Influence of the ionization process on characteristics of spatial relaxation of the average electron energy in inert gases in a uniform electric field

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Theoretical studies of the characteristics of the spatial relaxation of the average energy of electrons in inert gases (He, Ne, Ar, Kr, Xe) in a uniform electric field had been carried out. Conditions were considered when spatial relaxation has the character of damped oscillations. The calculations were performed using the Monte Carlo technique both without and taking into account the secondary electrons that appeared due to the ionization of atoms by electron impact. It was shown that the inclusion of secondary electrons leads to a noticeable decrease in the spatial relaxation length even in the case when the contribution of the ionization process to the electron energy balance is relatively small. For these gases, the upper boundary of the the electric field strength was determined, where the spatial relaxation of the average energy has the character of damped oscillations.

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I. INTRODUCTION

In the absence of an electric field, electrons in a gas are in thermal equilibrium with atoms and molecules, i.e., the electron temperature is equal to the gas temperature, and the energy distribution function of electrons is Maxwellian. If the gas is acted upon a uniform electric field, then the electrons, drifting in the gas under the action of the field, get energy from the field, and lose it in elastic and inelastic collisions with atoms and molecules. As a result, the energy distribution function is established, which is different from the Maxwell function, and the average energy of the electrons becomes greater than the average energy of the gas particles. The form of the distribution function under such conditions is described by the Boltzmann equation [1–3]. If the electric field is not very strong, then the distribution function is weakly anisotropic, and the Boltzmann equation is solved in the so-called two-term approximation for the distribution function. In this case, the spherically symmetric part of the electron energy distribution function (EEDF) $f_0(u)$ is calculated, which is normalized by the condition $2\pi(2/m)^{3/2}\int_0^\infty f_0(u)\sqrt{u}du = n_e$, where u is the electron energy, m is the electron mass, and n_e is the electron concentration. Knowing the distribution function and the cross sections for the electron scattering by atoms and molecules, one can calculate various transport and kinetic characteristics of electrons. In the general case, the form of the distribution function depends on many parameters: the strength of the electric field, the type of gas, the temperature and pressure of the gas, the population of the electronic levels of atoms, and the concentration of electrons. If the degree of excitation of the gas is low and the concentration of electrons is small, then in the Boltzmann equation, superelastic collisions of electrons with excited atoms and electron-electron collisions can be neglected. In this case, for a given gas, the steady-state EEDF will depend only on the local value of the reduced electric field E/N, where E is the electric field strength and N is the number of atoms per unit volume. In the present work, it is just such a situation that is considered.

If in some place in the gas a "perturbed" distribution function is created, which differs from the steady state distribution function for a given E/N value, then spatial/temporal relaxation of the perturbed EEDF to its steady state form will occur. For example, if in some place there is a stationary source of electrons with a certain energy spectrum, then the EEDF near the source will differ from steady-state distribution function in a given electric field. Accordingly, there will be a transition region within which the spatial relaxation of the EEDF occurs from its initial shape to the established one. In experiments, the source can be an electrode emitting electrons. Such a situation is realized, for example, in the well-known experiments by Franck and Hertz [4] (see also Refs. [5–8]), in which evidence was obtained for the existence of quantized energy levels in atoms, and in the experiments of Holst and Oosterhuis [9] and Druyvesteyn [10] (see also Ref. [11]), in which glowing layers were observed in a low-current discharge in inert gases.

A large number of works [12–41] are devoted to the theoretical study of the nature of the spatial relaxation of the EEDF and the associated transport and kinetic characteristics of electrons in a uniform electric field. It was found that in atomic gases, under certain conditions, spatial relaxation of transport and kinetic characteristics takes the form of damped oscillations. In Refs. [12–14], this effect was investigated analytically in the "black-wall" approximation. A detailed description of the spatial relaxation of the EEDF and the transport characteristics of electrons in real gases was carried out either by simulating the motion of electrons with the use of the Monte Carlo technique [15–26] or by solving the nonlocal Boltzmann equation for electrons [18–20,27–40]. Several works [16,19,22,24,38–39] analyzed the situation correspond-

ing to the experimental conditions of Franck and Hertz, Holst and Oosterhuis. In Refs [27–28,32–34,37]. the spatial relaxation of plasma parameters was analyzed in connection with the study of striations in the discharge positive column. In Ref. [38], using a model atomic gas as an example, it was shown that spatial relaxation of plasma parameters in the form of damped oscillations takes place in a certain range of values of *E*/*N*; there are lower and upper limits on the *E*/*N* values for this effect. A similar prediction was also made in Ref. [41] within the framework of the developed hydrodynamic model.

In inert gases, studies were carried out for He [15,16,20,23,27–31,35], Ne [16,19,23,32,35–37], Ar [15–17,21,23–26], Kr [16,23,30,31] and Xe [16,23]. For definiteness, we will talk about the average electron energy $u_m(z)$, where z is the coordinate in the direction opposite to the direction of the electric field. It was shown that spatial relaxation has the form of damped oscillations when energy losses in inelastic processes (excitation of electronic levels) prevail in the electron energy balance. In this case, the period of spatial oscillations is approximately equal to $\Lambda \approx \varepsilon^*/eE$, where ε^* is the energy of the lowest electronic level of the atom, and e is the absolute value of the electron charge. The characteristic length of spatial relaxation L can be determined from the exponent $\exp(-z/L)$, which describes the decrease in the oscillation amplitude. For generality, we will talk about the reduced relaxation length LN, which for a given gas depends only on the value of the reduced electric field E/N. It was shown (see, for example, Ref. [23]) that for all inert gases the dependences of LN on E/N have a maximum at some values of $(E/N)^*$. At $E/N < (E/N)^*$, the decrease in LN is due to an increase in the fraction of energy losses in elastic collisions in the energy balance of electrons. At low E/N, the spatial relaxation of the plasma parameters becomes aperiodic. At $E/N > (E/N)^*$, higher electronic levels are efficiently excited, which leads to suppression of the periodic structure (which is formed due to the predominant excitation of the lowest level) and, accordingly, to a decrease in LN.

In our previous work [23], for all the above inert gases, the dependences $u_m(z)$ were calculated in a wide range of E/N values, and the corresponding LN values were determined. There, the maximum values of E/N, up to which the calculation was carried out, were chosen so that the fraction of the ionization process in the energy balance of electrons was relatively small (<10%), since the calculations did not take into account the secondary electrons generated in the ionization process. In Ref. [23], in particular, the values $(E/N)^*$ at which the relaxation length is maximum, and the lower boundaries of the ranges of E/N values, where the spatial relaxation of the average electron energy has the form of damped oscillations, were determined. As for the upper boundaries of these intervals, this issue has practically not been studied. It was indicated in Ref. [19] that the upper bound for neon is $(E/N)_{up} = 300$ Td, but the corresponding dependences $u_m(z)$ were not presented. For a correct calculation of the spatial relaxation of plasma parameters at high E/N, it is necessary to take into account the secondary electrons which appeared in the ionization process. Note that in most of the works in which the calculations were carried out by solving the Boltzmann equation, secondary electrons were not taken into account, and the ionization process was considered as the excitation of an electronic level. We know only the work [35], in which, within the framework of the Boltzmann equation, the calculations for helium were carried out both without and taking into account secondary electrons. It was shown that the inclusion of secondary electrons leads to a noticeable decrease in the spatial relaxation length even in the case when the fraction of the ionization process in the electron energy balance is relatively small. In several works in which the Monte Carlo method was used, secondary electrons were taken into account in the calculations (see, for example, Refs. [16,19,24]), but a detailed analysis of their effect on the characteristics of spatial relaxation of plasma parameters was not carried out. We also mention Ref. [42], in which the variation of the EEDF in inert gases in a spatially modulated electric field $E(z) = E_0 + \delta E \exp(ikz)$ was analyzed in the limit $\delta E/E_0 \ll 1$, $k = 2\pi/\lambda$, where λ is the wavelength of the electric field disturbance. As a result, the dependence of the amplitude and phase of the electron concentration disturbance on the wavelength value was calculated. At that, the influence of various processes (in particular, ionization processes) on the calculated dependencies was studied.

In the present work, the spatial relaxation of the average electron energy in inert gases (He, Ne, Ar, Kr, Xe) in a uniform electric field was investigated. The calculations were carried out by the Monte Carlo method in two versions. In the first case, the secondary electrons appearing in the ionization processes were taken into account. Hereafter, the term "strict ionization" is used to denote this option. In the second case, secondary electrons were not taken into account, and the ionization process was considered as the excitation of an electronic level. The term "excitation approximation" is used further to refer to this option. The study pursues two goals: (a) to determine at what E/N values the inclusion of secondary electrons noticeably affects the spatial relaxation length of the plasma parameters; (b) to determine the upper limit of the range of E/N values, where spatial relaxation has the character of damped oscillations.

II. CALCULATION METHOD

The calculations were carried out by the Monte Carlo method, which is similar to that used in our previous work [23]. The difference is that in the present work, the calculations took into account the secondary electrons that appear in the process of ionization of atoms by electron impact. The initial electrons were launched at a point with the coordinate $z = 0$ and moved in a uniform electric field directed along the negative direction of the Z axis. Accordingly, the electrons drifted in the positive direction. The motion of electrons in gas was simulated in three-dimensional space, and all characteristics were calculated depending on the z coordinate, i.e., it was assumed that the distribution of electrons is uniform in the plane perpendicular to the Z axis. In a uniform electric field, the trajectory of an electron and the change in its energy along the trajectory were determined by integrating the corresponding equation of motion. The mean free path along the trajectory between two successive collisions with atoms was determined randomly from the solution of the equation

$$\int_{s_0}^{s_1} N Q_{tot}(u(s))ds = -\ln(1-\xi), \qquad (1)$$

where s is the coordinate along the electron trajectory, and s_0 and s_1 are the coordinates of the previous and subsequent collisions, respectively. $Q_{\rm tot}(u)$ is the sum of the cross sections of all processes taken into account in the calculation, ξ is a random number uniformly distributed over the interval (0,1).

The type of collision process was determined randomly. In this case, the probability of each process was assumed to be proportional to the value of the corresponding cross section at the energy $u = u_{\rm b}$, where $u_{\rm b}$ is the electron energy at the moment of collision. It was assumed that spherically symmetric scattering occurs for all types of collisions. In the case of elastic scattering, the electron energy $u_{\rm a}$ after the collision was calculated as a function of the scattering angle. If the process of excitation of an electronic level was chosen, then the electron energy after the collision was determined as $u_{\rm a} = u_{\rm b} - \Delta_{\rm e}$, where $\Delta_{\rm e}$ is the excitation energy.

During ionization, the remainder of the energy $u_{\rm b} - \Delta_i$ (where $\Delta_{\rm i}$ is the ionization energy) was distributed between the primary and secondary electrons. Opal *et al.* [43] measured the energy distribution of secondary electrons for various gases, and it was shown that the shape of the measured distributions is fairly well described by the empirical formula

$$\sigma(u_{\rm pr}, u_{\rm sec}) \sim \frac{1}{1 + (u_{\rm sec}/\overline{E})^{2.1}}, \tag{2}$$

where $u_{\rm pr}$ and $u_{\rm sec}$ are the energies of the primary and secondary electrons. Upon that, the secondary electron was considered the one of two (ionized and appeared in the process of ionization), which has less energy. The parameter $\overline{E}$ depends only on the type of gas, and for He, Ne, Ar, Kr, and Xe is 15.8, 24.2, 10.0, 9.6, and 8.7 eV, respectively. In our calculations, the energy of the secondary electron was determined randomly following distribution (2). The angular distribution of secondary electrons was assumed to be isotropic. The tragectory of each secondary electron was simulated in the same way as it was done for the initial electrons. For comparison, calculations were also carried out in which the ionization process was considered as the excitation of the electronic level, and the generation of secondary electrons was not taken into account.

The average electron energy u_m was calculated at various distances $z = z_i$ from the source similarly to [15,23,44]:

$$u_m(z_i) = \left(\sum_{j=1}^{M} u_j \Delta t_j\right) \Bigg/ \left(\sum_{j=1}^{M} \Delta t_j\right), \tag{3}$$

where $u_{\rm j}$ is the energy of the jth electron crossing a narrow interval $(z_i - \Delta z/2, z_i + \Delta z/2)$, $\Delta t_{\rm j}$ is the time of its stay in this interval, M is the total number of electron crossings the interval (it was taken into account that the same electron can cross a given interval several times). The quantity

$$n(z_i) = \sum_{j=1}^{M} \Delta t_j \tag{4}$$

describes the change in the concentration of electrons with distance in relative units.

In Ref. [23], comparative calculations were carried out using isotropic stationary sources of electrons with different energy spectra: with an energy of 1 eV; with an energy of 4 eV; with energy uniformly distributed in the range 0–4 eV. It was shown that in the case when the spatial relaxation of the average electron energy has the form of damped oscillations, the characteristic damping length and the period of spatial oscillations do not depend on the type of the energy spectrum of the electron source. All the main calculations were performed using a monochromatic source with an energy of 4 eV. The source with the same energy spectrum was used in the present work.

The calculations were carried out using a personal computer. (Core i7 CPU 2.8 GHz, 16 GB RAM, Windows 10). For a given gas pressure and temperature, the calculation time depends on the number of launched electrons, on the electric field strength, and on the distance along the Z axis to which the trajectory of each electron is simulated. For the conditions under study the number of launched electrons was varied within $5 \times 10^4 - 10^5$, and the computation time ranged from 1 to 5 h.

III. SETS OF CROSS SECTIONS USED IN THE CALCULATIONS

In the calculations, the same sets of cross sections for the scattering of electrons by He, Ne, Ar, Kr, and Xe atoms as in Ref. [23] were used. A brief description of these sets is presented in Table I. All the cross sections used are specified in the energy range up to 1000 eV.

IV. RESULTS AND DISCUSSION

The calculations were performed for a gas pressure of $P =$ 1 Torr and a temperature of $T = 273$ K. Let us first consider the results obtained for helium. The studies were carried out for values of $E > 3$ V/cm, at which the spatial relaxation of the average electron energy has the form of damped oscillations [23]. According to calculations, at $E \leqslant 6$ V/cm ($E/N \leqslant$ 17 Td), the "strict ionization" and "excitation approximation" models give exactly the same results, i.e., the appearance of secondary electrons does not affect the shape of the $u_{\rm m}(z)$ dependence. With an increase in E/N and, accordingly, with an increase in the rate constant of ionization of atoms by electron impact, the situation changes. Figure 1 shows the dependences $u_{\rm m}(z)$ calculated in the range of values $E = 8 - 100$ V/cm ($E/N = 22.6$ Td $- 282.6$ Td) both with and without secondary electrons consideration. As shown earlier in Ref. [35], the inclusion of secondary electrons leads to a decrease in the characteristic relaxation length of the average energy.

The influence of secondary electrons on the characteristics of spatial relaxation can be qualitatively commented in terms of electron trajectories in the phase space (w, z), where w is the total electron energy (kinetic + potential). This approach is often used for a qualitative analysis of the motion of electrons in constant and spatially periodic electric fields (see, for example, Ref. [14]). Actually, when the elastic collisions of electrons with atoms and the processes of excitation of several electronic levels are taken into account, the trajectories of electrons will have a rather complex form. But for qualitative reasoning, we can use a simplified model. If we neglect the energy losses of electron in elastic collisions, then in a constant electric field the trajectory of an electron between

TABLE I. Processes taken into account in the calculations and comments on the choice of the corresponding sets of cross sections for scattering of electrons by atoms.

Gas	Processes	Refs. and comments
He	Elastic scattering, excitation of the four lower electronic levels, excitation of the effective electronic level (uniting the upper levels), ionization.	The set of cross sections was created using the data from Ref. [45]. Earlier this set was used in Ref. [20].
Ne	Elastic scattering, excitation of the nine electronic states, ionization.	The set of cross sections was taken from Ref. [46]. A more detailed description of this set is given in Ref. [47].
Ar	Elastic scattering, excitation of the four lower electronic states, excitation of the two effective levels, ionization.	The set of cross sections was taken from Ref. [48] and extended to 1000 eV using the data from Ref. [49].
Kr	Elastic scattering, excitation of the four electronic states, ionization.	The set of cross sections was taken from Ref. [46]. A more detailed description of this set is given in Ref. [50].
Xe	Elastic scattering, excitation of the four electronic states, ionization.	The set of cross sections was taken from Ref. [46]. A more detailed description of this set is given in Ref. [50].

two successive inelastic collisions will have the form of a straight line $w =$ constant. If the only inelastic process is the excitation of an electronic level, then the electron trajectory will have a repetition pattern in the form of straight line segments, the distance between which along the w axis is equal to ε^*. Taking into account the appearance of a secondary electron in the ionization process and energy sharing between two electrons leads to a disturbance of the repetition pattern of trajectories and, accordingly, to a change in the characteristics of spatial relaxation.

Before proceeding to consider the influence of secondary electrons on the characteristic relaxation length of the average electron energy, it is important to note some other effects. As can be seen from Fig. 1, the inclusion of secondary electrons leads to a decrease in the steady-state value of the average electron energy. The difference increases with increasing the *E*/*N* value. This can be qualitatively explained as follows. If in a certain region of space Δz an electron with energy u_b has ionized an atom, and the appearance of a secondary electron is not taken into account, then one electron with energy $u_\mathrm{b} - \Delta_\mathrm{i}$ will remain in the region under consideration. If the generation of the secondary electron is taken into account, then in the considered region there will be two electrons with the total energy $u_\mathrm{b} - \Delta_\mathrm{i}$, and the average energy per electron will be two times less. Accordingly, the calculated average energy u_m will also be smaller. The higher the electric field strength, the more often the ionization processes occur, and the greater the difference between the steady-state values of u_m, calculated in the framework of the "strict ionization" and "excitation approximation" approaches.

The formulation of the problem considered by us corresponds to the conditions of steady-state Townsend experiments. In these experiments, at some distance from the cathode, an exponential increase in the electron concentration was formed with a distance $n_\mathrm{e}(z) \sim \exp(\alpha_i z)$, where α_i is the first Townsend coefficient. Figure 2 shows the dependence $n_\mathrm{e}(z)$ calculated for $E = 20\,\mathrm{V/cm}$ $(E/N \approx 56.5\,\mathrm{Td})$ within the "strict ionization" approach.

As can be seen from Fig. 2, starting from $z \approx 6$ cm, the $n_\mathrm{e}(z)$ dependence becomes exponential. From the presented data, it is possible to calculate the corresponding value of the first Townsend coefficient for $E/N \approx 56.5$ Td: $a_i/N \approx 4 \times 10^{-18}\,\mathrm{cm}^2$. The obtained value is in good agreement with the experimental data presented in Ref. [51].

At relatively low values of E (Fig. 1), the $u_\mathrm{m}(z)$ dependence has the form of damped oscillations relative to the mean value corresponding to the steady-state value of u_m. For example, for the case of the "strict ionization" approach, $u_\mathrm{m}(z)$ has such a form up to $E \approx 70\,\mathrm{V/cm}$ $(E/N \approx 200\,\mathrm{Td})$, the steady-state value of the average energy is approximately equal to $u_\mathrm{ms} \approx 16.5\,\mathrm{eV}$. At higher values of E, the character of the $u_\mathrm{m}(z)$ dependence changes. At $E \approx 90\,\mathrm{V/cm}$ $(E/N \approx 255\,\mathrm{Td})$, oscillations of $u_\mathrm{m}(z)$ with a small amplitude (with the first maximum excluded from consideration) occur against the background of a monotonically increasing average value, which tends to $u_\mathrm{ms} \approx 19.7\,\mathrm{eV}$ [Fig. 1(g)]. And at $E \approx 100\,\mathrm{V/cm}$ $(E/N \approx 280\,\mathrm{Td})$, the aperiodic form $u_\mathrm{m}(z)$ is even more pronounced [Fig. 1(h)]. It can be concluded that the upper boundary of the range of E (*E*/*N*) values, where the character of spatial relaxation of the average electron energy has the form of damped oscillations relative to the average value, for helium is $E \approx 70 - 100\,\mathrm{V/cm}$ $(E/N \approx 200$–$280\,\mathrm{Td})$. For definiteness, we will assume that the upper limit is the value $E_\mathrm{up} \approx 70\,\mathrm{V/cm}$ $((E/N)_\mathrm{up} \approx 200\,\mathrm{Td})$.

It is also worth noting that the periodic character of spatial relaxation changes to aperiodic at such electric field values, at which the steady-state value of u_m approaches the value of the energy of the lowest electronic level ε^* (in helium, $\varepsilon^* \approx 19.8\,\mathrm{eV}$). The last fact seems to be quite logical. Spatial oscillations $u_\mathrm{m}(z)$ occur due to the fact that low-energy electrons, moving in an electric field, acquire energy sufficient to excite electronic levels and then, having lost energy in acts of excitation, again become low energy (see more detailed explanations in Ref. [23]). At that, spatial fluctuations of the average energy occur relative to the value of u_ms, which is the steady-state value of the average energy in a given

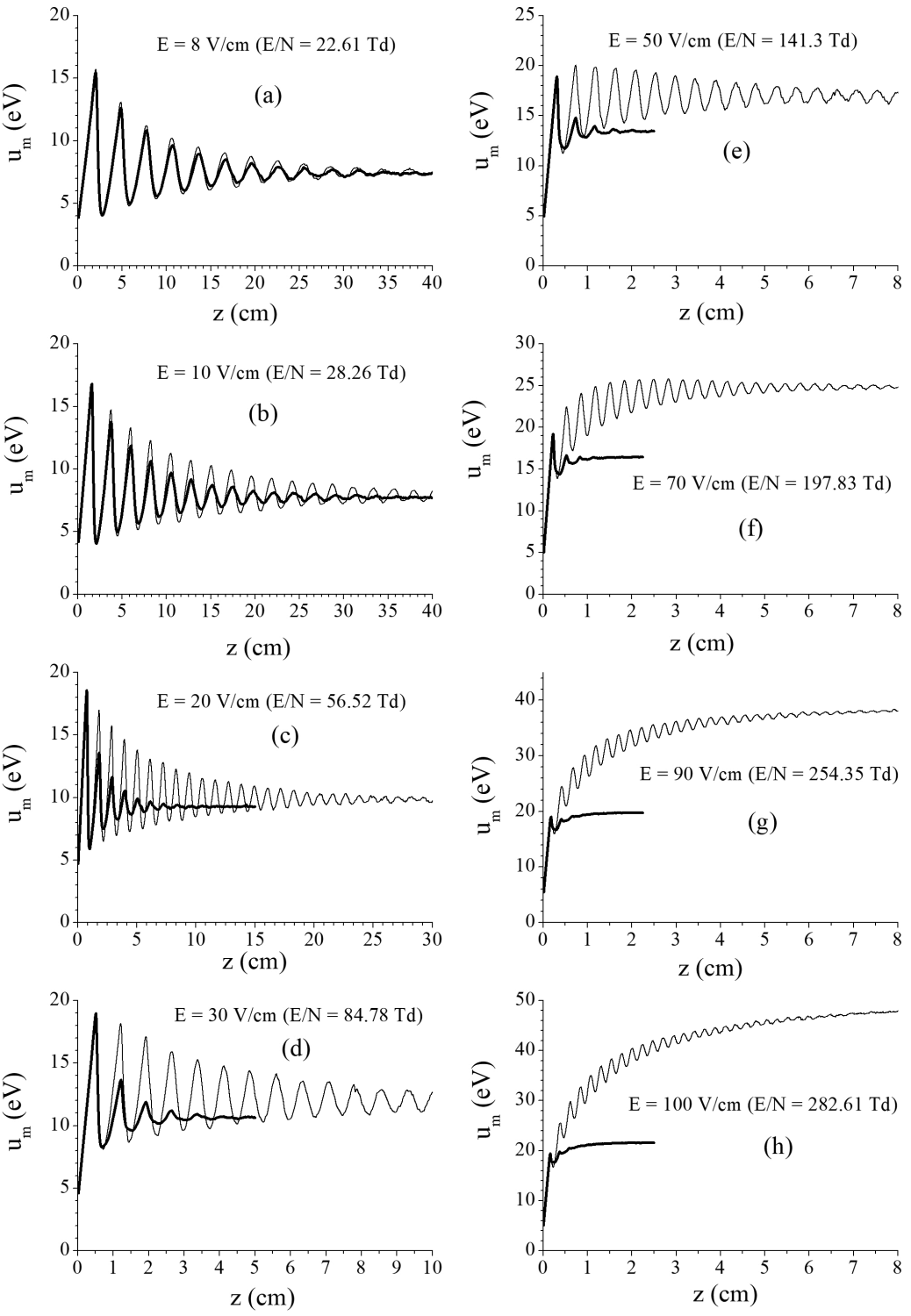

FIG. 1. Spatial relaxation of the average electron energy in helium in a uniform electric field. $P = 1$ Torr, $T = 300$ K. (a) $E = 8$ V/cm, (b) $E = 10$ V/cm, (c) $E = 20$ V/cm, (d) $E = 30$ V/cm, (e) $E = 50$ V/cm, (f) $E = 70$ V/cm, (g) $E = 90$ V/cm, (h) $E = 100$ V/cm. Thick lines show the results of calculations using the "strict ionization" model, thin lines show the results of calculations using the "excitation approximation" model.

electric field. And the initial amplitude of the oscillations is less than ε^*. The higher the electric field strength, the higher the value of $u_{\rm ms}$. When the value of $u_{\rm ms}$ approaches ε^*, the oscillations become poorly pronounced, since most of the electrons have an energy about ε^*. Besides, at $u_{\rm ms} \sim \varepsilon^*$, the rate of ionization is rather high, and the appearance of secondary electrons leads to additional suppression of oscillations.

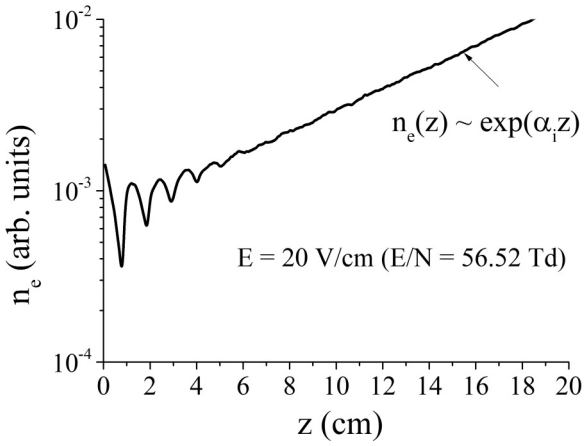

FIG. 2. Dependence of the electron concentration on the distance, calculated for $E = 20\,\mathrm{V/cm}$ $(E/N \approx 56.5\,\mathrm{Td})$ within the "strict ionization" approach. He, $P = 1\,\mathrm{Torr}$, $T = 273\,\mathrm{K}$.

It can be seen from Fig. 1 that the accounting of secondary electrons leads to a decrease in the characteristic relaxation length of the average electron energy, and this effect is enhanced with an increase in the electric field strength. For a quantitative description of the characteristic relaxation length, we applied the same method that was previously used in Ref. [23]. The calculated dependences $u_{\mathrm{m}}(z)$ were approximated by the expression

$$u_m(z) = Ae^{-\frac{z}{L}} \sin\left(\frac{2\pi}{\Lambda}z + \varphi\right) + u_{ms}, \qquad (5)$$

and the parameters A, L, Λ, φ, and u_{ms} were found from the condition of the minimum standard deviation of the values $u_{\mathrm{m}}(z)$ obtained by formula (5) from the calculated results. The parameter Λ describes the period of spatial oscillations, and the value of L in the exponent $\exp(-z/L)$ is the characteristic relaxation length. The values of L for helium obtained from calculations within the framework of the "strict ionization" approach are shown in Fig. 3(a) as a solid line. For comparison, the same figure shows the L values obtained within the "excitation approximation" model. These data were taken mostly from our previous work [23] (dashed line) and partially (for large values of E) were calculated in the present work (dotted line). As can be seen from Fig. 3(a), the accounting of secondary electrons leads to a decrease in the relaxation length, starting from the value $E \approx 8\,\mathrm{V/cm}$ $(E/N \approx 22.6\,\mathrm{Td})$, and at $E \approx 50\,\mathrm{V/cm}$ $(E/N \approx 141\,\mathrm{Td})$, the L values calculated with and without secondary electrons differ by a factor of 8. Fig. 3(a) also shows the value of L estimated by us from the dependence $u(z)$ taken from Ref. [35] where it was calculated by solving the nonlocal Boltzmann equation assuming that the remainder of electron energy after ionization process is divided between the primary and secondary electrons in a ratio of 1:9.

Calculations similar to those performed for helium were carried out for other inert gases: Ne, Ar, Kr, and Xe. Based on the results obtained within the "strict ionization" model, we determined (approximately, see the comments above for the case of helium) the upper boundary of the range of E (E/N), where the spatial relaxation of the average electron energy has the character of damped oscillations of the form (5) (see Table II).

As follows from the obtained results, for all gases under consideration, the character of spatial relaxation $u_{\mathrm{m}}(z)$ becomes aperiodic at such values of E (E/N) when the steady-state value of the average energy u_{ms} approaches the energy of the lowest electronic level ε^*. Note also that the value $(E/N)_{\mathrm{up}} \approx 185\,\mathrm{Td}$ obtained for Ne is less than that indicated in Ref. [19] (300 Td). Apparently, this is due to the approximate criterion for choosing the value of $(E/N)_{\mathrm{up}}$ (see the above comments on the choice of $(E/N)_{\mathrm{up}}$ for helium).

The dependences $L(E)$ calculated for Ne, Ar, Kr, and Xe within the framework of both approaches used are shown in Figs. 3(b)–3(e), respectively. Figure 3(b) also shows the values of L obtained from the $u_{\mathrm{m}}(z)$ dependences for neon, calculated in Ref. [19] by the Monte Carlo method with allowance for secondary electrons. Unfortunately, the used approximation for the energy sharing between two electrons in the ionization process is not indicated in Ref. [19]. It can be seen that obtained L values are in good agreement with those calculated in the present work.

As can be seen from Fig. 3(a), in the case of helium, the inclusion of secondary electrons in calculations leads to a decrease in the maximum value of L, and the position of the maximum shifts to the region of lower E values. In neon and argon [Figs. 3(b) and 3(c)], the inclusion of secondary electrons leads to a decrease in L, starting from the values of E at which the spatial relaxation length of electrons is maximum. In krypton and xenon [Figs. 3(d) and 3(e)], the effect of taking into account secondary electrons becomes noticeable in the region of the decreasing part of the $L(E)$ dependence. The values of the electric field E_s (and corresponding $(E/N)_{\mathrm{s}}$ values), at which the influence of secondary electrons on the relaxation length begins are shown in Table II.

As noted in the Introduction, in Ref. [35], based on the results of calculations for helium, it was qualitatively concluded that the inclusion of secondary electrons leads to a noticeable decrease in the spatial relaxation length even in the case when the fraction of the ionization process in the electron energy balance is relatively small. In this regard, it is of interest to correlate our results with the contribution of the ionization process to the electron energy balance. In the transition region of space, where the $u_{\mathrm{m}}(z)$ dependence has an oscillating character, the contribution of various processes (elastic scattering, excitation of electronic levels, ionization) to the electron energy balance also varies with z coordinate. A detailed study of the structure of the energy balance as a function of z is not the purpose of this work. Therefore, for the estimates, the energy balance of electrons was calculated by the local (established in space) EEDF, which for a particular gas depends only on the E/N value. The calculations were performed by solving the local Boltzmann equation [52] using the same sets of cross sections as in calculations by the Monte Carlo method. Figure 4 shows the fractions of ionization prosess in the energy balance of electrons, calculated depending on the E/N value (lower axis) and/or on the E value (upper axis) at a pressure of $P = 1\,\mathrm{Torr}$ and a temperature of $T = 273\,\mathrm{K}$. The symbols on the curves show the fraction values at $E = E_s$ $(E/N = (E/N)_{\mathrm{s}})$.

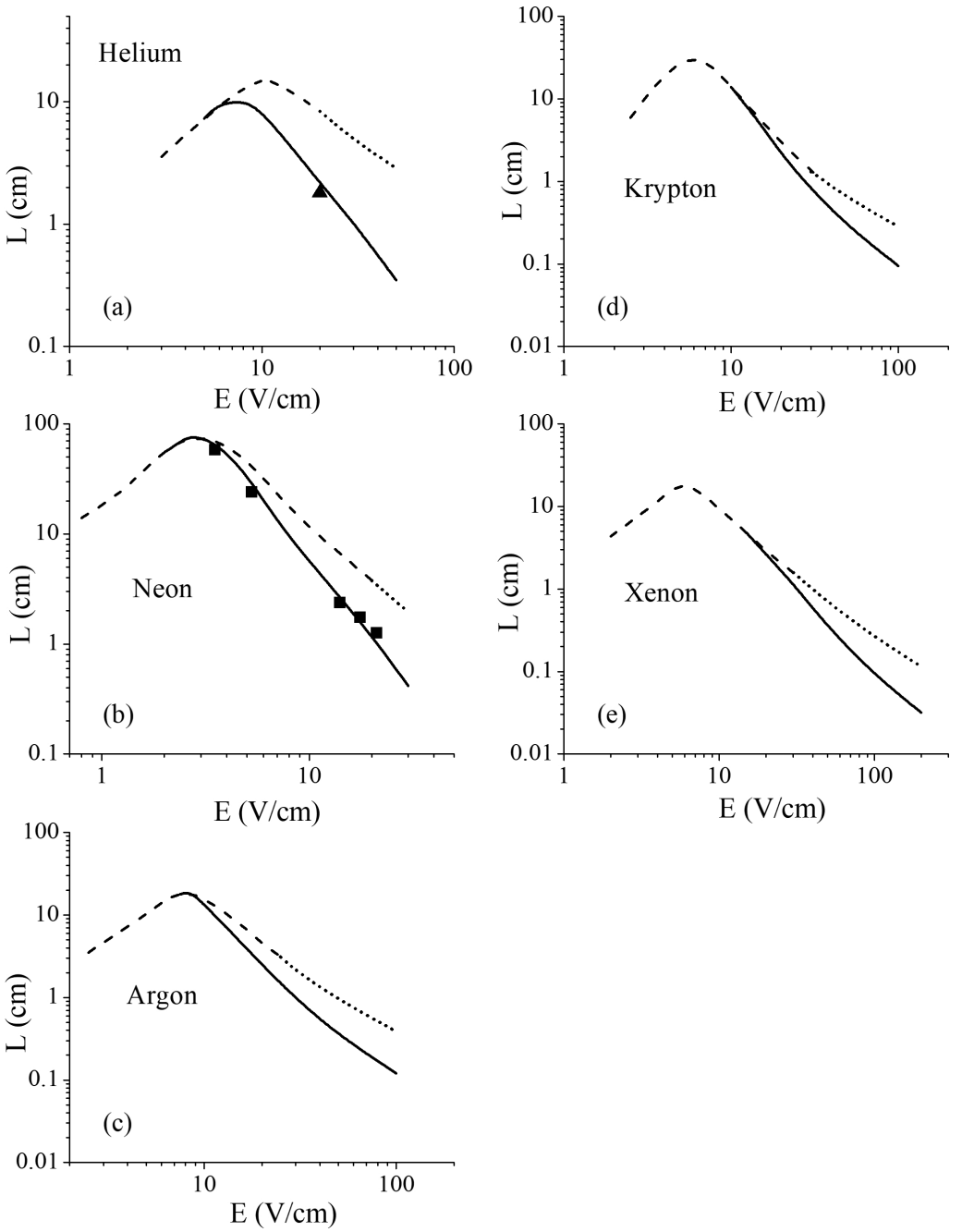

FIG. 3. Dependence $L(E)$ in various inert gases: (a) He, (b) Ne, (c) Ar, (d) Kr, (e) Xe. Solid lines represent results of calculations using the "strict ionization" approach, dashed and dotted lines represent results obtained within the "excitation approximation" model in [23] and in the present work, respectively. $P = 1$ Torr, $T = 273$ K. Symbols in Fig. (a) is the estimate of L from the data of Ref. [35], and in (b) from the data of Ref. [19].

TABLE II. The energy of the lowest electronic level ε^*; ionization energy I; the upper limit of the electric field strength E_{up} (at $P = 1$ Torr, $T = 273$ K); corresponding $(E/N)_{up}$ value; the steady-state value of the average electron energy u_{ms} at $E = E_{up}(E/N = (E/N)_{up})$; the value of the electric field E_s at which the influence of secondary electrons on the relaxation length begins; corresponding $(E/N)_s$ value.

Gas	ε^* (eV)	I (eV)	E_{up}(V/cm)	$(E/N)_{up}$ (Td)	u_{ms} (eV)	E_s (V/cm)	$(E/N)_s$ (Td)
He	19.8	24.5	70	200	16.5	6	17
Ne	16.6	21.6	65	185	15.6	3.5	10
Ar	11.5	15.8	150	425	9.1	9	25
Kr	9.9	14.0	150	425	8.1	15	42
Xe	8.3	12.1	250	700	7.1	19	54

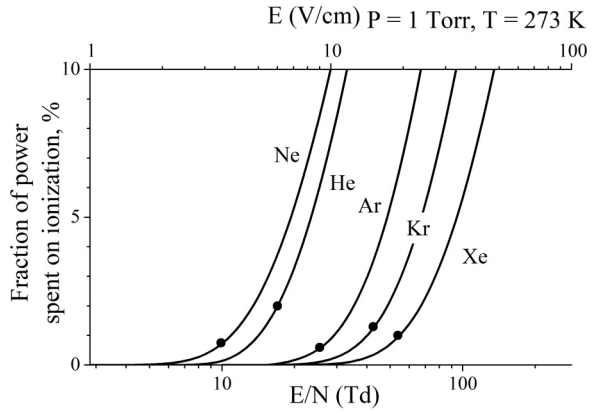

FIG. 4. The fraction of the power in the energy balance of electrons spent on ionization processes in the gases under consideration. The fraction was calculated depending on the magnitude of the electric field by solving the local Boltzmann equation. The symbols on the curves show the fraction values at $E = E_s(E/N = (E/N)_s)$.

As can be seen from Fig. 4, at $E = E_s$, the fraction of energy spent on ionization for all considered gases is ~1%. That is, the influence of secondary electrons on the relaxation length begins already at such values of the electric field strength, at which the contribution of ionization process to the energy balance of electrons is still very small. With an increase in the fraction of energy spent on ionization (with an increase in the electric field strength), this influence sharply increases. For example, at E values, at which the mentioned fraction is only 10% (11.5, 10, 23.5, 33, and 47.5 V/cm for He, Ne, Ar, Kr, and Xe, respectively, see Fig. 4), taking into account secondary electrons leads to a twofold decrease in the relaxation length [Figs. 3(a)–3(e)].

As noted above, the expression for the energy distribution of secondary electrons was obtained in Ref. [43] by approximating the experimental data. To understand how sensitive the calculation results are to the value of the exponent 2.1 in formula (2), we carried out test calculations in which the value of the exponent was equal to 1.9 and 2.3 (i.e., it was varied within 10%). According to calculations, such a variation of the exponent did not lead to noticeable changes in the spatial variation of the mean electron energy, $u_m(z)$, and, respectively, in the relaxation length, $L(E)$.

V. CONCLUSION

The Monte Carlo method was used to study the characteristics of the spatial relaxation of the average electron energy in He, Ne, Ar, Kr, and Xe in a uniform electric field. The studies were carried out for a gas pressure of 1 Torr and a temperature of 273 K in the range of values of the electric field strength, where the spatial relaxation of the average electron energy has the character of damped oscillations. The calculations were carried out taking into account the secondary electrons generated in the course of ionization. For comparison, calculations were also performed, in which secondary electrons were not taken into account, and the ionization process was considered as the process of excitation of the electronic level of an atom. It was shown that the inclusion of secondary electrons in calculations leads to a noticeable decrease in the spatial relaxation length even in the case when the fraction of the ionization process in the electron energy balance is relatively small. For example, when this fraction is 10%, the inclusion of secondary electrons leads to a twofold decrease in the relaxation length.

At relatively low electric fields, spatial fluctuations of the average electron energy occur relative to the value of the steady-state average energy u_{ms} in a given electric field, and the amplitude of the oscillations is less than the energy of the lowest electronic level of the atom ε^*. The steady-state average electron energy increases with an increase in the electric field. At high electric fields when the value of u_{ms} is close to ε^*, the oscillations become poorly pronounced, since most of the electrons have energy about ε^*. As a consequence, the periodic nature of spatial relaxation changes to aperiodic. For all the gases under study, the upper boundary of the range of values of the electric field strength is determined, where the spatial relaxation of the average energy has the character of damped oscillations.

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