Spatially periodic and kinklike distortions in microsized nematic volumes

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The purpose of this article is to describe the physical mechanism responsible for the appearance of both spatially periodic and kinklike distortions in a homogeneously aligned microsized nematic volume under the effect of crossed electric and magnetic fields. Numerical studies were carried out to describe the dynamic reorientation of the director′s field in a thick liquid crystal (LC) cell ($\sim$200 μm) under the effect of a large electric field **E** ($\sim$1.0 V/μm) directed at an angle α close to a right angle to magnetic field **B** ($\sim$7.0 T). It is shown that under the effect of **E** directed at $\alpha \sim 89.96^\circ$ to **B**, at least two scenarios of reorientation of the director field can be realized. First, in response to the suddenly applied electric field, spatially periodic patterns can appear in an initially uniformly aligned nematic domain. Second, when the same crossed external fields are applied at the smaller angle $\alpha \sim 88.81^\circ$ to each other, the mode of uniform director reorientation is dominated. In the case when the electric field $E \gg E_{\rm th}$ is applied orthogonally to both horizontal bounding surfaces of the LC cell and the magnetic field is turned off, in the microsized nematic volume the distortion in the form of the kinklike wave spreading normally to the horizontal bounding surfaces with the velocity in a few meters per second can be excited.

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I. INTRODUCTION

Liquid crystal (LC) materials have been referred to as a curious soft matter, but their impact on modern technology is very impressive [1]. Composed of anisotropic molecules, LC materials interact with external fields and restricted surfaces, which strongly influence their structure and optical properties. The problem of externally driven manipulation of LC materials, especially by an externally applied electric field, has brought an increasing number of integrated small-scale microdevices not only in the field of displays but also in LC sensors (LCSs) and LC actuators (LCAs) [2]. The widely used flat-panel LC displays (LCDs) consist of a LC film sandwiched between two glasses of plastic surfaces on the scale of the order of micrometers across which a voltage may be applied independently to each pixel of the LCD. This applied electric field may alter the molecular configuration of the LC layer and thus alter the optical characteristics of the LCD [3].

It should be noted that the problem associated with the description of the distortion of the orientational ordering of molecules described by director field $\hat{\mathbf{n}}$ under the effect of an external electric field **E** has been studied quite well [4]. When the electric field **E** is applied, for instance, perpendicularly to a homogeneously aligned nematic (HAN) cell, then this field can distort the molecular orientation $\hat{\mathbf{a}}$ with respect to director $\hat{\mathbf{n}}$ at a critical threshold field [4] $E_{\rm th} = \frac{\pi}{d}\sqrt{\frac{K_1}{\epsilon_0\epsilon_a}}$, where d is the thickness of the microsized LC cell, K_1 is the splay elastic constant, ϵ_0 is the absolute dielectric permittivity of free space, and ϵ_a is the dielectric anisotropy of the nematic phase. This form for the critical field is based on the assumption that the director remains strongly anchored (in our case, homogeneously) at the two horizontal bounding surfaces and that the physical properties of the LC are uniform over the entire sample for $E < E_{\rm th}$. When the electric field is switched on with a magnitude E greater than $E_{\rm th}$, the director $\hat{\mathbf{n}}$, in the splay geometry, reorients as a simple monodomain [5,6]. The nature of the reorientational process of the director $\hat{\mathbf{n}}$ to its stationary orientation $\hat{\mathbf{n}}_{\rm st}$ in the microsized nematic cell under the effect of the external electric field $E > E_{\rm th}$ has been investigated both experimentally and theoretically. The theoretical analysis is based on the hydrodynamic theory including the director motion and fluid flow, excited by the director reorientation [7–12]. The voltage applied across the HAN cell makes backflow possible in the microvolume, and backflow is additionally influenced by $\hat{\mathbf{n}}$. In turn, the experimental progress in understanding both structural and dynamic properties of LC materials confined in the microsized volume is connected with

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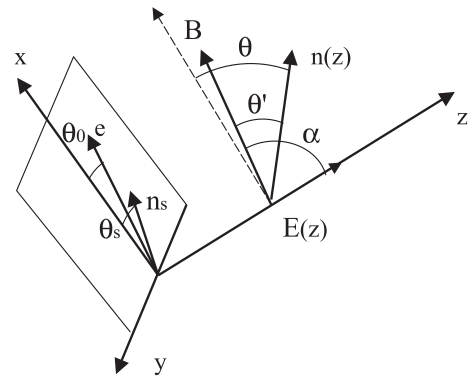

FIG. 1. The coordinate system used for theoretical analysis.

the progress in the time-resolved deuterium nuclear magnetic resonance (NMR) spectroscopic measurements [6,13–15]. For the case of a nematic phase in which the director is initially aligned by external magnetic field **B**, the deuterium NMR spectrum originating from a group of equivalent deuterons consists of a doublet whose the quadrupolar splitting is denoted by $\Delta\nu_0$. When a sufficiently strong electric field **E** is applied with respect to **B**, the director tends to be aligned parallel to **E**, and the quadrupolar splitting $\Delta\nu(\theta)$ is connected with $\Delta\nu_0$ as $\Delta\nu(\theta) = \Delta\nu_0 P_2(\cos\theta)$ [13], where $P_2(\cos\theta)$ is the second Legendre polynomial and θ is the angle made by $\hat{\mathbf{n}}$ with **B** (see Fig. 1). As the director moves from being parallel to **B**, the splitting $\Delta\nu(\theta)$ is going to decrease, pass through zero, and then increase to one-half of $\Delta\nu_0$ when $\hat{\mathbf{n}}$ is parallel to **E**. Thus, the deuterium NMR spectroscopy is found to be a powerful method to investigate the dynamic behavior of the director field in the microsized nematic volume.

In turn, when the electric field $E \gg E_{\rm th}$ ($\sim$1 V/μm) is applied, the state of the nematic system becomes unstable and the misalignment of the director with respect to the direction imposed by the aligning magnetic field increases so much that the reorientation caused by strong **E** manifests itself in the growth of one particular Fourier mode. In this case, the spectral line shape characterizing the initially aligned nematic sample broadens with time-dependent splitting while the initial steady doublet with constant splitting progressively vanishes [5,6]. Thus, the application of large **E** gives rise to the appearance of a doublet with vanishing amplitude that progressively grows with constant splitting so the total spectral intensity is transferred from the initial doublet to the new one, with half the quadrupolar splitting. Analysis of these results strongly suggests that the intermediate state is inhomogeneous and perturbed by thermal fluctuations, so in response to the suddenly applied electric field, spatially periodic patterns may appear in initially uniformly aligned nematic domains [7–11]. These nonuniform rotational modes involve additional internal elastic distortions of the conservative nematic system and, as a result, these deformations decrease the viscous contribution to the total energy of the nematic phase [16].

In turn, decreasing the viscous contribution leads to the decrease of the effective rotational viscosity coefficient and gives a faster response of the director rotation than for a uniform mode, as observed in the time-resolved deuterium NMR spectroscopic measurements [5,6]. It is, therefore, necessary to analyze the nematic response to an initial state exhibiting some thermal fluctuations of the director under the influence of the strong electric field. Such a response of the initially HAN domain to the suddenly applied electric field with the appearance of spatially periodic patterns will be analyzed on the basis of the Ericksen-Leslie theory [17,18] supplemented by the charge balance equation. The situation will also be analyzed when, under the effect of externally applied electric field, the kink or double π- forms of the distortion wave propagating along the normal to the bounding surfaces may occur in the microsized nematic volume [19].

Since anomalous changes in the shape of spectral lines do not provide any information about the average orientation of the director, it is necessary to carry out additional numerical studies of the system, which include both the reorientation of the director and the fluid flow.

The layout of this article is as follows. In the next section, we will give the theoretical background for describing the physical mechanism responsible for the field-induced spatially periodic and kinklike distortions of the director field in microsized nematic volumes, as well as for the field-induced reorientation of the director field as a simple LC monodomain. The numerical description of the occurrence of periodic structures in a homogeneously aligned domain under the effect of crossed electric and magnetic fields, as well as the reorientation of the director field as a simple monodomain is given in Sec. III. The description of the kinklike response of the homogeneously aligned microsized nematic volume under the effect of the externally applied electric field is also given in Sec. III. Our conclusions are given in Sec. IV.

II. FORMULATION OF THE BALANCE EQUATIONS FOR MICROSIZED NEMATIC VOLUME

We are primarily interested in describing the physical mechanism responsible for the excitation of spatially periodic distortions in the microsized nematic volume under the effect of crossed electric and magnetic fields. We will consider the response of the nematic volume composed of cyanobiphenyl molecules confined in the microsized volume delimited by two horizontal and two vertical surfaces at mutual distances of $2L$ and $2d$ ($L \gg d$) on a scale of the order of tens of micrometers. The coordinate system defined by our task assumed that electric field **E** is applied normally (or close to normal) to magnetic field **B**, which is directed parallel to horizontal surfaces. This problem will be considered within the framework of the extended Ericksen-Leslie theory [17,18], supplemented by the charge balance equation, and the geometry of this system can be considered as two-dimensional, since the director $\hat{\mathbf{n}}$ lies in the plane xz (or yz), determined by electric **E** and magnetic **B** fields. In this geometry, the system may be seen as three-dimensional, where the unit vector $\hat{\mathbf{i}}$ is directed parallel to the horizontal bounding surfaces and coincides with the direction of the vector **B**, while the unit vector $\hat{\mathbf{k}}$ coincides with the vector **E** and $\hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{i}}$, respectively. We can suppose that the components of the director $\hat{\mathbf{n}} = n_x\hat{\mathbf{i}} + n_z\hat{\mathbf{k}} = \cos(x, z, t)\hat{\mathbf{i}} + \sin(x, z, t)\hat{\mathbf{k}}$ (see Fig. 1) depend only on x, z variables and time t. Our analysis of the effect of the strong electric field on the reorientation of the director's field suggests that to describe the dynamical evolution of $\hat{\mathbf{n}}$ correctly, we do not need to include a proper treatment of backflow [5]. This means that the main role is played by electric and magnetic forces, and the role of viscous forces becomes negligible compared to the aforementioned contributions to the torque balance equation. In our case, this equation takes the form

$$\left[\frac{\delta\mathcal{R}_{\text{vis}}}{\delta\hat{\mathbf{n}}_{,t}}+\frac{\delta\psi_{\text{elast}}}{\delta\hat{\mathbf{n}}}+\frac{\delta\psi_{\text{el}}}{\delta\hat{\mathbf{n}}}+\frac{\delta\psi_{\text{mg}}}{\delta\hat{\mathbf{n}}}+\frac{\delta\psi_{\text{flex}}}{\delta\hat{\mathbf{n}}}\right]\times\hat{\mathbf{n}}=0, \quad (1)$$

where $\mathcal{R}_{\text{vis}}=\frac{1}{2}\gamma_1(\hat{\mathbf{n}}_{,t})^2$ is the dissipation function, γ_1 is the rotational viscosity coefficient, $\hat{\mathbf{n}}_{,t}=\frac{d\hat{\mathbf{n}}}{dt}=\frac{\partial\hat{\mathbf{n}}}{\partial t}+\mathbf{v}\cdot\nabla\hat{\mathbf{n}}$, $\frac{\partial\hat{\mathbf{n}}}{\partial t}$ is the partial derivative of $\hat{\mathbf{n}}$ with respect to time t, $\psi_{\text{elast}}=\frac{1}{2}[K_1(\nabla\cdot\hat{\mathbf{n}})^2+K_3(\hat{\mathbf{n}}\times\nabla\times\hat{\mathbf{n}})^2]=\frac{1}{2}[K_1(n_{x,x}+n_{z,z})^2+K_3(n_{z,x}-n_{x,z})^2]$ is the elastic energy density, $\psi_{\text{el}}=-\frac{1}{2}\epsilon_0\epsilon_a(\hat{\mathbf{n}}\cdot\mathbf{E})^2=-\frac{1}{2}\epsilon_0\epsilon_a(n_xE_x+n_zE_z)^2$ is the electric energy density, $\psi_{\text{mg}}=-\frac{1}{2}\frac{\chi_a}{\mu_0}(\hat{\mathbf{n}}\cdot\mathbf{B})^2=-\frac{1}{2}\frac{\chi_a}{\mu_0}(n_xB)^2$ is the magnetic energy density, and $\psi_{\text{flex}}=-\mathbf{P}\cdot\mathbf{E}$ is the flexoelectric energy density, respectively. Here $n_{x,\alpha}=\partial n_x/\partial\alpha$ $(\alpha=x,z)$ is the partial derivative of n_x with respect to space variables x or z, respectively, K_1 and K_3 are the splay and bend elastic constants, μ_0 is the magnetic constant, and χ_a is the magnetic anisotropy of the nematic sample, while the vector $\mathbf{P}$ is equal to $e_1\hat{\mathbf{n}}(\nabla\cdot\hat{\mathbf{n}})+e_3(\nabla\times\hat{\mathbf{n}})\times\hat{\mathbf{n}}$, and e_1 and e_3 are the flexoelectric constants. In our case, electric field $\mathbf{E}=E_x\hat{\mathbf{i}}+E_z\hat{\mathbf{k}}=E(z)(\cos\alpha\hat{\mathbf{i}}+\sin\alpha\hat{\mathbf{k}})$ makes the angle α with magnetic field $\mathbf{B}$, the value of which is varied in the vicinity of $\pi/2$ (see Fig. 1), whereas the torque balance equation takes the form

$$\begin{aligned}\gamma_1\theta_{,t}&=K_1[(2-\mathcal{M})\theta_{,xx}+\mathcal{G}(\theta)(\theta_{,zz}-\theta_{,xx})]\\&\quad+\frac{1}{2}K_1\mathcal{M}\big[\big(\theta_{,x}^2-\theta_{,z}^2-2\theta_{,xz}\big)\sin 2\theta-\theta_{,x}\theta_{,z}\cos 2\theta\big]\\&\quad+\frac{1}{2}\epsilon_0\epsilon_aE^2\sin 2(\alpha-\theta)-\frac{1}{2}\frac{\chi_a}{\mu_0}\sin 2\theta B^2\\&\quad-\frac{1}{2}(e_1+e_3)E_{,z}\sin\alpha\sin 2\theta.\end{aligned} \quad (2)$$

Here $\theta_{,t}=\partial\theta/\partial t$ and $\theta_{,x}=\partial\theta/\partial x$ denote the partial derivative of the angle θ with respect to time t and space coordinate x, respectively, $\mathcal{G}(\theta)=\cos^2\theta+K_{31}\sin^2\theta$, $K_{31}=K_3/K_1$, $\mathcal{M}=1-K_{31}$, and $E_{,z}=\partial E/\partial z$. In the following, we consider the microsized nematic volume confined between two electrodes when the director is weakly anchored to the horizontal bounding surfaces with the anchoring energy [20]

$$W^{\text{an}}=\tfrac{1}{2}A\sin^2(\theta_s-\theta_0), \quad (3)$$

where A is the anchoring strength, θ_s and θ_0 are the angles corresponding to the director orientation on the bounding surface, $\hat{\mathbf{n}}_s$, and easy axis, $\hat{\mathbf{e}}$, respectively (see Fig. 1). In this case, the torque balance transmitted to the bounding surfaces assumes that the direction angle must satisfy the boundary conditions [5],

$$\mathcal{G}(\theta)(\theta_{,z}(z))_{z=\mp d}=\frac{A}{2K_1}\sin 2\Delta\theta^{\mp}, \quad (4)$$

where $\Delta\theta^{\mp}=\theta_s^{\mp}-\theta_0^{\mp}$, while $\theta_s^{\mp}$ and $\theta_0^{\mp}$ are the pretilt angles of the surface director and the easy axes at $z=-d$ and $z=d$, respectively. In turn, the remaining two lateral surfaces assume that the direction angle must satisfy the strong anchoring conditions,

$$\theta(x=\mp L,-d<z<d)=0, \quad (5)$$

while the initial orientation of the director is directed parallel to both horizontal surfaces, with $\theta(x,z,\tau=0)=0$, and then allowed to relax to its stationary value $\theta_{\text{st}}(x,z)$.

The application of the voltage across the microsized nematic volume results in a variation of $E(z)$ through the film [5] which is obtained from

$$\big[\mathcal{B}(\theta)\overline{E}(z)\big]_{,z}=0,\quad\int_{-d}^{d}\overline{E}(z)dz=1, \quad (6)$$

where the function $\mathcal{B}(\theta)$ is equal to $\frac{\epsilon_\perp}{\epsilon_a}+\sin^2\theta$, $\overline{E}(z)=\frac{E(z)}{E}$, $E=\frac{U}{d}$, and U is the voltage applied across the microsized film.

In turn, the dimensionless torque balance equation describing the reorientation of $\hat{\mathbf{n}}$ to its stationary orientation $\hat{\mathbf{n}}_{st}$ can be written as

$$\begin{aligned}\theta_{,\tau}&=\delta_1[(2-\mathcal{M}-\mathcal{G}(\theta))\theta_{,xx}+\mathcal{G}(\theta)\theta_{,zz}]\\&\quad+\delta_1\mathcal{M}\left[\frac{\sin 2\theta}{2}\big(\theta_{,x}^2-\theta_{,z}^2-\theta_{,xz}\big)-\cos 2\theta\theta_{,x}\theta_{,z}\right]\\&\quad+\left[\delta_3\mathcal{F}_{,z}\sin\alpha-\frac{1}{2}\delta_2\right]\sin 2\theta+\frac{1}{2}\mathcal{F}^2(\theta)\sin 2(\alpha-\theta),\end{aligned} \quad (7)$$

where x and z denote the dimensionless space variables (i.e., scaled by d) corresponding to x and z axes, respectively, $\theta_{,x}=\partial\theta/\partial x$, $\theta_{,z}=\partial\theta/\partial z$, $\theta_{,\tau}=\partial\theta/\partial\tau$ is the partial derivative of the angle θ with respect to the dimensionless time $\tau=\frac{\epsilon_0\epsilon_a}{4\gamma_1}E^2t$, the function $\mathcal{F}(\theta)$ is equal to $(\mathcal{C}-\delta_3\sin 2\theta\theta_{,z})/\mathcal{B}(\theta)$, while the constant $\mathcal{C}$ is equal to $[\int_{-1}^{1}(\mathcal{B}(\theta))^{-1}dz]^{-1}$, and $\delta_1=\frac{4K_1}{\epsilon_0\epsilon_ad^2}\frac{1}{E^2}$, $\delta_2=\frac{4\chi_a}{\mu_0\epsilon_0\epsilon_a}(\frac{B}{E})^2$, and $\delta_3=\frac{e_1+e_3}{d\epsilon_0\epsilon_a}\frac{1}{E}$ are three parameters of the system. In the following calculations, the ratio L/d is chosen to be equal to 10.

In this case, the dimensionless analog of the Eq. (4) assumes that the angle θ has to satisfy the weak boundary conditions

$$\theta_{,z}(-10<x<10,z=\pm 1)=\pm\delta_4\theta(-10<x<10,z=\pm 1), \quad (8)$$

where $\delta_4=\frac{Ad}{K_1}$ is an extra one parameter of the LC system, while the remaining two lateral surfaces assume the strong anchoring conditions:

$$\theta(x=\pm 10,-1<z<1)=0. \quad (9)$$

Now the reorientation of the director in the microsized nematic volume imposed by crossed electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields can be obtained by solving the system of nonlinear partial differential equations (6) and (7), with appropriate boundary (8)and (9) and initial

$$\theta(x,z,\tau=0)=0 \quad (10)$$

conditions (hereafter referred to as case A).

In the case when the dielectric and magnetic anisotropies of the LC phase are positive [case of deuterated $4-\alpha, \alpha - d_2 -$ pentyl $- 4' -$ cyanobiphenyl ($5CB - d_2$)], and when a strong electric field **E** is applied at an angle α close to the right angle to magnetic field **B**, the director moves from parallel to the magnetic field to parallel to the electric field (the switching process). This reorientation process is characterized by the value of the relaxation time $\tau_{\rm on}(\alpha)$, and has a typical deuterium NMR signature: The initial quadrupolar doublet characterizing the initially aligned LC sample gives rise to an additional broad doublet with time-dependent slitting while, simultaneously, the initial steady doublet with constant splitting progressively vanishes. Thus, the process of reorientation of the director field in the microsized LC volume under the effect of the strong transverse electric field proceeds as a uniform reorientation of the simple LC monodomain. After the electric field is switched off, the director relaxes back to being parallel to the magnetic field.

Now the reorientation of the director in the microsized nematic volume under the influence of the external forces can be obtained by solving the nonlinear differential equations (6) and (7) with appropriate boundary (8) and (9) and initial (10) conditions.

In turn, the recent time-resolved deuterium NMR spectroscopic measurements of field-induced director reorientations show that in $4-\alpha\alpha - d_2 -$ pentyl $- 4' -$ cyanobiphenyl ($5CB - d_2$) at the temperature of 300 K and the density of 10^3 kg/m^3, the application of the strong electric field ($\sim$1.03 V/μm) at angle α close to the right angle to the magnetic field ($\sim$7.05 T) leads to the appearance of a doublet with vanishing amplitude, which gradually increases with constant splitting, so the total spectral intensity is transferred from the initial doublet to the new one, with half a quadrupolar splitting [6,16]. It is also shown that the values of the relaxation time $\tau_{\rm on}(\alpha)$, during the turn-on process, monotonically grow with increasing angle α, up to the maximum value of $\tau_{\rm on}^{\rm max}(\alpha)$. With further growth of α, up to the right angle $\alpha \sin \frac{\pi}{2}$, $\tau_{\rm on}(\alpha \sim \frac{\pi}{2})$ decreases rapidly with a few milliseconds, relative to $\tau_{\rm on}^{\rm max}(\alpha)$. Analysis of these results strongly suggests that the intermediate state is inhomogeneous and perturbed by thermal fluctuations, so in response to the suddenly applied electric field, spatially periodic patterns may appear in the initially uniformly aligned nematic domains [7–11]. These nonuniform rotational modes involve additional internal elastic distortions of the conservative nematic system and, as a consequence, these deformations reduce the viscous contribution to the total energy of the nematic phase [16]. This decrease in the viscous contribution leads to a decrease in the effective coefficient of rotational viscosity and provides a faster rotation response of the director than for the uniform mode, as observed in measurements of deuterium NMR spectroscopy [6,16]. Therefore, it is necessary to analyze the nematic response to the initial state, which exhibits some thermal fluctuations of the director under the influence of the strong electric field.

To find the role of the director fluctuations in maintaining spatially periodic patterns in the microsized nematic volume under the action of the strong orthogonal electric field, we performed the numerical study of the Eqs.(6) and (7) with the mixed boundary condition for angle θ, which reads in the dimensionless form as (hereafter referred to as case B)

$$\theta(-10 < x < 10, z = \pm 1) = 0,$$
$$\theta(x = \pm 10, -1 < z < 1) = 0, \tag{11}$$

and with the weak anchoring condition for angle θ, which reads in the dimensionless form as (hereafter referred to as case C)

$$\theta_{,z}(-10 < x < 10, z = \pm 1)$$
$$= \pm\delta_4\theta(-10 < x < 10, z = \pm 1),$$
$$\theta(x = \pm 10, -1 < z < 1) = 0, \tag{12}$$

and with the initial condition which reads in the dimensionless form as [7,8,10,11,21]

$$\theta(x, z, 0) = \theta_{\rm fl}\cos(q_x x)\cos(q_z z). \tag{13}$$

To observe the formation of spatially periodic patterns in nematic volume excited by the strong orthogonal electric field, we consider the initial condition $\theta(x, z, 0)$ in the form of Eq. (13), which determines the fluctuations of the director field over the nematic volume with amplitude $\theta_{\rm fl}$ and the wavelengths q_x and q_z of a separate Fourier modulation component.

In case B, they are described as $q_\kappa = \frac{\pi}{2}(2k+1)$, where $\kappa = x, z$ and $k = 0, 1, 2, \ldots$, while in the case C, as $q_x = \frac{\pi}{2}(2k+1)$, where $k = 0, 1, 2, \ldots$ and $q_z = \mp(\delta_4 \cot q_z)_{z=\pm1}$, respectively.

The optimal dimensionless wavelengths $q_x(\alpha)$ and $q_z(\alpha)$, corresponding to the angle α, can be obtained from the minimum of the total energy $W(\alpha)$, where

$$W(\alpha) = \frac{\delta_1}{2}\int dxdz \sum_{i=1,2}\mathcal{D}_i(x, z). \tag{14}$$

Here $\mathcal{D}_1(x, z) = \mathcal{D}_{11}(x, z) + \mathcal{D}_{12}(x, z)$, where $\mathcal{D}_{11}(x, z) = [-\frac{d}{L}\sin\hat{\theta}(\hat{\theta})_{,x} + \cos\hat{\theta}(\hat{\theta})_{,z}]^2$ and $\mathcal{D}_{12}(x, z) = K_{31}[\frac{d}{L}\cos\hat{\theta}(\hat{\theta})_{,x} + \sin\hat{\theta}(\hat{\theta})_{,z}]^2$ are the elastic contributions, respectively, while $\mathcal{D}_2(x, z) = \frac{1}{2}(\sin^2\hat{\theta} + \delta_2\cos^2\hat{\theta})$ is the electric and magnetic contributions to the total energy. Here $\hat{\theta}$ is equal to $\theta_{st}(x, z)$.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Reorientation of the director field in a microsized nematic volume imposed by crossed electric and magnetic fields as a simple monodomain

First, we investigate the process of reorientation of the director field as a single monodomain. For this purpose, we will consider a sample of $5CB - d_2$, at temperature 300 K and density 10^3 kg/m^3, the measured data for the elastic constants are $K_1 = 8.7$, and $K_3 = 10.2$ pN [22], respectively. For this LC compound, the calculated value of the dielectric anisotropy is equal to $\epsilon_a = 11.5$ [23], while the experimental value of the rotational viscosity coefficient γ_1 is equal to 0.136 Pa s [24], respectively. The voltage value across the nematic film thickness $2d = 194.7$ μm was chosen to be equal to $U = 200$ V, the same as for NMR measurements [5,6,16], the value of the anchoring strength A is equal to 10^{-6} J/m^2, and the ratio of L/d is equal to 10, while the calculated data for both flexoelectric coefficients, e_1 and e_3, are equal to

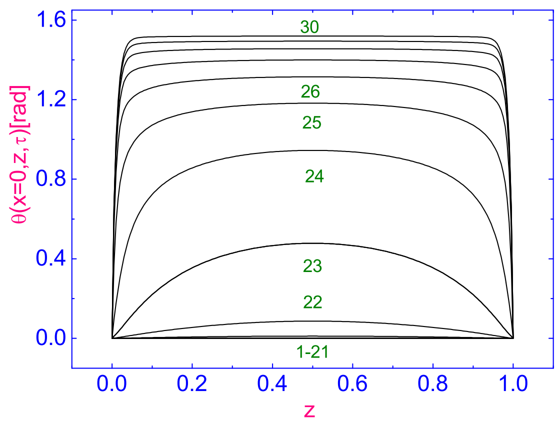

FIG. 2. The evolution of the angle $\theta(x=0,z,\tau)$ to its stationary distribution $\theta_{\rm st}(x=0,z)$ across the dimensionless thickness of the LC film for the case A. The first 30 curves $\theta(x=0,z)$ are plotted as solid lines and the neighboring lines are separated by the time interval $\Delta\tau=1.0$ up to $\tau_R=30$. Here $\alpha=\frac{\pi}{2}$

$-11.6\ pC/\mathrm{m}$ and $-4.3\ pC/\mathrm{m}$ [25], respectively. The value of the magnetic anisotropy is equal to $\chi_a=1.17\times10^{-6}$, while the value of the magnetic field is equal to $B=7.05$ T. Now the set of δ parameters, which is involved in Eqs. (7) and (8), takes the values $\delta_1=8.5\times10^{-6}$, $\delta_2=0.43$, $\delta_3=-0.0008$, and $\delta_4=22.4$.

The nonlinear partial differential equations (6) and (7), together with the boundary conditions (8) and the initial condition (10) (case A) have been solved by the numerical relaxation method [26]. The relaxation criterion $\epsilon=|(\theta(\tau_R)-\theta_{\rm eq})/\theta_{\rm eq}|$ for the calculating procedure was chosen to be equal to 5×10^{-4}, and the numerical procedure was then carried out until a prescribed accuracy was achieved.

In this case, the evolution of the polar angle $\theta(x=0,z,\tau)$ to its stationary distribution $\theta_{\rm st}(x=0,z,\tau=\tau_R(A))$, under the effect of the electric field ($\sim1.03\ V/\mu$m) applied at the angle $\alpha=1.57$ ($\sim89.96^\circ$) with respect to the magnetic (~7.05 T) field, with the initial condition presented as the Eq. (10), is shown in Fig. 2. In this case, for the nematic phase confined between the two electrodes with the voltage in $U=200$ V and the film thickness $2d=194.7\ \mu$m, the relaxation regime is characterized by a monotonic increase of the angle $\theta(x=0,z,\tau)$ up to the stationary distribution across the LC film $\theta_{\rm st}(x=0,z)$ with the relaxation time $\tau_R=30$ (~38.1 ms). Physically, this means that the electric field produces an alignment of the director away from the magnetic field, caused by the applied voltage, and that the field has a strong influence, at least in our case, compared with the other torques, on the relaxation process. Our calculations reveal the weak effect of viscous force on the relaxation process in the nematic cell under the influence of the electric field. The calculations also show that the electric, magnetic, elastic, and viscous torques exerted are vanishingly small for the case when the electric field is almost orthogonal to the magnetic field after a scaled relaxation time of 30 which corresponds to a relaxation time of 38.1 ms. Our recent time-resolved deuterium NMR spectroscopic measurements of field-induced director reorientations show that in $4-\alpha\alpha-d_2-$pentyl$-4'-$cyanobiphenyl ($5\mathrm{CB}-d_2$) at a temperature of 300 K and density of 10^3 kg/m^3, the application of the strong electric field (~1.03 V/μm) at angle $\alpha\sim85.1^\circ$ to the magnetic field (~7.05 T) leads to transferring the total spectral intensity from the initial doublet to the new one, with half a quadrupolar splitting, after approximately 20 ms [15], which is two times less than the calculated relaxation time.

To understand the nature of this discrepancy between the experimental results and the calculated values of the relaxation time of the director field in the microscopic volume of a nematic formed by 5CB molecules, we assume that the intermediate state is inhomogeneous and perturbed by thermal fluctuations, so in response to the sudden applied electric field, spatially periodic structures may appear in initially uniformly aligned nematic domains.

B. Spatially periodic patterns in a microsized nematic volume imposed by crossed electric and magnetic fields

Thus, now our main goal is to study the features observed during the dynamic reorientation of the director′s field in a thin ($\sim194.7\ \mu$m) $5\mathrm{CB}-d_2$ LC film, under the effect of the large electric field (~1.03 V/μm) directed at angle α close to a right angle to the magnetic field (~7.05 T) [15]. This may cause an appearance of stripes in the uniformly aligned microsized nematic volume.

The appearance of the periodic structure in an initially uniformly aligned LC domain under the effect of the strong electric field (~1.03 V/μm) directed at angle $\alpha=1.57$ ($\sim89.96^\circ$) to the magnetic field (~7.05 T) will be investigated using the system of nonlinear partial differential equations (6) and (7), together with the boundary conditions (11) (case B) or (12) (case C), and the initial condition (13), respectively. It has been solved by the numerical relaxation method with the relaxation criterion $\epsilon=|(\theta(\tau_R)-\theta_{\rm eq})/\theta_{\rm eq}|$ for calculating procedure. In this case, the relaxation criterion ϵ was chosen to be equal to 5×10^{-4} and the numerical procedure was then carried out until a prescribed accuracy was achieved.

The reorientation of the angle $\theta(x,z=0,\tau)$ to its stationary value $\theta_{\rm st}(x,z=0,\tau=\tau_R(B))$ under the effect of the electric field (~1.03 V/μm) applied at the angle $\alpha=1.57$ ($\sim89.96^\circ$) to the magnetic (~7.05 T) field for case B, with the initial condition presented as in Eq. (13), for two amplitude values $\theta_{\rm fl}$ equal to 0.01 ($\sim1.1^\circ$) (case 1) and 0.001 ($\sim0.1^\circ$) (case 2), respectively, are shown in Figs. 3(a) and 3(b). In case 1, it is shown that only for the values $q_x=0.79$ and $q_z=64.34$, the director reorientation process is characterized by a well-developed periodic structure with lattice points in $x=-10.0,-7.74,-5.74,-3.30,-1,27,1.21,3.21,5,65,7,68,\ 10,0$, and the relaxation time $\tau_1(A)$ is equal to 24.5 ($\sim31,1$ ms), while in case 2 the dependence of the angle profile $\theta(x,z=0,\tau)$ on time has a wavelike profile along the variable axis x growing in the positive direction. Only at the later stage of the reorientation process one deals with the completely convex profile growing in the positive direction, with the relaxation time $\tau_2(B)=25$ (~31.75 ms). Physically, this means that in case 2, the value of the electric field component E_z, directed perpendicular to the magnetic field, is not enough for maintaining of the spatially periodic patterns and is enough only for formation of the

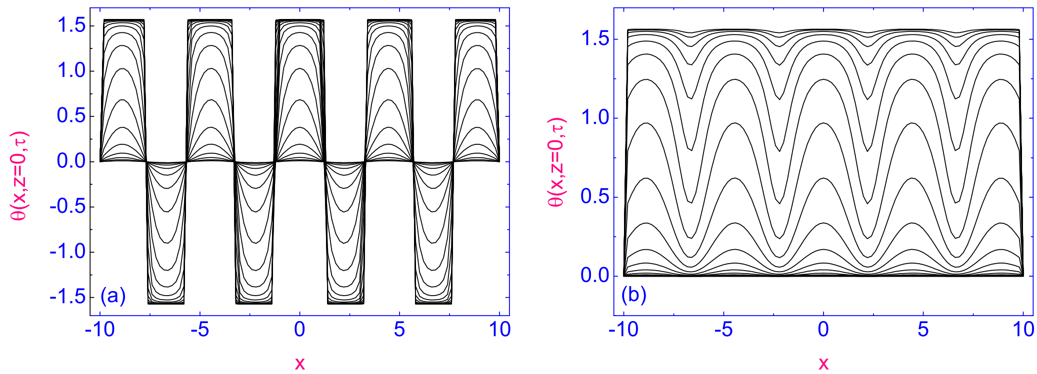

FIG. 3. Two scenarios of relaxation of the angle $\theta(x, z = 0, \tau)$ to its stationary distribution over the microsized nematic volume with the strong anchoring condition for the angle θ (case B) and under the effect of the electric field ($\sim$1.03 V/μm) applied at the angle $\alpha = 1.57$ to the magnetic field ($\sim$7.05 T). In case (a), the value of amplitude $\theta_{\rm fl}$ is equal to 0.01($\sim$1.1°), whereas in case (b) is equal to 0.001 ($\sim$0.11°), respectively.

wavelike deformations in the microsized nematic volume. With further decrease of angle α up to 1.565 ($\sim$ 88.81°), only for values of $q_x = 0.785$ and $q_z = 64.30$ and amplitudes $\theta_{\rm fl}$ greater or equal than 0.02, the nonuniform rotation mode rather than the uniform one [see Figs. 4(a) and 4(b) is provided. In this case, the reorientation of angle $\theta(x, z = 0, \tau)$ to its stationary value $\theta_{\rm st}(x, z = 0, \tau = \tau_R)$ under the effect of the same crossed electric and magnetic fields applied at angle $\alpha = 1.565$ ($\sim$ 88.81°), with two initial conditions for the amplitude values $\theta_{\rm fl} = 0.02$ ($\sim$2.2°) (case 3) and 0.01 ($\sim$1.1°) (case 4), respectively, are shown in Figs. 4(a) and 4(b). In case 3, the process of director reorientation is characterized by well-developed periodic structures with lattice points in $x = -10.0$, -7.42, -6.0, -2.94, $-1, 52$, 1.48, 2.88, 5.90,7.43, 10.0, and the relaxation time $\tau_3(B)$ is 20 ($\sim$ 25.4 ms), while in case 4 the time dependence of the angle profile $\theta(x, z = 0, \tau)$ has a wavelike profile along the variable axis x growing in the positive direction. Only at the later stage of the reorientation process one deals with the completely convex profile growing in the positive direction, with relaxation time $\tau_4(B) = 21.0$ ($\sim$26.7 ms).

Physically, this means that in case 3 with decrease of α up to 1.565 ($\sim$ 88.81°), the balance between the electric and magnetic forces provides maintaining only the nonperfect periodic patterns with lattice points in $x = -10.0$, -7.42, -6.0, -2.94, -1, 52, 1.48, 2.88, 5.90, 7.43, 10.0, and relaxation time $\tau_3(B) = 20.0$ ($\sim$25.4 ms), while in case 4, the time dependence of the $\theta(x, z = 0, \tau)$ profile resembles to the wavelike profile along the x axis growing in the positive direction, with relaxation time $\tau_4(B) = 21.0$ ($\sim$26.7 ms).

The process of reorientation of the director field $\hat{\mathbf{n}}(x = 0.0, z, \tau)$ to its stationary orientation $\hat{\mathbf{n}}_{\rm st}(x = 0.0, z, \tau_R = 24.5)$, which is described by angle $\theta(x = 0.0, z, \tau)$ along the dimensionless thickness $-1.0 < z < 1.0$, for a number of dimensionless times, starting from $\tau = 6$ ($\sim$ 7.8 ms) (curves (I)) and up to $\tau_1(B) = 24.5$ ($\sim$ 31.1 ms) [curve (IV)], is shown in Figs. 5(a) and 5(b). It is obvious from Figs. 3–5 that for certain values of the electric ($\sim$1.03 V/μm) and magnetic ($\sim$ 7.05 T) fields applied across the nematic film of thickness $\sim$194.7 μm, for values of angle α greater or equal to 1.565 ($\sim$ 88.81°), there is a threshold value of amplitude $\theta_{\rm fl}$ which provides the nonuniform rotation mode rather than the uniform one, while for the lower values both of amplitude $\theta_{\rm fl}$ and angle α the uniform mode dominates. Note that in both cases, strong anchoring conditions are assumed (case B).

Now let's study how the nature of anchoring with bounding surfaces affects the relaxation process of the director field. Consider the case of weak anchoring (case C) of the director with horizontal surfaces. The reorientation of the angle

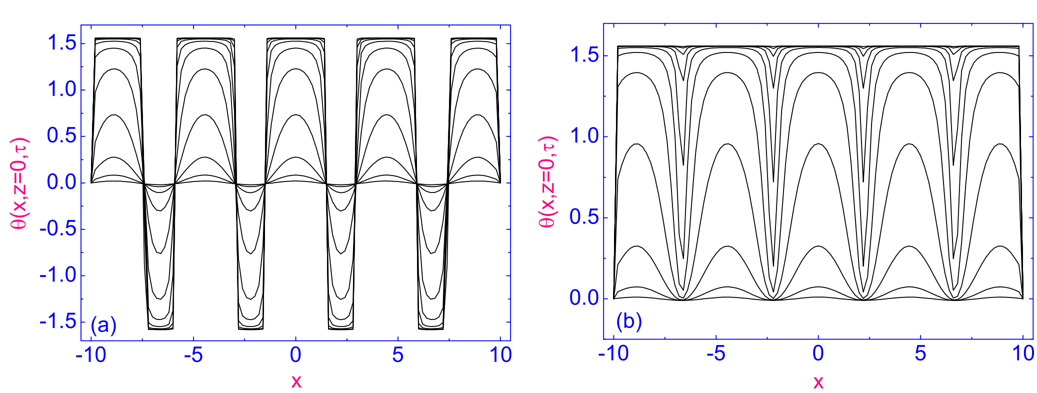

FIG. 4. The same as in Figs. 3(a) and 3(b) but the value of angle α is 1.565, while the values of $\theta_{\rm fl}$ are 0.02($\sim$ 2.2°) (a) and 0.01 ($\sim$1.1°) (b), respectively.

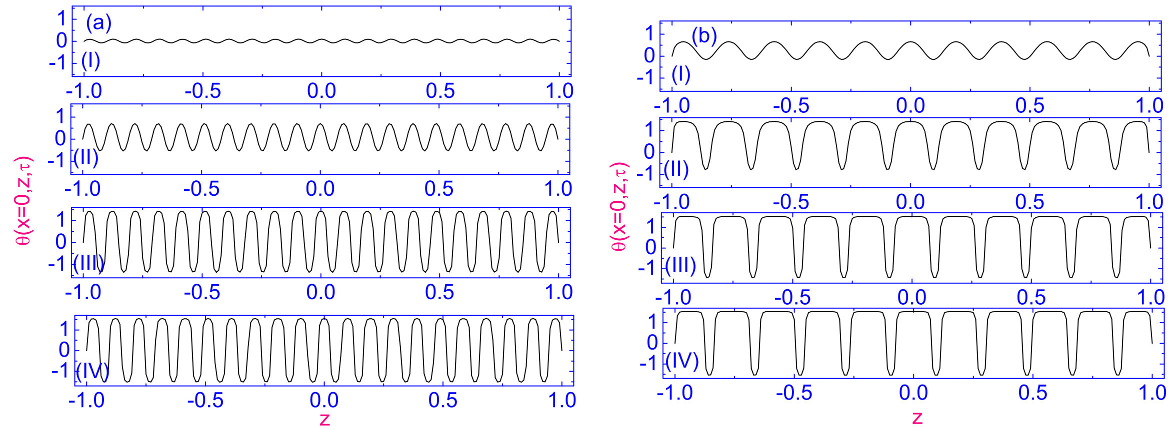

FIG. 5. (a) Formation of the periodic domain in the microsized nematic volume under the effect of crossed electric (∼1.03 V/μm) and magnetic (∼7.05 T) fields applied at angle $\alpha = 1.57$ and with amplitude $\theta_{\rm fl}$ equal to 0.01 (∼ 1.1°), starting from the dimensionless time $\tau = 6$ [curve (I)] and up to $\tau_1(B) = 24.5$ [curve (IV)], respectively. (b) Same as case (a), but both fields applied at angle $\alpha = 1.565$ and with the amplitude $\theta_{\rm fl}$ equal to 0.02 (∼ 2.°), respectively.

$\theta(x, z = 0, \tau)$ to its stationary value $\theta_{\rm st}(x, z = 0, \tau = \tau_R)$ under the effect of the electric field (∼1.03 V/μm) applied at the angle $\alpha = 1.57$ (∼ 89.96°) to the magnetic (∼7.05 T) field, for case C and with the initial condition in the form of the Eq. (13), for two amplitude values $\theta_{\rm fl}$ equal to 0.01 (∼1.1°) (case 5) and 0.001 (∼0.1°) (case 6), respectively, are shown in Figs. 6(a) and 6(b). It is shown in case 5 that only for values of $q_x = 0.785$ and $q_z = 64.3358$ is the director reorientation characterized by a well-developed periodic structure with lattice points in $x = -10.0, -7.71, -5.71, -3.27, -1, 27, 1.24, 3.24, 5.65, 7.68, 10.0$, and relaxation time $\tau_5(C) = 25.0$ (∼31.75 ms), while in case 6, the time dependence of the angle profile $\theta(x, z = 0, \tau)$, at early times up to $\tau = 12$ (∼15.24 ms) has a wavelike profile along the x axis growing in the positive direction. Only at the later stage of the reorientation process, after $\tau = 12$, one deals with the completely convex profile growing in the positive direction, with the relaxation time $\tau_6(C) = 26.0$ (∼33.0 ms). Physically, this means that in case 6, the value of the electric field component E_z, directed perpendicular to the magnetic field, is not enough for maintaining of the spatially periodic patterns, and enough only for the formation of the wavelike deformations in the microsized nematic volume.

With further decrease of angle α up to 1.565 (∼ 88.81°), only for values of $q_x = 0.79$ and $q_z = 64.336$, and amplitudes $\theta_{\rm fl}$ greater or equal than 0.02 the nonuniform rotation mode rather than the uniform one [see Figs. 7(a) and 7(b)] is provided. The reorientation of angle $\theta(x, z = 0, \tau)$ to its stationary value $\theta_{\rm st}(x, z = 0, \tau = \tau_R)$ under the effect of the electric field (∼1.03 V/μm) applied at angle α [equal to 1.565 (∼ 88.81°)] to the magnetic (∼7.05 T) field for case B with the initial condition presented as in Eq. (13), for two amplitude values $\theta_{\rm fl}$ equal to 0.02 (∼2.2°) (case 7) and 0.01 (∼1.1°) (case 8), respectively, are shown in Figs. 7(a) and 7(b)]. It is shown in case 7 that the director reorientation is characterized by a well-developed periodic structure with lattice points in $x = -10.0, -7.33, -6.12, -2.96, -1, 56, 1.53, 2.85, 6.06, 7.26, 10.0$, and relaxation time $\tau_7(C) = 20.5$ (∼26.0 ms), while in case 8 the time dependence of the angle profile $\theta(x, z = 0, \tau)$, at early times up to $\tau = 9$ (∼6.9 ms), has a wavelike profile along the x axis growing in the positive direction. Only at the later stage of the reorientation process, after $\tau = 20$, one deals with the completely convex profile growing in the positive direction, with the relaxation time $\tau_8(B) = 22.0$ (∼28.0 ms). Physically, this means that in case 8, the value of the electric

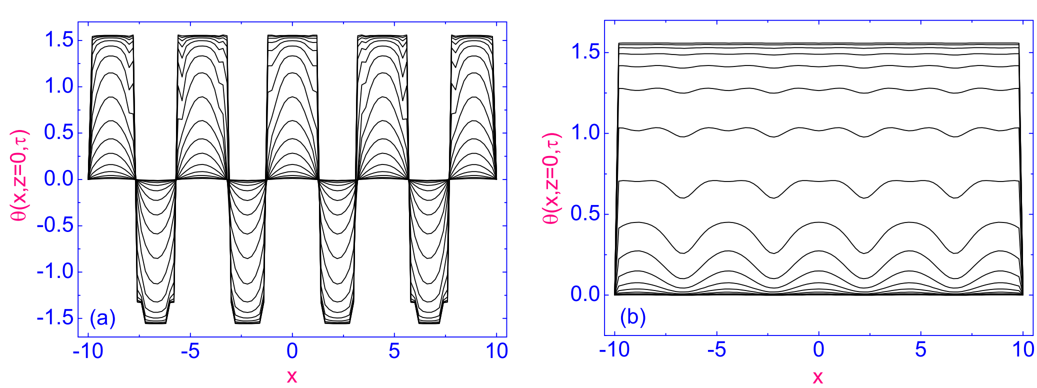

FIG. 6. The same as in Figs. 3(a) and 3(b) but for the case of the weak anchoring condition (case C). In case (a), the value of amplitude $\theta_{\rm fl}$ is equal to 0.01 (∼ 1.1°), while in case (b) is equal to 0.001 (∼ 0.11°), respectively.

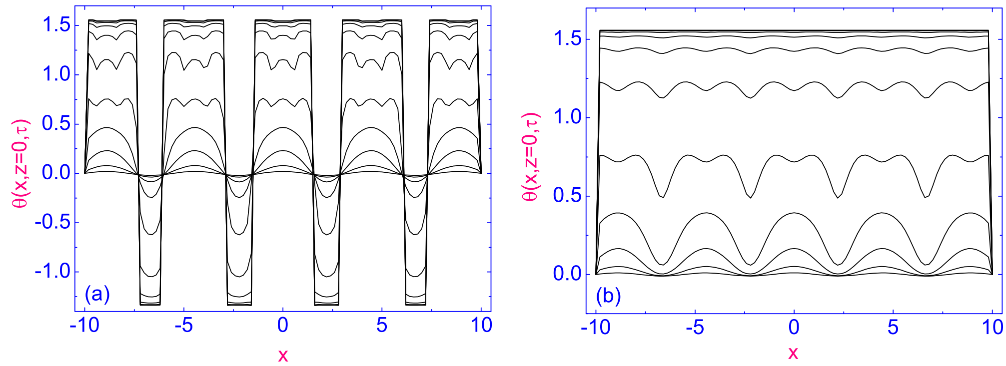

FIG. 7. The same as in Figs. 6(a) and 6(b) but the value of angle α is 1.565, whereas the values of θ_{fl} are 0.02 ($\sim 2.2^\circ$) (a) and 0.01 ($\sim 1.1^\circ$) (b), respectively.

field component E_z, directed perpendicular to the magnetic field, is not enough for maintaining the spatially periodic patterns and enough only for formation of the wavelike deformations in the microsized nematic volume.

Our calculations show that the effect of anchoring the director to horizontal bounding surfaces is reflected, first, in a slight change in the angular profiles of $\theta(x, z, \tau)$ during relaxation and, second, in a slight increase in the relaxation time of $\tau_i(B)$ $(i = 5, 6, 7, 8)$ compared to $\tau_i(A)$ $(i = 1, 2, 3, 4)$. Indeed, the calculated values of relaxation times $\tau_\delta(\Delta)$ $(\delta = 1, ..., 8$ and $\Delta = B, C)$ of the above LC system under the effect of crossed electric ($\sim$1.03 V/μm) and magnetic ($\sim$7.05 T) fields applied at two values of the angle $\alpha = 1.57$ ($\sim 89.96^\circ$) and 1.565 ($\sim 88.81^\circ$), which are collected in Table I, show the slight increase in the values of $\tau_i(B)$ $(i = 5, 6, 7, 8)$ relative to $\tau_i(B)$ $(i = 1, 2, 3, 4)$. In turn, a comparison of calculated and obtained using the NMR spectroscopic technique of relaxation time data $\tau_R(\alpha \sim 88.7^\circ) \sim 20$ ms [see Ref. [15], Fig. 5(a)] shows a good correspondence between these values. Analysis of these results shows that in $4-\alpha\alpha-d_2$ $-$ pentyl $-$ $4'$ $-$ cyanobiphenyl at temperature 300 K and density 10^3 kg/m^3, the application of the large electric field ($\sim$1.03 V/μm) applied at angle α close to the right angle to the magnetic field ($\sim$7.05 T) leads to the values $\tau_R(\alpha) \sim 20 \div 25$ ms for angles $\alpha \sim 88.7^\circ$ and higher.

Thus, our theoretical analysis of the numerical results, based on the predictions of the hydrodynamic theory, including both the director reorientation and charge balance equation, provides evidence for the appearance of spatially periodic patterns in response to the large electric field applied at an angle to the magnetic field. In turn, the large-amplitude periodic distortions modulated in the microsized nematic volume parallel to the horizontal restricted surfaces lead to the increase of the elastic energy of the conservative LC system and, as a result, it causes the decrease of the viscous contribution W_{vis} to the total energy of the LC system. In turn, the decrease of W_{vis} leads to a lower values of the rotational viscosity coefficient $\gamma_1(\text{eff})$ and should lead to the faster relaxation time $\tau_R(\alpha)$, as observed experimentally [5,6].

IV. KINKLIKE DISTORTIONS IN A MICROSIZED NEMATIC VOLUME IMPOSED BY STRONG ELECTRIC FIELD

In the case when the electric field $E(z) \gg E_{th}$ is applied orthogonally to both horizontal bounding surfaces, when angle α is equal to $\pi/2$ and the magnetic field is switched off, the evolution of the director field $\hat{\mathbf{n}}$ in the microsized HAN volume (HANV) described by angle $\theta(z, \tau)$ can be investigated using the dimensionless torque balance equation (7) which reduces to the equation

$$\theta_{,\tau}(z, \tau) = -\frac{E^2(z)}{2} \sin 2\theta(z, \tau). \tag{15}$$

Thus, now our aim is to describe the physical mechanism responsible for the appearance of the traveling distortion wave in the form of the kinklike distortion in the microsized HANV under the effect of electric field $E(z) \gg E_{th}$, when the magnetic field is turned off. In this case, we assume that the distortion in the form of the kinklike wave will propagate along the direction of the electric field with the velocity $w(z, t)$.

To be able to observe the evolution of the traveling disturbance in time with velocity $w(z, t)$, we transform Eq. (15) as follows: by substitutions $\Phi = 2\theta$ and $\kappa = E^2\tau$, the last equation takes the form

$$\Phi_{,\kappa}(z, \kappa) = -\sin \Phi(z, \kappa). \tag{16}$$

TABLE I. The calculated values of relaxation times $\tau_\delta(\Delta)$, where $\delta = 1, ..., 8$ and $\Delta = B, C$.

$\alpha = 1.57$	τ_1(B)	τ_2(B)	τ_5(C)	τ_6(C)
	24.5 ($\sim$31.1 ms)	25.0 ($\sim$31.75 ms)	24.5 ($\sim$31.1 ms)	26.0 ($\sim$33.0 ms)
$\alpha = 1.565$	τ_3(B)	τ_4(B)	τ_7(C)	τ_8(C)
	20.0 ($\sim$25.4 ms)	21.0 ($\sim$26.7 ms)	20.5 ($\sim$26.0 ms)	22.0 ($\sim$28.0 ms)

Among other solutions there exists an exact solution of Eq. (16) in the form of the traveling wave,

$$\Phi(z,\kappa) = \tan^{-1}[\sinh^{-1}(w\kappa - z + z_0)], \tag{17}$$

where z_0 is a constant, and w is the kink wave velocity along axis z.

Through a simple conversion $\partial_\kappa \Phi(z,\kappa) = -\cosh^{-1}(w\kappa - z + z_0)$, and taking into account the relation $\tan\Phi(z,\kappa) = \sinh^{-1}(w\kappa - z + z_0) = \mathcal{A}$, one has that $\sin\Phi(z,\kappa) = \mathcal{A}/\sqrt{1+\mathcal{A}^2} = \cosh^{-1}(w\kappa - z + z_0)$. This in turn allows us to rewrite Eq.(16) in the following way: $\partial_\kappa\Phi(z,\kappa) = -\cosh^{-1}(w\kappa - z + z_0) = -\sin\Phi(z,\kappa)$.

Thus, the solution (17) describes the kinklike wave $\Phi(z,\kappa)$ that propagates along the direction of the electric field with the velocity

$$w(z,t) = \frac{E^2\overline{E}(z)d\epsilon_a\epsilon_0}{\gamma_1} = \frac{\Delta K_1}{d\gamma_1}\overline{E}(z), \tag{18}$$

where $\Delta = (\frac{\pi E}{E_{\rm th}})^2$, $\overline{E}(z) = E(z)/E$, $E = U/d$, and U is the voltage applied across the microsized nematic volume.

Physically, this means that if $E \geqslant E_{\rm th}$ directed across the microsized HANV and the director field $\hat{\mathbf{n}}$ is initially disturbed, for example, at the bottom of the LC cell, with the condition $\theta(z_0 = 0, \tau = 0) = \frac{\pi}{2}$, then this perturbation must be propagated in the form of the kinklike wave along axis z with velocity w. For example, when the electric field $E = 200E_{\rm th} = 0.944$ V/μm is applied across the HANV with the thickness of 194.7 μm, then the kinklike wave velocity w along the z axis is equal to 2.87 m/s.

In the case when $E \to \infty$, we have that $\lim_{E\to\infty}\Delta \to \infty$, and the kink wave velocity is

$$\lim_{E\to\infty} w \to \infty. \tag{19}$$

Note that the reorientation of the director field $\hat{\mathbf{n}}$ in the form of the kinklike distortion wave spreading along the normal to both boundaries probably can be observed in polarized white light. Taking into account that the director reorientation takes place in the narrow area of the LC sample (the width of the kinklike distortion wave), under applied voltage ~200 V across the 5CB film with thickness of 200 μm, the kinklike distortion wave can be visualized in polarized white light as a dark strip running along the normal to both LC boundaries, with velocity $w \sim 3.0\ \mu$m/μs.

Keep in mind that high electric field may lead to avalanche electric breakdown in microsized nematic volumes.

V. CONCLUSION

This paper describes numerical advances in predicting the structural and dynamic behavior of the director field $\hat{\mathbf{n}}$ in a microsized nematic volume subjected to a strong electric field $\mathbf{E}$ applied almost orthogonally to magnetic field $\mathbf{B}$. Despite the fact that certain qualitative and quantitative advances have been made in the description of dynamic reorientation of the director′s field in a thick LC film (~200 μm) under the effect of the large electric field (~1.0 V/μm) directed at angle α to the magnetic field (~7.0 T) [5,6], there are still a number of questions concerning dissipative processes in the confined LC phase with complex boundaries of nematic cells that need to be clarified.

First, in the case when the strong electric field $E \gg E_{\rm th}$ (~1 V/μm) is applied at the angle α close to the right angle to the magnetic field (~7.05 T), our numerical results show that the state of the nematic system becomes unstable and the misalignment of the director with respect to the direction imposed by the aligning magnetic field increases so much that the reorientation caused by the strong $\mathbf{E}$ manifests itself in the growth of one particular Fourier mode. Analysis of these results strongly suggests that the intermediate state is inhomogeneous and perturbed by fluctuations of the nematic field, so in response to the suddenly applied electric field, spatially periodic patterns may appear in initially uniformly aligned nematic domains. These nonuniform rotational modes involve additional internal elastic distortions of the conservative nematic system and, as a result, these deformations decrease the viscous contribution to the total energy of the nematic phase. In turn, the large-amplitude periodic distortions modulated in the microsized volume lead to a lower values of the rotational viscosity coefficient and, as a result, lead to faster reorientation of the director field to its stationary orientation with respect to the electric field. This phenomena was observed experimentally using the time-resolved deuterium NMR spectroscopic measurements [6,15].

Second, when the electric field $E(z) \gg E_{\rm th}$ is applied orthogonally to both horizontal bounding surfaces and the magnetic field is switched off, in the microsized nematic volume the distortion in the form of the kinklike wave spreading normally to the horizontal bounding surfaces with the velocity in a few meters per second can be excited.

It should be noted that with another balance of forces and torques acting per unit LC volume, the formation of the spatially periodic pattern can be realized with another, not close to a right angle, value of the angle α between crossed electric and magnetic fields.

Therefore, further study of a wider range of problems related to understanding how elastic soft matter, such as LCs confined in a microsized volume, begins to deform under the influence of strong crossing electric and magnetic fields, requires additional effort, which will eventually lead to an increase in knowledge in the field of materials science.

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