Impact of magnetic field on the parallel resistivity

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The impact of magnetic field (MF) on the parallel resistivity $\eta_\parallel$ is studied for strongly magnetized plasmas with the electron thermal gyroradius ρ_{the} smaller than the Debye length λ_D but much larger than the Landau length λ_L. Two previous papers [P. Ghendrih *et al.*, Phys. Lett. A **119**, 354 (1987); S. D. Baalrud and T. Lafleur, Phys. Plasmas **28**, 102107 (2021)] found $\eta_\parallel$ to increase monotonically with MF. Unfortunately, both works used predetermined electron distribution functions and are thus not self-consistent. In this paper, we analyze the MF dependence of $\eta_\parallel$ self-consistently by solving the electron magnetized kinetic equation in a Lorentz gaslike approximation. It is found $\eta_\parallel$ decreases monotonically with MF, with λ_D in the usual Coulomb logarithm $\ln\Lambda = \ln(\lambda_D/\lambda_L)$ being replaced by ρ_{the}. The underlying physics is that the electrons affected only by the collisions with impact parameters between λ_L and ρ_{the} carry almost all the parallel current.

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I. INTRODUCTION

Resistivity [1], as a basic physical quantity, plays an important role in plasma physics related to space physics, astrophysics, and fusion physics. It describes the generation of electric currents [2], determines the efficiency of ohmic heating [3], and can induce many instabilities, such as the resistive tearing mode [4,5], resistive drift instability [6,7], resistive ballooning mode [8,9], etc. Therefore, accurate knowledge of its value is of great significance. For strongly magnetized plasmas with the electron thermal gyroradius ρ_{the} smaller than the Debye screening length λ_D, such as tokamak plasmas [10,11], ultracold neutral plasmas [12,13], and astrophysical plasmas [14,15], the resistivity should have a dependence on the magnetic field (MF) which drastically affects the electrons involved collisions with impact parameters p larger than ρ_{the} [16,17]. Ghendrih *et al.* [18] calculated the MF dependence of the parallel resistivity $\eta_\parallel$ by means of the rate of entropy production using the electron distribution function (EDF) pertinent to the Lorentz gas in the vanishing magnetic field limit and then extending it to a shifted Maxwellian EDF. In both cases, they found the ratio of $\eta_\parallel$ to its value without MF was $1 + \ln(\lambda_D/\rho_{\text{the}})/(2\ln\Lambda)$ for $\lambda_L < \rho_{\text{the}} < \lambda_D$ and $3/2$ for $\rho_{\text{the}} < \lambda_L$, with $\ln\Lambda = \ln(\lambda_D/\lambda_L)$ being the usual Coulomb logarithm and λ_L the Landau length, showing a monotonic increase of $\eta_\parallel$ with the MF. Baalrud and Lafleur [19] found $\eta_\parallel$ to increase by a factor of $3/2$ as well for $\rho_{\text{the}} < \lambda_L$ through calculating the ion friction force due to collisions with electrons with a shifted Maxwellian distribution. The above two works both used the predetermined EDFs and are thus not self-consistent. A self-consistent study is conducted for the impact of MF on $\eta_\parallel$ in this paper. Contrary to previous results, $\eta_\parallel$ is found to decrease monotonically with the MF, which is expected to have a notable impact on various plasma physics problems associated with $\eta_\parallel$, particularly on the resistive instabilities and transport phenomena.

The rest of the paper is organized as follows: In Sec. II, the physical model adopted in this paper is described. In Sec. III, the electron magnetized kinetic equation in the steady state in the presence of a static and uniform electric field along the MF is solved. Good agreement is achieved between the EDFs obtained analytically and numerically. Section IV is devoted to the research of the MF influence on $\eta_\parallel$ through calculating the parallel current. Finally, a brief conclusion is given in Sec. V.

II. PHYSICAL MODEL

We are concerned with a simple homogeneous plasma composed of electrons and one type of ions immersed in static and uniform electromagnetic fields $\mathbf{E} = E\hat{\mathbf{e}}_z$ and $\mathbf{B} = B\hat{\mathbf{e}}_z$ with $\hat{\mathbf{e}}_z$ being the unit vector in the z direction. The mass, charge, number density, temperature, thermal velocity, and gyrofrequency of the electrons are m_e, $-e$, n_e, T_e, $v_{\text{the}} \equiv \sqrt{k_B T_e/m_e}$ with k_B being the Boltzmann constant, and $\Omega_e \equiv eB/m_e$, respectively, and those of the ions are m_i, Ze, n_i, T_i, $v_{\text{thi}} \equiv \sqrt{k_B T_i/m_i}$, and $\Omega_i \equiv ZeB/m_i$, respectively. The MF strength is first assumed to satisfy $\lambda_L \ll \rho_{\text{the}} < \lambda_D < \rho_{\text{thi}}$, where $\rho_{\text{the}} \equiv v_{\text{the}}/\Omega_e$, $\lambda_D = \sqrt{\varepsilon_0 k_B T_e T_i/[n_e e^2(ZT_e + T_i)]}$ with ε_0 being the permittivity of the vacuum, and $\rho_{\text{thi}} \equiv v_{\text{thi}}/\Omega_i$ is the ion thermal gyroradius. This implies that the MF influence on the electron dynamics has to be taken into account during

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the collision process while its influence on the ion dynamics is ignorable. For simplicity, the electrons are supposed to collide only with background ions and not with other electrons. Since the current is mainly carried by the electrons, we can ignore the ion response to **E** and take its velocity distribution function f_i to be Maxwellian. Under these conditions, the evolution of the EDF f_e is governed by a magnetized Fokker-Planck (FP) equation [17,20],

$$\frac{\partial f_e}{\partial t} - \frac{eE}{m_e}\frac{\partial f_e}{\partial v_{ez}} = -\frac{\partial}{\partial \mathbf{v}_e}\cdot[\langle\Delta\mathbf{V}_e\rangle f_e] + \frac{1}{2}\frac{\partial^2}{\partial\mathbf{v}_e\partial\mathbf{v}_e}:[\langle\Delta\mathbf{V}_e\Delta\mathbf{V}_e\rangle f_e], \tag{1}$$

where $\mathbf{v}_e$ is the electron velocity and v_{ez} is its z component. $\langle\Delta\mathbf{V}_e\rangle$ and $\langle\Delta\mathbf{V}_e\Delta\mathbf{V}_e\rangle$ are the first- and second-order magnetized FP coefficients, respectively. In the static screening approximation, they are given by [16]

$$\langle\Delta\mathbf{V}_e\rangle = \Gamma\left\{\left(1+\frac{m_e}{m_i}\right)\ln\left(\frac{\rho_{\text{the}}}{\lambda_L}\right)\frac{\partial}{\partial\mathbf{v}_e}\left[\frac{1}{v_e}\text{erf}\left(\frac{v_e}{\sqrt{2}v_{\text{thi}}}\right)\right] + \frac{1}{2}\ln\left(\frac{\lambda_D}{\rho_{\text{the}}}\right)\left[\left(\frac{\partial}{\partial v_{ez}} - \frac{m_e v_{ez}}{m_i v_{\text{thi}}^2}\right)\mathcal{D}(v_{ez})\right]\hat{\mathbf{e}}_z\right\}, \tag{2}$$

$$\langle\Delta\mathbf{V}_e\Delta\mathbf{V}_e\rangle = \Gamma\left\{\ln\left(\frac{\rho_{\text{the}}}{\lambda_L}\right)\frac{\partial^2}{\partial\mathbf{v}_e\partial\mathbf{v}_e}\left[\left(v_{\text{thi}}^2\frac{\partial}{\partial v_e} + \frac{v_e^2+v_{\text{thi}}^2}{v_e}\right)\text{erf}\left(\frac{v_e}{\sqrt{2}v_{\text{thi}}}\right)\right] + \ln\left(\frac{\lambda_D}{\rho_{\text{the}}}\right)\mathcal{D}(v_{ez})\hat{\mathbf{e}}_z\hat{\mathbf{e}}_z\right\}, \tag{3}$$

where $\Gamma \equiv Z^2e^4n_i/(4\pi\varepsilon_0^2m_e^2)$ and

$$\mathcal{D}(v_{ez}) \equiv \frac{2v_{\text{thi}}^2}{v_{ez}^2}\left(-\frac{\partial}{\partial v_{ez}} + \frac{1}{v_{ez}}\right)\text{erf}\left(\frac{v_{ez}}{\sqrt{2}v_{\text{thi}}}\right), \tag{4}$$

with erf(x) being the error function, It can be seen that except $\langle\Delta V_{ez}\rangle$ and $\langle\Delta V_{ez}\Delta V_{ez}\rangle$, the only change of the other components of $\langle\Delta\mathbf{V}_e\rangle$ and $\langle\Delta\mathbf{V}_e\Delta\mathbf{V}_e\rangle$ compared to the no MF case is a replacement of λ_D in $\ln\Lambda$ by ρ_{the}. $\langle\Delta V_{ez}\rangle$ and $\langle\Delta V_{ez}\Delta V_{ez}\rangle$ have contributions from the collisions with $\rho_{\text{the}} < p < \lambda_D$, and depend sensitively on v_{ez}. In Figs. 1 and 2, we plot how the ratios $\mathcal{R}_f \equiv \langle\Delta V_{ez}\rangle/\langle\Delta v_{ez}\rangle$ and $\mathcal{R}_d \equiv \langle\Delta V_{ez}\Delta V_{ez}\rangle/\langle\Delta v_{ez}\Delta v_{ez}\rangle$ of $\langle\Delta V_{ez}\rangle$ and $\langle\Delta V_{ez}\Delta V_{ez}\rangle$ to their values $\langle\Delta v_{ez}\rangle$ and $\langle\Delta v_{ez}\Delta v_{ez}\rangle$ without MF vary with $|v_{ez}|$, respectively, for different ion to electron mass ratios m_i/m_e and $\alpha \equiv \ln(\lambda_D/\rho_{\text{the}})/\ln\Lambda$. As can be seen, for the electron perpendicular velocity $v_{e\perp}$ equal to v_{the}, both $\mathcal{R}_f$ and $\mathcal{R}_d$ are much greater than unity in the vicinity of $|v_{ez}| = v_{\text{thi}}$, indicating that the MF substantially enhances the electron parallel velocity friction and diffusion [16,21]. In addition, both $\mathcal{R}_f$ and $\mathcal{R}_d$ increase remarkably with m_i/m_e in this case. The most striking is the anomaly of $\mathcal{R}_f$ in the sense that its value becomes close to 900 when $v_{e\perp} = v_{\text{the}}$ and $|v_{ez}| = v_{\text{thi}}$ even for $m_i/m_e = 1836$ and $\alpha = 0.1$. $\mathcal{R}_f$ and $\mathcal{R}_d$ decrease with $|v_{ez}|$. When $|v_{ez}| \gg v_{\text{thi}}$, the contributions from the collisions with $\rho_{\text{the}} < p < \lambda_D$ become smaller and smaller and can be completely ignored as $|v_{ez}|$ far exceeds a critical value v_c. Then both $\mathcal{R}_f$ and $\mathcal{R}_d$ reduce to $1-\alpha$ as can be seen clearly from the enlarged view of the back segments of the curves in Figs. 1 and 2. v_c can be roughly estimated by comparing the two terms in the braces on the right-hand side (RHS) of Eq. (2) and determined by the relation $\alpha'\partial\mathcal{D}(v_c)/\partial v_c = -2v_c/v_e^3$, which gives $v_c = (3\alpha' v_e^3 v_{\text{thi}}^2)^{1/5}$ with $\alpha' \equiv \alpha/(1-\alpha)$.

To further simplify the model, the ion is assumed to be immobile in collisions with $p < \rho_{\text{the}}$ while its motion is allowed for in collisions with $\rho_{\text{the}} < p < \lambda_D$ to retain the feature of large $\mathcal{R}_f$ and $\mathcal{R}_d$ in the vicinity of $|v_{ez}| = v_{\text{thi}}$. We call it a Lorentz gaslike approximation. In this approximation, the first terms in the braces on the RHSs of Eqs. (2) and (3) are, respectively, reduced to $-\ln(\frac{\rho_{\text{the}}}{\lambda_L})\frac{\mathbf{v}_e}{v_e^3}$ and $\ln(\frac{\rho_{\text{the}}}{\lambda_L})\frac{v_e^2\mathbf{I}-\mathbf{v}_e\mathbf{v}_e}{v_e^3}$ with

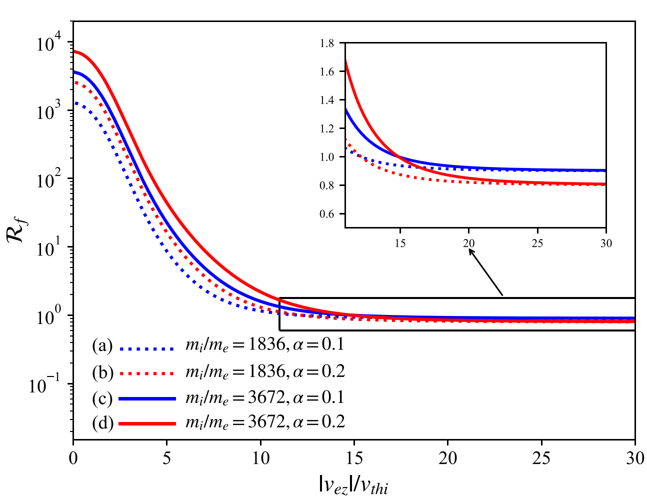

FIG. 1. The variation of $\mathcal{R}_f \equiv \langle\Delta V_{ez}\rangle/\langle\Delta v_{ez}\rangle$ with $|v_{ez}|$ for $T_i = T_e$, $v_{e\perp} = v_{\text{the}}$, and (a) $m_i/m_e = 1836$, $\alpha = 0.1$, (b) $m_i/m_e = 1836$, $\alpha = 0.2$, (c) $m_i/m_e = 3672$, $\alpha = 0.1$, and (d) $m_i/m_e = 3672$, $\alpha = 0.2$. The inset shows $\mathcal{R}_f$ in the region $11 < |v_{ez}|/v_{\text{thi}} < 30$.

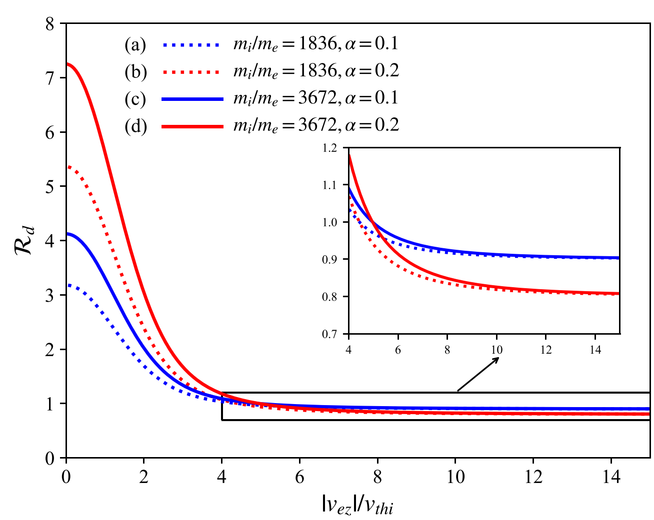

FIG. 2. The variation of $\mathcal{R}_d \equiv \langle\Delta V_{ez}\Delta V_{ez}\rangle/\langle\Delta v_{ez}\Delta v_{ez}\rangle$ with $|v_{ez}|$ with the same parameter settings as in Fig. 1 for the four cases. The inset shows $\mathcal{R}_d$ in the region $4 < |v_{ez}|/v_{\text{thi}} < 15$.

I being the unit tensor. Equation (1) thus becomes

$$\frac{\partial f_e}{\partial t}-\frac{eE}{m_e}\frac{\partial f_e}{\partial v_{ez}} =\frac{\Gamma}{2}\left\{\ln\left(\frac{\rho_{\rm the}}{\lambda_L}\right)\frac{\partial}{\partial \mathbf{v}_e}\cdot\left[\frac{v_e^2\mathsf{I}-\mathbf{v}_e\mathbf{v}_e}{v_e^3}\cdot\frac{\partial f_e}{\partial \mathbf{v}_e}\right] +\ln\left(\frac{\lambda_D}{\rho_{\rm the}}\right)\frac{\partial}{\partial v_{ez}}\left[\mathcal{D}(v_{ez})\left(\frac{\partial}{\partial v_{ez}}+\frac{m_e v_{ez}}{m_i v_{\rm thi}^2}\right)f_e\right]\right\}. \tag{5}$$

III. SOLVING THE ELECTRON MAGNETIZED KINETIC EQUATION ANALYTICALLY AND NUMERICALLY

$\eta_\parallel$ is defined by the parallel Ohm's law $\eta_\parallel \equiv E/J_z$, where $J_z \equiv -e\int v_{ez} f_e\, d^3\mathbf{v_e}$ is the parallel current. To calculate $\eta_\parallel$, f_e has to be determined first by solving Eq. (5). Despite its simple appearance, Eq. (5) is rather tricky to solve. We will seek for an approximate solution in the following. f_e can be viewed as composed of an equilibrium part f_{e0} of the Maxwellian distribution with $T_e = T_i$ and a perturbed part f_{e1} induced by $\mathbf{E}$, $f_e = f_{e0} + f_{e1}$. $\mathbf{E}$ is assumed to be very small so $f_{e1} \ll f_{e0}$ in general. Neglecting the nonlinear term involving the product of E and $\partial f_{e1}/\partial v_{ez}$ in Eq. (5), for the steady state $\partial f_{e1}/\partial t = 0$ we have

$$-\beta\frac{\partial f_{e0}}{\partial v_{ez}} =\frac{\partial}{\partial \mathbf{v}_e}\cdot\left[\frac{v_e^2\mathsf{I}-\mathbf{v}_e\mathbf{v}_e}{v_e^3}\cdot\frac{\partial f_{e1}}{\partial \mathbf{v}_e}\right] +\alpha'\frac{\partial}{\partial v_{ez}}\left[\mathcal{D}(v_{ez})\left(\frac{\partial}{\partial v_{ez}}+\frac{v_{ez}}{v_{\rm the}^2}\right)f_{e1}\right], \tag{6}$$

where $\beta \equiv 2eE/[m_e\Gamma \ln(\rho_{\rm the}/\lambda_L)]$. Considering that the second term on the RHS of the above equation plays a leading role for $|v_{ez}| \ll v_c$ but is trivial for $|v_{ez}| \gg v_c$, we split the whole v_{ez} space into two regions according to the magnitude of $|v_{ez}|$: region I for $0 < |v_{ez}| < v_c'$ and region II for $|v_{ez}| > v_c'$. v_c' is different from v_c and will be given later. In region I, the second term on the RHS of Eq. (6) is dominant. To deal with the possible situation of large derivatives of f_{e1} with respect to v_{ez} for $|v_{ez}|$ close to v_c' caused by the rapid variation of $\mathcal{D}(v_{ez})$ with v_{ez}, it is also necessary to retain the part proportional to $\partial^2 f_{e1}/\partial v_{ez}^2$ in the first term. So we have

$$-\beta\frac{\partial f_{e0}}{\partial v_{ez}} =\frac{1}{v_{e\perp}}\frac{\partial^2 f_{e1}^{\rm I}}{\partial v_{ez}^2} +\alpha'\frac{\partial}{\partial v_{ez}}\left[\mathcal{D}(v_{ez})\left(\frac{\partial}{\partial v_{ez}}+\frac{v_{ez}}{v_{\rm the}^2}\right)f_{e1}^{\rm I}\right]. \tag{7}$$

Integrating the above equation over v_{ez} twice yields

$$f_{e1}^{\rm I} = -\beta v_{e\perp}\xi^{-1}(v_{e\perp}, v_{ez}) \times\int_0^{v_{ez}}\frac{f_{e0}(v_{e\perp}, v_{ez}')+C_{\rm I}(v_{e\perp})}{1+\alpha' v_{e\perp}\mathcal{D}(v_{ez}')}\xi(v_{e\perp}, v_{ez}')dv_{ez}', \tag{8}$$

where $\xi(v_{e\perp},\, v_{ez}) \equiv \exp[\int_0^{v_{ez}}\gamma(v_{e\perp}, v_{ez}')dv_{ez}']$, $\gamma(v_{e\perp}, v_{ez}) \equiv \alpha' v_{e\perp} v_{ez}\mathcal{D}(v_{ez})/\{v_{\rm the}^2[1+\alpha' v_{e\perp}\mathcal{D}(v_{ez})]\}$, and $C_{\rm I}(v_{e\perp})$ is a function to be determined of $v_{e\perp}$. In region II, the first term on the RHS of Eq. (6) becomes dominant. In this case, it is more suitable to use the spherical coordinates (v_e, θ, ϕ) in velocity space with θ and ϕ being the polar and azimuthal angles, respectively. Considering the large value of $v_e\partial\mathcal{D}(v_e\cos\theta)/\partial\theta$ for $|v_{ez}|$ close to v_c', the part proportional to $\partial[\mathcal{D}(v_e\cos\theta)\partial f_{e1}/\partial\theta]/\partial\theta$ in the second term is kept as well. We thus have

$$-\beta\cos\theta\frac{\partial f_{e0}}{\partial v_e} =\frac{1}{v_e^3\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial f_{e1}^{\rm II}}{\partial\theta}\right] +\alpha'\frac{\sin\theta}{v_e^2}\frac{\partial}{\partial\theta}\left[\sin\theta\mathcal{D}(v_e\cos\theta)\frac{\partial f_{e1}^{\rm II}}{\partial\theta}\right]. \tag{9}$$

Multiplying both sides of the above equation by $\sin\theta$, taking $\sin^2\theta$ to be 1 in the second term on the RHS since this term is important only when $\theta \sim \pi/2$, and integrating the equation over θ twice yields

$$f_{e1}^{\rm II} = -\frac{\beta v_e^3 f_{e0}(v_e)}{2v_{\rm the}^2}\int_0^{v_{ez}}\frac{1}{1+\alpha' v_e\mathcal{D}(v_{ez}')}dv_{ez}' +C_{+(-)}^{\rm II}(v_e), \tag{10}$$

where $C_{\rm II}^{+(-)}(v_e)$ are functions to be determined of v_e with $C_{\rm II}^+$ for $v_{ez} > v_c'$ and $C_{\rm II}^-$ for $v_{ez} < -v_c'$. Continuities of f_{e1} and $\partial f_{e1}/\partial v_{ez}$ at $v_{ez} = \pm v_c'$ require

$$C_1^{\rm I} = \left(\frac{v_{e\perp}^2}{2v_{\rm the}^2}-1\right)f_{e0}(v_{e\perp}), \quad C_{\rm II}^+ = -C_{\rm II}^- = 0, \tag{11}$$

from which it follows that

$$f_{e1} = \begin{cases} -\beta v_{e\perp}\xi^{-1}(v_{e\perp}, v_{ez})\int_0^{v_{ez}}\frac{v_{e\perp}^2 f_{e0}(v_{e\perp})/(2v_{\rm the}^2)+f_{e0}(v_{e\perp}, v_{ez}')-f_{e0}(v_{e\perp})}{1+\alpha' v_{e\perp}\mathcal{D}(v_{ez}')}\xi(v_{e\perp}, v_{ez}')dv_{ez}' & \text{for } |v_{ez}| < v_c', \\ -\frac{\beta v_e^3 f_{e0}(v_e)}{2v_{\rm the}^2}\int_0^{v_{ez}}\frac{1}{1+\alpha' v_e\mathcal{D}(v_{ez}')}dv_{ez}' & \text{for } |v_{ez}| > v_c'. \end{cases} \tag{12}$$

v_c' remains to be determined. One can readily verify by using f_{e1} in the above equation that $\alpha' v_e\mathcal{D}(v_c') > 1$ is required to ensure the magnitude of the extra terms of Eq. (6) relative to Eq. (7) smaller than $|\beta\partial f_{e0}/\partial v_{ez}|$. Similarly, $\alpha' v_e\mathcal{D}(v_c') < 1$ is required to ensure the magnitude of the extra terms of Eq. (6) relative to Eq. (9) smaller than $|\beta\partial f_{e0}/\partial v_{ez}|$. Consequently, $\alpha' v_e\mathcal{D}(v_c') = 1$, which gives $v_c' = (2\alpha' v_e v_{\rm thi}^2)^{1/3}$.

From Eq. (12), we have

$$\frac{\partial f_{e1}}{\partial\theta} = \frac{\beta v_e^4\sin\theta f_{e0}}{2v_{\rm the}^2[1+\alpha' v_e\mathcal{D}(v_e\cos\theta)]} \tag{13}$$

for both regions I and II by retaining only the dominant terms.

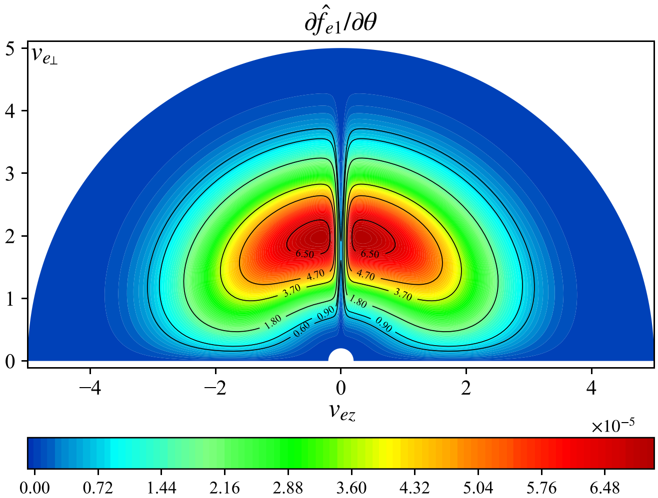

FIG. 3. Contour map of $\partial \hat{f}_{e1}/\partial\theta$ in the quasisteady state obtained numerically and the black contour lines from the theoretical result in Eq. (13) with $m_i/m_e = 3672$ and $\alpha = 0.2$.

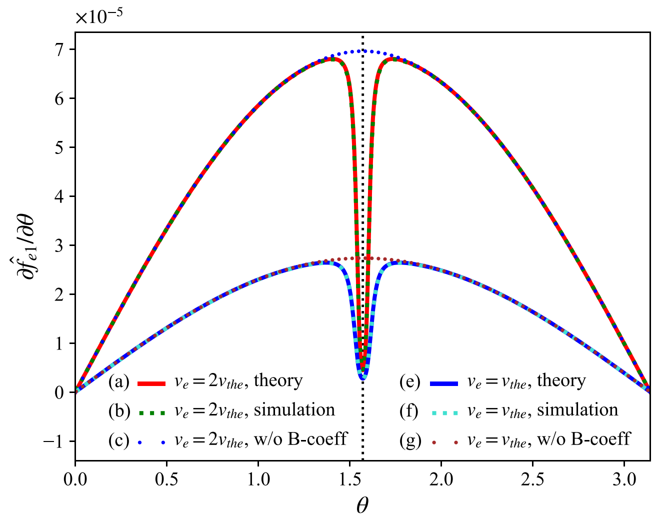

FIG. 4. Change of $\partial \hat{f}_{e1}/\partial\theta$ with θ for the two cases $\hat{v}_e = 1$ and $\hat{v}_e = 2$ with $m_i/m_e = 3672$ and $\alpha = 0.2$. The solid curves represent the theoretical results and the dashed curves the numerical results. The results without MF multiplied by $1/(1-\alpha)$ are shown for comparison by the dotted curves. The dashed black vertical line indicates the position of $\theta = \pi/2$.

To verify the theoretical results, Eq. (5) is solved numerically. Since f_e is axially symmetric, the numerical calculation is carried out in the two-dimensional (v_e, θ) velocity space. The calculation domain is chosen to be $(0.2 v_{\rm the} < v_e < 5 v_{\rm the}, 0 \leqslant \theta \leqslant \pi)$ to encompass the 99.8% contribution to J_z. The lower v_e truncation is introduced to avoid the huge waste of computing resources due to the small time step required by high collision frequency at small v_e. m_i/m_e is taken to be 3672 for a deuterium plasma and α is taken to be 0.2. f_e, $\mathbf{v}_e$, t, and E are normalized by $n_e/(2\pi v_{\rm the}^2)^{3/2}$, $v_{\rm the}$, ν_{ei}^{-1}, and E_c, respectively, where $\nu_{ei} \equiv \sqrt{2}\Gamma \ln\Lambda/(3\sqrt{\pi} v_{\rm the}^3)$ [22] is the electron-ion collision frequency and $E_c \equiv \nu_{ei} m_e v_{\rm the}/e$. The normalized quantities are labeled by the hat (ˆ) symbol in the following. $\hat{f}_e$ evolves under the action of $\hat{E}$ from the initial state $\hat{f}_{e0} = \exp(-\hat{v}_e^2/2)$, and reaches a quasisteady state after a few collision times. For the avoidance of the electron runaway to get a better comparison with the above-obtained linear theoretical results, a small value of $\hat{E} = 10^{-4}$ is taken. Figure 3 shows the contour map of $\partial \hat{f}_{e1}/\partial\theta$ in the quasisteady state obtained numerically and the black contour lines from the theoretical result in Eq. (13) with $m_i/m_e = 3672$ and $\alpha = 0.2$. A good agreement between the numerical and theoretical results is achieved. A remarkable feature in Fig. 3 is the sharp decrease of $\partial \hat{f}_{e1}/\partial\theta$ as $\hat{v}_{ez}$ approaches 0. To see this more clearly, the change of $\partial \hat{f}_{e1}/\partial\theta$ with θ is plotted in Fig. 4 for the two cases $\hat{v}_e = 1$ and $\hat{v}_e = 2$. The theoretical results represented by the solid curves and the numerical results by the dashed curves are in good agreement. In both cases, $\partial \hat{f}_{e1}/\partial\theta$ exhibits a nonmonotonic change with θ in the range $0 < \theta < \pi/2$ and has a minimum at $\theta = \pi/2$. This is quite different from the no MF case represented by the dotted curves in which $\partial \hat{f}_{e1}/\partial\theta$ changes sinusoidally with θ and reaches a maximum at $\theta = \pi/2$. The physics underlying this behavior is the large electron parallel velocity friction and diffusion around $v_{ez} = 0$ for $\rho_{\rm the} \ll \lambda_D$, reflected in the second term in the denominator on the RHS of Eq. (13). Compared to the $\hat{v}_e = 1$ case, the large parallel velocity friction and diffusion appear in a more narrow θ range for the $\hat{v}_e = 2$ case, leading to its steeper downtrend of $\partial \hat{f}_{e1}/\partial\theta$ around $\theta = \pi/2$.

IV. MF INFLUENCE ON $\eta_\parallel$

In spherical coordinates, J_z is given by

$$J_z = -2\pi e \int_0^\infty dv_e \int_0^\pi d\theta\, v_e^3 \sin\theta\cos\theta f_{e1}$$
$$= \pi e \int_0^\infty dv_e \int_0^\pi d\theta\, v_e^3 \sin^2\theta \frac{\partial f_{e1}}{\partial\theta}, \quad (14)$$

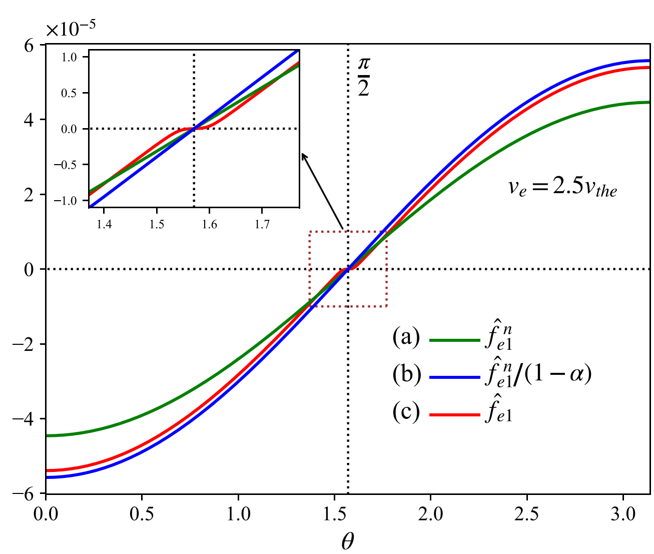

FIG. 5. Comparison of f_{e1} in Eq. (12) for $m_i/m_e = 3672$, $\alpha = 0.2$, and $v_e = 2.5 v_{\rm the}$ with its value f_{e1}^n without MF. The red curve represents $\hat{f}_{e1}$, the green curve $\hat{f}_{e1}^n$, and the blue curve $\hat{f}_{e1}^n/(1-\alpha)$.

TABLE I. Plasma parameters of three strongly magnetized environments. Columns list n_e, k_BT_e, B, $\ln\Lambda$, $\ln(\rho_{\rm the}/\lambda_L)$, and the relative correction of $\eta_\parallel$ due to the MF given by $-\alpha$.

	n_e (cm^{-3})	k_BT_e (eV)	B (T)	$\ln\Lambda$	$\ln(\rho_{\rm the}/\lambda_L)$	$-\alpha$
White dwarf atmospheres [26]	10^{16}	5	10^4	6.0	0.6	−90%
Ultracold neutral plasmas [27]	10^7	4×10^{-4}	0.01	2.2	0.3	−86%
Tokamak scrape-off layer plasmas (Alcator C-Mod) [10]	10^{13}	30	5	12.2	10.9	−10%

where integration over θ by parts is performed in the second step. Substituting Eq. (13) into the above equation and carrying out the integrals over θ and v_e yields

$$J_z = \frac{16\beta e n_e v_{\rm the}^3}{\sqrt{2\pi}}, \tag{15}$$

from which we finally arrive at

$$\eta_\parallel = \frac{\sqrt{2\pi}E}{16\beta e n_e v_{\rm the}^3} = \frac{Ze^2\sqrt{m_e}\ln(\rho_{\rm the}/\lambda_L)}{64\sqrt{2\pi}\varepsilon_0^2(k_BT_e)^{3/2}}. \tag{16}$$

It can be seen that the only difference from the conventional parallel resistivity [23] is the replacement of λ_D in $\ln\Lambda$ by $\rho_{\rm the}$. To explain the MF dependence of $\eta_\parallel$, we compare f_{e1} in Eq. (12) for $m_i/m_e = 3672$, $\alpha = 0.2$, and $v_e = 2.5v_{\rm the}$ with its value f_{e1}^n without MF in Fig. 5. It can be seen from the enlarged view of the curves around $\theta = \pi/2$ that for θ very close to $\pi/2$ corresponding to $|v_{ez}| \sim 0$, f_{e1} is much smaller than f_{e1}^n due to the MF-induced large electron parallel velocity friction and diffusion. For θ deviating slightly from $\pi/2$, f_{e1} becomes larger than f_{e1}^n and is about $1/(1-\alpha)$ times of f_{e1}^n when $|v_{ez}| \gg v_c'$ as a result of the vanishing of the effects of collisions with $\rho_{\rm the} < p < \lambda_D$. The large difference between f_{e1} and f_{e1}^n indicates that it is incorrect to calculate the MF dependence of $\eta_\parallel$ by employing f_{e1}^n. An explanation of $\eta_\parallel$ dependence on MF can be provided based on the feature of f_{e1}. For very small $|v_{ez}|$, although $f_{e1} \ll f_{e1}^n$, this kind of influence is not important to $\eta_\parallel$ since the correlated velocity region hardly contributes to J_z even in the no MF case. For the velocity region $|v_{ez}| \gg v_c'$ which contributes almost all J_z, $f_{e1} \approx f_{e1}^n/(1-\alpha)$. Therefore, J_z is about $1/(1-\alpha)$ times of its value without MF, indicating $\eta_\parallel$ becomes $1-\alpha$ times of the no MF case.

For even stronger MF satisfying $\lambda_L \ll \rho_{\rm the} < \rho_{\rm thi} < \lambda_D$, both the MF influence on the electron and ion dynamics has to be taken into account during the collision process. It is found that all the procedures and results described above apply to this case except that $\mathcal{D}(v_{ez})$ is changed to

$$\begin{aligned}\mathcal{D}(v_{ez}) \equiv& \frac{\alpha_1'}{\alpha'}\frac{2v_{\rm thi}^2}{v_{ez}^2}\left(-\frac{\partial}{\partial v_{ez}} + \frac{1}{v_{ez}}\right)\text{erf}\left(\frac{v_{ez}}{\sqrt{2}v_{\rm thi}}\right)\\ &+ \frac{\alpha_2'}{\alpha'}\sqrt{\frac{2}{\pi}}\frac{v_{\rm thi}}{\Omega_i^2\ln(\lambda_D/\rho_{\rm thi})}\int_{\lambda_D^{-1}}^{\rho_{\rm thi}^{-1}}dk\int_{-1}^{1}dx\\ &\times k|x|(1-x^2)\exp\left(-\frac{(kv_{ez}x+\Omega_i)^2}{2k^2v_{\rm thi}^2x^2}\right),\end{aligned} \tag{17}$$

with $\alpha_1' \equiv \ln(\rho_{\rm thi}/\rho_{\rm the})/\ln(\rho_{\rm the}/\lambda_L)$ and $\alpha_2' \equiv \ln(\lambda_D/\rho_{\rm thi})/\ln(\rho_{\rm the}/\lambda_L)$. $\eta_\parallel$ is still given by Eq. (16).

As can be seen, the electron-ion collisions with $\rho_{\rm the} < p < \lambda_D$ make no contribution to $\eta_\parallel$. Since the electron-electron collisions with $\rho_{\rm the} < p < \lambda_D$ are also negligible [24] in the evolution of f_e without considering reflections [25], we can conclude that collisions with $\rho_{\rm the} < p < \lambda_D$ are trivial for obtaining a MF-dependent $\eta_\parallel$. The result that λ_D in $\ln\Lambda$ in the expression of $\eta_\parallel$ without MF should be replaced by $\rho_{\rm the}$ when $\rho_{\rm the} < \lambda_D$ is thus universal in spite that it is obtained in a Lorentz gaslike approximation in this paper. Then the relative correction of $\eta_\parallel$ due to the MF is $-\alpha$ in this case. Table I presents the plasma parameters including $\ln\Lambda$, $\ln(\rho_{\rm the}/\lambda_L)$, and $-\alpha$ of three strongly magnetized environments. We note that the MF influence on $\eta_\parallel$ is significant in the white dwarf atmospheres and the ultracold neutral plasmas with the relative corrections close to 90%. In view of the small values of $\ln(\rho_{\rm the}/\lambda_L)$ in these two circumstances, $\eta_\parallel$ should be calculated in a more accurate manner beyond the logarithmic accuracy, which is beyond the scope of this paper. For the Alcator C-Mod tokamak scrape-off layer plasmas, the relative correction is about 10%. In the future, as the MF of tokamaks increases, its influence on $\eta_\parallel$ may become more pronounced.

V. CONCLUSION

In conclusion, by solving the electron magnetized kinetic equation theoretically and numerically in a Lorentz gaslike approximation, a monotonic decrease of $\eta_\parallel$ with the MF has been found with $\ln(\lambda_D/\lambda_L)$ in its expression without MF being replaced by $\ln(\rho_{\rm the}/\lambda_L)$ when $\rho_{\rm the} < \lambda_D$. This result is universal and of great significance for the research of $\eta_\parallel$-related physical processes.

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