Evolution of honesty in higher-order social networks

Aanjaneya Kumar*

Department of Physics, Indian Institute of Science Education and Research, Dr. Homi Bhabha Road, Pune 411008, India

Sandeep Chowdhary [†] Department of Network and Data Science, Central European University, 1100 Vienna, Austria

Valerio Capraro D‡

Department of Economics, Middlesex University, The Burroughs, London NW4 4BT, United Kingdom

Matjaž Perc 08

Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška Cesta 160, 2000 Maribor, Slovenia; Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan Alma Mater Europaea ECM, Slovenska Ulica 17, 2000 Maribor, Slovenia; and Complexity Science Hub Vienna, Josefstädterstraße 39, 1080 Vienna, Austria

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Sender-receiver games are simple models of information transmission that provide a formalism to study the evolution of honest signaling and deception between a sender and a receiver. In many practical scenarios, lies often affect groups of receivers, which inevitably entangles the payoffs of individuals to the payoffs of other agents in their group, and this makes the formalism of pairwise sender-receiver games inapt for where it might be useful the most. We therefore introduce group interactions among receivers and study how their interconnectedness in higher-order social networks affects the evolution of lying. We observe a number of counterintuitive results that are rooted in the complexity of the underlying evolutionary dynamics, which has thus far remained hidden in the realm of pairwise interactions. We find conditions for honesty to persist even when there is a temptation to lie, and we observe the prevalence of moral strategy profiles even when lies favor the receiver at a cost to the sender. We confirm the robustness of our results by further performing simulations on hypergraphs created from real-world data using the SocioPatterns database. Altogether, our results provide persuasive evidence that moral behavior may evolve on higher-order social networks, at least as long as individuals interact in groups that are small compared to the size of the network.

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I. INTRODUCTION

The flow of information from a source to its destination is ubiquitous and fundamental to life at all levels—from signaling at the cellular level for coordinating the activities of cells to communication between people in and across societies. Particularly crucial is communication and transmission of honest signals between different parts of a system for maintaining its efficient functioning as epitomized by colonies of ants and honey bees and also by the *supercooperating* humans [1]. However, the breakdown of communication and dishonest signaling can severely hamper the operations of a system and lead to undesirable consequences. A striking example of this is how misinformation and rumours can propagate fear and paranoia during times of crises, like the coronavirus disease 2019 (COVID-19) pandemic, and amplify the hardship that come along with them. Needless to say that tackling the problem of misinformation and fake news stands as one of the most important challenges of our time.

While the study of fake news has attracted a lot of attention over the last few years [2–4], the literature on signaling and deception has a much longer history [5]. Evolutionary biologists have developed several models to study diverse scenarios such as predator-prey signaling [6–8], sexual signaling [9,10], and the interactions between siblings [11] and between parents and offsprings [12]. On the other hand, the focus of economists and psychologists has been towards the development of tasks which would allow us to quantify (dis)honesty. Some examples include the die-rolling paradigm [13], the matrix search task [14], the Philip Sidney game [15], and the sender-receiver game [16]. The latter game, in particular, has received a great deal of attention in the last few years.

The sender-receiver game is a classic example of a game with asymmetric information, and it provides us with a paradigm to explore strategic interactions between two types of agents: *senders*, who possess a piece of information and can either honestly or dishonestly communicate it to the second

^{*}kumar.aanjaneya@students.iiserpune.ac.in

[†]chowdhary_sandeep@phd.ceu.edu

[‡]v.capraro@mdx.ac.uk

[§]matjaz.perc@gmail.com



FIG. 1. Schematic of different aspects of our work. (a) The hypergraph considered for our simulations-the hyperring. The one shown in the figure consists of 11 nodes with each hyperedge containing 4 nodes. Panels (b) and (c) denote two possible scenarios which exemplify the need to incorporate group interactionssuppose a sender sends a deceitful message to a group of receivers, which could possibly lead them to disregard the social distancing norms in the pandemic. The situation described in panel (b) depicts a scenario where none of the receivers were deceived (all marked in green), whereas panel (c) represents a scenario where a receiver who was not deceived (center, marked in green) is surrounded by receivers who were deceived. It is clear that the central receiver in panel (c) should obtain a lower payoff than the one in panel (b) as the pandemic is being facilitated (by the deceived receivers) and restrictions will be extended for everyone. This group effect is not captured by the pairwise models considered in literature.

type of agents, *receivers*, who can choose to either believe or not believe the message sent to them. A particularly interesting feature of this game is that it allows us to distinguish among different types of lies, based on whether lying has positive or negative consequences for the players [17]. Not surprisingly, it has attracted a lot of research attention from behavioral scientists [16–39] and, more recently, from physicists. Recent studies [40,41] using Monte Carlo simulations following the replicator dynamics have provided an extensive theoretical study of the sender-receiver game in well-mixed and structured populations to complement the massive amount of experimental data available.

Although important, these studies consider only one-to-one interactions, and a major assumption is that the payoffs of individual receivers in a population depend solely on their own strategy and the strategy of the sender they interacted with. However, since people in a society are interconnected, it is inevitable that the payoffs of each receiver would not only depend on whether they were deceived but also on whether the other receivers around them got deceived. In Figs. 1(b) and 1(c), we emphasize this point. This realization, though simple, has profound consequences for models of strategic interactions as the fitness of an individual in a population is entangled with the fitness of other people in the population. This calls for an alternative modeling framework, which goes beyond the simple consideration of only pairwise interactions between agents and takes into account the effects

of group interactions to provide an improved description of reality [42]. While this paradigm of including *higher-order interactions* has successfully been incorporated in the study of the evolution of cooperation in the form of the public goods game [43], there is a strong need for a systematic study of such group effects in other problems of strategic interactions, such as signaling games, where higher-order interactions are inevitable and must be accounted for. This encourages us to explore the role of group interactions in the evolution of honesty and lying in the sender-receiver game. We note that in several other contexts, higher-order interactions have already been shown to lead to collective phenomena as in the case of synchronization phenomena [44–47], random walks [48,49], consensus [50,51], and ecological dynamics [52,53].

The plan of our paper is as follows: In Sec. II, we extend the sender-receiver game to incorporate group effects. In Sec. III A, we study the evolution of different strategies in a well-mixed population and explore the parameter space of the game to unravel rich results. In Sec. III B, we explore the evolution of strategies on a class of hypergraphs, namely, the hyperrings. The hyperrings also allows us to investigate the role played by the size of the hyperedges on the evolution of the strategies. In Sec. III C we examine the evolution on four "real-world" hypergraphs, built using the SocioPatterns dataset. We conclude with a discussion and a brief outlook for future research in Sec. IV.

II. MATHEMATICAL MODEL

A. The sender-receiver game

We first define the canonical two-player sender-receiver game, as introduced by Erat and Gneezy [17]. In this game, the sender first rolls a die and observes the outcome of the roll which can be any of the six possible outcomes in $\{1, 2, 3, 4, 5, 6\}$. The sender then sends a message about the outcome, which can be the truth (T) or a lie (L), and the message is communicated to the receiver. After receiving the message, the receiver chooses a number between 1 and 6. If this number is equal to the actual outcome of the die, without loss of generality, we set the payoffs of the sender and the receiver, both, to be 0. In case this number is different from the actual outcome of the die, then the sender gets a payoff s and the receiver gets r. This game can be easily reduced to a game with two strategies for each player. Indeed, the payoff of the receiver essentially depends only on whether they choose to believe (B) the message sent by the sender, or not (N). We can rewrite the payoff bimatrix of this game as follows:

$$\begin{array}{c|cc}
B & N \\
\hline
T & 0,0 & s,r \\
L & s,r & \frac{4}{5}s, \frac{4}{5}r
\end{array}$$

where the ratio $\frac{4}{5}$ comes from the fact that, if the sender lies and the receiver does not believe the sender, then the receiver reports a wrong outcome of the die with the probability $\frac{4}{5}$.

The fact that the sender-receiver game is essentially a game with two players, each of which has only two strategies, makes it suitable to study using methods of statistical physics, and the Monte Carlo method, in particular. Another reason why the sender-receiver game is becoming increasingly popular is that it allows one to study different types of lies, depending on the values of s and r, as follows.

(i) *Pareto white lies* are those that benefit both the sender and the receiver: r, s > 0.

(ii) Altruistic white lies are those that benefit the receiver at a cost to the sender: r > 0, s < 0.

(iii) *Black lies* are those that benefit the sender at a cost to the receiver: r < 0, s > 0.

(iv) *Spiteful lies* are those that harm both the sender and the receiver: r, s < 0.

The distinction among different types of lies is useful also for the equilibrium analysis, which indeed depends on the type of lie. In the domain of spiteful lies, there are two equilibria in pure strategies—(T, B) and (L, N)—and one equilibrium in mixed strategies— $(\frac{1}{6}T + \frac{5}{6}L, \frac{1}{6}B + \frac{5}{6}N)$. In the domain of altruistic or black lies, there is only one equilibrium: $(\frac{1}{6}T + \frac{5}{6}L, \frac{1}{6}B + \frac{5}{6}N)$. In the domain of Pareto white lies, there are two equilibria in pure strategies—(T, N)and (L, B)—and one equilibrium in mixed strategies, once again, $(\frac{1}{6}T + \frac{5}{6}L, \frac{1}{6}B + \frac{5}{6}N)$. The cases r = 0 and/or s = 0are straightforward, because the corresponding players are indifferent between the two available strategies.

B. Introducing group interactions

We now extend the canonical sender-receiver game to allow for multiple receivers, who interact among themselves, meaning that the payoff of each receiver does not only depend on whether the receiver herself believes the message sent by the sender but also on whether the other receivers believe this message. In particular, we assume that the payoff obtained by each receiver is the sum of two components: the payoff due to the individual "pairwise" interaction with the sender (Π_I), and the payoff due to the group interaction with the other receivers (Π_G).

The pairwise payoff, Π_I , is defined to be identical to the payoff of the standard sender-receiver game described in Sec. II A.

To account for the group payoff, Π_G , we introduce some terminology and some notation. We say that a receiver is *deceived* by the sender if they report a wrong outcome of the die. We denote x_W as the fraction of receivers who are deceived by the sender (excluding the receiver for which we are calculating the payoff), where W stands for *wrong* as the receiver reports the wrong outcome of the die. When a receiver is not deceived by the sender, we define their group payoff as $\Pi_G = k_R x_W$, where k_R is a fixed real number, and R stands for *right*, because the receiver reports the wrong outcome of the die. When the receiver reports the wrong outcome of the die, we denote their group payoff as $\Pi_G = k_W x_W$, where k_W is, again, a real constant.

To summarize, the total payoff, $\Pi_I + \Pi_G$, obtained by each receiver can be tabulated as

$$\frac{B}{T \quad 0 + k_R x_W} \quad r + k_W x_W} \\
L \quad r + k_W x_W \quad \frac{1}{5} k_R x_W + \frac{4}{5} (k_W x_W + r)$$

Of course, as r, k_R , and k_W vary in \mathbb{R}^3 , one can have different prototypical cases. A detailed discussion of the different types of group interactions captured by this model is presented in our discussion section. In principle, the model allows for various choices of group payoffs Π_G received by individuals, which can be defined to be general functions $f_R(x_W)$ and $f_W(x_W)$, depending on whether the receiver was deceived by the sender or not. For simplicity, we make the choice of assuming the payoffs to be linear functions of the fraction of deceived receivers in the group $[f_R(x_W) = k_R x_W]$ and $f_W(x_W) = k_W x_W$]. Clearly, setting $k_R = k_W = 0$, one reduces the game to the standard sender-receiver game.

So far, we have defined only the payoff of the receiver. We define the payoff of the sender to be simply the sum of the payoffs that they obtain in their interactions with each receiver.

C. The Monte Carlo method for well-mixed populations

We consider a sender-receiver game among n agents. Initially, each agent is randomly assigned one of the four pure strategy profiles (T, B), (T, N), (L, B), and (L, N). Then one agent is randomly selected to play as the sender. The n agents then receive a payoff from the n-player sender-receiver game played with the selected agent in the role of sender and all the other agents in the role of receivers. At the end of this interaction, another agent is selected to play in the role of sender. We repeat this procedure n times so that, at the end of this Monte Carlo step, each agent has played the role of sender exactly once. This concludes one Monte Carlo step.

At the end of a Monte Carlo step, for each agent we randomly select another agent. Then the first agent copies the strategy of the second agent with a probability that depends on the difference between their accumulated payoffs in the last *n* rounds. In particular, if player P_1 and its randomly selected pair, player P_2 , collect payoffs Π_{P_1} and Π_{P_2} , respectively, then P_1 copies the strategy of P_2 with the probability

$$w = \frac{1}{1 + \exp\left[\left(\frac{\Pi_{P_1}}{n} - \frac{\Pi_{P_2}}{n}\right)/K\right]},$$
(1)

where *K* quantifies the uncertainty or error in strategy adoptions. In real-world settings, one would expect that agents would try to copy their neighbors who are performing better than them. However this imitation would have its limitations and imperfections, and with a small probability, agents can also copy the strategy of poorly performing neighbors. In our simulations, we choose K = 0.1, unless explicitly stated otherwise.

To reach sufficient accuracy, we simulate large system sizes, with n = 500, as well as long enough thermalization and sampling times, of 10^5 Monte Carlo steps.

D. The Monte Carlo method for hypergraphs

To study the effect of spatial correlations on the evolution of strategies in the *n*-player sender-receiver game, we simulate the game on hypergraphs. A hypergraph is a generalization of a graph, where edges, instead of connecting two nodes, can connect any number of nodes. Formally, a hypergraph is a pair H = (X, E), where X is a nonempty set of nodes and E is a family of nonempty subsets of X, called hyperedges. Nodes that belong to the same hyperedge are said to be neighbors. The sender-receiver game is simulated on hypergraphs as follows.

We treat each hyperedge as a well-mixed group. Within one Monte Carlo step, we first select a hyperedge and all players in that hyperedge play the group sender-receiver game with each other such that each player in the hyperedge gets to play the role of a sender exactly once. To account for heterogeneities in sizes of the hyperedge, the payoff obtained by each agent in the hyperedge is divided by the number of agents in the hyperedge. This is repeated until we go over all hyperedges. Players accumulate payoffs from all hyperedges they are part of. This brings us to the imitation step, where we go over each of the *n* players in the hypergraph one by one, and each player P_1 copies the strategy of a randomly chosen neighbor P_2 with the probability

$$w = \frac{1}{1 + \exp\left[\left(\frac{\Pi_{P_1}}{K_1} - \frac{\Pi_{P_2}}{K_2}\right)/K\right]},$$
 (2)

where \mathcal{K}_i denotes the number of hyperedges node *i* is part of.

III. RESULTS

The results section is structured as follows. In Sec. III A, we simulate the evolutionary group sender-receiver game on well-mixed populations, and we compare the results with those already obtained for the standard pairwise senderreceiver game [40]. We see that, already in the well-mixed case, several features emerge with the addition of group interactions. Then, in Sec. III B we study the evolution in the group sender-receiver game on a particular class of hypergraphs, hyperrings, and we see that, compared to the well-mixed populations, further features emerge. Finally, in Sec. III C we explore the evolution of the group sender-receiver game in four real-world hypergraphs, and we show that the results qualitatively confirm those obtained in the hyperrings. The main result is that the spatial structure leads to the evolution of the moral strategy (T, B), whereby the sender sends a truthful message and the receiver trusts the message sent by the sender, even for small values of k_R and independently of k_W . Moreover, the moral strategy (T, B) can more easily emerge in small groups than in large ones.

A. Well-mixed populations

We start by reporting the densities in the stationary state of the four strategy profiles (T, B), (T, N), (L, B), and (L, N), as a function of the parameters s, r, k_R , and k_W in a well-mixed population consisting of 500 agents.

First, we isolate the effect of k_W by setting $k_R = 0$. We report two simulations, one for $k_W = 10$ and one for $k_W = 0.25$. We choose these two parameter values as they provide key insights into the behavior of the model and how evolutionary dynamics is affected by incorporation of higher-order interactions. Since the absolute values of *s* and *r* (the pairwise payoffs) lie between 0 and 1, choosing the value of the group payoff strength to be $k_W = 0.25$ allows us to probe the model behavior in the case in which the payoffs associated with the group interactions. Similarly, the value $k_W = 10$ allows us to explore the behavior of the model when group interactions

bring a much higher payoff than pairwise interactions and thus are the dominant factor in determining the evolution of the strategies.

When $k_W = 10$, it is extremely beneficial for each receiver to be deceived, whenever at least another receiver is deceived. This suggests that the believing strategy B should quickly vanish and, at the stationary state, receivers should never believe the message sent by the sender. This intuition is confirmed by the simulations [Fig. 2(a)]. The strategy played by senders can also be easily determined. It suffices to observe that, when s > 0, senders would want to maximize their chances of deceiving the receivers and hence would choose to tell the truth (T) to a population of nonbelievers (N), making (T, N)the strategy profile with maximal payoff. On the contrary, when s < 0, senders have an incentive to lie (L) to a population of nonbelievers (N), making (L, N) the stable strategy profile. Hence, for s > 0, (T, N) is the evolutionarily stable strategy, while otherwise it is (L, N). We report this result in Fig. 2(a), where we plot the stationary frequencies of different strategies, for s and r taking values between -1 and 1.

When $k_W = 0.25$, it is still beneficial for each receiver to be deceived, whenever the other receivers are deceived. However, since this time both group interaction parameters k_R and k_W are small, we observe a more nuanced evolution that looks similar to the one already reported for pairwise interactions (see Fig. S1 in Ref. [54], and also Fig. 2 in Ref [40]), apart from a small shift. This result in reported in Fig. 2(b). Next, we isolate the effect of k_R , by setting $k_W = 0$. We report two simulations, one for $k_R = 10$ and one for $k_R = 0.25$.

When $k_R = 10$, it is extremely beneficial for a receiver not to be deceived whenever the other receivers are deceived. Starting from random initial conditions, it is easy to argue that believers have an initial advantage as they are less likely to be deceived. If, in addition, s < 0, then the "moral strategy profile" (T, B), whereby the sender tells the truth and the receiver believes the message sent by the sender, would have the highest fitness. Therefore, in this case we would expect a quick convergence to (T, B). Figure 2(c) confirms this intuition. The case of s > 0 is more nuanced. A theoretical analysis suggests that the system undergoes a shift in the optimal strategy profile from (L, B) to (T, N). Indeed, since believers have an initial advantage and since now lying is beneficial to the sender, (L, B) will tend to proliferate at the initial stages of the evolution. However, as L evolves, it is no longer beneficial for receivers to believe the message sent by the sender and, therefore, they would start playing N. Since when receivers do not believe the message sent by the sender, the difference in the sender's payoff when the sender plays T compared to when she or he plays L is very little, this would lead the evolution to stabilize around the strategy profile (T, N), with a small residual of the other strategy profiles. This nuanced evolution is reported in Fig. S2 in the Supplemental Material [54], which highlights that we do indeed see the anticipated initial increase in (L, B); however, soon after, (T, N) takes over. It is interesting to remind, at this stage, that in this regime it is extremely beneficial for an agent to not be deceived whenever others are deceived. Reminiscent of the tragedy of the commons [55–57], the ultimate fate here is unfavorable to all, as no one earns the much coveted group payoff even for the slightest temptations to lie (s > 0). Even



FIG. 2. Final densities of the four strategy profiles, (T, B), (T, N), (L, B), and (L, N), in well-mixed populations for (a) $k_R = 0$ and $k_W = 10$, (b) $k_R = 0$ and $k_W = 0.25$, (c) $k_R = 10$ and $k_W = 0$, and (d) $k_R = 0.25$ and $k_W = 0$. We show that the presence of group interactions can lead to evolution which is quite different from the case where only pairwise interactions are considered (compare with Fig. S1 in Ref. [54], and Fig. 2 of Ref. [40]).

the sender's payoff for lying, which instigates the transition from (T, B) to (T, N), is not enjoyed in the end. See Fig. 2(c), for a heat map of stationary frequencies of the four strategies for $k_R = 10$ and $k_W = 0$, for various values of *s* and *r*.

When $k_R = 0.25$, it is still beneficial for receivers not to be deceived whenever other receivers are deceived. However, since both group interaction parameters k_R and k_W are small, this time we see an evolution that more closely portrays the evolution in the pairwise game, apart from a shift to the right upon the introduction of k_R as opposed to the effect of k_W making (T, B) more prevalent for s < 0 and promoting not believing (N) for s > 0. We report this result in Fig. 2(d).

To conclude this section, where we isolated the effects of k_R and k_W , we now mention some of our results for the cases when they both act together. For $k_R = k_W$, the group payoffs of each round act like a constant addition of payoff to each strategy in the simple pairwise game, and hence, we expect that the evolution of strategies in this case resembles the pairwise sender-receiver game. We confirm our expectation in Fig. S1 in the Supplemental Material [54]. We also report the cases (k_R , k_W) = (-0.25, 0) and (0, -0.25) in the Supplemental Material [54]. They are all very similar to the cases just reported.

B. Hyperring

Next, we conduct simulations on a hyperring consisting of 500 nodes. The hyperring can be thought of as a lattice equivalent for hypergraphs because of its uniform structure (see Fig. 1). Apart from the total number of nodes, only one parameter has to be predetermined to uniquely define a hyperring, the size of each hyperedge. We start by analyzing the case in which hyperedges have size 4. At the end of this subsection, we discuss the dependence on the size of the hyperedge.

As a first step, we simulate the evolution for $(k_R, k_W) =$ (0.25, 0) and for $(k_R, k_W) = (0, 0.25)$, so that we can make a direct comparison with the results presented in the previous subsection, regarding well-mixed populations. In Fig. 3, we report the evolution for $(k_R, k_W) = (0.25, 0)$ [panel (a)] and $(k_R, k_W) = (0, 0.25)$ [panel (b)]. In both cases, we notice that the evolution is far more nuanced on the hyperring, compared to the well-mixed populations. In particular, we notice that the moral strategy profile (T, B), whereby the sender sends a truthful message and the receiver believes the message sent by the sender, which in well-mixed populations evolves only in the trivial case of spiteful lies (r, s < 0), in the hyperring evolves with nonzero frequency also in the domain of black lies (r < 0, s > 0) and in the domain of altruistic white lies (r > 0, s < 0). On the other hand, in both cases, the evolution of the immoral strategy profile' (L, N) is disfavored, compared to the well-mixed case. We note that a similar enhancement of moral strategies has already been observed in the canonical (pairwise) sender-receiver game played on networks, compared to the case of well-mixed populations [41]. However, we point out that higher-order effects have a more nuanced effect on the evolutionary dynamics-the evolution of the strategy (T, B) on pairwise networks can be further enhanced, or completely hindered, depending on the nature of group interactions, i.e., the value of the parameters k_R and k_W (see Fig. S5 in Ref. [54]).



FIG. 3. Final densities of the four strategy profiles, (T, B), (T, N), (L, B), and (L, N), in the hyperring where each hyperedge has size 4, for (a) $k_R = 0.25$ and $k_W = 0$ and (b) $k_R = 0$ and $k_W = 0.25$. We also provide a comparison between $(k_R, k_W) = (0.25, -0.25)$ for (c) well-mixed populations and (d) hyperrings. Here we note that there is an emergence of the moral profile on the hyperring, which is not present in well-mixed populations for (s > 0, r < 0) and (s < 0, r > 0).

Figures 3(c) and 3(d) provide a comparison between the evolution on the hyperring with that on the well-mixed populations for $(k_R, k_W) = (0.25, -0.25)$. The heat map in Fig. 3(c) is for the well-mixed population, and the heat map in Fig. 3(d) is for the hyperring with four nodes in each hyperedge. We show that in this case, too, the spatial structure provided by the hyperring favors the evolution of the moral strategy profile (T, B) and disfavors the evolution of the immoral strategy profile (L, N), both in the domain of altruistic white lies and in the domain of selfish black lies. We point out that the two evolutions are fundamentally different. While the moral strategy profile (T, B) evolves with almost zero frequency in well-mixed populations, it evolves with almost 70% frequency in the hyperring. On the other hand, the immoral strategy profile evolves to a frequency close to 30%.

In the Supplemental Material [54], we also report the results of the simulations for $(k_R, k_W) = (-0.25, 0), (0, -0.25),$ (-0.25, 0.25), and (0.25, 0.25). The emergence of the moral strategy profile (T, B) in regions of r < 0, s > 0, s < -r(black lies) and r > 0, s < 0, r < -s (altruistic white lie) is an especially interesting feature and can be observed in all cases, albeit with varying frequencies. Evidently, the evolution of (T, B) in the regime of black lies is enhanced by a positive value of k_R and a negative value of k_W , corroborated by the high frequency of the moral profile for $(k_R, k_W) =$ (0.25, -0.25) and significantly less frequency for parameter values $(k_R, k_W) = (-0.25, 0.25)$. Our results convincingly point towards the conclusion that spatial correlations provide a route for the emergence of honest signaling in groups, at least when the population consists of groups of small size. We now analyze the dependence on the size of the hyperedges. Figure 4 reports the stationary frequencies of the four strategy profiles for parameters s = 0.2, r = -0.3, $k_R = 0.25$, and $k_W = -0.25$ as a function of the number of elements in a hyperedge. We see that the group size is unfavorable to



FIG. 4. Stationary frequencies of the four strategy profiles for parameters s = 0.2, r = -0.3, $k_R = 0.25$, and $k_W = -0.25$ as a function of number of elements in a hyperedge. We show that the emergence of the moral profile due to the spatial correlations provided by the hypergraph structure is only significant when group sizes are small, and the effect dies out as we increase the group size—the relative density of (T, B) in the figure is the highest when each group contains four elements, and monotonically decreases as the group sizes increase.

the evolution of moral behavior, as the stationary frequency of the moral strategy profile (T, B) decreases as a function of the group size and becomes nearly zero for group size greater than or equal to 10. This is, *a posteriori*, not surprising, as, when the group size increases, the hyperring converges to a well-mixed population, which we already know to be unfavorable to the evolution of (T, B) (see Sec. III A). In the Supplemental Material [54], we also report the final densities for several other values of k_R , k_W , r, and s (with r and s either in the domain of black lies or in the domain of altruistic lies). In all cases, we found that the final density of (T, B)decreases with the group size. In the domain of spiteful and Pareto white lies, clearly (T, B) either always evolves with 100% frequency (spiteful lies) or with 0% (Pareto white lies). This case is trivial and thus excluded from the numerical analysis.

C. Real-world hypergraphs

The key result of the previous section is that the hyperring structure promotes the evolution of the moral strategy profile (T, B), at least when the group size of the hyperedges is relatively small. One might wonder whether this is a particular feature of hyperrings with a small group size or whether it holds also for other hypergraphs and, in particular, for more heterogenous hypergraphs, like the ones created from real-world data.

To answer this question, we created four hypergraphs describing real-world interactions using the publicly available SocioPatterns dataset. SocioPatterns is "an interdisciplinary research collaboration formed in 2008 that adopts a datadriven methodology to study social dynamics and human activity. Since 2008, [they] have collected longitudinal data on the physical proximity and face-to-face contacts of individuals in numerous real-world environments" [58]. In particular, the SocioPatterns datasets record face-to-face interactions with a temporal resolution of 20 s. This allows us to check whether individuals are truly interacting as a group, and this allows us to create a hypergraph as follows. For every 20-s window, we create a network of interactions and catalog all the maximal cliques beyond size 2. Repeated appearance of a clique, beyond a specified threshold, is treated as a group interaction and we create a hyperedge consisting of the nodes in the recurring clique. A similar approach of creating hypergraphs has already been adopted to study opinion dynamics on hypergraphs [59], as well as to study the temporal nature of higher-order interactions [60].

In doing so, we created four hypergraphs starting from the following datasets that we downloaded from SocioPatterns:

(i) primary Schools dataset, formed by school children and teachers at a primary school in France;

(ii) conference dataset, formed by participants at the 2009 SFHH conference in Nice, France;

(iii) high school dataset, formed by students at a high school in Marseilles;

(iv) workplace dataset, formed by the staff at an office building in France.

These datasets represent a diverse variety of social situations and thus provide the ideal setting for our simulations. Figure 5 reports the time evolution of the four strategy profiles for each of these four real-world hypergraphs. We note that, in each hypergraph, the moral strategy profile (T, B) persists with a nonzero frequency. In the primary school hypergraph [Fig. 5(a)], it evolves with frequency close to 17%; in the high school hypergraph [Fig. 5(b)], it evolves with frequency around 30%; in the workplace hypergraph [Fig. 5(c)], it evolves with frequency around 32%; and in the conference hypergraph [Fig. 5(d)], it even evolves with frequency above 60% [61].

IV. DISCUSSION

In our work, we provided a natural extension to the senderreceiver game which allowed us to study the impact of group interactions on the evolution of honesty. Different combinations of parameter values in our model correspond to different types of higher-order effects in strategic interactions among agents. Some particularly relevant choices of parameters, from a practical perspective, include the following.

(i) r < 0 and $0 > k_R > k_W$. It is beneficial for each receiver if they do not get deceived and also the other receivers in the group do not get deceived.

Example. During the COVID-19 crisis, there have been several incidents caused by the spreading of fake news. For example, in India a rumor spread about some trains being arranged by the government for the laborers that were stuck away from home. This caused a gathering of thousands of people at a major railway station [62]. We can formalize this situation with the following: k_R , $k_W < 0$, because the more receivers get deceived, the more social distancing rules are violated, leading to an increase in the COVID-19 transmission, with potentially negative consequences also for receivers that were not deceived; $k_R > k_W$, because, everything else being equal, a person is more likely to get infected if being deceived (i.e., gathering at the railway station), than if not; and r < 0, because receivers who go to the railway station spend money and time for no reason.

(ii) r < 0, $k_R > k_W$, and $k_R > 0$. It is beneficial for each receiver if they do not get deceived, but other receivers in the group get deceived.

Example. At the beginning of the COVID-19 crisis, there has been a shortage of essential goods, including groceries and household items of daily use. Suppose there is a rumor spreading that these goods are available at shop X and not at shop Y (where they are actually available). Then, it is in the receiver's best interest if they do not get deceived but other people do, in order to minimize the competition for the utilities in shop Y.

(iii) r > 0 and $0 > k_W > k_R$. It is beneficial for each receiver if they get deceived but the other receivers in the group do not get deceived.

Example. Any white lie being told in a population of competing receivers belongs to this case. For instance, suppose that two sport teams are competing for an important match and that, right before the match, the presidents of the two teams are informed about some tragic event that might lower the performance of the teams, for example, the sudden death of someone well known in their field. The presidents have to decide whether to tell the truth to the coaches and the teams before the match or not. In this case, for each team, it is better



FIG. 5. Persistence of honest signaling and believing behavior (T, B), demonstrated on four hypergraphs generated using real-world interaction patterns, namely, primary school (a), conference (b), high school (c), and workplace (d) datasets (descriptions in text) for $k_R = 0.25$, $k_W = -0.25$, s = 0.2, and r = -0.3. These simulations provide evidence for our observation on hyperrings that higher-order interactions, coupled with spatial correlations, allow for the emergence of the moral profile (T, B).

to believe the white lie, while the other team does not. Indeed, in this case, one team is more likely to win the game, because the other team lowers its performance.

(iv) r > 0, $k_W > k_R$, and $k_W > 0$. It is beneficial for each receiver if they get deceived and if also the other receivers in the group get deceived.

Example. This is a very general case, because there are a number of examples where cooperative behavior, which benefits everyone, emerges from collective beliefs which are not supported by evidence. The examples include various cultural beliefs.

By performing Monte Carlo simulations of the proposed model, we first explored in detail the stationary frequencies of the different strategy profiles in well-mixed populations across the parameter space of the model. In doing so, several features emerged out of group interactions, which were not observed in the canonical pairwise sender-receiver game. Also by isolating the group effects, we gained further insight into how the evolution is modified upon the inclusion of group interactions in well-mixed populations.

But, of course, real populations are often structured, and, in many cases, individuals do not interact in pairs, but in groups. These higher-order structures are thus important for a realistic modeling of any complex system. To probe them, we used the formal paradigm of hypergraphs, which allowed us to model networked systems with group interactions. We first performed simulations on hyperrings, which, owing to their uniform structure, can be thought of as the hypergraph version of a lattice. We observed that the evolution of strategies changes drastically, and, in particular, we identified the regions in the parameter space where the moral strategy profile, whereby senders tell the truth and receivers believe the message sent by the senders, evolves, which was not observed in well-mixed populations. In particular, we find that new values of the parameters emerge, which allows agents to overcome the temptation to lie (s > 0), even for comparatively small values of k_R and k_W . We further established that this effect, which was not present in well-mixed populations, is strongest when the group size is small and decays very quickly with the increasing sizes of groups. This decrease in morality as a function of the group size is not surprising, as our model of the hyperring converges to well-mixed populations as the size of the hyperedges increases. What is surprising is the fact that the moral strategy profile does evolve in small groups. Moreover, this is not just a mathematical curiosity due to the special hyperring structure, as we found a qualitatively similar result in all real-world hypergraphs that we have built from the SocioPatterns dataset: the strategy profile (T, B) persists in all real-world hypergraphs that we have built, although with different final densities.

Recent works have explored the evolution of honesty in only the canonical, pairwise sender-receiver game in well-mixed populations and in networks [40,41], as part of a research direction to study the evolution of moral *behavior* [63,64]. This work constitutes a systematic study of group effects in signaling games. As any emerging study, also this study has some limitations that can suggest directions for future work. For example, we considered only the case in which there is one sender and multiple receivers. However, in reality, sometimes there are multiple senders that compete among themselves, such as news outlets which compete for who has more receivers. Future work could extend our formalization to include competition among senders. Also, as real higher-order networks, we considered only four hypergraphs downloaded from the SocioPatterns database, which refer to physical interactions in a primary school, a high school, a conference, and an office. It is possible that these interactions are spatially different from virtual interactions that happen online. Understanding the effects of the network structure on the spread of misinformation online is certainly an important direction for future work.

This study also contributes to our understanding of the effect of group size on the evolution of morality. Previous work has mainly focused on another form of morality, cooperative behavior [65]. In this context, a line of work using several techniques, ranging from numerical simulations to mathematical analyses and behavioral experiments, has found that the relationship between group size and cooperation is very nuanced and much depends on the particular experimental paradigm being used to formalize cooperative behavior [66–77].

Cooperation, one of the most well-studied topics in the field of evolutionary game theory [78–83], is only one specific type of moral behavior among several others [65], among which honesty is also present [29]. However, unlike cooperation, other moral behaviors have not received significant theoretical attention, and certainly not from the point of view of group interactions. In a society as complex as ours, it is expected that the phenomena that emerge out of it cannot be explained solely through pairwise interactions between individuals as many interactions naturally happen in groups. We hope that our work provides a stepping stone towards bridging this gap and encourages further studies devoted to group interactions. An interesting future direction also emerges on the experimental side, where behavioral changes in strategic interactions between individuals can be explored in situations where their rewards are entangled with each other.

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