Enhancing the performance of an open quantum battery via environment engineering

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We investigate the charging process of open quantum battery in the weak system-environment coupling regime. A method to improve the performance of open quantum battery in a reservoir environment, which described by a band-gap environment model or a two-Lorentzian environment model, is proposed by manipulating the spectral density of environment. We find that the optimal quantum battery, characterized by fast charging time and large ergotropy, in the band-gap environment can be obtained by increasing the weights of two Lorentzians and the spectral width of the second Lorentzian, which is in sharp contrast to the quantum battery in two-Lorentzian environment. Then we extend our discussion to multiple coupled reservoir environments, which are composed of N coupled dissipative cavities. We show that, the performance of quantum battery can be enhanced by increasing the coupling strength between the nearest-neighbor environments and decreasing the size of the environments. In particular, to fully charge and extract the total stored energy as work for quantum battery can be achieved by manipulating the coupling strength between the nearest-neighbor environments. Our results provide a practical approach for the realization of the optimal quantum batteries in future experiments.

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I. INTRODUCTION

In recent years, with the development of quantum technology promoted by the potential power of quantum mechanics [1–12], quantum effects have been considered by people to realize the miniaturization of technology, which opens up new prospects for sensing [13–16], computing [17], and other quantum technologies [18,19]. As an important part of the research on technology miniaturization of nanodevices, quantum batteries play an important role on the quantum level [20–23]. Compared with classical batteries, the charging power of a quantum battery can be greatly improved by using collective quantum resources. Therefore, people have conducted extensive research on quantum batteries [24,25], especially how to use quantum resources to obtain the optimal quantum battery [26–28] that not only has high charging efficiency but also can transfer the stored energy to the consumption center to the greatest extent.

In the beginning, many researchers regard quantum battery as a closed system [29–31], that is, the charger and the quantum battery are not influenced by the environment. For example, by using closed Dicke quantum battery and closed Rabi quantum battery as examples [29], the quantum advantage of the charge power of Dicke quantum battery is demonstrated. Then for a two-photon closed Dicke quantum battery, the authors showed that the two-photon coupling leads to better performances both in the charging times and average charging power of the quantum battery compared to the single-photon case [30]. Closed quantum battery consisting of an ensemble of two-level atoms has also been investigated [31]. It has been found that noninteracting atoms can be fully charged by a harmonic charging field. Furthermore, charging a closed quantum system composed of N independent two-level atoms through a time dependent classical resource has been reported. The above studies focus on how to charge the closed quantum battery faster so as to obtain the optimal quantum battery.

Because the actual systems interact with the environment [32], it is very important to study open quantum batteries [33–40]. Recently, a number of studies [41–44] have shown that, with proper design, the negative effects of the environment on the performance of the quantum batteries can be greatly reduced. In certain cases, environment can even help to improve the performance of the quantum battery. For instance, the authors introduce an open quantum battery protocol using dark states to achieve both superextensive capacity and power density, with noninteracting spins coupled to a reservoir [24]. Salimi *et al.* [44,45] has shown the strong system-environment couplings can enhance the charging performance of quantum battery. However, in most cases, the coupling between the system and the environment is very weak, which causes the charging performance of the quantum battery to be greatly reduced. Therefore, in the weak system-environment coupling regime, how to improve the performance of quantum battery is an important problem.

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In this paper, we study the charging process of the quantum battery in the weak system-environment coupling regime. We first consider the band-gap environment model and two-Lorentzian environment model. Currently, these two environmental models can be implemented experimentally. More specifically, the band-gap environment model can be realized by a two-level atom embedded in a photonic crystal cavity with the periodic dielectric structures forming photonic band gaps [46,47]. Two noise environmental sources faced by a single nitrogen-vacancy (NV) spin can be modeled as a two-Lorentzian environment [48]. Based on these actual physical scenarios, it is reasonable for us to consider the band-gap environment model and the two-Lorentzian environment model. Both models are formed by adding a new Lorentzian spectrum to the original single Lorentzian spectrum. The charging performance of the quantum battery in the band-gap environment can be enhanced by increasing the weights of two Lorentzians and the spectral width of the second Lorentzian. However, for the two-Lorentzian environment, the manipulation the weights of two Lorentzians and the spectral width of the second Lorentzian cannot change the fact that the ergotropy of the quantum battery is always zero. Therefore, compared with the two-Lorentzian environment, the situation of the quantum battery in the band-gap environment should be selected to obtain the optimal quantum battery. Then among other factors, the coupling strength between the environments and the size of the environment are a significant knob to control the charging performance of the quantum battery. We, thus, explore the coupling strength between the nearest neighbor environments and the scaling effects of the environment on the charging process of the quantum battery by considering the situation that the quantum battery is in the multiple coupled reservoir environments. We find that the storage energy of the quantum battery, the average charging power, and the ergotropy can be promoted with the increase of the coupling strength between the environments and the decrease of the size of the environment. In particular, the maximum internal energy and maximum extractable work of a quantum battery can be obtained by increasing the coupling strength between the environments.

This paper is organized as follows. In Sec. II we review the energy stored of the quantum battery, average charging power, and the extractable work. Section III discusses the influence of the weights of two Lorentzians and the spectral width of the second Lorentzian on the performance of the quantum battery from two cases that the quantum battery is in band-gap environment or two-Lorentzian environment, respectively. In Sec. IV we discuss the scaling effects of the environment and the coupling strength between the environments on the charging process of the quantum battery. The conclusions drawn from the present study are given in Sec. V.

II. QUANTUM BATTERY

A good quantum battery should have two conditions: one is to store the maximum energy in the shortest time, and the other is the ability to discharge the energy sufficiently in the required time. To get a good quantum battery, we study the performance of the quantum battery, i.e., the energy stored, average charging power, and the extractable work.

At t time, the energy stored of the quantum battery is defined as

$$E_B = \mathrm{Tr}\,[H_B\rho_B(t)] - \mathrm{Tr}\,[H_B\rho_B(0)], \tag{1}$$

where H_B is the Hamiltonian of the quantum battery, $\rho_B(s)$ ($s = 0$ or t) is state of the quantum battery at time s. The average charging power at time t is given by

$$P_B = \frac{E_B}{t}. \tag{2}$$

In both cases, we focus on maximum storage energy and storage power. Therefore, we introduce $E_{\max} \equiv \max_t[E_B(t)] \equiv E_B(t_E)$, $P_{\max} \equiv \max_t[P_B(t)] \equiv P_B(t_P)$.

To define quantum battery's ability to produce useful work, the ergotropy is introduced as

$$W_B = \mathrm{Tr}\,(\rho_B(t)H_B) - \mathrm{Tr}\,(\sigma_{\rho_B}H_B), \tag{3}$$

in which σ_{ρ_B} is called as passive states where no amount of work can be extracted from the quantum battery in a cyclic unitary process. The maximum ergotropy can be obtained by $W_{\max} \equiv \max_t[W_B(t)] \equiv W_B(t_W)$.

In the following, we use the storage energy E_B, the average charging power P_B and the ergotropy W_B to evaluate the performance of the quantum battery. The larger E_B, P_B, and W_B are expected to be.

III. THE CHARGING PERFORMANCE OF THE QUANTUM BATTERY IN A SINGLE ENVIRONMENT

Our total system consists of two two-level systems (a quantum battery B and a quantum charger C) interacting with each other, and the structure electromagnetic reservoirs. Then to get the conditions under which quantum batteries can charge more efficiently, we considered two situations that the quantum battery in a two-Lorentzian environment or a band-gap environment.

A. Case of the quantum battery in band-gap environment

We first consider that the quantum battery interacts resonantly with its structural environment. The Hamiltonian of the system under the rotational wave approximation can be given as $H = H_0 + H_{\mathrm{I}}$, where

$$H_0 = \frac{\omega_0}{2}\sigma_z^{\mathrm{B}} + \frac{\omega_0}{2}\sigma_z^{\mathrm{C}} + \sum_k \omega_k a_k^\dagger a_k, \tag{4}$$

$$H_{\mathrm{I}} = \Omega(\sigma_+^{\mathrm{B}}\sigma_-^{\mathrm{C}} + \sigma_-^{\mathrm{B}}\sigma_+^{\mathrm{C}}) + \sum_k (g_k^B\sigma_+^{\mathrm{B}}a_k + g_k^{*\mathrm{B}}\sigma_-^{B}a_k^\dagger), \tag{5}$$

here the first two terms in Eq. (4) represent the Hamiltonian of the quantum battery and the quantum charger, as well as the last term is the Hamiltonian of the reservoir. The interaction Hamiltonian of the system is represented in Eq. (5), where σ_+^{j} and σ_-^{j} represent the raising and lowering Pauli operators of the jth qubit. For the sake of analysis, we assume that the spectral density function of the reservoir is $D(\omega) = W_1\Gamma_1/[(\omega-\omega_c)^2 + (\Gamma_1/2)^2] - W_2\Gamma_2/[(\omega-\omega_c)^2 + (\Gamma_2/2)^2]$, which is composed of positive weight Lorentzian spectrum and negative weight Lorentzian spectrum. For an ideal band-gap model, the two Lorentzian spectrums are centered at same frequency ω_c. To ensure the

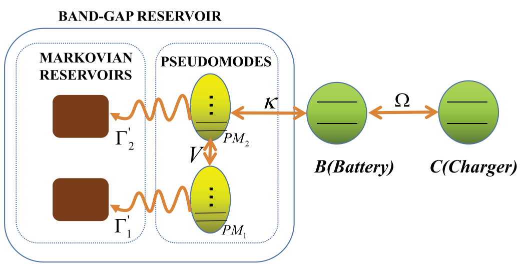

FIG. 1. Schematic representation of the quantum charger C, quantum battery B, and pseudomodes system in band-gap environment.

positivity of the spectral density $D(\omega)$, the weights and the widths of the two Lorentzian spectrums are satisfied $W_1 - W_2 = 1$ and $\Gamma_2 < \Gamma_1$. In particular, for a perfect band-gap model, $D(\omega_c) = 0$, i.e., $W_1/\Gamma_1 = W_2/\Gamma_2$. In the following, we use pseudomodes approach [49–53] to study the dynamical behavior of quantum battery in the reservoir environment. This approach relies on the relationship between quantum battery dynamics and the shape of the spectral distribution of the reservoir. More accurately speaking, the key quantities affecting the time evolution of the quantum battery are the poles of the spectral distribution in the lower half complex plane. Each pole is associated with a pseudomode. For the band-gap reservoir environment, the reservoir can be represented by two nondegenerate pseudomodes that leak into the Markovian reservoir with dissipation rates $\Gamma_1' = W_1\Gamma_2 - W_2\Gamma_1$ and $\Gamma_2' = W_1\Gamma_1 - W_2\Gamma_2$, as described in Fig. 1. Then the quantum battery only interacts with the second pseudomode PM_2 (the strength of the coupling κ) which is in turn coupled to the first pseudomode PM_1 [the strength of the coupling $V = \sqrt{W_1W_2}(\Gamma_1 - \Gamma_2)/2$].

The dynamics of the total system can be given by the exact pseudomode master equation

$$\frac{d\rho}{dt} = -i[H_0^1, \rho] - \frac{\Gamma_1'}{2}(a_1^\dagger a_1\rho - 2a_1\rho a_1^\dagger + \rho a_1^\dagger a_1) - \frac{\Gamma_2'}{2}(a_2^\dagger a_2\rho - 2a_2\rho a_2^\dagger + \rho a_2^\dagger a_2), \tag{6}$$

where $H_0^1 = \omega_0\sigma_z^B/2 + \omega_0\sigma_z^C/2 + \omega_c a_1^\dagger a_1 + \omega_c a_2^\dagger a_2 + \Omega(\sigma_+^B\sigma_-^C + \sigma_-^B\sigma_+^C) + \kappa(\sigma_+^B a_2 + \sigma_-^B a_2^\dagger) + V(a_1^\dagger a_2 + a_1 a_2^\dagger)$, here $a_j(a_j^\dagger)$ represents the annihilation (creation) operators of the jth pseudomode. We consider quantum charger C, quantum battery B, and pseudomodes (PM_1, PM_2), respectively, in the excited state $|1_C\rangle$ and the vacuum state $|0_B0_{\mathrm{PM}_1}0_{\mathrm{PM}_2}\rangle$, i.e., $\rho(0) = |1_C0_B0_{\mathrm{PM}_1}0_{\mathrm{PM}_2}\rangle\langle 1_C0_B0_{\mathrm{PM}_1}0_{\mathrm{PM}_2}|$. According to Eq. (6), the solution of the total system dynamics can be written as $\rho(t) = \lambda(t)|0000\rangle\langle 0000|_{\mathrm{CBPM_1PM_2}} + |\widetilde{\varphi}(t)\rangle\langle\widetilde{\varphi}(t)|$, with $|\widetilde{\varphi}(t)\rangle = u(t)|1000\rangle_{\mathrm{CBPM_1PM_2}} + v(t)|0100\rangle_{\mathrm{CBPM_1PM_2}} + m_1(t)|0010\rangle_{\mathrm{CBPM_1PM_2}} + m_2(t)|0001\rangle_{\mathrm{CBPM_1PM_2}}$, where $\lambda(t)$ is the vacuum state $|0000\rangle\langle 0000|_{\mathrm{CBPM_1PM_2}}$ population, and satisifies $\dot{\lambda}(t) = \Gamma_1'|m_1(t)|^2 + \Gamma_2'|m_2(t)|^2$. $u(t)$, $v(t)$, and $m_n(t)$ $(n = 1, 2)$ correspond to probability amplitudes for the quantum charger, battery, and pseudomodes, respectively. Substitute the expression for $\rho(t)$ into Eq. (6), the time-dependent amplitudes $u(t)$, $v(t)$, $m_1(t)$, $m_2(t)$ are determined by a set of differential equations as $idu(t)/dt = \omega_0 u(t) + \Omega v(t)$, $idv(t)/dt = \kappa m_2(t) + \Omega u(t) + \omega_0 v(t)$, $idm_1(t)/dt = (\omega_c - i\Gamma_1'/2)m_1(t) + Vm_2(t)$, $idm_2(t)/dt = (\omega_c - i\Gamma_2'/2)m_2(t) + \kappa v(t) + Vm_1(t)$. The above differential equations can be solved by combining standard Laplace transform with numerical simulation. The dynamics of the quantum battery B can be obtained by tracing the quantum charger C and two pseudomodes, i.e., $\rho_B = \mathrm{Tr}_{C,\mathrm{PM}_1,\mathrm{PM}_2}\,\rho$. The matrix elements of ρ_B are $\rho_{ee}^B(t) = |v(t)|^2$, $\rho_{gg}^B(t) = 1 - |v(t)|^2$, $\rho_{ge}^B(t) = \rho_{ge}^B(0)v(t)$, $\rho_{eg}^B(t) = \rho_{eg}^B(0)v^*(t)$.

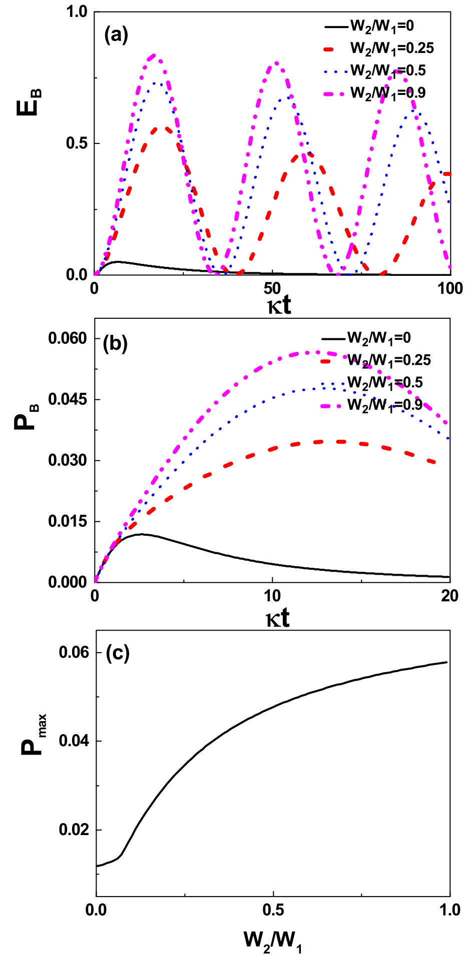

FIG. 2. (a, b) The internal energy E_B and the charging power P_B of the quantum battery as a function of the dimensionless quantity κt for different values of the weight of the two Lorentzians W_2/W_1. (c) Maximum charging power $P_{\max}$ of the quantum battery as a function of the weight W_2/W_1 of the two Lorentzians. The parameters are: (a–c) $\Gamma_1 = 5\kappa$, $\Omega = 0.1\kappa$.

According to Eqs. (1), (2), and (3), we can get the internal energy $E_B = \omega_0|v(t)|^2$ of the quantum battery, charging power $P_B = \omega_0|v(t)|^2/t$, and the ergotropy $W_B = \omega_0(2|v(t)|^2 - 1)\Theta(|v(t)|^2 - \frac{1}{2})$, where $\Theta(x - x_0)$ is the Heaviside function. Then, we can analyze the influences of the weights of the two Lorentzians and the spectral width of the second Lorentzian in the perfect band-gap environment on the internal energy E_B, charging power P_B and ergotropy W_B of the quantum battery.

For a single-Lorentzian spectrum environmental model $D(\omega) = \Gamma/[(\omega - \omega_c)^2 + (\Gamma/2)^2]$ (i.e., $W_2/W_1 = 0$ in Fig. 2), when the spectral width Γ of the Lorentzian and the coupling strength κ between the system and the pseudomode satisfies $\Gamma > 2\kappa$, the system and the environment is in the weak coupling regime, leading to a lower charging power of the quantum battery. The band-gap spectrum environmental model is formed by adding a negative weight Lorentzian spectrum to the original single Lorentzian spectrum. Therefore, when the first Lorentzian spectrum in the band-gap environment meets the weak coupling regime (i.e., $\Gamma_1 = 5\kappa$), the effect of additional negative weight Lorentzian spectrum on the charging performance of quantum battery should be explored. Our aim is to improve the charging power of quantum battery in the weak system-environment coupling regime (i.e., $\Gamma_1 = 5\kappa$ in Fig. 2) by manipulating the weight W_2/W_1 of the two Lorentzians and the spectral width Γ_2. In perfect band-gap environment, due to the relationship between the weights (W_1, W_2) of the two Lorentzians and the spectral width (Γ_1, Γ_2), for a fixed $\Gamma_1 = 5\kappa$, the spectral width is $\Gamma_2 = W_2\Gamma_1/W_1$. Thus, Γ_2 and W_2/W_1 have the same influence on the charging performance of the quantum battery. For convenience, let us study the influence of W_2/W_1 on the internal energy E_B and charging power P_B. We find that E_B and P_B of a quantum battery can be improved by increasing the weight W_2/W_1 of the two Lorentzians, as shown in Figs. 2(a) and 2(b). Then to comprehensively understand the influence of W_2/W_1 on the charging process of the quantum battery, we plot the change of the maximum charging power $P_{\max}$ with W_2/W_1 in Fig. 2(c). It is shown that the maximum charging power $P_{\max}$ of a quantum battery increases monotonically as W_2/W_1 increases. This means that, in the weak system-environment coupling regime, to achieve the optimal charging power, a larger the weight W_2/W_1 of the two-Lorentzians and a larger the spectral width Γ_2 should be consider.

A good quantum battery would also be able to transfer stored energy completely to the center of consumption in a useful way. Here we also investigate the influence of the weight W_2/W_1 of the two Lorentzians on the ergotropy W_B. As depicted in Fig. 3(a), a larger W_2/W_1 will give a larger ergotropy W_B, meaning that an increase in W_B allows us to extract more energy from the battery. Then Fig. 3(b) shows the dependence of maximum ergotropy $W_{\max}$ on W_2/W_1. The $W_{\max}$ will monotonically increase as W_2/W_1 is over certain threshold W_{2c}/W_{1c}. That is to say, to improve the charging performance of quantum battery in perfect band-gap environment, a larger the weight of the two Lorentzians W_2/W_1 and a larger the spectral width Γ_2 are required.

One may wonder why the increase of W_2/W_1 improves the charging performance of quantum battery. Here we explain this problem from the perspective of the energy flow between the pseudomodes and the quantum battery. To witness the direction of energy flow between the quantum battery and the pseudomodes (i.e., PM_1 and PM_2), the compensation rate for the population change of the pseudomodes is used, which is defined as [49]

$$M(t) \equiv \frac{d\sum_{n=1}^{2}|m_n(t)|^2}{dt} + \sum_{n=1}^{2}\Gamma_n'|m_n(t)|^2, \tag{7}$$

where $|m_n(t)|^2$ is the excited state population of the pseudomodes PM_1 and PM_2. If the pseudomodes energy decreases (i.e., $d\sum_{n=1}^{2}|m_n(t)|^2/dt < 0$) and this decrease cannot be compensated for by the dissipation term $\sum_{n=1}^{2}\Gamma_n'|m_n(t)|^2$ of the pseudomodes, then we can determine that the energy flow is from the pseudomode PM_2 to the quantum battery (i.e., $M(t) < 0$). In other words, the negative value of $M(t)$ represents the energy flow from the pseudomode PM_2 to the quantum battery B. In Fig. 4, we plot the witness $M(t)$ as a function of κt for different W_2/W_1. We find that, when $W_2/W_1 = 0$, $M(t)$ is always positive, which means that the energy has been flowing from the quantum battery B to the pseudomode PM_2, resulting in poor charging performance of

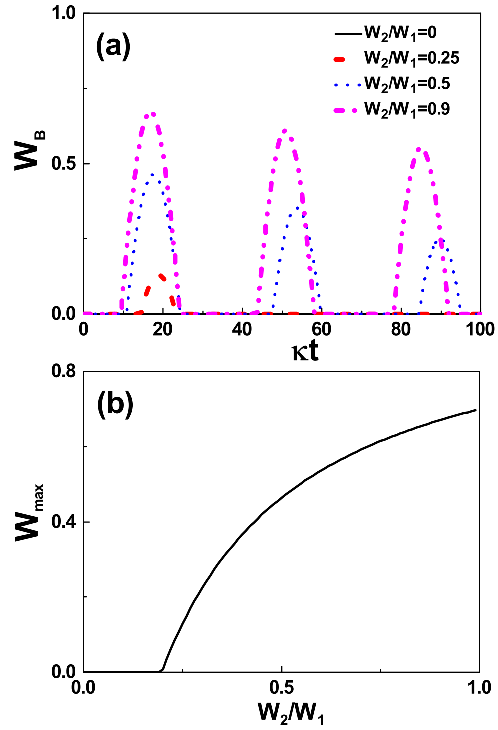

FIG. 3. (a) Ergotropy W_B as a function of the dimensionless quantity κt for different values of the weight W_2/W_1 of the two Lorentzians. (b) Maximum ergotropy $W_{\max}$ of the quantum battery as a function of the weight W_2/W_1 of the two Lorentzians. The parameters are: (a, b) $\Gamma_1 = 5\kappa$, $\Omega = 0.1\kappa$.

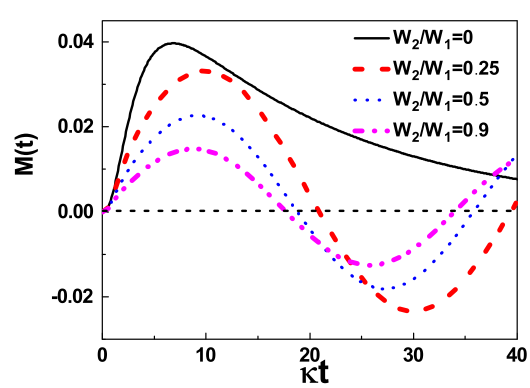

FIG. 4. The witness $M(t)$, Eq. (7), as a function of the dimensionless quantity κt for different values of the weight W_2/W_1 of the two Lorentzians. The parameters are: $\Gamma_1 = 5\kappa$, $\Omega = 0.1\kappa$.

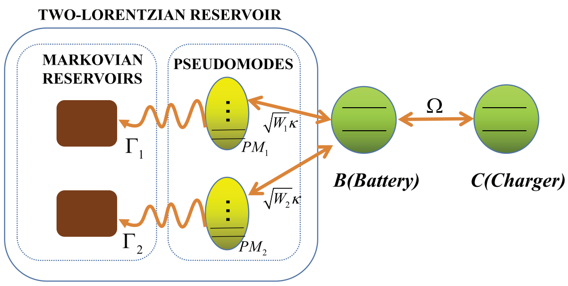

FIG. 5. Schematic representation of the quantum charger, quantum battery and pseudomodes system in two-Lorentzian environment.

the quantum battery. While $W_2/W_1 \neq 0$, the negative value of $M(t)$ begins to appear, and the larger W_2/W_1 is, the earlier the negative value of $M(t)$ appears. This implies that with the increase of the weight W_2/W_1 of the two Lorentzians, the energy will flow from the pseudomode PM_2 to the quantum battery earlier, leading to better charging performance of the quantum battery. That is to say, the fundamental reason, for improving the charging power of a quantum battery by adjusting W_2/W_1, is the energy flow from the pseudomode PM_2 to the quantum battery.

B. Case of the quantum battery in two-Lorentzian environment

Now let us consider a quantum battery B in a two-Lorentzian environmental model, which the spectrum density is $D(\omega) = W_1\Gamma_1/[(\omega-\omega_c)^2+(\Gamma_1/2)^2] + W_2\Gamma_2/[(\omega-\omega_c)^2+(\Gamma_2/2)^2]$, where the weight of the two Lorentzian spectrum satisfies $W_1 + W_2 = 1$. This spectrum environmental model is formed by adding a positive weight Lorentzian spectrum to the original single Lorentzian spectrum. The total Hamiltonian of the system is $H = H_0 + H_\text{I}$, where

$$H_0 = \frac{\omega_0}{2}\sigma_z^\text{B} + \frac{\omega_0}{2}\sigma_z^\text{C} + \sum_k \omega_k b_k^\dagger b_k, \tag{8}$$

$$H_\text{I} = \Omega(\sigma_+^\text{B}\sigma_-^\text{C} + \sigma_-^\text{B}\sigma_+^\text{C}) + \sum_k (g_k\sigma_+^\text{B} b_k + g_k^*\sigma_-^B b_k^\dagger), \tag{9}$$

here $b_k^\dagger$ (b_k) represents the raising (lowering) operators of the kth mode of the reservoir. The pseudomode method is also used to represent the two-Lorentzian reservoir as shown in Fig. 5. The exact master equation for the quantum battery-environment and charger dynamics in the two-Lorentzian environmental model can be written as

$$\frac{d\rho}{dt} = -i[H_0^2, \rho] - \frac{\Gamma_1}{2}(b_1^\dagger b_1\rho - 2b_1\rho b_1^\dagger + \rho b_1^\dagger b_1) - \frac{\Gamma_2}{2}(b_2^\dagger b_2\rho - 2b_2\rho b_2^\dagger + \rho b_2^\dagger b_2), \tag{10}$$

where $H_0^2 = \omega_0\sigma_z^\text{B}/2 + \omega_0\sigma_z^\text{C}/2 + \Omega(\sigma_+^\text{B}\sigma_-^\text{C} + \sigma_-^\text{B}\sigma_+^\text{C}) + \omega_c b_1^\dagger b_1 + \omega_c b_2^\dagger b_2 + \kappa\sqrt{W_1}(\sigma_+^\text{B} b_1 + \sigma_-^\text{B} b_1^\dagger) + \kappa\sqrt{W_2}(\sigma_+^\text{B} b_2 + \sigma_-^\text{B} b_2^\dagger)$. We assume that only quantum charger C is in the excited state $|1_C\rangle$, quantum battery B and other pseudomodes (PM_1, PM_2) are in the vacuum state $|000\rangle$. Then according to the same steps of quantum battery in band-gap environment, the dynamics of the quantum battery in the basis ($|e\rangle, |g\rangle$) can be obtained

$$\rho_B(t) = \begin{bmatrix} |c(t)|^2 & \rho_{eg}(0)c(t) \\ \rho_{ge}(0)c^*(t) & 1-|c(t)|^2 \end{bmatrix}. \tag{11}$$

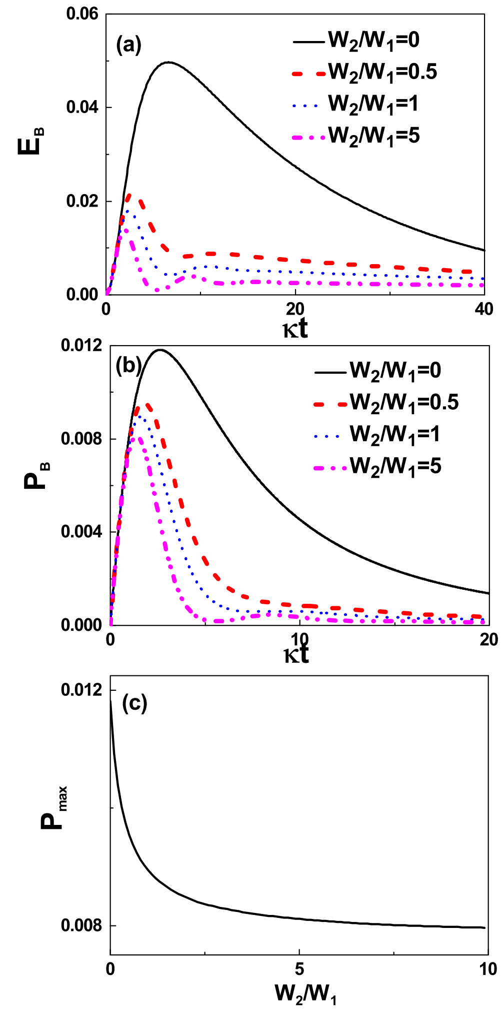

FIG. 6. (a, b) The internal energy E_B and the charging power P_B of the quantum battery as a function of the dimensionless quantity κt for different values of the weight of the two Lorentzians spectrum W_2/W_1. (c) Maximum charging power P_max of the quantum battery as a function of the weight of the two Lorentzians spectrum W_2/W_1. The parameters are: (a–c) $\Gamma_1 = 5\kappa$, $\Gamma_2 = \kappa$, $\Omega = 0.1\kappa$.

To get how to improve the charging power of quantum battery B in two-Lorentzian environment, we analyzed the influence of the weight W_2/W_1 of the two Lorentzians and the spectral width Γ_2 on the charging power of the quantum battery. According to Eqs. (1), (2), (3), and (11), the internal energy $E_B = \omega_0|c(t)|^2$ of the quantum battery, charging power $P_B = \omega_0|c(t)|^2/t$, the ergotropy $W_B = \omega_0(2|c(t)|^2-1)(|c(t)|^2 > 1/2)$ and $W_B = 0$ for $|c(t)|^2 \leqslant 1/2$ can be obtained.

Unlike the situation of the quantum battery in band-gap environment, the internal energy E_B and charging power P_B of quantum battery would be decreased with the increase of W_2/W_1, as described in Figs. 6(a) and 6(b). To understand

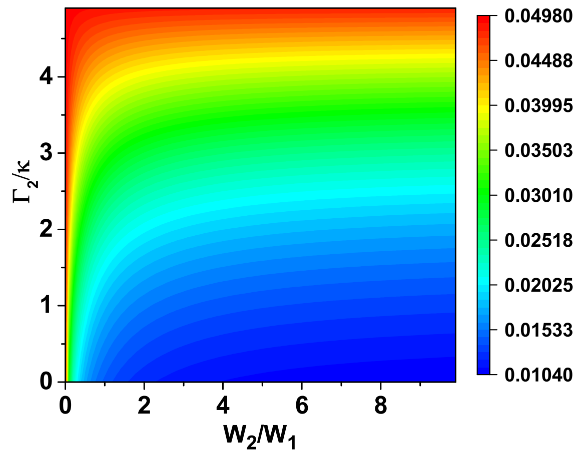

FIG. 7. Maximum internal energy $E_{\max}$ of the quantum battery as a function of the weight W_2/W_1 of the two Lorentzians and the spectral width Γ_2/κ. The parameters are: $\Gamma_1 = 5\kappa$, $\Omega = 0.1\kappa$.

the impact of W_2/W_1 on the charging process of quantum batterys more clearly, Fig. 6(c) displays the change of maximum charging power $P_{\max}$ for different W_2/W_1. It is worth noting that, the maximum charging power $P_{\max}$ of a quantum battery decreases monotonically as W_2/W_1 increases, meaning that the charge power of the quantum battery cannot be enhanced by increasing W_2/W_1. This may be the result of that more energy are lost in the pseudomodes environment when W_2/W_1 is large. We also analyze the influence of the spectral width Γ_2 on the charging power of the quantum battery. A comprehensive picture for the dependence of $E_{\max}$ on Γ_2 and W_2/W_1 is shown in Fig. 7, where we can see the maximum internal energy $E_{\max}$ of the quantum battery could be enhanced as W_2/W_1 decreases and Γ_2 increases. Here, by fixing $\Gamma_1 = 5\kappa$ and $\Omega = 0.1\kappa$, the optimal internal energy $E_{\max} = |c(t)|^2 = 0.0498$ of the quantum battery is less than 1/2, resulting in $W_B = 0$. In other words, when the first Lorentzian spectrum satisfies the weak coupling mechanism (i.e., $\Gamma_1 = 5\kappa$), the quantum battery in a two-Lorentzian environment cannot transfer energy to the consumption center by manipulating W_2/W_1 and Γ_2, leading to the poor performance of the quantum battery.

IV. THE CHARGING PERFORMANCE OF QUANTUM BATTERY IN THE MULTIPLE COUPLED ENVIRONMENTS

In the previous section, we considered the charging performance of the quantum battery in a structure reservoir environment. In this section, we extend our results to multiple coupled reservoir environments to explore the the coupling strength between the nearest-neighbor environments and the scaling effects of the environments on the performance of the quantum battery. For this purpose, the quantum battery is in a controlled environment consisting of N coupled reservoir environment. We model each reservoir environment as a bosonic mode, m_n, decaying to a Markovian reservoir. In this general setup, as shown in Fig. 8, the quantum battery is coupled with strength κ to the mode m_n, which decay to their respective Markovian reservoirs with rates $\Gamma_n = \Gamma$. The coupling strength between the two nearest-neighbor cavities

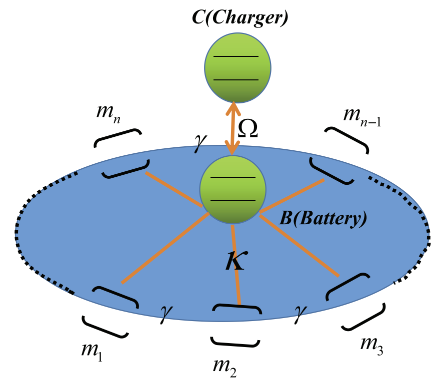

FIG. 8. Diagrammatic representation of the quantum battery in multiple coupled environments.

is γ. It is then valuable to point out that this multi-coupled environments can be restored to the situation of the band-gap environment and two-Lorentzian environment by reducing the size of the environment and manipulating the coupling of various parts between the system and the environment. The total Hamiltonian of the system is given by $H = H_0 + H_I$ and reads

$$H_0 = \frac{\omega_0}{2}\sigma_z^B + \frac{\omega_0}{2}\sigma_z^C + \sum_{n=1}^{N}\omega_n d_n^\dagger d_n,$$

$$H_I = \Omega(\sigma_+^B\sigma_-^C + \sigma_-^B\sigma_+^C) + \sum_{n=1}^{N}\kappa(\sigma_-^B d_n^\dagger + \sigma_+^B d_n) + \sum_{\langle ij\rangle}\gamma(d_i^\dagger d_j + d_j^\dagger d_i), \quad (12)$$

where $d_n^\dagger$ (d_n) is the creation (annihilation) operator of mode m_n ($n = 1, 2, ..., N$). $\langle ij\rangle$ means the nearest-neighbor cavities. The density operator of the total system can be written as

$$\frac{d\rho}{dt} = -i[H,\rho] - \sum_{n=1}^{N}\frac{\Gamma}{2}(d_n^\dagger d_n\rho - 2d_n\rho d_n^\dagger + \rho d_n^\dagger d_n). \quad (13)$$

Here, we consider quantum charger C in the excited state $|1_C\rangle$, quantum battery B and other cavity modes m_n in the vacuum state $|00...0\rangle_{B,m_1,...m_n}$. Then in accordance with the same steps of quantum battery in band-gap environment, we can obtain the elements of the density matrix of the quantum battery as $\rho_{ee}^B(t) = |r(t)|^2$, $\rho_{gg}^B(t) = 1 - |r(t)|^2$, $\rho_{ge}^B(t) = \rho_{ge}^B(0)r(t)$, $\rho_{eg}^B(t) = \rho_{eg}^B(0)r^*(t)$. The internal energy $E_B = \omega_0|r(t)|^2$, power $P_B = \omega_0|r(t)|^2/t$, and extractable work $W_B = \omega_0(2|r(t)|^2 - 1)(|r(t)|^2 > 1/2,\ W(t) = 0 \text{ for } |r(t)|^2 \leqslant 1/2)$ of the quantum battery can be obtained from Eqs. (1), (2), and (3). In the following, we mainly focus on how the performance of the quantum battery can be enhanced by controlling the scaling effects N of the environments and the coupling strength γ between environments.

If there are no other lossy cavities, then the quantum battery's dynamics mainly depends on the parameters Γ and κ in such a way that $\Gamma > 4\kappa$ ($\Gamma < 4\kappa$), identified as the weak-coupling (strong-coupling) regime, leads to the poor charging performance of quantum battery (the strong charging performance of quantum battery). In the case of adding the lossy

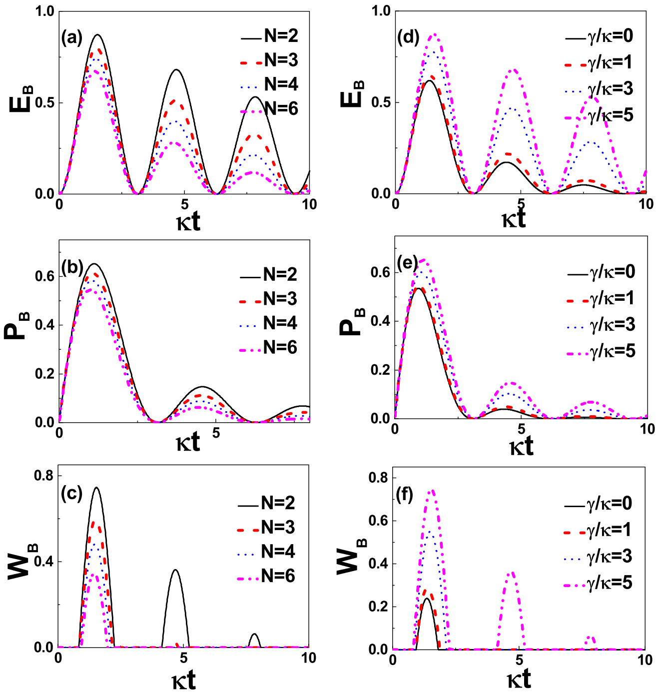

FIG. 9. (a–c) The internal energy E_B, the charging power P_B, and the ergotropy W_B of the quantum battery in the weak system-environment coupling regime as a function of the dimensionless quantity κt for different values of the number of cavity modes N. (d–f) The internal energy E_B, the charging power P_B, and the ergotropy W_B of the quantum battery in the weak system-environment coupling regime as a function of the dimensionless quantity κt for different values of the coupling strength γ/κ between the two nearest-neighbor cavity modes. The parameters are: (a–c) $\Gamma = 5\kappa$, $\Omega = \kappa$, $\gamma = 5\kappa$; (d–f) $\Gamma = 5\kappa$, $\Omega = \kappa$, $N = 2$.

cavities, the charging performance of the quantum battery would be considered in the weak system-environment coupling regime. In Fig. 9, the internal energy E_B, the charging power P_B, and the ergotropy W_B of the quantum battery in the weak system-environment coupling regime as a function of the dimensionless quantity κt have been plotted. In Figs. 9(a), 9(b) and 9(c), the charging performance (i.e., E_B, P_B, and W_B) of the quantum battery decreases with the increase of the number of the cavities. This may be due to the increase in the number of dissipative environments resulting in more energy dissipation of the quantum battery in the environment. However, in Figs. 9(d), 9(e) and 9(f), the charging performance of the quantum battery can be improved as the increase of the coupling strength γ. That is to say, in the weak system-environment coupling regime, the larger the value γ and the smaller value N can be requested to trigger the stronger charging performance of the quantum battery.

To fully understand the influence of the number of dissipative cavities N and the coupling strength γ between the nearest dissipative cavities on the maximum internal energy $E_{\max}$ and maximum extraction work $W_{\max}$ of the quantum battery, we plot the changes of $E_{\max}$ and $W_{\max}$ with N and γ in Figs. 10 and 11, respectively. In Fig. 10, the maximum internal energy $E_{\max}$ of the quantum battery can be enhanced in the weak system-environment coupling regime by

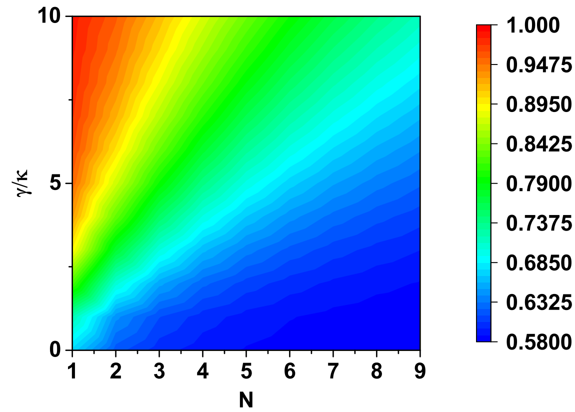

FIG. 10. Maximum internal energy $E_{\max}$ of the quantum battery as a function of the number of cavities N and the coupling strength γ/κ between the two nearest-neighbor cavity modes. The parameters are: $\Gamma = 5\kappa$, $\Omega = \kappa$.

increasing γ and decreasing N. Here, it is worth noting that, the coupling strength γ increase makes it possible to achieve the fully charged for the quantum battery. Similarly, the small number of dissipative cavities N and large coupling strength γ can improve the maximum ergotropy of the quantum battery. We find that extract the total stored energy of the quantum battery as work can be gotten by manipulating the coupling strength γ. To obtain the optimal quantum battery, the larger γ and the smaller N in the weak system-environment coupling regime are required to improve the charging performance of the quantum battery.

V. CONCLUSION

In summary, we have studied the charging performance of the quantum battery in a single environment and the multiple coupled environments for the weak system-environment coupling regime. To be more specific, we first consider the quantum battery in a reservoir environment, which described by a band-gap environment model or a two-Lorentzian model. We show that, the performance of the quantum battery in

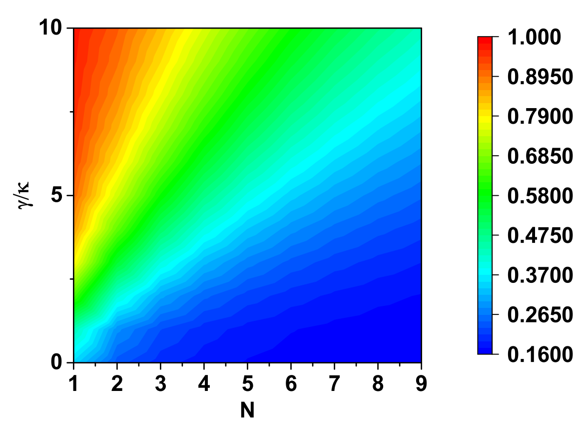

FIG. 11. Maximum ergotropy $W_{\max}$ of the quantum battery as a function of the number of cavities N and the coupling strength γ/κ between the two nearest-neighbor cavity modes. The parameters are: $\Gamma = 5\kappa$, $\Omega = \kappa$.

band-gap environment can be enhanced with the increase of the weights W_2/W_1 of the two Lorentzians and the spectral width Γ_2, when the first Lorentzian spectrum meets the weak-coupling regime. However, in a two-Lorentzian environment, the performance of the quantum battery cannot be improved by increasing the weight W_2/W_1 of the two Lorentzians. And in this case, no available energy can be extracted from the quantum battery in the weak system-environment coupling regime by manipulating W_2/W_1 and Γ_2. Therefore, to obtain the best performance of the quantum battery in a reservoir environment, two conditions have to be met: (1) Select the situation of the quantum battery in the band-gap environment. (2) The larger the weights W_2/W_1 of two Lorentzians and the larger the spectral width Γ_2 of the second Lorentzian are required. Here it is worth noting that, in the experiment, the spectral width and the weights of the two Lorentzians can be controlled. For example, by adjusting the correlation time of the environment and the coupling strength between a single NV spin and environment, the spectral width and the weights of the two Lorentzians can be manipulated [48]. Thus, it is significant to improve the charging performance of quantum batteries by regulating the weights of two Lorentzians and the spectral width of the second Lorentzian.

We have also extended our results to the multiple coupled reservoir environments, which are composed of N coupled dissipative cavities. Then we explore the coupling strength between the nearest-neighbor cavities and the scaling effects of the environment on the performance of the quantum battery. We find that, the optimized quantum battery can be obtained by decreasing the number of the cavities and increasing the coupling strength between the nearest-neighbor cavities. Particularly, the quantum battery can be fully charged by manipulating the coupling strength between the nearest-neighbor cavities. The obtained results in this paper provide possible ways to acquire the optimal charging performance of open quantum battery in the weak system-environment coupling regime.

ACKNOWLEDGMENTS

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[1] M. F. Riedel, D. Binosi, R. Thew, and T. Calarco, The European quantum technologies flagship programme, Quantum Sci. Technol. **2**, 030501 (2017).
[2] T. Baumgratz, M. Cramer, and M. B. Plenio, Quantifying Coherence, Phys. Rev. Lett. **113**, 140401 (2014).
[3] A. Streltsov, G. Adesso, and M. B. Plenio, Colloquium: Quantum coherence as a resource, Rev. Mod. Phys. **89**, 041003 (2017).
[4] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, Rev. Mod. Phys. **81**, 865 (2009).
[5] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, The classical-quantum boundary for correlations: Discord and related measures, Rev. Mod. Phys. **84**, 1655 (2012).
[6] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, Quantum cryptography, Rev. Mod. Phys. **74**, 145 (2002).
[7] V. Scarani, H. Bechmann-Pasquinucci, N. J. Cerf, M. Dušek, N. Lütkenhaus, and M. Peev, The security of practical quantum key distribution, Rev. Mod. Phys. **81**, 1301 (2009).
[8] R. Raussendorf and H. J. Briegel, A One-Way Quantum Computer, Phys. Rev. Lett. **86**, 5188 (2001).
[9] J.-W. Pan, Z.-B. Chen, C.-Y. Lu, H. Weinfurter, A. Zeilinger, and M. Żukowski, Multiphoton entanglement and interferometry, Rev. Mod. Phys. **84**, 777 (2012).
[10] H.-P. Breuer, E.-M. Laine, and J. Piilo, Measure for the Degree of Non-Markovian Behavior of Quantum Processes in Open Systems, Phys. Rev. Lett. **103**, 210401 (2009).
[11] M. Thorwart, J. Eckel, J. H. Reina, P. Nalbach, and S. Weiss, Enhanced quantum entanglement in the non-Markovian dynamics of biomolecular excitons, Chem. Phys. Lett. **478**, 234 (2009).
[12] E.-M. Laine, J. Piilo, and H.-P. Breuer, Measure for the non-Markovianity of quantum processes, Phys. Rev. A **81**, 062115 (2010).
[13] M. P. Almeida, F. de Melo, M. O. Hor-Meyll, A. Salles, S. P. Walborn, P. H. S. Ribeiro, and L. Davidovich, Experimental observation of environment-induced sudden death of entanglement, in *SPIE Proc. of the Noise and Fluctuations in Photonics, Quantum Optics, and Communications, Florence, Italy*, Vol. 6603, edited by L. Cohen (SPIE, 2007), p. 660318.
[14] O. Jiménez Farías, C. Lombard Latune, S. P. Walborn, L. Davidovich, and P. H. Souto Ribeiro, Determining the dynamics of entanglement, Science **324**, 1414 (2009).
[15] J. F. Poyatos, J. I. Cirac, and P. Zoller, Quantum Reservoir Engineering with Laser Cooled Trapped Ions, Phys. Rev. Lett. **77**, 4728 (1996).
[16] A. R. R. Carvalho, P. Milman, R. L. de Matos Filho, and L. Davidovich, Decoherence, Pointer Engineering, and Quantum State Protection, Phys. Rev. Lett. **86**, 4988 (2001).
[17] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2010).
[18] R. Alicki and M. Fannes, Entanglement boost for extractable work from ensembles of quantum batteries, Phys. Rev. E **87**, 042123 (2013).
[19] T. P. Le, J. Levinsen, K. Modi, M. M. Parish, and F. A. Pollock, Spin-chain model of a many-body quantum battery, Phys. Rev. A **97**, 022106 (2018).
[20] F. C. Binder, S. Vinjanampathy, K. Modi, and J. Goold, Quantacell: Powerful charging of quantum batteries, New J. Phys. **17**, 075015 (2015).
[21] F. Campaioli, F. A. Pollock, F. C. Binder, L. Céleri, J. Goold, S. Vinjanampathy, and K. Modi, Enhancing the Charging Power of Quantum Batteries, Phys. Rev. Lett. **118**, 150601 (2017).
[22] G. M. Andolina, D. Farina, A. Mari, V. Pellegrini, V. Giovannetti, and M. Polini, Charger-mediated energy transfer

in exactly solvable models for quantum batteries, Phys. Rev. B **98**, 205423 (2018).

[23] I. Henao and R. M. Serra, Role of quantum coherence in the thermodynamics of energy transfer, Phys. Rev. E **97**, 062105 (2018).

[24] J. Q. Quach and W. J. Munro, Using Dark States to Charge and Stabilize Open Quantum Batteries, Phys. Rev. Appl. **14**, 024092 (2020).

[25] F. Pirmoradian and K. Mølmer, Aging of a quantum battery, Phys. Rev. A **100**, 043833 (2019).

[26] Y. Huangfu and J. Jing, High-capacity and high-power collective charging with spin chargers, Phys. Rev. E **104**, 024129 (2021).

[27] C.-K. Hu, J. Qiu, P. J. P. Souza, J. Yuan, Y. Zhou, L. Zhang, J. Chu, X. Pan, L. Hu, J. Li, Y. Xu, Y. Zhong, S. Liu, F. Yan, D. Tan, R. Bachelard, C. J. Villas-Boas, A. C. Santos, and D. Yu, Optimal charging of a superconducting quantum battery, arXiv:2108.04298v1.

[28] W. Lu, J. Chen, L.-M. Kuang, and X. Wang, Optimal state for Tavis-Cummings quantum battery via Bethe ansatz method, Phys. Rev. A **104**, 043706 (2021).

[29] D. Ferraro, M. Campisi, G. M. Andolina, V. Pellegrini, and M. Polini, High-Power Collective Charging of a Solid-State Quantum Battery, Phys. Rev. Lett. **120**, 117702 (2018).

[30] A. Crescente, M. Carrega, M. Sassetti, and D. Ferraro, Ultrafast charging in a two-photon Dicke quantum battery, Phys. Rev. B **102**, 245407 (2020).

[31] Y.-Y. Zhang, T.-R. Yang, L. Fu, and X. Wang, Powerful harmonic charging in a quantum battery, Phys. Rev. E **99**, 052106 (2019).

[32] H.-P. Breuer and F. Petruccione, *Theory of Open Quantum Systems* (Oxford University Press, New York, 2002).

[33] F. Barra, Dissipative Charging of a Quantum Battery, Phys. Rev. Lett. **122**, 210601 (2019).

[34] S.-Y. Bai and J.-H. An, Floquet engineering to reactivate a dissipative quantum battery, Phys. Rev. A **102**, 060201(R) (2020).

[35] F. Zhao, F.-Q. Dou, and Q. Zhao, Quantum battery of interacting spins with environmental noise, Phys. Rev. A **103**, 033715 (2021).

[36] S. Gherardini, F. Campaioli, F. Caruso, and F. C. Binder, Stabilizing open quantum batteries by sequential measurements, Phys. Rev. Research **2**, 013095 (2020).

[37] D. Farina, G. M. Andolina, A. Mari, M. Polini, and V. Giovannetti, Charger-mediated energy transfer for quantum batteries: An open-system approach, Phys. Rev. B **99**, 035421 (2019).

[38] R. Alicki, A quantum open system model of molecular battery charged by excitons, J. Chem. Phys. **150**, 214110 (2019).

[39] C. L. Latune, I. Sinayskiy, and F. Petruccione, Energetic and entropic effects of bath-induced coherences, Phys. Rev. A **99**, 052105 (2019).

[40] M. T. Mitchison, J. Goold, and J. Prior, Charging a quantum battery with linear feedback control, Quantum **5**, 500 (2020).

[41] S. Seah, M. Perarnau-Llobet, G. Haack, N. Brunner, and S. Nimmrichter, Quantum Speed-Up in Collisional Battery Charging, Phys. Rev. Lett. **127**, 100601 (2021).

[42] D. Rosa, D. Rossini, G. M. Andolina, M. Polini, and M. Carrega, Ultra-stable charging of fast-scrambling SYK quantum batteries, J. High Energy Phys. 11 (2020) 067.

[43] S. Ghosh, T. Chanda, and A. Sen(De), Enhancement in the performance of a quantum battery by ordered and disordered interactions, Phys. Rev. A **101**, 032115 (2020).

[44] F. H. Kamin, F. T. Tabesh, S. Salimi, F. Kheirandish, and A. C. Santos, Non-Markovian effects on charging and self-discharging process of quantum batteries, New J. Phys. **22**, 083007 (2020).

[45] F. T. Tabesh, F. H. Kamin, and S. Salimi, Environment-mediated charging process of quantum batteries, Phys. Rev. A **102**, 052223 (2020).

[46] A. Goban, C. L. Hung, S. P. Yu, J. D. Hood, J. A. Muniz, J. H. Lee, M. J. Martin, A. C. McClung, K. S. Choi, D. E. Chang, O. Painter, and H. J. Kimble, Atom-light interactions in photonic crystals, Nat. Commun. **5**, 3808 (2014).

[47] D. Lindley, Landmarks—The birth of photonic crystals, Phys. Online J. **6**, 94 (2013).

[48] Y. Romach, C. Müller, T. Unden, L. J. Rogers, T. Isoda, K. M. Itoh, M. Markham, A. Stacey, J. Meijer, S. Pezzagna, B. Naydenov, L. P. McGuinness, N. Bar-Gill, and F. Jelezko, Spectroscopy of Surface-Induced Noise Using Shallow Spins in Diamond, Phys. Rev. Lett. **114**, 017601 (2015).

[49] L. Mazzola, S. Maniscalco, J. Piilo, K.-A. Suominen, and B. M. Garraway, Pseudomodes as an effective description of memory: Non-Markovian dynamics of two-state systems in structured reservoirs, Phys. Rev. A **80**, 012104 (2009).

[50] B. M. Garraway, Decay of an atom coupled strongly to a reservoir, Phys. Rev. A **55**, 4636 (1997).

[51] B. M. Garraway, Nonperturbative decay of an atomic system in a cavity, Phys. Rev. A **55**, 2290 (1997).

[52] B. J. Dalton, S. M. Barnett, and B. M. Garraway, Theory of pseudomodes in quantum optical processes, Phys. Rev. A **64**, 053813 (2001).

[53] G. Pleasance, B. M. Garraway, and F. Petruccione, Generalized theory of pseudomodes for exact descriptions of non-Markovian quantum processes, Phys. Rev. Research **2**, 043058 (2020).