{T}_{\rm vis} = 0$ is composed of electric $\mathbf{T}_{\rm el} = \epsilon_0\epsilon_a\hat{\mathbf{n}} \times \mathbf{E}(\hat{\mathbf{n}} \cdot \mathbf{E})$, elastic $\mathbf{T}_{\rm elast} = \hat{\mathbf{n}} \times \mathbf{h}$, and viscous $\mathbf{T}_{\rm vis} = -\gamma_1\hat{\mathbf{n}} \times \frac{\partial\hat{\mathbf{n}}}{\partial t}$ torques. Here $\mathbf{h} = -K_2[\mathcal{A}\nabla \times \hat{\mathbf{n}} + \nabla \times (\mathcal{A}\hat{\mathbf{n}})]$ is the twist component of the molecular field, $\mathcal{A} = \hat{\mathbf{n}} \cdot \nabla\hat{\mathbf{n}}$, γ_1 is the rotational viscosity coefficient, $\epsilon_a = \epsilon_\parallel - \epsilon_\perp$, $\epsilon_\parallel$ and $\epsilon_\perp$ are the dielectric

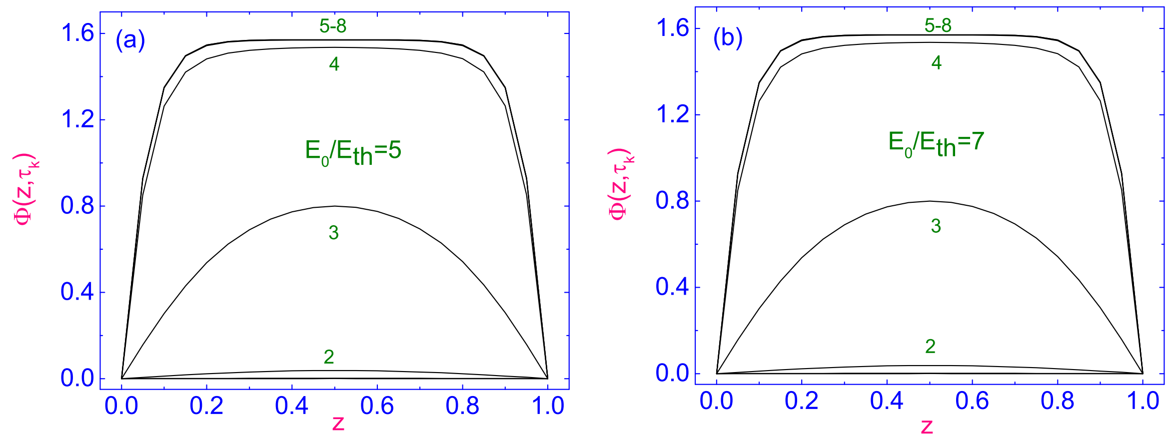

FIG. 2. (a) Evolution of the azimuthal angle $\Phi(z, \tau_k)$ ($k = 1, \dots, 8$) vs the dimensionless distance z counted from the lower bounding surface to its stationary distribution across the microsized HAN film under the effect of the electric field $E_0 = 5.0E_{\rm th}$. (b) Same as in (a) but with the value of $E_0 = 7.0E_{\rm th}$. In both these cases the boundary conditions is given by Eq. (2) (case A) and the first eight curves are plotted as solid lines. The neighboring curves are separated by the time intervals (a) $\Delta\tau = 0.01$ and (b) $\Delta\tau = 0.0063$.

constants parallel and perpendicular to the director $\hat{\mathbf{n}}$, respectively, and ϵ_0 is the dielectric permittivity of free space.

In the case of the microsized HAN geometry and in the absence of flow, the dimensionless torque balance equation takes the form [1,2]

$$\Phi_{,\tau}(z,\tau) = \Phi_{,zz}(z,\tau) + \tfrac{1}{2}\sin 2\Phi(z,\tau), \tag{1}$$

where $\Phi_{,\tau}(z,\tau) = \frac{\partial \Phi(z,\tau)}{\partial \tau}$ is the derivative of the azimuthal angle Φ with respect to the dimensionless time $\tau = \frac{\epsilon_0 \epsilon_a E_0^2}{\gamma_1} t$ and $\Phi_{,zz}(z,\tau) = \frac{\partial^2 \Phi(z,\tau)}{\partial z^2}$ is the second derivative of the angle Φ with respect to the dimensionless space variable z (i.e., scaled by the film thickness d). In the case of the microsized HAN geometry (see Fig. 1) and strong anchoring, the torque balance equation transmitted to the bounding surfaces assumes that the azimuthal angle Φ has to satisfy the boundary condition (case A)

$$\Phi(z)_{z=0} = 0, \quad \Phi(z)_{z=1} = 0. \tag{2}$$

In turn, in the case of the HAN geometry (see Fig. 1) and weak $W_{\rm an} = \frac{1}{2} A \sin^2(\Phi_s - \Phi_0)$ anchoring [16], the torque balance equation transmitted to the bounding surfaces assumes that the azimuthal angle Φ has to satisfy the boundary condition (case B)

$$\left(\frac{\partial \Phi(z)}{\partial z}\right)_{z=0,1} = \mathcal{W}\Delta\Phi, \tag{3}$$

where $\mathcal{W} = \frac{Ad}{K_2}$ is the dimensionless anchoring strength, $\Delta\Phi = \Phi_s - \Phi_0$, and Φ_s and Φ_0 are the azimuthal angles corresponding to the director orientation on the bounding surface and easy axis $\hat{\mathbf{e}}$, respectively. In this case the elastic torque $T_{\rm elast} = \frac{K_2}{d}(\frac{\partial \Phi(z)}{\partial z})_{z=0,1}$ tends to align $\hat{\mathbf{n}}_s$ along $\mathbf{E}$ and the opposed anchoring torque $T_{\rm an} = -\frac{\partial W_{\rm an}}{\partial \Phi_s}$ rotates $\hat{\mathbf{n}}_s$ toward $\hat{\mathbf{e}}$.

Now the distortion of the director field in the microsized HAN film under the externally applied electric field $\mathbf{E}$ from the initial state to its stationary orientation, being initially disturbed orthogonally to $\mathbf{E}$ with $\Phi(z, \tau = 0) = 0$, can be investigated by a standard numerical relaxation method [17], with the boundary conditions in the form of cases A and B.

III. NUMERICAL AND ANALYTICAL RESULTS FOR HOMOGENEOUSLY ALIGNED NEMATIC FILM

Our main goal is to investigate how the HAN film is converted to TN film under the effect of the external electric field $\mathbf{E} = E_0\hat{\mathbf{j}}$ directed along the unit vector $\hat{\mathbf{j}}$, while the director on both bounding surfaces is aligned parallel to the unit vector $\hat{\mathbf{i}}$ (see Fig. 1). Numerical and analytical studies will be carried out to describe both the nontraveling and traveling dynamic reorientations of the director's field, from the initial state to its stationary orientation, in the HAN film under the effect of $\mathbf{E}$ suddenly applied parallel to both boundaries. For this purpose, we will consider a sample of 5-cyanobiphenyl LCs, at the temperature 300 K and density 10^3 kg/m^3, confined between two bounding surfaces at a distance of $d = 10$ μm. The measured value of the elastic constant K_2 of this compound is equal to 5.4 pN [18], the calculated value of the dielectric anisotropy is equal to $\epsilon_a = 11.5$ [19], and the experimental value of the rotational viscosity coefficient γ_1 is equal to 0.136 Pa s [20].

A. Numerical results for nontraveling distortions in the HAN film

In this case, the external electric field E_0 varied from $E_0 = 3.0E_{\rm th}$ ($\sim 2.3\times 10^{-5}$ C/m^2) to $E_0 = 7.0E_{\rm th}$ ($\sim 5.25\times 10^{-5}$ C/m^2) is applied parallel to the j axis, for cases A and B, respectively, and the initial condition $\Phi(z, \tau = 0) = 0$ is chosen in such a way that the director $\hat{\mathbf{n}}(z, \tau = 0)$ is parallel to the i axis. In the calculations, the standard numerical relaxation method [17], with the boundary conditions in the form of cases A and B, is used. The criterion $\epsilon = |[\Phi_{(m+1)}(z) - \Phi_{(m)}(z)]/\Phi_{(m)}(z)|$ of convergence of the iterative process is chosen to be equal to 10^{-4} and the numerical procedure is then carried out until a prescribed accuracy is achieved. Here m is the iteration number.

The results of calculations of distortion of the director field in the microsized HAN film, which is described by the azimuthal angle $\Phi(z, \tau)$, when the external electric fields are equal to $E_0 = 5.0E_{\rm th}$ ($\sim 3.75\times 10^{-5}$ C/m^2) and $E_0 = 7.0E_{\rm th}$ ($\sim 5.25\times 10^{-5}$ C/m^2) and the director is strongly (case A) anchored to the bounding surfaces, are shown in Figs. 2(a) and

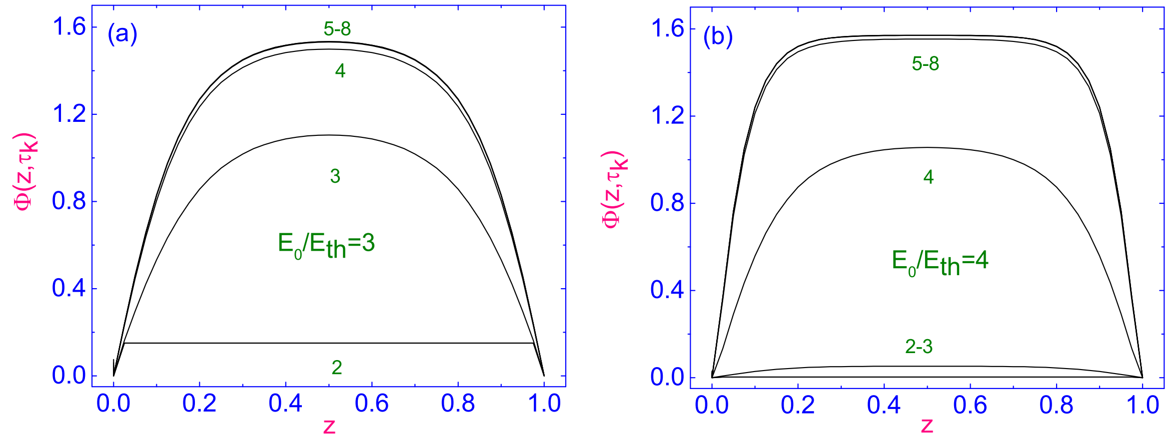

FIG. 3. (a) Evolution of the azimuthal angle $\Phi(z, \tau_k)$ $(k = 1, \dots, 8)$ vs the dimensionless distance z counted from the lower bounding surface to its stationary distribution across the microsized HAN film under the effect of the electric field $E_0 = 3.0E_{\rm th}$, while the director is weakly (case B) anchored to the bounding surfaces with the values of $\frac{Ad}{K_2}\Delta\Phi$ equal to 0.1. (b) Same as in (a) but with the value of $E_0 = 4.0E_{\rm th}$. In both these cases the first eight curves are plotted as solid lines and neighboring curves, separated by the time intervals (a) $\Delta\tau = 0.0094$ and (b) $\Delta\tau = 0.0075$.

2(b), respectively. Calculations show that the initial perturbation of the director $\hat{\mathbf{n}}(\tau = 0)$ under the effect of the electric field $E_0 = 5.0E_{\rm th}$, in the case of the HAN geometry (see Fig. 1) and strong anchoring (case A), reaches the stationary orientation, described by the azimuthal angle $\Phi_{\rm st}(z, \tau_R = \tau_8)$, after the relaxation time $\tau_R = \tau_8 = 0.063$, while in the case of $E_0 = 7.0E_{\rm th}$, after the relaxation time $\tau_R = \tau_8 = 0.049$. In both these cases the boundary condition is given by Eq. (2) (case A) and the first eight curves are plotted as solid lines. The neighboring curves are separated by the time intervals $\Delta\tau = 0.01$ [Fig. 2(a)] $\Delta\tau = 0.0063$ [Fig. 2(b)]. These calculations show that the reorientation process of the director field in the microsized HAN volume proceeds as a uniform reorientation of a simple nematic monodomain.

Now let us consider the same relaxation process in the microsized HAN volume, under the effect of the externally applied electric field, when the director $\hat{\mathbf{n}}$ is weakly anchored to both bounding surfaces as described by Eq. (3) (case B). For the homogeneously aligned LCs at an indium tin oxide surface, the experimental data for A are varied between 10^{-4} and 10^{-6} J/m^2, and thus the combination of $\frac{Ad}{K_2}$ values varies between 2 and 200. In the case of the small $\Delta\Phi$, for instance, $\Delta\Phi \in [0.01, 0.1]$, the value of $\frac{Ad}{K_2}\Delta\Phi$ varies between 0.02 and 20.0.

The results of calculations of distortion of the director field in the microsized HAN film, which is described by the azimuthal angle $\Phi(z, \tau)$, when the external electric fields are equal to $E_0 = 3.0E_{\rm th}$ ($\sim 3.8\times10^{-5}$ C/m^2) and $E_0 = 4.0E_{\rm th}$ ($\sim 6.1\times10^{-5}$ C/m^2) and the director is weakly (case B) anchored to the bounding surfaces with the value of $\frac{Ad}{K_2}\Delta\Phi$ equal to 0.1, are shown in Figs. 3(a) and 3(b) respectively. With a further increase in the magnitude of the electric field E_0 from $E_0 = 4.0E_{\rm th}$ to $E_0 = 7.0E_{\rm th}$, the relaxation process of the director field $\hat{\mathbf{n}}$, in the case of the weak (case B) anchoring condition, accelerates (see Fig. 4) from $\tau_R(E_0 = 4.0E_{\rm th}) \sim 0.06$ to $\tau_R(E_0 = 7.0E_{\rm th}) \sim 0.05$. In the last case $\tau_R(E_0 = 7.0E_{\rm th}) \sim 0.05$, the first eight curves are plotted as solid lines and neighboring curves are separated by the time interval $\Delta\tau \sim 0.0063$. Calculations also show that the effect of the external electric field $E_0/E_{\rm th}$ on the relaxation times $\tau_R(\omega)$ (ω = A and B) of the director field $\hat{\mathbf{n}}$ to its stationary orientation $\hat{\mathbf{n}}_{\rm st}$ in the microsized HAN film, corresponding to cases A and B, decreases with an increase in the magnitude of the electric field (see Table I).

B. Numerical and analytical results for traveling distortion in the microsized HAN film

Thus, our goal now is to describe the physical mechanism responsible for the appearance of the traveling distortion wave in the microsized HAN film under the effect of the electric field $\mathbf{E} = E_0\hat{\mathbf{j}}$. In this case, we assume that the distortion in the form of the running wave front will propagate across the nematic film at the speed of $v(z, t)$.

In order to be able to observe the formation of the traveling distortion moving at a velocity v in the microsized HAN

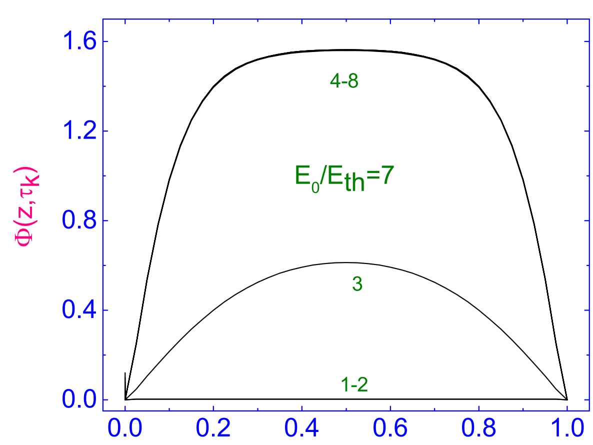

FIG. 4. Same as in Fig. 3 but under the effect of the electric field $E_0 = 7.0E_{\rm th}$. In this case the first eight curves are plotted as solid lines and neighboring curves are separated by the time interval $\Delta\tau = 0.0063$.

TABLE I. Calculated dimensionless $\tau_R(\omega)$ and dimension $t_R(\omega)$ values of relaxation times, where ω = A and B, corresponding to cases A and B, respectively.

	Case A		Case B	
E_0/E_{th}	τ_R(A)	t_R(A)$\times 10^4$ (s)	τ_R(B)	t_R(B)$\times 10^4$ (s)
3.0			0.075	5.7
4.0			0.06	4.6
5.0	0.063	4.8		
7.0	0.049	3.7	0.05	3.8

volume under the effect of the externally applied electric field **E**, consider the dimensionless analog of Eq. (1) in the form [1,2]

$$\Phi_{,\tau}(z,\tau) = \Phi_{,zz}(z,\tau) + \tfrac{1}{2}\sin 2\Phi(z,\tau)$$
$$= \Phi_{,zz}(z,\tau) + f(\Phi), \quad (4)$$

where $\tau = \frac{\epsilon_0\epsilon_a E_0^2}{\gamma_1} t$ is the dimensionless time and z is the dimensionless space variable scaled by the correlation length $\kappa = \frac{dE_{\text{th}}}{\pi E_0}$ of the electric field E_0. Equation (4) is a nonlinear diffusion equation, of the type investigated in the classical work by Kolmogorov *et al.* [21]. They predicted the qualitative behavior of $\Phi(z,\tau)$ and showed that an initial disturbance bounded between stable $[\Phi(z=z_3)=\frac{\pi}{2}]$ and unstable $[\Phi(z=z_{1,2})=0]$ states can be described by the traveling wave $\Phi(z-v\tau)$ propagating at speed v. It was also shown that the velocity v is satisfied to

$$\lim_{\tau\to\infty} 2\sqrt{\left(\frac{df(\Phi)}{d\Phi}\right)_{\Phi=0}} \leqslant v \leqslant \lim_{\tau\to\infty} 2\sqrt{\sup_{\Phi\in[0,1]}\left(\frac{f(\Phi)}{\Phi}\right)} \quad (5)$$

and both limits are equal to 2, giving the result for v, and the traveling wave $\Phi(z-v\tau)$ is defined in an infinitely large interval $(-\infty,+\infty)$. However, in our case, Eq. (4) is defined in the limiting interval [0,1] and the traveling front $\Phi(z-v\tau)$ starts to move away from one unstable $\Phi(\xi=\xi_1)=0$ [or $\Phi(\xi=\xi_2)=0$] state to the stable state $\Phi(\xi=\xi_3)=\frac{\pi}{2}$, where $\xi_3=(\xi_1+\xi_2)/2$.

Thus, we will investigate the evolution of the traveling distortion in time with the dimensionless velocity v and consider the dimensionless analog of Eq. (4) in the coordinate system $\xi = z/\kappa - v\tau$. Now Eq. (4) takes the form

$$\Phi_{,\tau}(\xi) = v\Phi_{,\xi}(\xi) + \Phi_{,\xi\xi}(\xi) + \tfrac{1}{2}\sin 2\Phi(\xi). \quad (6)$$

In the following the strong anchoring of the director to both bounding surfaces will be considered

$$\Phi_{\xi=\xi_1} = 0, \quad \Phi_{\xi=\xi_2} = 0, \quad (7)$$

where $\xi_1 = -v\tau$ and $\xi_2 = \frac{\pi E_0}{E_{th}} - v\tau$ are the positions for lower and upper boundaries, respectively.

Now the evolution of distortion of the director field in the microsized HAN film under the externally applied electric field **E**, from the initial state to its stationary orientation, being initially disturbed as

$$\Phi(\xi,\tau=0) = 0, \quad (8)$$

can be investigated by a standard numerical relaxation method [9] with strong boundary conditions. In this case, the external electric field with $E_0 = 6.0E_{\text{th}}$ ($\sim 3.9\times10^{-5}$ C/m^2) and $E_0 = 7.0E_{\text{th}}$ ($\sim 4.6\times10^{-5}$ C/m^2) (same as in the nontraveling case) is applied parallel to the y axis with the strong boundary and initial conditions.

In the calculations, the standard numerical relaxation method [17], with the boundary conditions in the form of case A, is used. The criterion $\epsilon = |[\Phi_{(m+1)}(z) - \Phi_{(m)}(z)]/\Phi_{(m)}(z)|$ of convergence of the iterative process is chosen to be equal to 10^{-4} and the numerical procedure is then carried out until a prescribed accuracy is achieved. Calculations show [see Figs. 5(a) and 5(b)] that in a HAN film under the effect of the externally applied electric fields $E_0 = 6.0E_{\text{th}}$ and $7.0E_{\text{th}}$, two traveling distortion waves running in different directions along the z axis to both bounding surfaces can be formed. The center of formation of these waves is located in the vicinity of the center of the HAN film (near the point $\xi_3 = 0.0$), while the bounding surfaces are located at the points $\xi_{1,2} = \pm1.3$, respectively. The relaxation times $\tau_R(E_0/E_{\text{th}})$ for these separated processes are equal to 0.0342 ($E_0/E_{\text{th}} = 6$), or approximately 250 μs, and 0.0375 ($E_0/E_{\text{th}} = 7$), or approximately 290 μs, respectively. The process of reorientation of

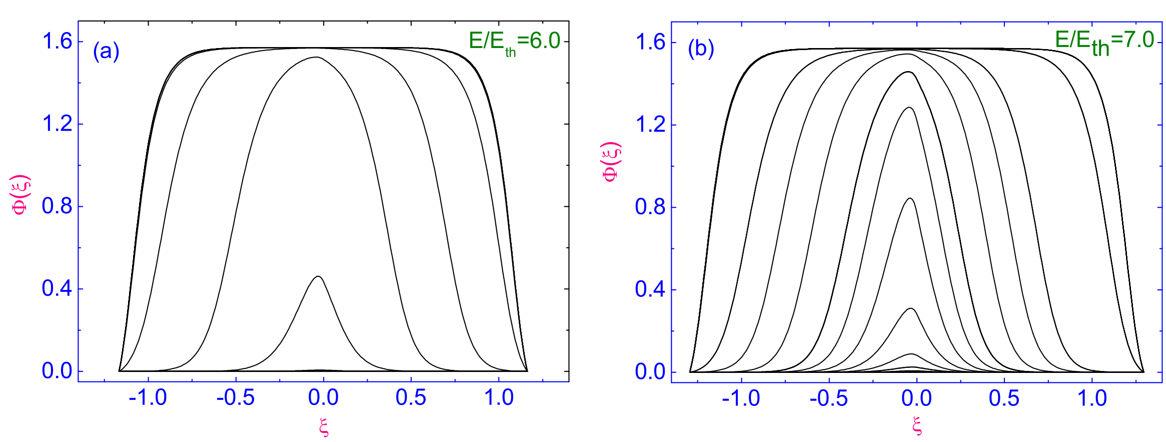

FIG. 5. Formation of two traveling distortion waves $\Phi(\xi)$ in the HAN film running in opposite directions along the z axis to both bounding surfaces under the effect of electric field (a) $E_0/E_{\text{th}} = 6.0$ and (b) $E_0/E_{\text{th}} = 7.0$. The first 12 curves are plotted as solid lines and neighboring curves are separated by the time intervals (a) $\Delta\tau = 0.0029$ and (b) $\Delta\tau = 0.0031$.

the director's field in this case begins with the fact that an area of initial reorientation is formed in the center of the HAN film, which is rapidly growing. In the case of $E_0 = 6.0E_{\text{th}}$, a small LC domain with $\hat{\mathbf{n}} \parallel \mathbf{E}$ is reached after the time interval $\tau_4 = 0.012$, while in the case of $E_0 = 7.0E_{\text{th}}$, after the time interval $\tau_7 = 0.022$, it is almost twice slower. Then, in both these cases, waves of reorientation of the director's field, running in opposite directions, begin to be excited. Finally, after the time $\tau_R(E/E_{\text{th}})$, the entire volume of the nematic LC is oriented along the direction of the electric field $\mathbf{E}$. Thus, the physical mechanism that is responsible for the electric-field-induced distortion of the director field $\hat{\mathbf{n}}$ in the form of traveling wave provides, in the case of the HAN geometry, a much faster relaxation regime than in the nontraveling mode.

IV. CONCLUSION

This paper has described some numerical advances in predicting the structural and dynamic behavior of the director field $\hat{\mathbf{n}}$ in a microsized homogeneously aligned nematic volume subjected to a strong electric field $\mathbf{E}$ applied parallel to the horizontal bounding surfaces. an electrically driven mechanism formation of both the traveling and nontraveling distortions modes in the HAN films has been proposed. Calculations showed that the physical mechanism responsible for the electric-field-induced distortion of the director field $\hat{\mathbf{n}}$ in the form of the traveling wave provides much faster relaxation than in the nontraveling mode, when the reorientation process of the director field in the microsized HAN volume proceeds as a uniform reorientation of a simple nematic monodomain. Indeed, in the case of traveling distortion mode, for $E/E_{\text{th}} = 7$, the dimensionless value of the relaxation time $\tau_R(E/E_{\text{th}})$ is equal to 0.0375, or approximately 290 μs, while in the case of the nontraveling mode the dimensionless value of the relaxation time $\tau_R(E/E_{\text{th}})$ is equal to 0.05, or approximately 380 μs.

Here a natural question arises: Which mechanism of torsional deformation is preferable in the case of a homogeneously aligned nematic film? The answer to this question can only be given by experiment. As possible options, we can offer two experiments.

First, it should be noted that the reorientation of the director field $\hat{\mathbf{n}}$ in the form of the traveling distortion wave spreading along the normal to both boundaries probably can be observed in polarized white light. Taking into account that the director reorientation takes place in the narrow area of the LC sample, upon applying the large electric field $\mathbf{E}$ ($\sim$1 V/μm) directed parallel to the bounding surfaces, the traveling distortion wave can be visualized in polarized white light as a dark strip running along the normal to both LC boundaries, with the velocity $v \sim 1.0$ μm/μs.

If this is the case, then we can measure the relaxation time of the director field corresponding to the traveling distortion wave and compare it with the time needed to reorient the director field as a monodomain. In this case, the nuclear magnetic resonance technique [7] can be used to measure the relaxation time.

Second, another possibility of experimental detection of traveling distortion waves of the director field in TN films under the effect of the electric field has been proposed [2]. It was found that the torques acting on the director $\hat{\mathbf{n}}$ may excite the pressure traveling wave spreading along the normal to both boundaries, whose resemblance to a kinklike wave increases with the increasing value of the applied electric field greater than some critical value E_{cr}. If so, then in the case of the HAN film, additional pressure will also be applied to bounding surfaces for a very short time. Thus, by fixing this additional pressure, it is possible to measure, using the appropriate setting, the relaxation time corresponding to the evolution of the disturbance state into a stationary orientation in the form of a kinklike (traveling) wave. Therefore, the further study of a wider range of problems related to understanding how elastic soft matter, such as liquid crystals confined in a microsized volume, begins to deform under the influence of strong electric field requires additional effort, which will eventually lead to an increase in our knowledge in the field of materials science.

ACKNOWLEDGMENT

This study was funded by RFBR (Russia) and DFG (Germany), Project No. 20-52-12040, and by the Russian Science Foundation, Project No. 22-22-00736.

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