Intervention strategies for epidemic spreading on bipartite metapopulation networks

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Intervention strategies are of great significance for controlling large-scale outbreaks of epidemics. Since the spread of epidemic depends largely on the movement of individuals and the heterogeneity of the network structure, understanding potential factors that affect the epidemic is fundamental for the design of reasonable intervention strategies to suppress the epidemic. So far, most of previous studies mainly consider intervention strategies on the network composed of a single type of locations, while ignoring the movement behavior of individuals to and from locations that are composed of different types, i.e., residences and public places, which often presents heterogeneous structure. In addition, the transmission rate in public places with different population flows is heterogeneous. Inspired by the above observation, we build a bipartite metapopulation network model and propose intervention strategies based on the importance of public places. With the Markovian Chain approach, we derive the epidemic threshold under intervention strategies. Experimental results show that, compared with the uniform intervention to residences or public places, nonuniform intervention to public places is more effective for suppressing the epidemic with an increased epidemic threshold. Specifically, interventions to public places with large degree can further suppress the epidemic. Our study opens a new path for understanding the spatial epidemic spread and provides guidance for the design of intervention strategies for epidemics in the future.

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I. INTRODUCTION

Rapid spread of epidemic, such as SARS [1,2], H1N1 influenza [3–5], and the ongoing novel coronavirus (COVID-19) [6–9], has led to serious threats for human beings, leading to a significant and widespread socioeconomic disruption [10]. Since the epidemic universally takes a short time from initial infections to a global outbreak, it is necessary to design effective nonpharmacological intervention strategies before the vaccine is successfully developed.

In recent years, a great effort has been devoted to the design of intervention strategies for suppressing the epidemic with the metapopulation model [11–14]. Relying on the reaction-diffusion process, each node represents a patch composed of a population of individuals and each edge describes the route that individuals migration between patches. Studies found that network structure and mobility patterns may affect the geographical diffusion of epidemics [15,16]. For instance, Colizza *et al.* presents a thorough analysis on the metapopulation model characterized by heterogeneous network structure, which can promote the epidemic spread [12]. Balcan *et al.* explored the recurrent mobility patterns, which hinders the epidemic spread with an increased the epidemic threshold [17]. Considering that individuals typically only visit a limited number of places [18–20], mainly commuting between residences and public places such as work location and supermarket, Granell *et al.* proposed a metapopulation model on the bidirectional mobility of individuals between their specific locations and verify the key properties of the epidemic process [21]. Chang *et al.* proposed a metapopulation model by considering bidirectional movements of individuals, verifying that people who tend to visit denser locations provide insightful information for the design of effective interventions for the spatial epidemics spread [22]. In reality, considering the heterogeneous connection structure of public places and the heterogeneous distribution of transmission rate in them, it is necessary to develop reasonable interventions in order to effectively control the epidemic. For instance, Matsuki *et al.* proposed an intervention strategy by considering the risk of each patch and classifies the patches as high-risk and low-risk ones. It shows that intervening low-risk patches is effective to prevent a global epidemic outbreak [23]. Recently, Wang *et al.* proposed a nonuniform intervention strategy by giving priority to patches based on their importance [24]. In addition, interventions based on individual behavior such as isolation [25], detection and contact tracing [26–29], and vaccination [30,31] can also mitigate transmission. For instance, Aleta *et al* shows that travel restrictions of individuals as well as the deployment of vaccines is able to delay the peak of the epidemic and reduce the attack rate.

In real life, individuals migrate between two types of locations and the connections of public places are often heterogeneous. Studies show heterogeneous networks accelerate the epidemic spread [11,15]. However, most of present studies on interventions are based on the assumption that individuals migrate between locations of the same type [32–36], while ignoring different types of locations [20,37], such as residences and public places. Although a few studies consider

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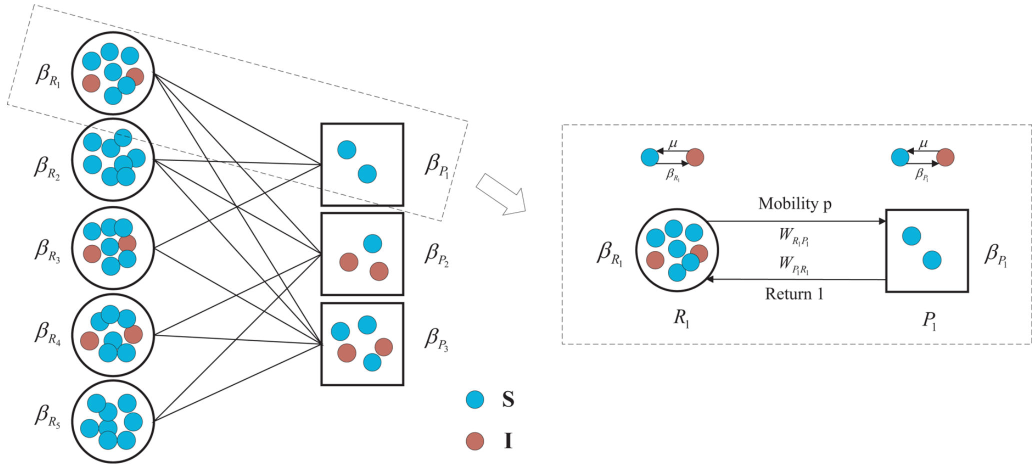

FIG. 1. Schematic framework of the metapopulation model with heterogeneous distribution of transmission rate. The bipartite network is composed of $N_r = 5$ residences and $N_p = 3$ public places. For the reaction diffusion process, there are two different stages at each time step t. In the day stage, individuals in residence R_i move to public place P_j with probability p according to the transfer matrix $W_{R_iP_j}$. They mix uniformly in residences or public places. In the night stage, after returning to their residences, individuals mix again in their residences.

different types of locations [21–24], they simplify the bipartite network as a complete graph, neglecting the impact of the network heterogeneity on the epidemic spread. Based on the above discussion, we build a metapopulation model on heterogeneous bipartite networks to explore how to design reasonable intervention strategies to suppress the epidemic spread. In addition, the contact rate in public places with different population flows is heterogeneous, resulting in the difference of the transmission rate in public places [22]. Therefore, each public place should implement appropriate intervention strategies based on the population flow, that is, the implementation of intervention to public places is nonuniform. Prioritizing intervention to high-traffic public places would be beneficial to suppress the epidemic spread. Therefore, we propose a nonuniform intervention strategy based on the importance of public places. With the Markovian chain approach [38,39], we derive the epidemic threshold under intervention strategies. Experiment results show that interventions to public places with large degree are more effective to hinder epidemics.

This paper is organized as follows. In Sec. II, we first propose a metapopulation model with recurrent mobility patterns on a bipartite graph, and then uniform and nonuniform interventions are introduced. Next, we demonstrate the susceptible-infective-susceptible (SIS) model on bipartite networks under interventions and derive the epidemic threshold under different interventions. The effectiveness of different interventions on the epidemic spread is verified in Sec. III. We summarize our work in Sec. IV.

II. A BIPARTITE METAPOPULATION MODEL WITH INTERVENTIONS

A. Model description

To study intervention strategies for suppressing the epidemic, we start by establishing a metapopulation model on bipartite networks, as shown in Fig. 1. First, we consider a bipartite network composed of N_r residences and N_p public places, satisfying the condition $N_r > N_p$. Due to the different migration behavior and the heterogeneity connections of public places, the bipartite network is assumed to be weighted and heterogeneous, encoded in an adjacency matrix, whose entry $w_{R_iP_j}$ defines the weight between residence R_i and public place P_j. Then residence R_i is the home of a number of individuals n_{R_i}, so that the total population of the network is $V = \sum_{i=1}^{N_r} n_{R_i}$. To reflect the heterogeneous distribution of the transmission rate, we assume that the transmission rate of residences and public places are β_{R_i} for $i = 1, \dots, N_r$ and β_{P_j} for $j = 1, \dots, N_p$, respectively.

To simulate the real observation that individuals leave their residences to public places in the day and return to their residences in the night, the dynamical process is described as two different stages at each time step t, that is, the day stage and the night stage. First, in the day stage, with probability p, individuals in residence R_i move to public place P_j according to the matrix $W_{R_iP_j}$, or with probability $(1 - p)$, they remain in their original residence R_i. With the SIS model, each susceptible individual gets infected by contacting with infectious individuals in residence R_i (public place P_j) with probability β_{R_i} (β_{P_j}). Next, in the night stage, all the individuals who left their residences return to their residences, where susceptible individuals get infected on contact with infected individuals with probability β_{R_i} and infectious individuals recover with probability μ. Here we assume the day stage and the night stage as one time step [40]. It is possible to take them as two separate steps. Due to the migration of individuals to public places in the day time, the number of individuals in residences changes in the day time, which is not suitable for the Markov chain method.

B. Network structure-based intervention strategies

Facing with pandemic outbreaks, how to design rapid and effective interventions to hinder the epidemic spread is a complicated problem. Based on the observation that a highly connected public place usually plays a more fundamental role in suppressing the epidemic spread [41], we propose a nonuniform intervention strategy based on the importance of public places by adjusting the transmission rate of public places. To simplify the analysis, the efficient transmission rate of all residences is assumed constant as β_r and that of public places is assumed as β_p, which can be described as $\beta_p = m\beta_r$ with $m > 0$. After intervening public place P_j, the transmission rate of public place P_j is updated as β_{P_j}, expressed as

$$\beta_{P_j} = c_p\theta_{P_j}\beta_p,\ j = 1, 2, \ldots, N_p, \tag{1}$$

where c_p $(0 \leqslant c_p \leqslant 1)$ represents the intervention factor for public places. The parameter θ_{P_j} represents the intervention priority factor for public place P_j, expressed as a function of public place P_j's degree, k_{P_j}, and tuned by parameter α as follows:

$$\theta_{P_j}(k_{P_j}, \alpha) = \begin{cases} 1, & \text{if} \quad k_{P_j}^{\alpha} > \langle k_p^{\alpha}\rangle, \\ \frac{k_{P_j}^{\alpha}}{\langle k_p^{\alpha}\rangle}, & \text{if} \quad k_{P_j}^{\alpha} < \langle k_p^{\alpha}\rangle, \end{cases} \tag{2}$$

for $j = 1, 2, \ldots, N_p$. $\langle k_p^{\alpha}\rangle = \sum_{k_p} k_p^{\alpha} P(k_p)$ represents the αth-order moment of degree k_p and $P(k_p)$ represents the degree distribution of public places. It indicates that intervening a public place or not is determined by comparing the $k_{P_j}^{\alpha}$ with the αth-order moment of degree k_p, $\langle k_p^{\alpha}\rangle$. If the degree of public place P_j, k_{P_j}, satisfies the condition $k_{P_j}^{\alpha} < \langle k_p^{\alpha}\rangle$, then it will be intervened with proportion factor $\frac{k_{P_j}^{\alpha}}{\langle k_p^{\alpha}\rangle}$; otherwise, there is no need to be intervened. By doing so, not all the public places are necessary to be intervened.

Specifically, if $\alpha = -1$, then public places are classified as the ones with degree larger than the average degree $\langle k_p\rangle$ and the other ones with degree less than the average degree $\langle k_p\rangle$. The intervention factor is expressed as follows:

$$\theta_{P_j}(k_{P_j}, \theta_0) = \begin{cases} 1, & \text{if} \quad k_{P_j} > \langle k_p\rangle, \\ \theta_0, & \text{if} \quad k_{P_j} < \langle k_p\rangle, \end{cases} \tag{3}$$

for $j = 1, 2, \ldots, N_p$. That is, only patches with degree larger than the average degree are intervened with factor θ_0. We will present a simple example of Appendix A for details.

In order to understand how the nonuniform intervention implements in the network and what kind of patches are priorly intervened, we show the intervention factor θ_{P_j} as a function of public place P_j's degree k_{P_j} for different values of α in Fig. 2. If $\alpha < 0$ [Fig. 2(a)], then only public places with large degree are intervened with priority with more intensive scale. Therefore, the transmission rate in public places with large degree is less than that with lower degree, while with $\alpha > 0$ [Fig. 2(b)], it is converse. Additionally, the greater the absolute of α, the smaller the transmission rates after interventions will be.

To compare with nonuniform intervention to public places, we also propose a uniform intervention to public places by setting $\theta_{P_j} = 1 (j = 1, 2, \ldots, N_p)$ in Eq. (1). Specifically, with $c_p = 1$ and $\theta_{P_j} = 1 (j = 1, 2, \ldots, N_p)$, it indicates that no intervention is performed in public places.

Except for the intervention to public places, it is also possible to intervene residences. Since the connections of residences to public places are homogeneously distributed, we consider a uniform intervention to residences. After intervening residence R_i, the transmission rate in residences R_i is updated as β_{R_i}, expressed as

$$\beta_{R_i} = c_r\beta_r,\ i = 1, 2, \ldots, N_r, \tag{4}$$

where c_r $(0 \leqslant c_r \leqslant 1)$ represents the intervention factor for residences. Specifically, when $c_r = 1$, it indicates that no intervention is performed in residences.

According to the intervention strategies proposed above, we next implement two types of intervention scenarios. One is to intervene one type of location, that is, nonuniform or uniform intervention to public places, and unform intervention to residences. The other is to simultaneously intervene two types of locations, that is, uniform intervention to both residences and public places and nonuniform intervention to public places combined with uniform intervention to residences. To make the comparison reasonable, the transmission rates after intervention has to be tuned the same way, which we will introduce in detail in Sec. III.

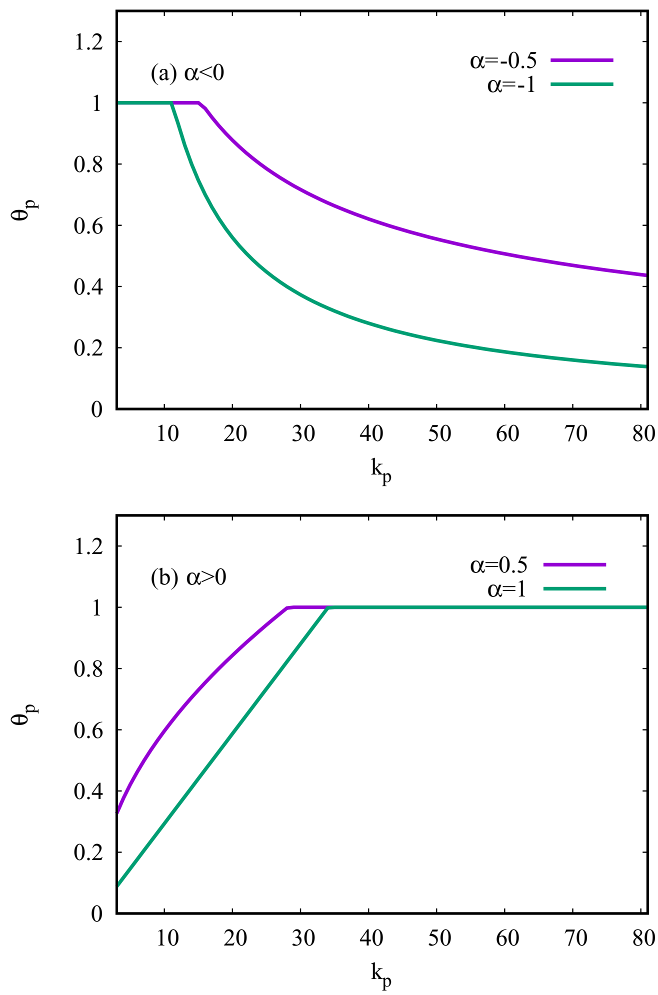

FIG. 2. Intervention priority factor $\theta_p(k_p, \alpha)$ versus degree k_p when nonuniform intervention to public places in Eq. (2). (a) $\alpha < 0$; (b) $\alpha > 0$.

C. The SIS model on bipartite networks under interventions

To calculate the epidemic threshold λ_c under different interventions, we use the Markovian chain approach to solve this problem. For the SIS model, we have a set of N_r variables,

$\rho_{R_i}(t)$, $i = 1, 2, \dots, N_r$, denoting the proportion of infected individuals in residence R_i at time t. Therefore, the proportion of susceptible individuals associated with residence R_i at time t is $1-\rho_{R_i}(t)$. The time evolution of $\rho_{R_i}(t)$ can be described as follows:

$$\rho_{R_i}(t+1) = \rho_{R_i}(t)(1-\mu) + [1-\rho_{R_i}(t)]\Pi_{R_i}(t), \tag{5}$$

for $i = 1, 2, \dots, N_r$, where $\rho_{R_i}(t+1)$ represents the proportion of infected individuals in residence R_i at time $t+1$. The first term of Eq. (5) represents the proportion of infected individuals associated to R_i that did not get recovered at time t. The second term represents the probability that susceptible individuals get infected at time t with probability $\Pi_{R_i}(t)$, which is written as

$$\begin{aligned}\Pi_{R_i}(t) = {} & (1-p)\Phi^{\circledcirc}_{R_i}(t) + p\sum_{j=1}^{N_p} W_{R_iP_j}\Phi^{\circledcirc}_{P_j}(t) \\ & + p\sum_{j=1}^{N_p} W_{R_iP_j}[1-\Phi^{\circledcirc}_{P_j}(t)]\Phi^{\bigstar}_{R_i}(t) \\ & + (1-p)[1-\Phi^{\circledcirc}_{R_i}(t)]\Phi^{\bigstar}_{R_i}(t),\end{aligned} \tag{6}$$

where $\Phi^{\circledcirc}_{R_i}(t)[\Phi^{\bigstar}_{R_i}(t)]$ represents the probability that individuals get infected in residence R_i in the day (night) stage. $\Phi^{\circledcirc}_{P_j}(t)$ represents the probability that individuals get infected in public place P_j. The first term of Eq. (6) represents the probability that individuals remain at residence R_i and get infected. The second term represents the probability that individuals move from residence R_i to public place P_j and get infected with probability $\Phi^{\circledcirc}_{P_j}(t)$. The third term represents the probability that individuals did not get infected in public place P_j but get infected after returning to residence R_i. The last term represents the probability that individuals who remained at residence R_i are not infected in the day stage but get infected in the night stage. $\Phi^{\circledcirc}_{R_i}(t)$ and $\Phi^{\bigstar}_{R_i}(t)$ can be expressed as

$$\Phi^{\circledcirc}_{R_i}(t) = 1-[1-\beta_{R_i}\rho_{R_i}(t)]^{n_{R_i\to R_i}}, \tag{7}$$

$$\Phi^{\bigstar}_{R_i}(t) = 1-[1-\beta_{R_i}\rho_{R_i}(t)]^{n_{R_i}}. \tag{8}$$

Taking Eq. (7) as an example, the second term on the right-hand side $[1-\beta_{R_i}\rho_{R_i}(t)]^{n_{R_i\to R_i}}$ quantifies that the probability that an individual who remained in residence R_i does not get infected by infected individuals. Therefore, $1-[1-\beta_{R_i}\rho_{R_i}(t)]^{n_{R_i\to R_i}}$ represents the probability that an individual gets infected in residence R_i in the day stage. Similarly, $\Phi^{\circledcirc}_{P_j}(t)$ is expressed as

$$\Phi^{\circledcirc}_{P_j}(t) = 1-\prod_{i=1}^{N_r}[1-\beta_{P_j}\rho_{R_i}(t)]^{n_{R_i\to P_j}}, \tag{9}$$

$j = 1, 2, \dots, N_p$, where $[1-\beta_{P_j}\rho_{R_i}(t)]^{n_{R_i\to P_j}}$ quantifies the probability that an individual does not get infected by contacting infected individuals migrated to public place P_j. So $1-[1-\beta_{R_i}\rho_{R_i}(t)]^{n_{R_i}}$ represents the probability that an individual gets infected in public place P_j in the day stage.

In Eqs. (7) and (9), $n_{R_i\to R_i}$ represents the number of individuals who remained in residence R_i, and $n_{R_i\to P_j}$ represents the number of individuals who moved from residence R_i to public place P_j. Since individuals stochastically leave the residence with probability p, the probability of k individuals who left residence R_i follows a binomial distribution, expressed as

$$P(X=k) = C^k_{n_{R_i}} p^k (1-p)^{n_{R_i}-k}, \tag{10}$$

for $i = 1, \dots, N_r$, the expectation of which is $E(X) = n_{R_i}p$.

Therefore, the number of individuals who remained in residence R_i is $n_{R_i\to R_i} = n_{R_i}(1-p)$, and the number of individuals who left residence R_i is $n_{R_i}p$. The number of individuals who moved to public place P_j from residence R_i is $n_{R_i\to P_j} = n_{R_i}pW_{R_iP_j}$, where $W_{R_iP_j} = \frac{w_{R_iP_j}}{w_{R_i}}$ and $w_{R_i} = \sum_j w_{R_iP_j}$. $w_{R_iP_j}$ represents the edge weight between residence R_i and public place P_j.

D. The epidemic threshold under intervention

In this section, the analysis of the epidemic threshold is carried out under the scenario of nonuniform interventions to public places, while uniform intervention to residences. In Eq. (5), when the system reaches the steady state, we assume that $\rho_{R_i}(t+1) = \rho_{R_i}(t) = \rho_{R_i}$. Since the number of infected individuals is negligible near the epidemic threshold, we have $\rho_{R_i} = \epsilon_{R_i} \ll 1$. Equation (5) can then be simplified as

$$\epsilon_{R_i} = \epsilon_{R_i}(1-\mu) + (1-\epsilon_{R_i})\Pi_{R_i}. \tag{11}$$

Substituting Π_{R_i} into Eq. (11), it can be rewritten as

$$\begin{aligned}\epsilon_{R_i} = {} & (1-\mu)\epsilon_{R_i} + (1-\epsilon_{R_i})[(1-p)\Phi^{\circledcirc}_{R_i} \\ & + p\sum_{j=1}^{N_p} W_{R_iP_j}\Phi^{\circledcirc}_{P_j} + p\sum_{j=1}^{N_p} W_{R_iP_j}(1-\Phi^{\circledcirc}_{P_j})\Phi^{\bigstar}_{R_i} \\ & + (1-p)(1-\Phi^{\circledcirc}_{R_i})\Phi^{\bigstar}_{R_i}].\end{aligned} \tag{12}$$

Substituting Eqs. (7), (8), and (9) into Eq. (12), we have

$$\begin{aligned}\epsilon_{R_i} = {} & (1-\mu)\epsilon_{R_i} + (1-\epsilon_{R_i})[(1-p)[1-(1-\beta_{R_i}\epsilon_{R_i})^{n_{R_i\to R_i}}] \\ & + p\sum_{j=1}^{N_p} W_{R_iP_j}\left[1-\prod_{k=1}^{N_r}(1-\beta_{P_j}\epsilon_{R_k})^{n_{R_k\to P_j}}\right] \\ & + p\sum_{j=1}^{N_p} W_{R_iP_j}\left(1-\left\{\left[1-\prod_{k=1}^{N_r}(1-\beta_{P_j}\epsilon_{R_k})^{n_{R_k\to P_j}}\right]\right\}\right) \\ & \times\{1-[1-\beta_{R_i}\epsilon_{R_i}(t)]^{n_{R_i}}\} \\ & + (1-p)[1-(\{1-[1-\beta_{R_i}\epsilon_{R_i}(t)]^{n_{R_i\to R_i}}\})] \\ & \times\{1-[1-\beta_{R_i}\epsilon_{R_i}(t)]^{n_{R_i}}\}],\end{aligned} \tag{13}$$

and when $\epsilon_{R_i} \ll 1$, we can get $(1-\rho_{R_i})^n \approx 1-n\rho_{R_i}$, and then $\prod_{i=1}^{N_r}(1-\rho_{R_i})^n \approx 1-\sum_i n\rho_{R_i}$. Equation (13) can be rewritten as

$$\begin{aligned}\epsilon_{R_i} = {} & (1-\mu)\epsilon_{R_i} + (1-\epsilon_{R_i})[(1-p)^2 n_{R_i}\beta_{R_i}\epsilon_{R_i} \\ & + p\sum_{j=1}^{N_p} W_{R_iP_j} n_{R_i}\beta_{R_i}\epsilon_{R_i} \\ & + p^2\sum_{j=1}^{N_p}\sum_{k=1}^{N_r} W_{R_iP_j}W_{R_kP_j}n_{R_k}\beta_{P_j}\epsilon_{R_k} + (1-p)n_{R_i}\beta_{R_i}\epsilon_{R_i}].\end{aligned} \tag{14}$$

By replacing $\beta_{R_i} = c_r\beta_r$ and $\beta_{P_j} = c_p\theta_{P_j}m\beta_r$, we can obtain

$$\begin{aligned}\epsilon_{R_i} = (1-\mu)\epsilon_{R_i} + (1-\epsilon_{R_i})[(1-p)^2 n_{R_i} c_r\beta_r\epsilon_{R_i} \\ + p\sum_{j=1}^{N_p} W_{R_iP_j} n_{R_i} c_r\beta_r\epsilon_{R_i} \\ + p^2\sum_{j=1}^{N_p}\sum_{k=1}^{N_r} W_{R_iP_j}W_{R_kP_j} n_{R_k} c_p\theta_{P_j} m\beta_r\epsilon_{R_k} \\ + (1-p)n_{R_i}c_r\beta_r\epsilon_{R_i}]. \end{aligned} \quad (15)$$

Equation (15) is expressed in the form of vector ϵ^* as

$$\frac{\mu}{\beta_r}\epsilon^* = M\epsilon^*. \quad (16)$$

Thus, the propagation threshold β_r^c can be expressed as the form of the maximum eigenvalue of the matrix M,

$$\beta_r^c = \frac{\mu}{\Lambda_{\max}(M)}, \quad (17)$$

where the matrix M reads as

$$\begin{aligned}M_{R_iR_k} = [(1-p)^2 c_r n_{R_i} + c_r n_{R_i}]\delta_{R_iR_k} \\ + p^2\sum_{j=1}^{N_p} W_{R_iP_j}W_{R_kP_j} m c_p\theta_{P_j} n_{R_k},\end{aligned} \quad (18)$$

where $\delta_{R_iR_k} = 1$ if $i = k$; otherwise, $\delta_{R_iR_k} = 0$. We see that the entry $M_{R_iR_k}$ of the matrix M is determined by the initial population distribution n_{R_i}, the mobility rate p, the transfer matrix W, and the intervention factors for public places, c_p and θ_{P_j}, and that for residence c_r.

If $\theta_{P_j} = 1$, $j = 1, 2, \ldots, N_p$ and $0 < c_r = c_p < 1$, then it represents uniform intervention to both residences and public places. Then the matrix M can be rewritten as

$$\begin{aligned}M_{R_iR_k} = [(1-p)^2 c_r n_{R_i} + c_r n_{R_i}]\delta_{R_iR_k} \\ + p^2\sum_{j=1}^{N_p} W_{R_iP_j}W_{R_kP_j} m c_p n_{R_k}.\end{aligned} \quad (19)$$

If $\theta_{P_j} = 1$, $j = 1, 2, \ldots, N_p$ and $c_r = c_p = 1$, then it indicates that no intervention is implemented. Then the matrix M can be rewritten as

$$\begin{aligned}M_{R_iR_k} = [(1-p)^2 n_{R_i} + n_{R_i}]\delta_{R_iR_k} \\ + p^2\sum_{j=1}^{N_p} W_{R_iP_j}W_{R_kP_j} m n_{R_k},\end{aligned} \quad (20)$$

which is consistent with the results in Ref. [21].

Based on the above discussion, we see that the epidemic threshold under intervention depends on different choices of the intervention factors c_r and c_p and the intervention priority θ_{P_j}. To further understand the impact of interventions on the epidemic threshold, we analyze the effect of different interventions on the epidemic threshold in detail by providing a simple network in Appendix A.

III. SIMULATION RESULTS

In order to explore the impact of different intervention strategies on the epidemic spread, we perform Monte Carlo (MC) simulations and verify the Markovian equations for the bipartite metapopulation network model with interventions. The detailed MC simulation process is provided in Appendix B. In reality, since the number of residences is usually more than that of public places, we assume that the network is composed of $N_r = 200$ residences and $N_p = 100$ public places. The total population is $V = 5000$. The degree distribution of public places $P(k_p)$ follows $P(k_p) \propto k_p^{-2.1}$, where the maximum and the minimum degree are $k_{p,\max} = 81$ and $k_{p,\min} = 2$. The average degrees of residence and public place are $\langle k_r\rangle = 17$ and $\langle k_p\rangle = 34$, respectively. Since the initial population distribution affects the epidemic spread, we consider two types of initial population distributions, that is, homogeneous distribution (HOD), where each residence has the same number of individuals, i.e., $n_{R_i} = 25$, and heterogeneous distribution (HED), where the number of population in the residence is proportional to its weight. The flux weight between residence R_i and pubic places P_j, $w_{R_iP_j}$, is randomly distributed in the range [1,50]. The initial transmission rate of residences and public places is $\beta_p = m\beta_r$ with $m = 1.25$, which is consistent with the real observation that the transmission rate in public places is higher than that in residences. At the beginning of the simulation, there are 50 infected individuals and no recovered individuals. The mobility rate is $p = 0.4$. The recovery rate is $\mu = 0.2$ in simulations. Here we take the day-night cycle within a day, as a time step may cause the deviation of the discrete-time model from the continuous-time model [40], while the smaller transmission rate and the recovery rate may reduce the deviation, which is further verified in the MC simulation results.

A. Interventions to public places or residences alone

In this section, we will explore the effectiveness of intervention to public places or residences alone. First, we study nonuniform (NUP) and uniform (UP) intervention strategies to public places alone. Then we analyze uniform intervention to residences alone (UR), since the connections of residences are homogeneous distribution.

To study the influence of priority and intensity of NUP, we perform experiments under the conditions of the HOD and HED for different choices of α in Fig. 3. The dots represent the Monte Carlo simulation results and the curves are the results of the Markovian equations. The vertical lines are the theoretical threshold derived from Eqs. (17) and (18) for different choices of α. In Fig. 3(a), we see that for different α, the epidemic threshold and the final proportion of infections show obvious difference. Specifically, compared with no intervention ($\alpha = 0$), intervening public places with large degree ($\alpha < 0$) reduces the proportion of infections and increases the epidemic threshold. For instance, the epidemic threshold increases from 0.004 to 0.006 with $\alpha = -1$. Moreover, a larger absolute value of α results in a higher epidemic threshold. However, it is ineffective to intervene public places with small degree ($\alpha > 0$). This is because only a few

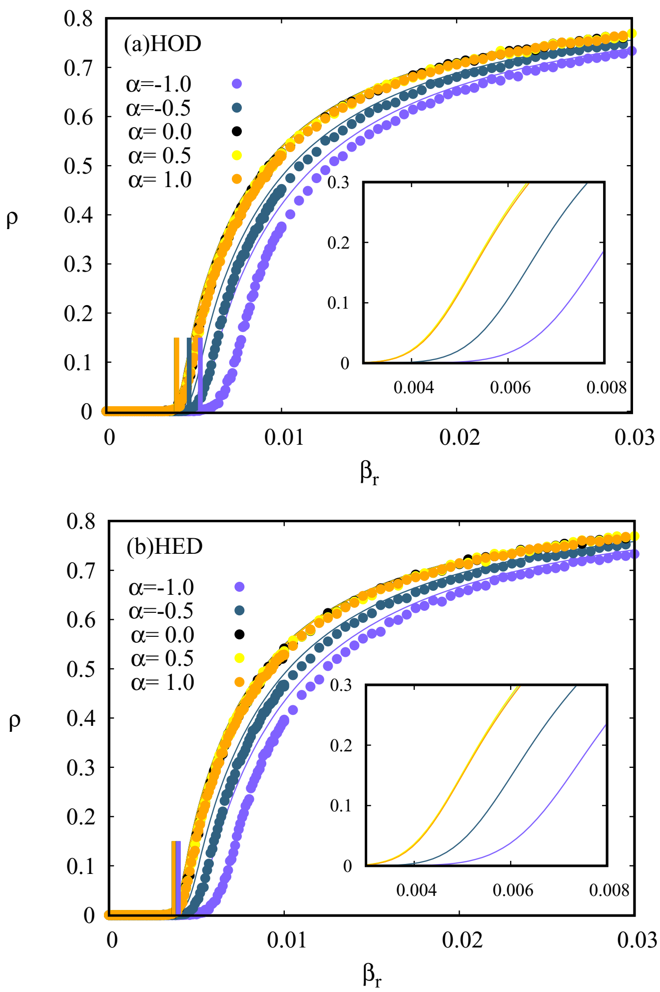

FIG. 3. NUP tuned by parameter α under different initial population distributions. (a) HOD. (b) HED. The migration probability is $p = 0.4$. The recovery probability is $\mu = 0.2$. The intervention factors for residences and public places are $c_r = 1$ and $c_p = 1$, respectively. The inset amplifies the curves when β is in the range of [0.002, 0.008].

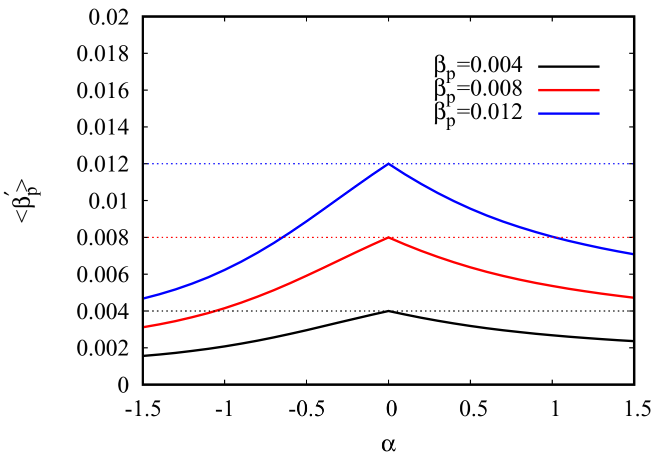

FIG. 4. The average transmission rate of public places after NUP $\langle \beta_p' \rangle$ for different α. The initial infection rates in public places are $\beta_p = 0.004$ (purple), $\beta_p = 0.008$ (green), and $\beta_p = 0.012$ (blue), respectively. The dotted lines represent $\langle \beta_p' \rangle$ with $\alpha = 0$ for $\beta_p = 0.004$ (purple), $\beta_p = 0.008$ (green), and $\beta_p = 0.012$ (blue).

individuals reach public places with small degree. Thus, the infections before and after intervention do not change much. In summary, intervening public places with large degree can more effectively control epidemics. The epidemic threshold increased and the final proportion of infected individuals decreased after intervention with the increase of the absolute of the parameter $\alpha (\alpha > 0)$ under the condition of the HOD [Fig. 3(a)] and the HED [Fig. 3(b)], since the connections of the residence are homogeneous, it leads to the uniform migration of individuals from residences to public places. However, the gap between the thresholds before and after interventions for the HED is less than that for the HOD. It notes that NUP for the HED is not as effective as that for the HOD. Since the results of the HOD and HED are similar, in the following, we will take the HOD as an example.

To compare the effectiveness of UP and NUP, it is necessary to keep the average transmission rate of public places after intervention $\langle \beta_p' \rangle$ the same. Figure 4 shows the relationship between $\langle \beta_p' \rangle$ and α. When $\alpha = 0$, all the public places have the same infection rate for a given β_p, i.e., $\langle \beta_p' \rangle = \beta_p$. In addition, $\langle \beta_p' \rangle$ decreases as the increase of the absolute value of α. The average transmission rate after intervention to public places with large degree ($\alpha < 0$) is less than that with small degree ($\alpha > 0$), since intervention to public places with large degree is more intensive.

Next, we explore the effectiveness of UP alone in suppressing the epidemic. Figure 5(a) shows the impact of UP with different factors c_p. Clearly, compared with no intervention ($c_p = 1$ and $\alpha = 0$), the more intensive the intervention to public places (small c_p) is, the larger the epidemic threshold will be. It means that intervening to pubic places with same intensity can effectively delay the epidemic, while it has no obvious effect on the final outbreak size.

To compare the difference between UP and NUP in suppressing the epidemic, $\alpha = 1, c_p = 0.67$ and $\alpha = -1, c_p = 0.52$ are set to satisfy the condition of the same transmission rate after intervention, $\langle \beta_p' \rangle$. In Fig. 5(b), compared with no intervention ($\alpha = 0$ or $c_p = 1$), both UP and NUP with large degree ($\alpha < 0$) can increase the epidemic threshold. Under the condition of the same average transmission rate after interventions, i.e., $\alpha = -1$ and $c_p = 0.52$, NUP with large degree ($\alpha < 0$) can more effectively control the epidemic. In contrast, when $\alpha = 1$ and $c_p = 0.67$, NUP with small degree ($\alpha > 0$) is less effective than uniform intervention. In summary, intervening public places with large degree has a better effect on the control of epidemic. In reality, redistributing the flow of public places with large degree is expected to the efficient control of epidemic.

Next, to fully understand the effect of intervention strategies on the epidemic, we explore UR alone. Figure 6 shows the impact of UR with different factor c_r. Compared with no intervention ($c_r = 1$, black curve), more intensive intervention to residences (controlled by c_r) can suppress the epidemic in terms of a larger epidemic threshold and a smaller infection scale. In addition, compared with UP, UR is more effective

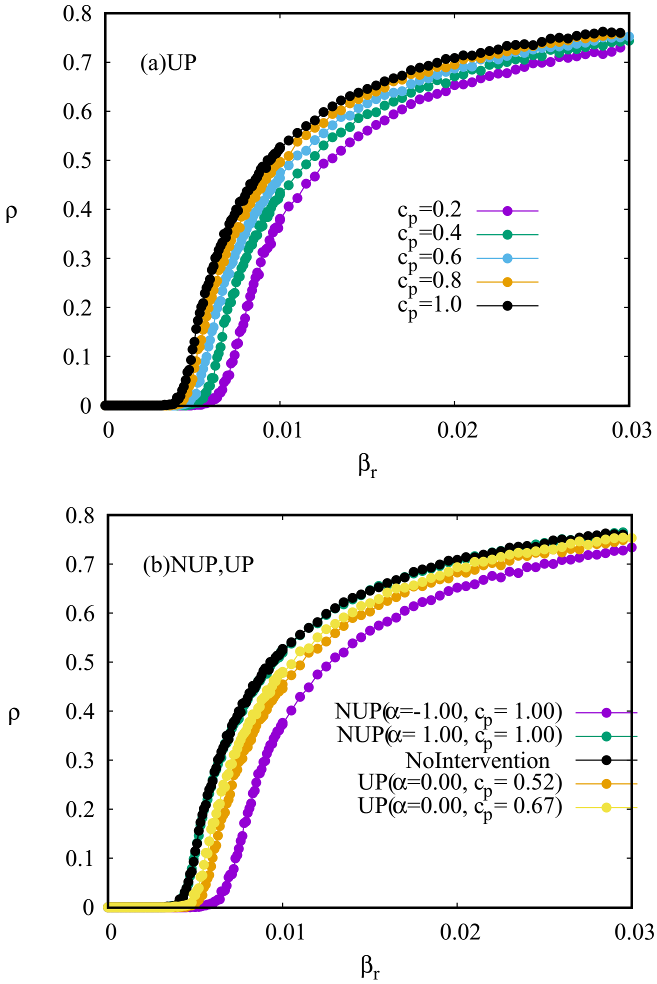

FIG. 5. (a) Uniform intervention to public places for different intensity c_p with $\alpha = 0$, while residences are not intervened with $c_r = 1$. (b) Comparison of UP and NUP. The intervention factor for residences is $c_r = 1$. Other parameters are set same as in Fig. 3.

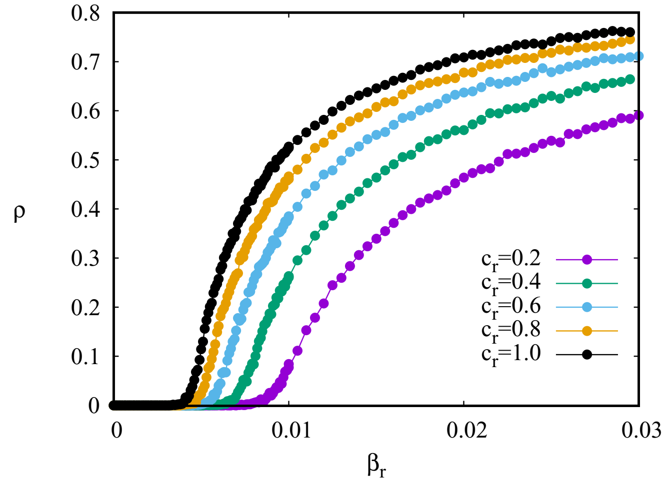

FIG. 6. Uniform intervention to residences for different intervention intensity c_r with $\alpha = 0$, while public places are not intervened with $c_p = 1$ and $\alpha = 0$. Other parameters are set same as in Fig. 5.

in suppressing the epidemic. This result is expected, since the number of residences is more than that of public places, resulting in the reduction of the final proportion of infected individuals in the network.

In all, when intervening one type of locations alone, NUP with large degree ($\alpha < 0$) is more effective in curbing the epidemic spread, compared with UP or UR alone. Moreover, both UP and UR have no obvious difference in the terms of epidemic scales.

B. Simultaneous interventions to both residences and public places

To further explore the effectiveness of intervention to both residences and public places, we also implement two intervention strategies: One scenario is uniform intervention to both public places and residences (UR + UP), and the other scenario is nonuniform intervention to public places combined with uniform intervention to residences (NUP + UR).

To make the comparison reasonable, we introduce the average transmission rate of the network after intervention $\langle \beta' \rangle$ expressed as

$$\langle \beta' \rangle = \frac{N_r c_r \beta_r + N_p c_p \langle \beta_p \rangle \beta_p}{N_r + N_p}, \tag{21}$$

where $N_r + N_p$ is the total number of residences and public places in the network. The first and second terms of the numerator in Eq. (21) represents the sum of transmission rates of residences and public places after intervention.

To explore the effectiveness of uniform intervention to both public places and residences (UR + UP), we compare it with UR or UP with the same $\langle \beta' \rangle$. We take no-intervention ($c_r = 1$ and $c_p = 1$, black curve) as the baseline model. Different intervention strategies are represented by different values of c_r and c_p, as shown in Fig. 7. We see that compared with no intervention, UR + UP ($c_r = 0.84$ and $c_p = 0.84$, $c_r = 0.89$, dark blue and $c_p = 0.89$, red curves) slightly increases the epidemic threshold. Compared with UR (orange curve) or UP

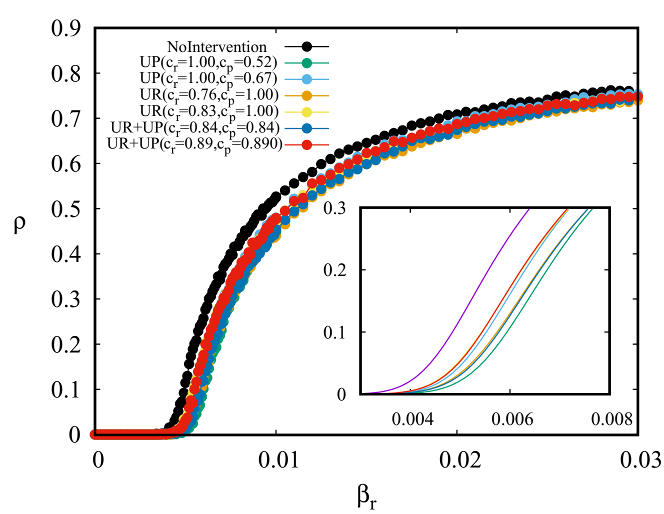

FIG. 7. UR + UP. The intervention factors for public places are $\alpha = 0$. Other parameters are set same as in Fig. 5. The inset amplifies the curves for β in [0.004, 0.008].

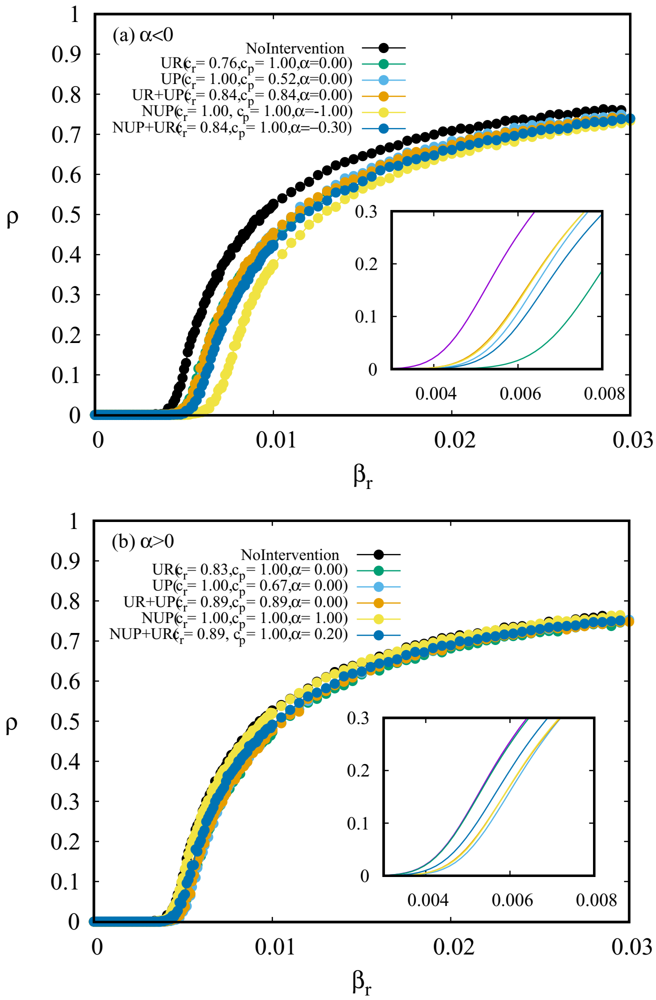

FIG. 8. NUP + UR. (a) $\alpha < 0$; (b) $\alpha > 0$. Other parameters are set same as in Fig. 5. The inset amplifies curves when β is from 0.004 to 0.008.

(green curve), UR + UP (dark blue curve) plays similar role in suppressing the epidemic.

Next, we study the effectiveness of UP, UR, NUP, UR + UP, and NUP + UR in curbing the epidemic spread. First, we analyze the impact of nonuniform intervention to public places with large degree combined with uniform intervention to residences (NUP + UR) in suppressing the epidemic spread in Fig. 8(a). It can be seen that compared with UP (blue curve), UR (orange curve), or UR + UP (yellow curve), NUP (green curve) with large degree ($\alpha < 0$) performs best in terms of a higher epidemic threshold, while NUP with small degree ($\alpha > 0$) does not show obvious advantage and all the above-mentioned interventions show similar performance, see Fig. 8(b). Moreover, in Fig. 8(a) or Fig. 8(b), there is no obvious difference in curbing the epidemic between NUP + UR (oxford blue curve) and UR + UP (yellow curve). It indicates that in reality, intervening public places with more population flux would be more effective in the control of the epidemic.

In summary, when intervening two type of locations, the effectiveness of the scenario of UR + UP and the scenario of NUP + UR performs similarly to that of UR or UP and does

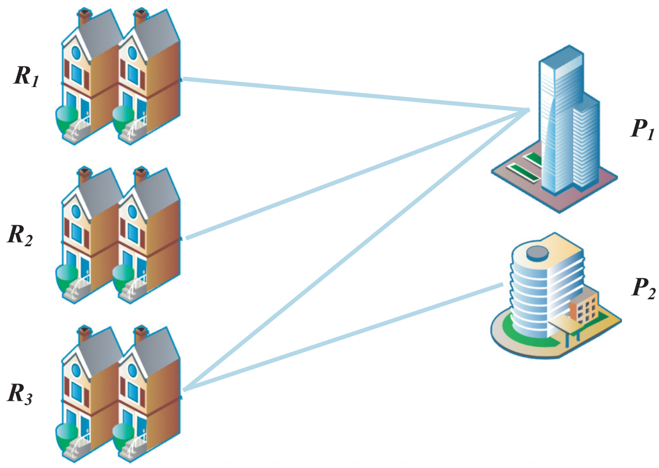

FIG. 9. A metapopulation network with three residences and two public places.

not show obvious advantage both in terms of the epidemic threshold as well as the infection scale. In all, NUP with large degree ($\alpha < 0$) plays the most effective role in suppressing the epidemic compared with the above mentioned interventions. In contrast, NUP with small degree ($\alpha > 0$) is less effective.

IV. DISCUSSION AND CONCLUSION

So far, nonpharmacological intervention strategies are still the most effective methods to suppress the epidemic until sufficient available of vaccine. Therefore, it is important for designing effective intervention strategies to suppress the epidemic spread. In addition, mobility of individuals mainly goes to and from locations of two types, that is, residences and public places, further complicates the design of effective interventions to the spatial spread of epidemic. So far, it has found that public places with higher flow contain a larger number of infected individuals. Therefore, designing intervention strategies according to the importance of locations is fundamental to curb the epidemic spread.

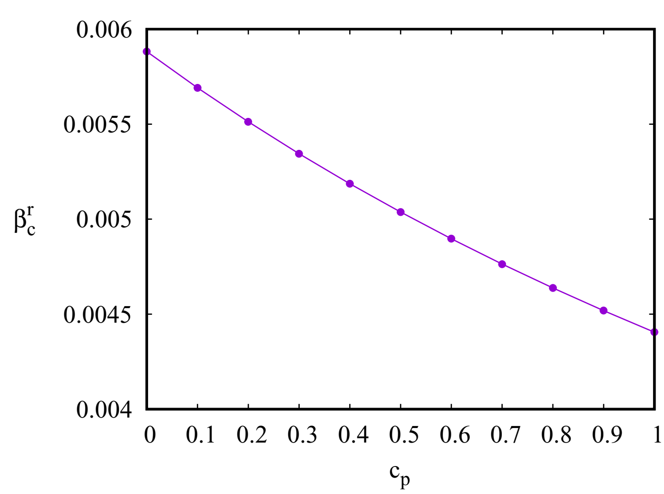

FIG. 10. Case 1: Uniform intervention to all public places. The number of individuals in a residence is $n = 25$ and the mobility rate is $p = 0.4$.

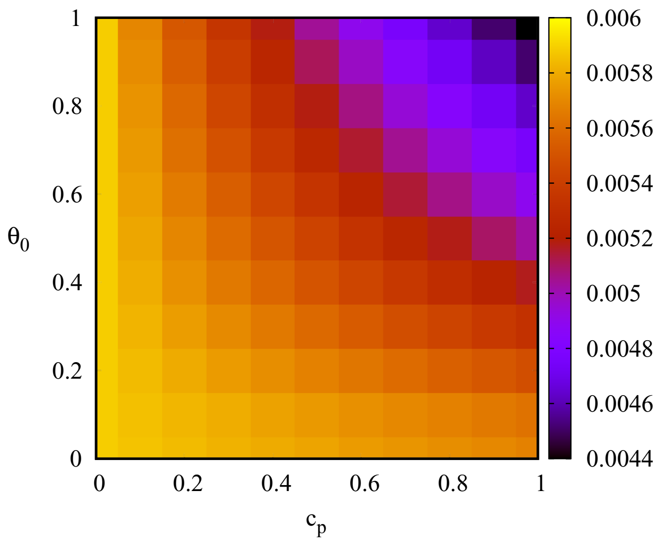

FIG. 11. Case 2: Intervention to public places with large degree. Other parameters are set same as in Fig. 10.

To solve such a problem, we propose an intervention strategy based on the flow of public places by using a bipartite metapopulation model with the SIS process. The flow of public places is assumed to be proportional to the degree of public places. The implementation of intervention is relevant with the degree of the locations and the priority parameter. Our study shows that NUP with large degree ($\alpha < 0$) can most effectively curb the epidemic, while NUP with small degree ($\alpha < 0$) has little effect on suppressing the epidemic. Moreover, the uniform intervention strategies to public places or residences have similar effect in curbing the epidemic, but they are not as effective as NUP with large degree ($\alpha < 0$). It indicates that in reality, redistributing the population flow from highly connected public places to the ones with less flow is expected to be helpful for the control of epidemics [41].

The current work still has some shortcomings. On the one hand, the contact pattern between individuals within a location is assumed to be homogeneously mixed, while in reality, this is not true. For example, the contact patterns within each patch are heterogeneous and the number of contacts is limited [42]. On the other hand, intervention strategies should be time dependent on the dynamics of the epidemic, which has been omitted in the present work. In spite of shortcoming, our work provides some insights on how to effectively control the epidemic spread. As for further work, the impact of contact patterns between individuals on the dynamics of the epidemic deserves further study.

ACKNOWLEDGMENTS

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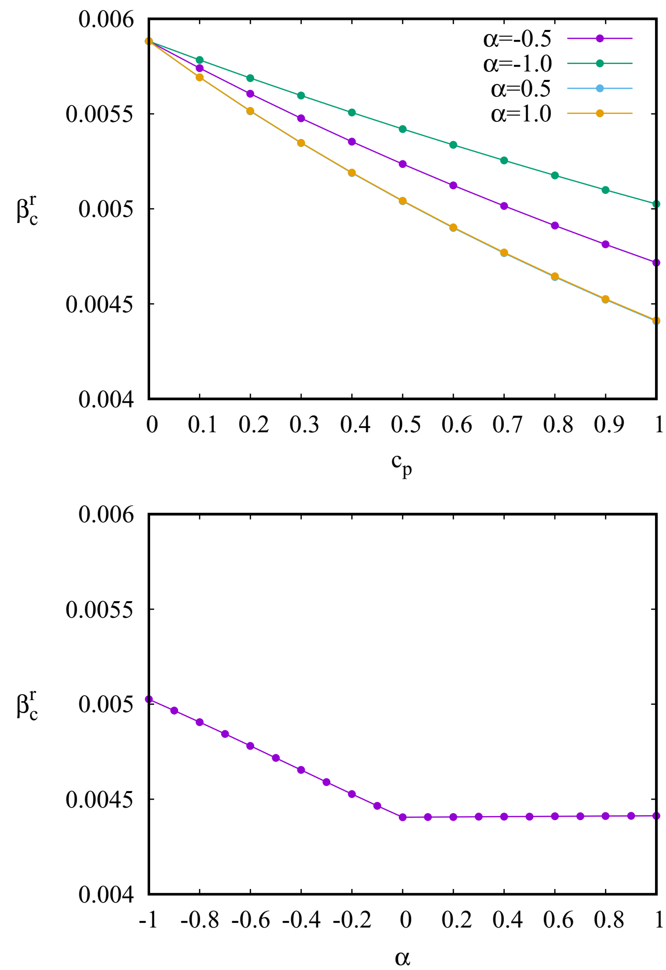

FIG. 12. Case 3: NUP. (a) The epidemic threshold β_c^r versus c_p for different α; (b) The epidemic threshold β_c^r versus α. The intervention factor is $c_p = 1$. Other parameters are set same as in Fig. 10.

APPENDIX A: A SIMPLE EXAMPLE FOR THE INTERVENTION ON THE BIPARTITE NETWORKS

To help explain the impact of intervention strategies on the epidemic threshold, we build a simple metapopulation network with three residences and two pubic places in Fig. 9, where the degrees of two public places are three and one, respectively.

To simplify analysis, we assume that $n_{R_i} = n, i = 1, 2, 3$ and the weight matrix is $W = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0.5 & 0.5 \end{pmatrix}$. The initial transmission rate between residences and public places is set as $\beta_p = m\beta_r$ with $m = 1.25$. In the main text, the epidemic threshold β_c^r on this network is obtained by Eqs. (17) and (18).

Under this assumption, the matrix M can be rewritten as

$$M = \begin{Bmatrix} [(1-p)^2+1]nc_r + \frac{5}{4}p^2\theta_{P_1}nc_p & \frac{5}{4}p^2\theta_{P_1}nc_p & \frac{5}{8}p^2\theta_{P_1}nc_p \\ \frac{5}{4}p^2\theta_{P_1}nc_p & [(1-p)^2+1]nc_r + \frac{5}{4}p^2\theta_{P_1}nc_p & \frac{5}{8}p^2\theta_{P_1}nc_p \\ \frac{5}{8}p^2\theta_{P_1}nc_p & \frac{5}{8}p^2\theta_{P_1}nc_p & [(1-p)^2+1]nc_r + \frac{5}{16}p^2(\theta_{P_1}+\theta_{P_2})nc_p \end{Bmatrix} \quad \text{(A1)}$$

Next, we analyze the epidemic thresholds with some special cases.

Case 1: Uniform intervention to all public places. By setting $c_r = 1, 0 < c_p < 1, \theta_{P_j} = 1, j = 1, 2$, the matrix M is rewritten as

$$M = \begin{Bmatrix} [(1-p)^2+1]n + \frac{5}{4}p^2nc_p & \frac{5}{4}p^2nc_p & \frac{5}{8}p^2nc_p \\ \frac{5}{4}p^2nc_p & [(1-p)^2+1]n + \frac{5}{4}p^2nc_p & \frac{5}{8}p^2nc_p \\ \frac{5}{8}p^2nc_p & \frac{5}{8}p^2nc_p & [(1-p)^2+1]n + \frac{5}{8}p^2nc_p \end{Bmatrix}, \tag{A2}$$

from which we see that only the intervention intensity for public places c_p affect the epidemic threshold. For clarity, we perform a numerical iteration of Eqs. (17) and (A2) to obtain the epidemic threshold, as shown in Fig. 10. It can be seen that the epidemic threshold decreases with c_p, i.e., more intensive intervention to public places will delay the epidemic with a larger epidemic threshold.

Case 2: Intervention to public places with large degree. Intervention to public places with degree larger than the average degree $\langle k_p \rangle$ with the parameter $\alpha = -1$, that is,

$$\theta_{P_j} = \begin{cases} 1, & k_{P_j} < \langle k_p \rangle \\ \theta_0, & k_{P_j} > \langle k_p \rangle \end{cases}, \tag{A3}$$

where θ_0 represents the intervention factor for pubic places. The transmission rate of public place P_j will be written as $\beta_{P_j} = c_p\theta_{P_j}\beta_p$. By setting $c_r = 1, 0 < c_p < 1$, the matrix M is expressed as

$$M = \begin{Bmatrix} [(1-p)^2+1]n + \frac{5}{4}p^2\theta_0 nc_p & \frac{5}{4}p^2\theta_0 nc_p & \frac{5}{8}p^2\theta_0 nc_p \\ \frac{5}{4}p^2\theta_0 nc_p & [(1-p)^2+1]n + \frac{5}{4}p^2\theta_0 nc_p & \frac{5}{8}p^2\theta_0 nc_p \\ \frac{5}{8}p^2\theta_0 nc_p & \frac{5}{8}p^2\theta_0 nc_p & [(1-p)^2+1]n + \frac{5}{16}p^2(\theta_0+1)nc_p \end{Bmatrix}. \tag{A4}$$

In order to analyze the effect of the intervention intensity parameter c_p and the priority intervention parameter θ_0 on the epidemic threshold, we perform numerical iterations of Eqs. (17) and (A4). As shown in Fig. 11, the smaller the value of c_p and θ_0, the greater the epidemic threshold. This result is expected, since a more intensive intervention to public places would lead to lower transmission rates, resulting in the reduction of the infection scale. Therefore, intensively intervening public places with large degree is effective for suppressing the epidemic.

Case 3: Nonuniform intervention to public places. Intervening to public places with degree satisfies the condition $k_{P_j}^\alpha < \langle k_p^\alpha \rangle$ with the proportion factor $\frac{k_{P_j}^\alpha}{\langle k_p^\alpha \rangle}$, that is,

$$\theta_{P_j} = \begin{cases} 1, & k_{P_j}^\alpha > \langle k_p^\alpha \rangle \\ \frac{k_{P_j}^\alpha}{\langle k_p^\alpha \rangle}, & k_{P_j}^\alpha < \langle k_p^\alpha \rangle \end{cases}. \tag{A5}$$

By setting $c_r = 1$ and $0 < c_p < 1$, the matrix M is expressed as

$$M = \begin{Bmatrix} [(1-p)^2+1]n + \frac{5}{4}p^2\frac{k_{P_1}^\alpha}{\langle k_p^\alpha \rangle}nc_p & \frac{5}{4}p^2\frac{k_{P_1}^\alpha}{\langle k_p^\alpha \rangle}nc_p & \frac{5}{8}p^2\frac{k_{P_1}^\alpha}{\langle k_p^\alpha \rangle}nc_p \\ \frac{5}{4}p^2\frac{k_{P_1}^\alpha}{\langle k_p^\alpha \rangle}nc_p & [(1-p)^2+1]n + \frac{5}{4}p^2\frac{k_{P_1}^\alpha}{\langle k_p^\alpha \rangle}nc_p & \frac{5}{8}p^2\frac{k_{P_1}^\alpha}{\langle k_p^\alpha \rangle}nc_p \\ \frac{5}{8}p^2\frac{k_{P_1}^\alpha}{\langle k_p^\alpha \rangle}nc_p & \frac{5}{8}p^2\frac{k_{P_1}^\alpha}{\langle k_p^\alpha \rangle}nc_p & [(1-p)^2+1]n + \frac{5}{16}p^2\frac{k_{P_1}^\alpha}{\langle k_p^\alpha \rangle} + \frac{k_{P_2}^\alpha}{\langle k_p^\alpha \rangle}nc_p \end{Bmatrix}. \tag{A6}$$

Finally, we explore how the intervention intensity parameter c_p and the selection parameter α affect the epidemic threshold. In Fig. 12(a), the epidemic threshold decreases with c_p. Intervention to public places with large degree ($\alpha < 0$) is more effective to curb the epidemic in reducing the infection scale and increasing the epidemic threshold. As shown in Fig. 12(b), the epidemic threshold increases with the absolute value of α, since a larger α represents a more intensive intervention. In addition, for $\alpha < 0$, the epidemic thresholds under different value of α show obvious differences. Based on the above analysis, when an emergent epidemic breaks out, intensive intervention to public places with large degree with priority can better curb the epidemic spread.

APPENDIX B: THE MONTE CARLO SIMULATION ON METAPOPULATION NETWORKS WITH SIS PROCESS

In order to explore the impact of different intervention strategies on the epidemic spread, we perform MC simulations as follows:

(1) *Initialize the metapopulation network.*

(a) *Build the metapopulation network.* The relevant parameters are the degree distribution $P(k_p)$, the number of residences N_r and public places N_p, and the number of individuals in residence R_i, n_{R_i}.

(b) *Set intervention parameters. The relevant parameters are the intervention factors for public places c_p and for residences c_r, the parameter α.*

(2) *Reaction-diffusion process at one time step.*

(a) *Individuals' migration.* Individuals leave residences with probability p or remain in residences with probability $(1-p)$.

(b) *SIS process in the day stage.* Susceptible individuals in residence R_i or public place P_j get infected with probability $\Phi^{\circledcirc}_{R_i}$ or $\Phi^{\circledcirc}_{P_j}$.

(c) *Return.* Individuals who left return to the original residences.

(d) *SIS process in the night stage.* Susceptible individuals in residence R_i get infected with probability $\Phi^{\star}_{R_i}$, and infected individuals in residence R_i recover with probability μ.

(3) *Count the proportion of infected individuals in the network at steady state.*

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