Letter

Absorbing phase transitions in systems with mediated interactions

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Experiments of periodically sheared colloidal suspensions or soft amorphous solids display a transition from reversible to irreversible particle motion that, when analyzed stroboscopically in time, is interpreted as an absorbing phase transition with infinitely many absorbing states. In these systems, interactions mediated by hydrodynamics or elasticity are present, causing passive regions to be affected by nearby active ones. We show that mediated interactions induce a universality class of absorbing phase transitions distinct from conserved directed percolation, and we obtain the corresponding critical exponents. We do so with large-scale numerical simulations of a minimal model for the stroboscopic dynamics of sheared soft materials and we derive the minimal field theoretical description.

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Soft materials under cyclic shear often show an intriguing phase transition in their microscopic dynamics, termed reversible-irreversible transition (RIT). At low enough driving amplitudes, the system reaches a reversible state where its configuration is strictly unchanged when observed stroboscopically (once per cycle), whereas at large amplitudes the stroboscopic dynamics is diffusive. This transition is observed in systems varying considerably in their microscopics, including non-Brownian suspensions [1,2], granular materials [3], microemulsions [4], and soft glasses [5]. While for dilute suspensions reversibility is borne from time-reversible Stokes hydrodynamics [6], for jammed systems it is argued to come from a repeated sequence of plastic events [7,8].

The RIT is a form of absorbing phase transition (APT). The order parameter is the activity A, which measures the fraction of the system which evolves irreversibly during a cycle; reversible states correspond to $A = 0$. APTs arise in many nonequilibrium contexts, among them the spreading of infectious diseases, reaction-diffusion problems, and fracture propagation [9]. In the case of RIT, there exist infinitely many absorbing states not related by any symmetry (e.g., all configurations leading to contact-free cycles for dilute suspensions), and the particle number N is conserved. It has therefore been argued [6,10,11] to belong to the conserved directed percolation (CDP) or Manna class [9,12–15]. The CDP field theory [10,13–16] involves the local density $\rho(\boldsymbol{r}, t)$ (with $\int d\boldsymbol{r}\rho(\boldsymbol{r}, t) = N = V\rho_0$) and activity $A(\boldsymbol{r}, t)$, with dynamics

$$\partial_t \rho = D_\rho \nabla^2 A, \tag{1}$$

$$\partial_t A = f(A) + D_A \nabla^2 A + \sigma\sqrt{A}\eta\,, \tag{2}$$

where $f(A) = f_{\mathrm{CDP}}(A) \equiv (-\alpha + k\rho)A - \lambda A^2$ and $\eta(\boldsymbol{r}, t)$ is a Gaussian noise with $\langle\eta(\boldsymbol{r}, t)\rangle = 0$ and $\langle\eta(\boldsymbol{r}, t)\eta(\boldsymbol{r}', t')\rangle = \delta(\boldsymbol{r} - \boldsymbol{r}')\delta(t - t')$. The APT is at $\rho_0^c = \alpha/k$ where the mean activity vanishes as $\langle A\rangle \propto (\rho_0 - \rho_0^c)^\beta$. In mean field, $\beta_{\mathrm{CDP}}^{\mathrm{MF}} = 1$, while in two dimensions $\beta_{\mathrm{CDP}} \approx 0.64$ [17].

However, while CDP has been argued to capture some realizations of RIT [6,18], several results challenge the CDP classification of RIT. Some experiments report a convex behavior ($\beta > 1$) close to the transition, both above [19] (also in numerics [20]) and below jamming [4]. A first-order transition is reported in some semidilute systems [21], as well as in numerics on dense systems [22–26]. Some of these CDP-incompatible behaviors have been argued to stem from hydrodynamic interactions [4]. More generally, a natural expectation is that, due to hydrodynamic or elastic interactions, a region of local activity can impact nearby passive ones. This mechanism is absent from both CDP field theory and minimal models implementing it [6,11,27].

In this Letter we report a universality class for APTs with infinitely many absorbing states, distinct from CDP, which arises when passive regions are affected by active ones. We do so by introducing a generalization of the minimal model studied in Ref. [27] for the stroboscopic dynamics of periodically sheared suspensions. Mediated interactions are mimicked at a mean-field level by a diffusion of passive particles depending on the total activity. We characterize the RIT for our model in simulations and show that activity-induced diffusion of passive particles makes it either a convex second-order transition with $\beta > 1$ or even a first-order transition. Furthermore, by a coarse-graining of our minimal model we show that the CDP's normal form (2) is replaced by

$$f(A) = f_{\mathrm{CDP}}(A) + f_{\mathrm{p}}(A) = (-\alpha + \tilde{k}\rho)A - \mu\rho A^{3/2} - \lambda A^2, \tag{3}$$

where $\tilde{k}$ is a renormalized coefficient. The presence of a $A^{3/2}$ term, with $\mu > 0$, is key to the universality class we describe. Such a nonanalytic term eludes symmetry or conservation arguments; this property is shared by other nonequilibrium phase transitions [28] and is probably the reason this universality class eluded detection.

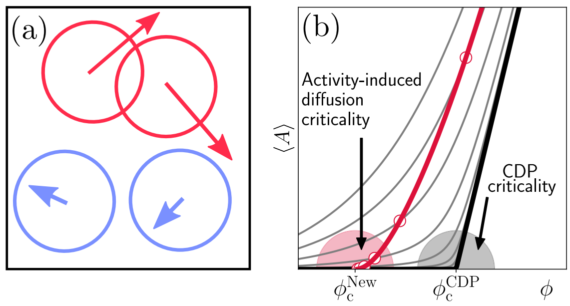

FIG. 1. (a) Time step in our model. Particles in overlap ("active," shown in red in the upper portion of the panel) are randomly moved by a typical distance Δ^a, while others ("passive,' shown in blue in the lower portion of the panel) are randomly moved by a typical distance Δ^p function of the total activity. (b) Mean-field scenario. For CDP an APT occurs at $\phi = \phi_c^{CDP}$ (black curve). Thermal diffusion rounds the APT (gray isotherms, increasing diffusion from bottom to top). If activity controls diffusion, the CDP critical point is avoided: the APT occurs at a distinct critical point [red (thick gray) curve].

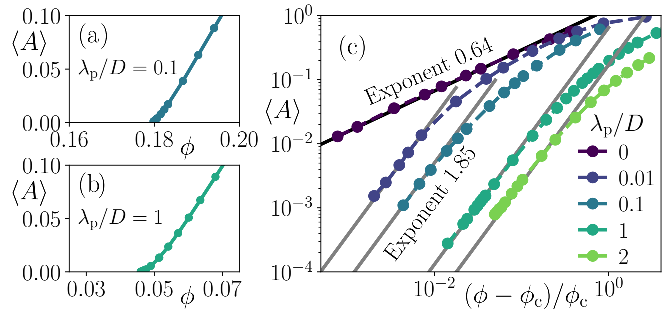

FIG. 2. Left panels: Average activity $\langle A\rangle$ versus mean area fraction ϕ for $c = 0$ and (a) $\lambda_p/D = 0.1$ or (b) $\lambda_p/D = 1$. (c) $\log\langle A\rangle$ versus $\log(\phi - \phi_c)/\phi_c$, showing the critical behavior at $c = 0$, for several values of λ_p/D.

Minimal particle model. Our approach to the modeling of the RIT in systems with mediated interactions follows a now well-accepted strategy which considers interactions in a minimal way, without attempting a detailed description of interactions present in an experimental system. This approach has led to celebrated models for RITs, most notably the random organization model [6,27,29,30], where contacts during cyclic shear of suspensions are modeled via a simple pairwise activity in a discrete-time dynamics. This approach has proven to be influential and has since been extended to take into account multiparticle collisions which necessarily arise in dense suspensions [11]. In the random organization model [6] in its isotropic form (here abbreviated as the IRO model) [27,31], a set of N disks of diameter D are distributed in space (we here consider only the two-dimensional case); those overlapping with others are called "active," and the others are called "passive." With reference to a dilute suspension, active particles are interpreted as colliding during a shear cycle. At each time step (taken as a time unit), active particles move with Gaussian-distributed random displacements with standard deviation Δ_a. In the original IRO model, passive particles are kept fixed. We instead assume that passive particles are randomly displaced over a distance that depends on the overall activity at time t, $\bar{A}(t) = N_a(t)/N$ (with N_a being the number of active particles) [Fig. 1(a)]. Indeed, in a real system a displacement $\boldsymbol{\Delta}_i^a$ of an active particle i induces a displacement $\boldsymbol{\Delta}_j^p$ on a passive particle j separated by $\boldsymbol{r}_{ij}$ via a tensorial propagator $\boldsymbol{G}(\boldsymbol{r}_{ij})$ (which may be long-ranged), such that $\boldsymbol{\Delta}_j^p = \boldsymbol{G}(\boldsymbol{r}_{ij})\cdot\boldsymbol{\Delta}_i^a$. Assuming that the total displacement of a passive particle generated by several active particles is additive, and that active displacements are uncorrelated, one obtains that the variance of passive displacements $\langle(\boldsymbol{\Delta}_i^p)^2\rangle = \Delta_a^2 K\rho_0\bar{A}$, with $K = \int d\boldsymbol{r}\,\boldsymbol{G}(\boldsymbol{r}){:}\boldsymbol{G}(\boldsymbol{r})$, assuming this integral converges. This holds even for long-ranged interactions [$G_{\alpha\beta}(\boldsymbol{r}) \propto 1/r^\mu$ at large distance r] as far as $\mu > \frac{d}{2}$, a condition satisfied by the elastic propagator ($\mu = d$) and by hydrodynamic interactions caused by force dipoles such as particle contacts. (More details are given in Ref. [32].) More generally, correlated active motion leads to corrections to the variance of order $\bar{A}^2$. We thus account for the effect of mediated interactions in a mean-field spirit by assuming Gaussian-distributed displacements of passive particles with a standard deviation $\Delta_p = s(\bar{A}) \equiv \lambda_p\sqrt{\bar{A}}[1 + c\bar{A}]$, where $\lambda_p > 0$ and $c > -1$ to ensure positivity. For $\lambda_p = 0$, our model reduces to the IRO model, which is expected to belong to the CDP universality class. To focus on the potential deviations to CDP, we here restrict to $\Delta_a/D = 1$. Dimensionless model parameters are then (i) the area fraction $\phi = \pi D^2\rho_0/4$, (ii) the ratio λ_p/D, and (iii) the coefficient c of the correction to the scaling $\Delta_p \propto \sqrt{\bar{A}}$.

At the mean-field level, adding a small thermal diffusivity $\mathcal{D} = \Delta_p^2$ close to ϕ_c^{CDP} [30] gives rise to the activity $\langle A\rangle_{CDP}(\phi, \mathcal{D})$ as sketched in Fig. 1(b). Close to an APT the activity is small, so one might expect that activity-induced diffusion amounts to adding an infinitesimal Δ_p close to the CDP critical point. However, Δ_p is not a control parameter in our model, as it is controlled by activity. Enforcing $\sqrt{\mathcal{D}} = s[\langle A\rangle_{CDP}(\phi, \mathcal{D})]$ (red open circles), one finds that $\mathcal{D}$ is finite at $\phi = \phi_c^{CDP}$ and actually vanishes at a smaller density [red (thick gray) curve]. Thus, activity-induced diffusion does not trivially lead to a thermally rounded CDP criticality, which is indeed never approached; we show below that it actually creates a distinct critical point away from the CDP one [Fig. 1(b)].

Numerical analysis. We use system sizes up to $N = 2^{24}$ to get as close as possible to the APT. We first focus on the case $c = 0$, and we briefly discuss later the effect of $c \neq 0$.

We plot in Figs. 2(a) and 2(b) the average activity in steady state $\langle A\rangle$ as a function of ϕ, for two different values of λ_p/D, for $c = 0$. We observe an APT at the value ϕ_c which decreases when increasing λ_p/D, that is, with stronger mediated interactions. Importantly, the convexity of $\langle A\rangle(\phi)$ observed in Figs. 2(a) and 2(b) indicates that $\beta > 1$, in contrast with the CDP value $\beta_{CDP} \approx 0.64$. This is confirmed on a log scale showing $\langle A\rangle$ as a function of $\varepsilon \equiv (\phi - \phi_c)/\phi_c$ in Fig. 2(c). (We show in Ref. [32] how we determined ϕ_c.) For $\lambda_p/D = 0$ (IRO model), we recover $\beta_{CDP} \approx 0.64$. For $\lambda_p/D > 0$, a crossover is observed between a regime compatible with β_{CDP} (at least for small λ_p/D, for which ϕ_c is close to ϕ_c^{CDP}) far enough from the critical point, and a new critical behavior with $\beta \approx 1.85$ in an interval close to the critical point, which

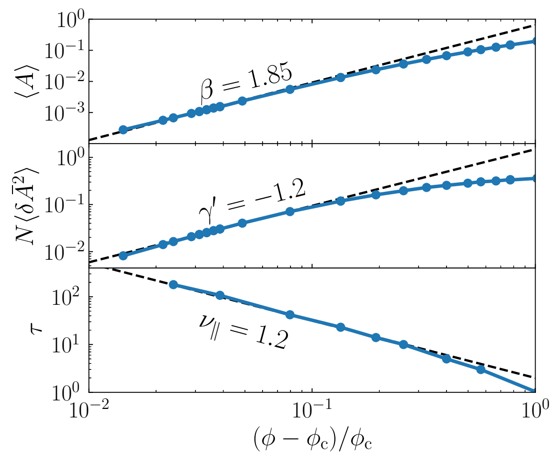

FIG. 3. The average activity $\langle A \rangle$ (top panel), the normalized variance $N\langle(A - \langle A \rangle)^2\rangle$ of activity fluctuations (middle panel), and the correlation time δt of activity fluctuations (bottom panel) versus $(\phi - \phi_c)/\phi_c$ for $c = 0$ and $\lambda_p/D = 1$.

widens upon increase of λ_p/D. It thus appears clearly that a tiny motion of passive particles mimicking the effect of mediated interactions modifies the universality class of the APT. Quite importantly, the value $\beta \approx 1.85$ is much larger than the mean-field value $\beta_{\rm CDP}^{\rm MF} = 1$, showing that, although we have included mediated interactions in a mean-field manner, the model is not following mean-field directed percolation, which rather happens when $\Delta_a/D \to \infty$ [17].

We further characterize the universality class in Fig. 3, showing the critical behavior of the variance of the activity $N\langle\delta\bar{A}^2\rangle \propto \varepsilon^{-\gamma'}$, where $\delta\bar{A} = \bar{A} - \langle A \rangle$, and of the activity correlation time $\tau \propto \varepsilon^{-\nu_\parallel}$, for $\lambda_p/D = 1$. Here we define τ as $\langle\delta\bar{A}(t)\delta\bar{A}(t+\tau)\rangle/\langle\delta\bar{A}(t)^2\rangle = 1/e$. We find $\gamma' \approx -1.2$ and $\nu_\parallel \approx 1.2$, in contrast to CDP values $\gamma'_{\rm CDP} \approx 0.37$ and $\nu_{\parallel\rm CDP} \approx 1.23$. We performed in Ref. [32] a finite-size scaling analysis which supports these values. The sign of γ' even differs from the one of CDP; in our model, activity fluctuations vanish at the transition. This is due to the large β value, as shown by the hyperscaling relation $\gamma' = d\nu_\perp - 2\beta$ [17], with d being the dimension of space. Similar behavior is observed in a variant of contact process [33] or active yielding [34].

An important feature of the CDP class is hyperuniformity, characterized by a scaling of the structure factor $S(\mathbf{q}) \propto q^\kappa$ when $q \to 0$ $(0 < \kappa < 1)$ at ϕ_c [27,29]: large-scale density fluctuations are much weaker than those for an equilibrium system at the same density. Adding a small thermal diffusion near the CDP critical point even enhances hyperuniformity [30]. We show in Ref. [32] that when increasing λ_p/D the hyperuniform regime instead shrinks and a low-q plateau develops due to activity-induced diffusion. This further highlights the non-CDP nature of the RIT in our model.

We now turn to the case $c > 0$, shown in Fig. 4. Increasing c at fixed $\lambda_p/D = 1$, the curve $\langle A \rangle(\phi)$ steepens close to the transition (however, critical exponents are unaffected [32]), eventually becoming discontinuous [Fig. 4]. Moreover, after a first decay to a pseudosteady plateau, the relaxation to an absorbing state is discontinuous in time. Finite-size effects, however, prevent us from deciding whether the transition is a genuine APT or rather a first-order transition inside the active phase in the vicinity of a continuous APT. It is, nonetheless, reminiscent of simulations of cyclically sheared particle model [22,23].

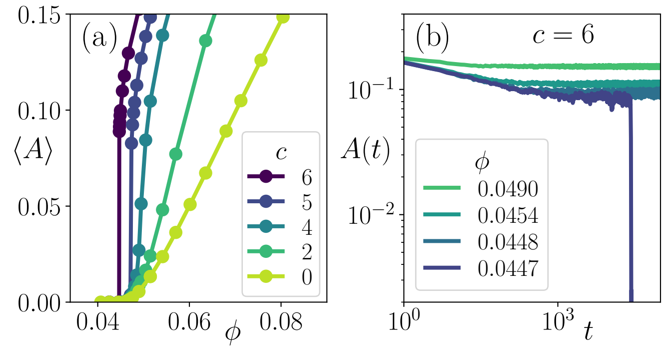

FIG. 4. (a) Average activity $\langle A \rangle$ as a function of area fraction ϕ for $\lambda_p/D = 1$ and several values of c. (b) Time series of the activity for the case $c = 6$, for several ϕ across the first-order transition.

Continuum description. To rationalize our findings, we now look for a continuum description in a local mean-field framework. The crucial ingredient is that passive particle diffusion creates activity, which adds a contribution $f_p(A)$ to the normal form in Eq. (2). Estimating the number of active particles created in a time step $(t, t + \delta t)$ from the radial distribution function of passive particles $g(r)$ leads to

$$N f_p(A)\delta t \simeq N_p\rho_p \int d^d\boldsymbol{r}\, g(r)\mathcal{P}_{\rm overlap}(\Delta_p, r), \tag{4}$$

where $r = |\boldsymbol{r}|$, $N_p = N - N_a$ and $\rho_p = N_p/N$ are the number of passive particles and their density, and $\mathcal{P}_{\rm overlap}(\Delta_p, r)$ is the probability that a couple of particles at distance r overlap in the next time step. Passing in polar coordinates and setting $r = D + \Delta_p x$, we have

$$\frac{\delta t\, f_p(A)}{2S_d\rho} = (1 - A)^2\Delta_p \int_0^2 dx[D + \Delta_p x]^{d-1} \times g(D + \Delta_p x)\mathcal{P}_O(\Delta_p, x),$$

where $\mathcal{P}_O(\Delta_p, x) = \mathcal{P}_{\rm overlap}(\Delta_p, D + \Delta_p x)$, we used $\mathcal{P}_{\rm overlap}[\Delta_p, r > 2(R + \Delta_p)] = 0$, and $S_d = d\pi^{d/2}/\Gamma(d/2 + 1)$ is the surface of the unit sphere. We show in Ref. [32] that for $d = 2$ the expansion of $\mathcal{P}_O(\Delta_p, x)$ in Δ_p is

$$\mathcal{P}_O(\Delta_p, x) = \mathcal{P}_O^{(0)}(x) - \frac{\Delta_p}{D}\mathcal{P}_O^{(1)}(x) + O(\Delta_p^2), \tag{5}$$

where $\mathcal{P}_O^{(i)}(x)$, with $i = 1$ and 2, are positive and given by model-parameter-independent integrals which can be readily evaluated numerically.

Estimating the radial distribution function $g(D + \Delta_p x)$ for small A is more subtle. To get insight, and assuming isotropy, we consider a minimal two-body description for the motion of two nearest-neighbor passive particles, p_0 and p_1. We fix p_0 at the origin and consider p_1 as a discrete-time random walker. The latter moves in an annular shape of radii D and L representing, respectively, p_0 and the second nearest particle to p_1. Whenever p_1 reaches one of the two boundaries, it is redistributed uniformly in the annulus. The pair correlation g of the original model corresponds, in this effective description, to the stationary state $P_s(r)$ of p_1. In three spatial dimensions, and for $L \to \infty$, $P_s(D + \Delta_p x) = \Delta_p Q_1(x)/(D + \Delta_p x)$, where $Q_1(x)$ is given in terms of an inverse Laplace transform [32] and is model parameter independent [35,36]. Surprisingly, the mathematical structure of the problem is very different in $d = 2$ and, to the best of our knowledge, no exact solution is available. Yet, numerical integration is straightforward and its results are reported in Ref. [32]. To a very good accuracy, $P_s(D + \Delta_p x) = \Delta_p Q(x)/(D + \Delta_p x)$ also for $d = 2$, and $Q(x)$ is independent of Δ_p and c. We thus conclude that

$$(D + \Delta_p x) g(D + \Delta_p x) \sim_{\Delta_p \to 0} \Delta_p\, g^{(0)}(x), \tag{6}$$

where, obviously, $g^{(0)}(x) > 0$ for all x. Combining Eqs. (4) and (7) we conclude that

$$f_p(A) = \alpha_p \rho A - \mu \rho A^{3/2} + O(A^2), \tag{7}$$

where $\alpha_p = (2S_d/\delta t)\lambda_p \int_0^2 dx\, g^{(0)} \mathcal{P}_O^{(0)} > 0$ and $\mu = (2S_d/\delta t)(\lambda_p^2/D) \int_0^2 dx\, g^{(0)} \mathcal{P}_O^{(1)} > 0$; the normal form in Eq. (2) should thus be

$$f(A) = (-\alpha + \tilde{k}\rho)A - \mu\rho A^{3/2} + \mathcal{O}(A^2)\,, \tag{8}$$

where $\tilde{k} = k + \alpha_p$. The leading term merely renormalizes the linear coefficient of Eq. (2) and does not change the critical properties. Mediated interactions, however, have a drastic effect, through the $A^{3/2}$ contribution. We get in mean field a continuous transition at $\rho_0^c = \alpha/\tilde{k}$ and, because $\mu > 0$, in the active phase $\langle A \rangle = (\tilde{k}/\mu\rho_0)(\rho_0 - \rho_0^c)^2$, so that $\beta_{MF} = 2$, slightly larger than the measured value $\beta \approx 1.85$.

This coarse-graining strongly supports the existence of a different universality class for APT whenever infinitely many absorbing states are present and local activity affects passive particles. By contrast, a small thermal diffusion of passive particles ($\Delta_p/D \ll 1$ independent from A) would modify the CDP normal form to $f(A) = \alpha_p' \rho \Delta_p^2 - \mu' \rho \Delta_p^3 + (-\alpha + k\rho)A - \lambda A^2 + O(A^3)$, hence, just rounding the transition. [Yet taking $\Delta_p = \lambda_p \sqrt{A}$ yields back Eq. (8).] Our analysis further provides the field-theoretical description within which the universality class described might be studied. This corresponds to replacing the CDP normal form (2) with Eq. (8). It should be also noted that the density evolution (1) is expected to be transformed into

$$\partial_t \rho = D_\rho \nabla^2 A + D_m \nabla^2 (A\rho) + \sigma_m \nabla \cdot (\sqrt{A\rho}\, \boldsymbol{\xi}), \tag{9}$$

with $\boldsymbol{\xi}$ being a vectorial Gaussian white noise. While dimensional analysis indicates that D_m and σ_m are irrelevant close to the upper critical dimension, a detailed renormalization group analysis or large-scale numerics of our continuum theory are needed to assess the importance of D_m and σ_m; this is left for future works. We finally observe that the presence of a discontinuous APT numerically found at high c values is likely explained by a change in the sign of μ (still assuming a stabilizing A^2 term). In fact, direct measurements of $g^{(0)}$ reported in Ref. [32] indicate that, in the many-body system, $g^{(0)}$ is not independent of A exactly: it can be shown that this adds a new contribution to μ which might change its sign.

Conclusion. We investigated a minimal model for the stroboscopic dynamics of periodically sheared soft matter, taking into account the effect of active regions on passive ones in a mean-field way. We showed, using numerical simulations and a local mean-field analytical argument to derive a continuum theory, that mediated interactions modify the universality class of an APT, which does not belong to CDP anymore. CDP is not the only universality class of APTs with infinitely many absorbing states; as such, the field-theoretical description proposed here is expected to describe APTs in many other contexts in which local activity affects nearby passive regions.

Here, the displacement of passive particles depends on the spatially averaged activity, in a mean-field spirit. This seems relevant in a realistic system with mediated interactions that are long ranged. It is indeed well known that for equilibrium systems close to a continuous transition long-ranged interacting systems can behave as mean-field ones (for strong long-range interactions) or the critical exponents can be modified perturbatively around mean-field ones (see, for instance, Ref. [37,38]). Our model could then be the starting point for a perturbation theory that tries to capture non-mean-field effects, if any. For short-range propagators (e.g., screened hydrodynamic interactions in dense suspensions), displacements of passive particles rather result from a more local activity. Whether this would turn the critical behavior reported here into a crossover (in the Renormalization Group (RG) sense) and CDP would be recovered in the infinite-size limit is an important question left for future work.

Let us also remark that, in practice, it is often difficult to distinguish active particles from passive ones because all particles move, even ever so slightly [18,21]. This could be a consequence of mediated interactions. A test of this could be to determine the distribution of particle displacements, which close to the APT is expected to become bimodal, with much smaller displacements for passive particles than for active ones. Finally, our results imply that hyperuniformity is suppressed by mediated interactions, a consequence of experimental relevance since scattering techniques may provide access to the structure factor.

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[1] D. J. Pine, J. P. Gollub, J. F. Brady, and A. M. Leshansky, Nature (London) **438**, 997 (2005).

[2] L. Corté, S. J. Gerbode, W. Man, and D. J. Pine, Phys. Rev. Lett. **103**, 248301 (2009).

[3] J. R. Royer and P. M. Chaikin, Proc. Natl. Acad. Sci. USA **112**, 49 (2015).

[4] J. H. Weijs, R. Jeanneret, R. Dreyfus, and D. Bartolo, Phys. Rev. Lett. **115**, 108301 (2015).

[5] D. Fiocco, G. Foffi, and S. Sastry, Phys. Rev. E **88**, 020301(R) (2013).
[6] L. Corte, P. M. Chaikin, J. P. Gollub, and D. J. Pine, Nat. Phys. **4**, 420 (2008).
[7] M. Mungan and T. A. Witten, Phys. Rev. E **99**, 052132 (2019).
[8] M. Mungan, S. Sastry, K. Dahmen, and I. Regev, Phys. Rev. Lett. **123**, 178002 (2019).
[9] H. Hinrichsen, Adv. Phys. **49**, 815 (2000).
[10] G. I. Menon and S. Ramaswamy, Phys. Rev. E **79**, 061108 (2009).
[11] C. Ness and M. E. Cates, Phys. Rev. Lett. **124**, 088004 (2020).
[12] S. Manna, J. Phys. A: Math. Gen. **24**, L363 (1991).
[13] A. Vespignani, R. Dickman, M. A. Munoz, and S. Zapperi, Phys. Rev. Lett. **81**, 5676 (1998).
[14] M. Rossi, R. Pastor-Satorras, and A. Vespignani, Phys. Rev. Lett. **85**, 1803 (2000).
[15] P. Le Doussal and K. J. Wiese, Phys. Rev. Lett. **114**, 110601 (2015).
[16] F. van Wijland, Phys. Rev. Lett. **89**, 190602 (2002).
[17] S. Lübeck, Int. J. Mod. Phys. B **18**, 3977 (2004).
[18] K. Hima Nagamanasa, S. Gokhale, A. K. Sood, and R. Ganapathy, Phys. Rev. E **89**, 062308 (2014).
[19] E. D. Knowlton, D. J. Pine, and L. Cipelletti, Soft Matter **10**, 6931 (2014).
[20] I. Regev and T. Lookman, J. Phys.: Condens. Matter **31**, 045101 (2018).
[21] R. Jeanneret and D. Bartolo, Nat. Commun. **5**, 3474 (2014).
[22] T. Kawasaki and L. Berthier, Phys. Rev. E **94**, 022615 (2016).
[23] K. Nagasawa, K. Miyazaki, and T. Kawasaki, Soft Matter **15**, 7557 (2019).
[24] A. D. S. Parmar, S. Kumar, and S. Sastry, Phys. Rev. X **9**, 021018 (2019).
[25] P. Das, H. A. Vinutha, and S. Sastry, Proc. Natl. Acad. Sci. USA **117**, 10203 (2020).
[26] W.-T. Yeh, M. Ozawa, K. Miyazaki, T. Kawasaki, and L. Berthier, Phys. Rev. Lett. **124**, 225502 (2020).
[27] E. Tjhung and L. Berthier, Phys. Rev. Lett. **114**, 148301 (2015).
[28] C. Aron and M. Kulkarni, Phys. Rev. Res. **2**, 043390 (2020).
[29] D. Hexner and D. Levine, Phys. Rev. Lett. **114**, 110602 (2015).
[30] D. Hexner, P. M. Chaikin, and D. Levine, Proc. Natl. Acad. Sci. USA **114**, 4294 (2017).
[31] E. Tjhung and L. Berthier, J. Stat. Mech.: Theory Exp. **2016**, 033501 (2016).
[32] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.105.L032602 for additional numerical results, among which finite-size scaling and hyperuniformity, as well as tests of the two-body description, which includes Refs. [39,40].
[33] C. Argolo, Y. Quintino, P. H. Barros, and M. L. Lyra, Phys. Rev. E **87**, 032141 (2013).
[34] C. Villarroel and G. Düring, Soft Matter **17**, 9944 (2021).
[35] S. N. Majumdar, A. Comtet, and R. M. Ziff, J. Stat. Phys. **122**, 833 (2006).
[36] R. M. Ziff, S. N. Majumdar, and A. Comtet, J. Phys.: Condens. Matter **19**, 065102 (2007).
[37] M. E. Fisher, S.-k. Ma, and B. G. Nickel, Phys. Rev. Lett. **29**, 917 (1972).
[38] D. Mukamel, in *Long-Range Interacting Systems*, Lecture Notes of the Les Houches Summer School Vol. 90, edited by T. Dauxois, S. Ruffo, and L. F. Cugliandolo (Oxford University, Oxford, 2009).
[39] S. Lübeck and P. C. Heger, Phys. Rev. E **68**, 056102 (2003).
[40] S. Lübeck, Phys. Rev. E **66**, 046114 (2002).