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Self-organized multistability in the forest fire model

Diego Rybski*
Potsdam Institute for Climate Impact Research–PIK, Member of Leibniz Association, P.O. Box 601203, 14412 Potsdam, Germany
Department of Environmental Science Policy and Management, University of California Berkeley, 130 Mulford Hall #3114, Berkeley, California 94720, USA; and Complexity Science Hub Vienna, Josefstädterstrasse 39, A-1090 Vienna, Austria

Van Butsic
Department of Environmental Science Policy and Management, University of California Berkeley, 130 Mulford Hall #3114, Berkeley, California 94720, USA

Jan W. Kantelhardt
Institute of Physics, Martin-Luther-University Halle-Wittenberg, 06099 Halle, Germany

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The forest fire model in statistical physics represents a paradigm for systems close to but not completely at criticality. For large tree growth probabilities p we identify periodic attractors, where the tree density ρ oscillates between discrete values. For lower p this self-organized multistability persists with incrementing numbers of states. Even at low p the system remains quasiperiodic with a frequency $\approx p$ on the way to chaos. In addition, the power-spectrum shows $1/f^2$ scaling (Brownian noise) at the low frequencies f, which turns into white noise for very long simulation times.

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I. INTRODUCTION

Based mainly on numerical studies, we find that the standard forest fire model (FFM) from statistical physics [1,2] shows a peculiar type of quasiperiodic oscillation. The behavior resembles a Feigenbaum map [3]; i.e., the average oscillation period duration increases on the way to chaos and critical behavior. However, there are no period-doubling bifurcation transitions. The characteristic oscillation frequency $f_{\rm max}$, as characterized by the peak in the power spectrum, remains well defined for very low tree-growth probabilities, i.e., when the timescales of tree growth, tree cluster ignition, and tree cluster burn down become very different (full separation of timescales [1]). $f_{\rm max}$ can be described using a model reformulation introduced for relating the FFM with wildfire models from ecology [4], allowing the calculation of effective model parameters. On long timescales, i.e., for frequencies well below $f_{\rm max}$, time-dependent tree and fire densities initially follow a random walk ($\approx 1/f^2$ scaling of the power spectral density versus frequency f) and approach white noise (approximately flat spectrum) for very long simulations. Critical behavior, as typically characterized by $1/f$ noise (pink noise), only appears as a transition.

Historically, a slightly different FFM [5] has been suggested as an example of self-organized criticality (SOC) [6] to explain the origin of $1/f$ noise. The concept of SOC means that, without fine-tuning of parameters, a system drives itself to a critical point characterized by temporal fluctuations of a major variable around its mean and $1/f$ scaling behavior of its power spectrum. SOC characterizes the dynamics of many devices and natural systems, e.g. electronic devices [7], biological systems [8], astrophysical processes [9], and climate [10].

However, it was soon realized that the model by Bak et al. [5] shows "no indication of a nontrivial critical behavior" [11]. Therefore, the slightly different Drossel-Schwabl FFM (DS-FFM) [1] was introduced shortly later, yielding critical behavior in the case of a double separation of timescales. However, this condition can be fulfilled more easily in the model version suggested independently by Henley [2,12] with instantaneous burning of connected tree clusters, since only the timescales of tree growth and tree cluster ignition have to be considered. We thus use Henley's FFM version in this paper (see model description below). It was also used in two independent studies with huge system sizes [13,14]. Following the early report that "typical states of the system are not critical in the sense of being marginally stable locally," [15] they concluded that "all proposed scaling laws seem to be just transient" [13] and "the DS-FFM is not critical in the sense of being free of characteristic scales" [14].

Nevertheless, the DS-FFM yields an effective power-law scaling behavior for the fire-frequency distribution versus fire size [16], which is confirmed in an analytical approach [17] and approximately consistent with observations of real forest fires [18]. It can also fit well indices describing landscape diversity versus average burnt area per year and several fire properties versus fire size classes [4]. In this sense the practical applicability of the DS-FFM is well comparable [4] with wildfire models from ecology, i.e., so-called landscape fire succession models, although vegetation growth is deterministic and fire spread is stochastic in most ecology models,

*ca-dr@rybski.de

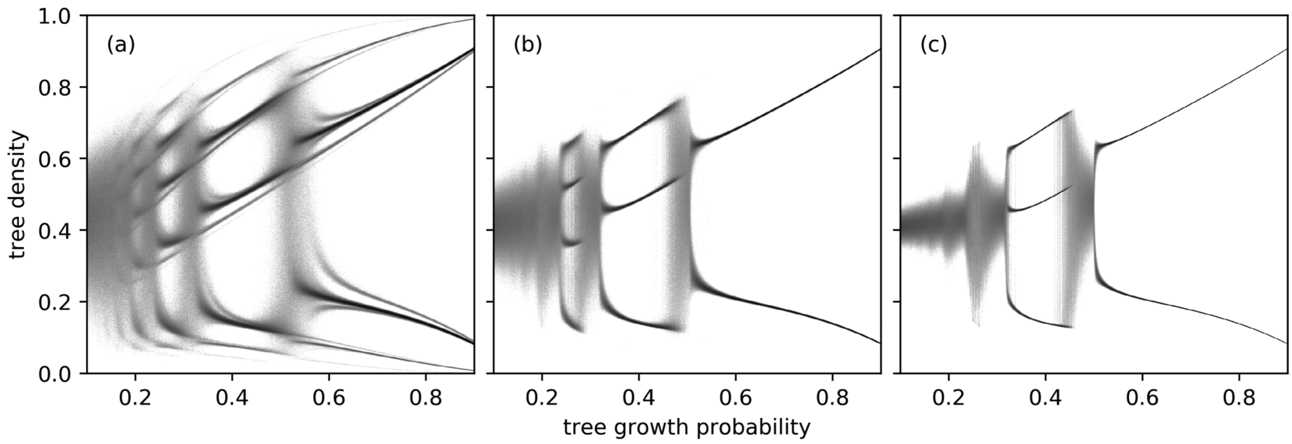

FIG. 1. Density state diagrams for the FFM (Henley's version [2]) with system sizes (a) $L = 200$, (b) $L = 400$, and (c) $L = 1000$. The tree densities ρ occurring along the temporal trajectories are plotted versus the tree growth probability p in the range $0.1 \leqslant p \leqslant 0.9$. The other model parameters are $q = 0.0001$, $\rho_0 = 0.5$, and $I = 8192$. Gray scale corresponds to logarithmic occurrence of each tree density; no occurrence is white.

contrary to the DS-FFM. Therefore, despite not yielding critical behavior in the strict sense of statistical physics—though a kind of "weak" criticality may apply [19]—the DS-FFM still seems a practically relevant and interesting model. We thus fully agree with the very recent conclusion by Palmieri and Jensen that "the peculiar scaling properties of the DS-FFM should be regarded as an asset rather than a limitation" [20]. Hence, we study the short-term and long-term dynamics of the DS-FFM with techniques from time series analysis and—in addition to clarifying the fluctuation behavior—identify a novel quasiperiodic behavior (a well-defined average oscillation period) that extends to the limit of separation of timescales and can be characterized as self-organized multistability (SOM).

II. MODEL DESCRIPTION

In this study, we use Henley's version of the FFM [2,12]. Each cell on a $L \times L$ square lattice can either be empty or occupied by a tree. Initially, trees are randomly placed with the density $\rho_0 = 0.5$. In each time step, new trees grow on empty cells with a probability p. In addition, lightning can strike each cell with a probability $q \ll p$ followed by the burning of the whole nearest-neighbor cluster of trees. Thus, all cells connected to the stricken cell via paths of trees become empty. In our simulations we consider linear system sizes L between 200 and 1000, p values between 0.01 and 0.9, and $q = 0.0001$. We note that comparisons with more detailed wildfire models from ecology show that the size of the cells should be roughly between 6 and 55 ha [4,21], so that each "tree" does in fact describe the behavior of larger areas of forest. Hence, a simulation with $L = 1000$ can represent a forest of up to 500 000 km^2, which is about 1/6 of the total forest area in the U.S.A. The simulations for Fig. 1(c) ($L = 1000$, 1000 different values of p) took approximately four days of single-processor CPU time.

Initially, nearest-neighbor clusters of trees are small, as characterized by the correlation length ξ from percolation theory [22,23], but when the density ρ of trees approaches the critical two-dimensional percolation threshold $\rho_c \approx 0.593$ on the square lattice, a so-called infinite cluster emerges, spanning the whole system. Thus, lightning strikes become much more efficient with increasing ρ. Rough approximations yielded astronomically large ratios $\theta = p/q \approx 10^{40}$ [13] or 10^{37} [20] for reaching the universality class of percolation, where the average ρ shall approach ρ_c and the fire-size distribution, i.e., the size distribution of lightning stricken clusters, shall approach the distribution of percolation clusters. Nevertheless, the fire-size distribution already shows a power-law scaling over several orders of magnitude for much smaller values of θ [4,16] closer to the values for real forests.

III. SELF-ORGANIZED MULTISTABILITY

The surprisingly rich dynamics of the model can be seen in Fig. 1, where the tree density ρ is plotted versus the tree growth probability p for the second halves of the considered $I = 8192$ time steps (to avoid possible transient behavior) and three system sizes. For $p \gtrapprox 0.5$ and the two larger systems [Figs. 1(b) and 1(c)], one can see that the system has reached an attractor with period two, as it jumps between a low $\rho \lessapprox \rho_c$ with only small isolated patches of trees and a rather high $\rho \gtrapprox \rho_c$, where most cells form a dense spanning cluster. In this situation, obviously, a large fire will occur each second time step, consuming the spanning cluster. However, there are also regimes with attractors of period three ($p \approx 0.4$) and four ($p \approx 0.25$), the latter apparently becoming unstable for the largest considered system [Fig. 1(c)]. For small systems, the periodic attractors are also not ideal, since weaker side lines appear next to the strong periodic lines in Fig. 1(a). We note that quite similar pictures emerge for the fire density ϕ, although only single lines appear in the (quasi)periodic regimes.

To understand the temporal dynamics of the FFM in more detail, we focus on examples of the tree-density trajectory $\rho(t)$ as shown in Fig. 2. For $p = 0.1$, Fig. 2(b), the share of tree cells fluctuates irregularly around $x_i \approx 0.4$. Visual inspection suggests quasiperiodic oscillations with a period T of approximately 10 iterations. Similarly, quasiperiodic oscillations with

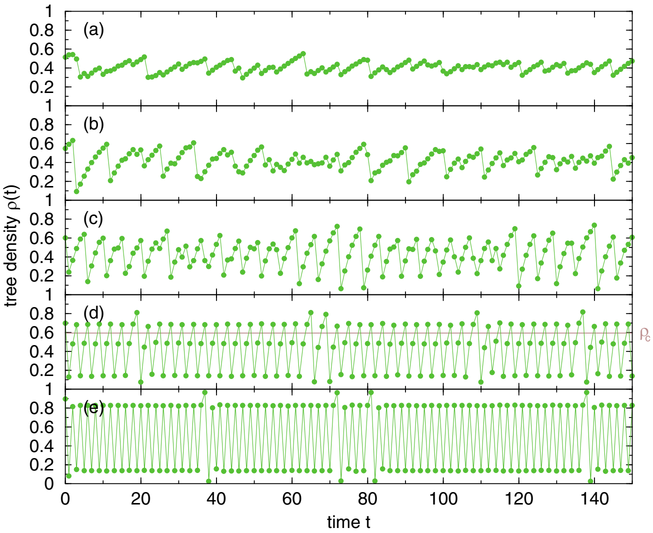

FIG. 2. Typical post-transient time series of tree density for growth probabilities (a) $p = 0.05$, (b) $p = 0.1$, (c) $p = 0.2$, (d) $p = 0.4$, and (e) $p = 0.8$. The other model parameters are $L = 200$, $q = 0.0001$ and $\rho_0 = 0.5$. The horizontal line in (d) indicates the theoretical percolation threshold $\rho_c \approx 0.593$. Points represent data—connecting lines are added for clarity. Animated simulations using the same parameters can be found under the following internet links: [29] (a), [30] (b), [31] (c), [32] (d), [33] (e) (trees shown in white, empty sites in black)

a period of approximately $T = 5$ iterations are observed for $p = 0.2$ in Fig. 2(c).

The corresponding frequencies can also be identified in the power spectra shown in Figs. 3(b)–3(f) for several values of p. The quasiperiodic oscillations yield peaks with local maxima at frequencies $f_{\max} = 1/T$. In particular the spectrum for $p = 0.05$ shown in Fig. 2(a) indicates quasiperiodic oscillations with a frequency $f_{\max} \approx 0.05$, corresponding to a period of $T = 20$ iterations, although the periodicity can hardly be seen with the naked eye in the time series in Fig. 2(a). These results suggest the relationship $f_{\max} \approx p$.

For Fig. 3(a) we have identified the frequency $f_{\max}$ of each peak and plotted it versus p. The local peak in each spectrum is located using a moving window of four values in which a parabola is fitted. $f_{\max}$ is then given by the position of the parabola with highest maximum; small f values are excluded to avoid the systematic increase at low frequencies. The initial approximately linear relationship for $p \ll 1$ can be described by the analytical formula derived by Zinck and Grimm [4] for representing the DS-FFM in the form of a landscape fire succession model as common in ecology. They calculated the fire ignition probability $P(t)$ of a cell depending on the time t since the last fire on the considered cell, $P(t) = 1 - [1 - p(1 - \rho_c)]^t$, so that

$$f_{\max}(p) = 1/T(p) = \frac{\ln[1 - p(1 - \rho_c)]}{\ln(1 - P)} \quad (1)$$

$$\approx -p(1 - \rho_c)/\ln(1 - P) \quad \text{for } p \ll 1. \quad (2)$$

Our fit to the data with slope ≈1.06 suggests that an average fire ignition probability $P \approx 0.31$ must be reached for the fire if we assume $\rho_c = 0.593$. The approximate identity between $f_{\max}$ and p makes sense as faster growth corresponds to a higher overall evolution of the system.

While the linear relation for $f_{\max}(p)$ extends up to $p \approx 0.25$, we find a set of discrete steps between $p \approx 0.25$ and $p = 0.9$. The time series in Figs. 2(d) and 2(e) for $p = 0.4$ and 0.8 show that quasiperiodic oscillations with periods $T = 3$ and 2 iterations occur. The corresponding plateaus at $f_{\max} = 1/T$ are clearly seen in Fig. 3(a). While the highest plateau for $p \geqslant 0.54$ has period 2, it can be seen in Fig. 3(b) that an only slightly weaker peak at $f_{\max} = 1/3$ also appears at the edge of the plateau. The share of trees mostly jumps in a regular fashion between three values. This peak dominates the second plateau from $p \approx 0.33$ to 0.52. The spectrum at $p = 0.31$ in Fig. 3(e) finally shows a typical result for the third plateau with $f_{\max} = 1/4$. Therefore, the system apparently drives itself not only towards fluctuations around critical behavior in one parameter range (low values of p, see also below), but also towards quasiperiodic attractors with different characteristic periods in another parameter range (large values of p)—a behavior that can be characterized as self-organized multistability.

To better understand the discrete steps in $f_{\max}(p)$, let us look again at Fig. 1. For the system size $L = 200$ also considered in the other figures, Fig. 1(a) shows a quite chaotic regime at low values of p up to ≈0.2, where many densities occur with similar probability. For $p = 0.4$ the system exhibits three main densities that form the period-3 attractor, and a few weaker ones, as also suggested by Fig. 2(d). For $p > 0.6$ only two main densities are visible. We note that implementing periodic boundary conditions does not seem to have a significant influence on the diagram.

Overall, with decreasing p, the number of densities increments one by one, whereas the regions in between seem chaotic. The p ranges with a larger number of densities become smaller and, for even lower p, indistinguishable from the chaotic behavior. The regions with a fixed number of characteristic densities match the steps in Fig. 3(a). These findings differ from what is know from the logistic map, because (i) there is no period-doubling on the route to chaos, but there are rather period increments by one, (ii) there seem to be smaller nearly chaotic regimes between the quasiperiodic regimes, and (iii) full chaos is never reached, since the system still retains quasiperiodic behavior with a characteristic frequency $f_{\max}$ down to the smallest considered values of p and described by Eq. (2), although the peak in the power spectrum becomes broader with decreasing p. In addition, we see nearly monotonous increases in tree densities as function of time followed by one sudden decrease (due to a large fire) in each period [Figs. 2(b)–2(e)], while densities usually jump between small and large values within periodic attractors of, e.g., the logistic equation in a Feigenbaum scenario describing the period-doubling transition of a dynamical system to chaos.

However, we would like to stress that quasiperiodic attactors and self-organized multistability are not externally triggered, since both processes, tree growth and lightning, are completely stochastic in our FFM. In a previous paper reporting (log-)periodic behavior in the FFM [24], the periodic attractors are caused by deterministically periodic lightnings.

For larger system sizes, $L = 400$ and 1000 in Figs. 1(b) and 1(c), the lines in the quasiperiodic regimes become

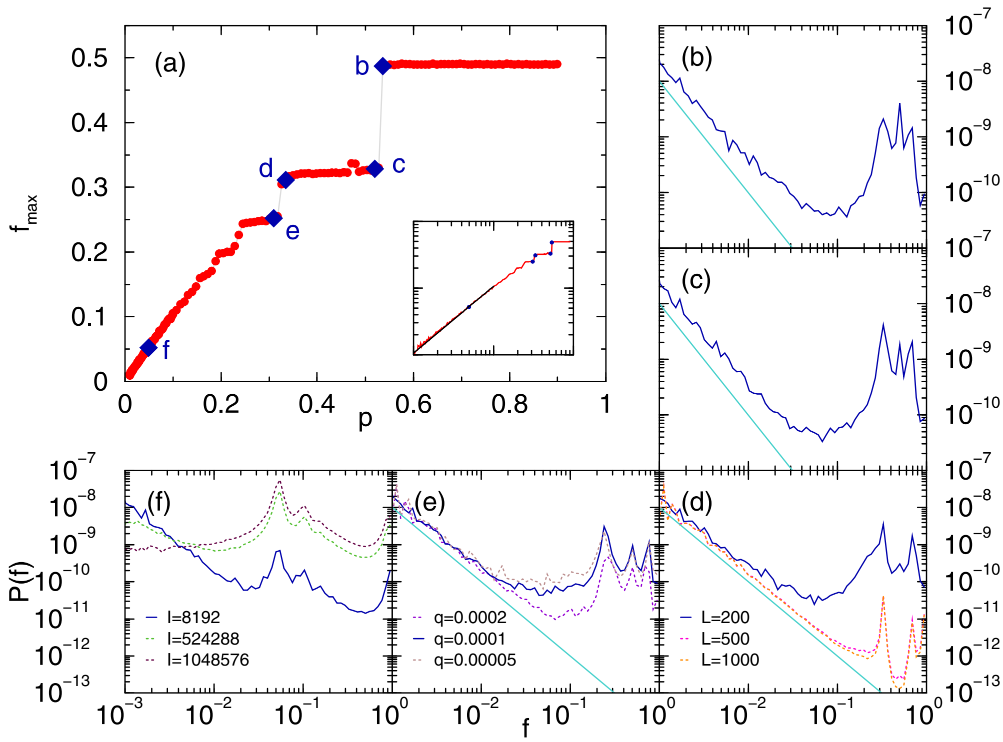

FIG. 3. (a) Dependence of the frequency $f_{\max} = 1/T$ of the quasiperiodic oscillations on the tree growth probability p. Based on our simulations with p values in the range $0.9 > p > 0.01 > q = 0.0001$, we have determined the peak frequencies from power spectra. The inset shows a double logarithmic plot of the same data with a linear fit (slope: 1.06 for $0.01 \leqslant p \leqslant 0.1$). The blue dots indicate the data, for which full power spectra are shown in panels (b–f), with $p = 0.54$ (b), 0.52 (c), 0.33 (d), 0.31 (e), and 0.05 (f). All spectra with blue lines have been calculated by fast Fourier transform, using the second halves of $I = 8192$ iterations simulated for systems with $L = 200$. Additional spectra with dotted lines are shown in (d) for larger L, in (e) for other q, and in (f) for larger I. We note that the spectra are not qualitatively influenced by the lightning parameter q nor by the system size L, while increasing the simulation time I alters the low-frequency behavior. In the double logarithmic plots of the spectra, the power-law $P(f) \sim f^{-2}$ for low frequencies f is indicated by the light blue lines.

sharper, and the weaker additional lines vanish there. These additional lines can thus be attributed to finite-size effects. However, the widths of the quasiperiodic regimes also become a bit smaller, so that the period-4 regime vanishes in the largest simulations. Nevertheless, at least the period-2 attractor will remain stable in the limit of infinite system size, and model variants with modified fire spreading (see, e.g., Ref. [21]) will probably retain attractors with higher periods for very large system sizes.

IV. LONG-TERM SCALING BEHAVIOR

At low frequencies in the power spectra in Figs. 3(b)–3(f), approximately between $f = 10^{-3}$ and $f = 10^{-2}$, we find a power-law regime following $P(f) \sim f^{-2}$ for simulations with $I = 8192$ iterations. In other words, at timescales beyond the quasiperiodicities, the system exhibits Brownian motion type of fluctuations—and not $1/f$ noise as previously believed. This initial Brownian motion type of fluctuations thus seems to come from decaying transient behavior, i.e., a random-walk like motion of the system towards fluctuations around a steady state. This seems consistent with the recent interpretation of the FFM showing “self-organized quasi criticality” [19]; i.e., the system fluctuates around the (theoretical) critical point without ever reaching it exactly.

However, for very long simulations, the low-frequency power-law is gradually changing towards a flat spectrum, $P(f) \sim f^0$, i.e., white noise, see dotted spectra in Fig. 3(f). This effect seems to be independent of the system size L. Initially, the fluctuations are like a random walk, but converge to white noise for asymptotically large times. Since the system needs memory to maintain long-term correlations, the vanishing of temporal correlations might be attributed to the limited memory in the finite-size system. Nevertheless, in the real world it seems possible that forests are in the initial transient regime, since the external parameters are not constant due to, e.g., climate change.

V. FLUCTUATIONS

Finally, we want to characterize the system’s fluctuations in the chaotic regime $p < 0.1$. Figure 4(a) shows that the average density is nearly independent of the system size. However, the densities’ standard deviation σ_ρ scales as

$$\sigma_\rho \sim p^{1/2}/L \tag{3}$$

as shown in Figs. 4(b) and 4(c). This makes sense since with increasing p the systems gains speed and correspondingly stronger fluctuations can be expected. In addition, fluctuations in a larger system become smaller proportional to one over the

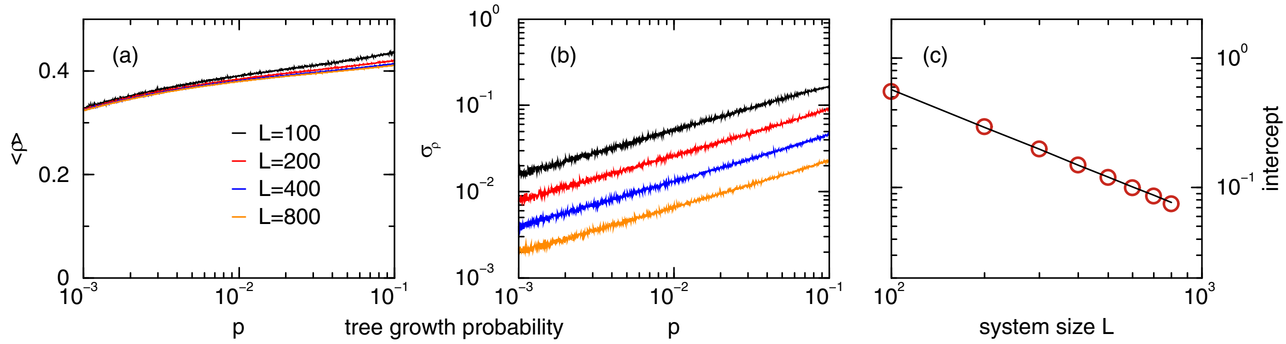

FIG. 4. (a) Average and (b) standard deviation of the tree density ρ as a function of p for systems with different linear sizes L [see legend in panel (a)] and $q = 0.0001$. Panel (c) shows the intercepts of linear regressions to $\ln \sigma_\rho(\ln p)$ as a function of the system size L. Fitted slopes in panel (b) are between 0.52 and 0.53. In panel (c) the straight line fit has the slope -0.97.

square root of the system size according to the central limit theorem, and since the system size is $N = L^2$ here, the $\sim 1/L$ scaling shown in Figs. 4(c) agrees with expectations.

VI. DISCUSSION

In summary, we find four—to our best knowledge—yet unknown properties of the FFM, the first two representing self-organized multistability: (i) periodic attractors for large p, (ii) quasiperiodic oscillations, where the frequency is approximately proportional to p for lower p, (iii) Brownian motion on even slower timescales, and (iv) square-root dependence of the standard deviation on p and the system size.

In the standard FFM (Drossel-Schwabl and Henley version) constant conditions and spatial homogeneity are assumed. In the real world though, these assumptions are challenged. Most wildfires are started by people, not lightning and the spatial occurrence is not random [25]. Wind, orography, weather, and climate all influence the direction and speed of wildfire spread [26]. Last but not least, forest management can modify forest growth and fuel accumulation and fire suppression efforts can limit the size of fires [27]. Together, these factors may make it difficult to confirm these results empirically.

Nevertheless, we conclude that the FFM exhibits complex spatio-temporal structures opening a perspective for further research. In particular, it will be relevant to find out to what extent the attractors of period three and two persist if the system sizes are increased and how attractors with even longer periods could be stabilized. Such stabilization can probably be achieved by considering stochastic fire spread in addition to stochastic tree growth as common in wildfire models from ecology [4,21]. We note that most ecosystems are in fact fire adapted [28]. That is, they have evolved to be burned at different time intervals. Therefore, the quasiperiodic oscillations in the FFM we find here can probably predict or at least fit these time intervals. It will also be interesting to see which of our findings can be confirmed for the original DS-FFM, where fires spread iteratively.

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