# Reflection and absorption of electromagnetic radiation by inhomogeneous photoionized plasma, produced by multiphoton ionization of inert gas atoms

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(Received 21 July 2021; revised 29 August 2021; accepted 27 September 2021; published 11 October 2021)

The interaction of electromagnetic radiation with inhomogeneous plasma formed by multiphoton ionization of inert gas atoms has been studied. In high-frequency and normal skin effects modes the field structure in plasma is described and reflection and absorption coefficients are found. It is shown that as the thickness of the plasma region, in which the photoelectron density grows linearly, increases, both the depth of field penetration and the absorption coefficient increase, too. It is found that, due to the Ramzauer-Townsend effect, there is a relative increase in the effective frequency of photoelectron collisions with atoms, which is accompanied by a significant increase in the absorption coefficient.

DOI: 10.1103/PhysRevE.104.045203

## I. INTRODUCTION

The study of the interaction between electromagnetic radiation and inhomogeneous plasma has attracted the specialists attention for a long time. A large number of works devoted to the study of the plasma inhomogeneity effect on various electromagnetic phenomena have been carried out only in recent years. Let us highlight some of them. In Refs. [1,2] the stimulated Raman scattering of inhomogeneous plasma has been studied. The reflection features of powerful radiation by a dense plasma layer are investigated [3,4]. Multidimensional numerical calculations of laser absorption and electron acceleration in inhomogeneous plasma have been performed [5]. The theory of Thomson scattering of electromagnetic radiation by an inhomogeneous plasma is developed [6]. The effect of radial plasma inhomogeneity on collision absorption of electromagnetic field in a silicon discharge is studied [7]. In the above papers fully ionized plasmas with relatively high electron temperatures were investigated. This communication deals with an inhomogeneous photoionized plasma with qualitatively different properties. A photoionized plasma with a strong nonequilibrium electron velocity distribution is formed when femtosecond laser pulses effect on gas. In particular, in multiphoton ionization of atoms the energy electrons distribution is characterized by the presence of narrow peaks at energies on the order of one electron volt [8–13]. In this case, the ionization degree of the relatively dense gas is low and the produced plasma properties are mainly determined by the collisions of photoelectrons with neutral atoms [14–18]. The photoionized plasma is localized in the focusing region of the laser pulse and the density of photoelectrons decreases from the center of focus to the periphery.

We consider the interaction of a probe electromagnetic wave with a photoionized plasma produced by multiphoton ionization of dense inert gas atoms. Assuming that the photoelectron density increases linearly from zero to a constant value at distances greater than $L$ as one moves away from the boundary, several modes of probe radiation penetration deep into the plasma have been studied. In each mode the field structure in the plasma is described and the absorption and reflection coefficients are found. First, the field penetration in high-frequency skin effect mode is studied, when the field frequency $\omega$ is much smaller than $\omega_L$—the plasma frequency of electrons in the constant density region but much larger than $\nu$—the collision frequency of photoelectrons. If the variable density layer width $L$ is smaller than $\delta$ (the depth of the skin layer at high frequency skin effect mode), then the absorption and reflection of the probe radiation is described by expressions obtained by assuming a sharp change in the electron density (see Ref. [18]). If $L$ is greater than $\delta$ and $z_0$—the distance from the plasma boundary to the point where the photoelectron density equals the critical one—is much smaller than $c/\omega$, then the field penetration depth and absorption factor increase by $(L/\delta)^{1/3} \gg 1$ times. If the distance to the critical density point is greater than $c/\omega$, then up to the point $z_0$ the field oscillates and its amplitude increases. The effective penetration depth is comparable to the distance to the critical density point. As a consequence of a large increase in the field penetration depth into the plasma, the absorption coefficient depends strongly exponentially on the electron collision frequency and can reach values close to unity, which corresponds to almost total absorption of the field in a variable density layer. Similar behavior patterns of the field and absorption coefficient are established in the normal skin effect regime, when the field frequency is relatively small $\nu(1-\alpha/3) \gg \omega$, where the $\alpha$ coefficient depends on the type of photoelectron scattering cross section on the inert gas atoms. For $L$ smaller than $\delta_n$ (the skin layer depth for the normal skin effect), the results obtained by assuming a sharp change in the photoelectron density (see Ref. [18]) can be applied. If $L \gg \delta_n$, but $|z_0| \ll c/\omega$, then the field penetration depth and absorption

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factor increase by a factor of $(L/\delta_n)^{1/3} \gg 1$. At $|z_0| \gg c/\omega$ the field penetrates the plasma to a distance of about $|z_0|$ and the absorption coefficient is close to unity.

Note an important property characteristic for a photoionized inert gas plasma. According to the Ramzauer-Townsend effect, at photoelectron energies typical for multiphoton ionization of atoms, their scattering cross section on neutral atoms increases with increasing energy [19,20]. As a consequence, our obtained expressions for the field, reflection, and absorption coefficients depend on the effective collision frequency, which is several times larger than in the absence of the Ramzauer-Townsend effect. In the high-frequency skin effect mode, the effective collision frequency is given by the expression $\nu(1+\alpha/3)$, and in the normal skin effect mode by the expression $\nu/(1-\alpha/3)$.

## II. MODEL AND FORMULATION

When laser pulses of moderate intensity $10^{12}$-$10^{14}\mathrm{W/cm^2}$ and with a duration of no more than 100 fs are applied to inert gases, a weakly ionized gas with a highly nonequilibrium distribution of photoelectrons in the focusing region is formed [10,14,21–23]. In the multiphoton ionization mode, the characteristic energy of photoelectrons is several electron volts [8–13]. In the case of ionization of inert gas atoms, the degree of ionization is relatively small ($10^{-4}$-$10^{-6}$) [10,14,21,23]. At atmospheric pressure, this degree of ionization corresponds to the concentration of photoelectrons ($10^{13}$-$10^{15}$) $\mathrm{cm^{-3}}$ [14,21,23]. Under these conditions, the frequency of electron-electron collisions is no more than $10^{10}$ $\mathrm{s^{-1}}$ and the collision frequency of photoelectrons with neutral atoms is significantly higher $\sim 10^{12}$ $\mathrm{s^{-1}}$. Due to the frequent collisions of photoelectrons with neutral atoms after the laser pulse exposure, an isotropic distribution of photoelectrons is formed during the time $\sim 1$ $\mathrm{ps^{-1}}$, which means that relaxation along the momentum directions occurs. The relaxation of photoelectron distribution over energy is mainly due to electron-electron collisions and occurs during a time greater than $\sim 100$ $\mathrm{ps^{-1}}$. Thus, there is an inhomogeneous photoionized plasma with a highly nonequilibrium energy distribution of photoelectrons over a relatively wide time interval. The photoelectron density changes at distances on the order of the focusing area size. In the time interval mentioned above the change in the density profile, caused by the expansion of the photoionized plasma, can be neglected, since the size of the focusing region is usually large enough. This approximation is justified by the fact that the spread of photoelectrons is restrained by the charge separation field, while the spread of ions is prevented by frequent collisions with neutral atoms. A photoionized plasma with an electron density $10^{13}$-$10^{15}$ $\mathrm{cm^{-3}}$ and a nonequilibrium energy distribution has unusual properties. In particular, in such a plasma there is a possibility of electron sound waves propagation with a frequency greater than the plasma frequency of electrons [24,25], and there is also a possibility of electromagnetic waves of terahertz frequency range amplification [14]. The problem considered below is of interest due to studies of the interaction of terahertz and ultrahigh-frequency waves with photoionized plasma, in which the plasma frequency of the electrons is comparable to or greater than the frequency of the incident radiation. Below, the theory of the probe wave interaction with a frequency smaller than the plasma frequency with the photoionized plasma formed during multiphoton ionization of a inert gas is developed. An important feature of further analysis is the consideration of an inhomogeneous distribution of photoelectrons on the scale of the focusing region.

### A. Photoelectron distribution function and permittivity of inert gas plasma

Let us consider the interaction of a monochromatic electromagnetic wave with an inhomogeneous plasma produced by multiphoton ionization of inert gases and occupying the $z > 0$ region of space. We assume that in the layer $0 < z < L$ the photoelectron density $n(z)$ increases linearly with increasing coordinate from $n = 0$ at $z = 0$ to $n_0$ at $z = L$, and remains constant $n = n_0$ at $z > L$. The degree of ionization of the inert gas is considered to be small enough that permits to take into account only the collisions of photoelectrons with neutral atoms when describing the interaction of a photoionized plasma with a probe monochromatic wave. At times longer than the momentum relaxation time, but shorter than the photoelectron energy relaxation time, the photoelectron velocity distribution function can be approximated by the expression $f(v,z) = [n(z)/4\pi v_0^2]\,\delta(v - v_0)$. Here $v_0 = \sqrt{2\epsilon_0/m}$, $m$ is the mass of electron, and $\epsilon_0$ is the photoelectron energy obtained by multiphoton ionization of a inert gas atom.

In the interaction of an electromagnetic wave with such a plasma, we consider that the frequency of the wave is much larger than the inverse time of energy relaxation. Moreover, we assume that the frequency of the wave satisfies the inequality $\omega \gg \omega_L(v_0/c)$, where $c$ is the speed of light, $\omega_L = \sqrt{4\pi n_0 e^2/m}$, $e$ is the electron charge. The field strength of the acting electromagnetic wave is represented as $\mathbf{E}(z,t) = (1/2)(E_0,0,0)\cdot\exp[-i\omega(t - z/c)] +$ c.c.. The electromagnetic wave produces an electric field $(1/2)(E(z),0,0)\exp(-i\omega t) + \mathrm{c.c.}$ along $ox$ axis in the plasma and causes a small perturbation of the photoelectron velocity distribution function $(1/2)\,\delta f(\mathbf{v},z)\exp(-i\omega t) +$ c.c.. To determine $\delta f(\mathbf{v},z)$ we will use a linearized kinetic equation with a collision integral describing the relaxation along the directions of photoelectron velocity without changing their energy

$$-i\omega\delta f(\mathbf{v},z) + \frac{ev_x E(z)}{mv}\frac{\partial f(v,z)}{\partial v} = -\nu(v)\left[\delta f(\mathbf{v},z) - \int \frac{d\Omega}{4\pi}\delta f(\mathbf{v},z)\right], \qquad (1)$$

where $d\Omega$ is the solid angle element. The collision frequency of photoelectrons with inert gas atoms is represented as $\nu(v) = N\sigma_{\mathrm{tr}}(v)v$, where $N$ is the concentration of neutral atoms, and $\sigma_{\mathrm{tr}}(v)$ is the transport cross section of electrons scattering on neutral atoms, which depends on the photoelectron velocity. It is justified to use such an expression for the collision frequency by the fact that in multiphoton ionization the typical energy $\epsilon_0$ does not exceed a few eV and in monoatomic inert gases the threshold for inelastic collisions of electrons with atoms is noticeably higher. The dependence of the neutral atom concentration on the coordinate $z$ is neglected, which is possible in a weakly ionized gas.

Using the solution of Eq. (1), we find the complex amplitude of the current density along the axis $ox$ at the frequency $\omega$

$$j(z) = e\int d\mathbf{v} v_x \delta f(\mathbf{v}, z) = \frac{e^2}{m}\int d\mathbf{v}\frac{v_x^2}{v}\frac{\partial f(v,z)}{\partial v}\frac{E(z)}{i\omega - \nu(v)}. \quad (2)$$

From Maxwell's equations we have the equation for $E(z)$ in plasma

$$\frac{d^2E(z)}{dz^2} + \frac{\omega^2}{c^2}E(z) = -\frac{4\pi i\omega}{c^2}j(z). \quad (3)$$

Taking the expression (2) into account, we rewrite Eq. (3) as

$$\frac{d^2E(z)}{dz^2} + \frac{\omega^2}{c^2}\varepsilon(\omega, z)E(z) = 0, \quad (4)$$

where the permittivity has the form

$$\varepsilon(\omega, z) = 1 - \frac{\omega_L^2(z)}{\omega(\omega + i\nu)}\left[1 - i\frac{\alpha}{3}\frac{\nu}{\omega + i\nu}\right]. \quad (5)$$

Here $\omega_L(z) = \sqrt{4\pi n(z)e^2/m}$, $\nu \equiv \nu(v_0)$, $\alpha = \partial \ln\nu/\partial \ln v_0$ is a quantity determined by the average photoelectron energy and the type of transport cross section dependence on energy. Note that in inert gases the transport cross section in the region of energies just below 1 eV has a minimum [19,20], and at the point corresponding to the average photoelectrons energy the derivative of the collision frequency is greater than zero. Therefore, the parameter $\alpha$ usually has positive values.

### B. Electric field in plasma and absorption coefficient

The way in which the photoelectron density depends on the coordinate makes it possible to divide the area occupied by the plasma into two parts. In the region $0 < z < L$ the electron density depends linearly on the coordinate, and in the region $z > L$ the density is constant. In the area $0 < z < L$, let us represent the expression (5) as

$$\varepsilon(\omega, z) = 1 + \frac{z}{L}\Delta\varepsilon, \quad (6)$$

where

$$\Delta\varepsilon = -\frac{\omega_L^2}{\omega(\omega + i\nu)}\left[1 - i\frac{\alpha}{3}\frac{\nu}{\omega + i\nu}\right] \quad (7)$$

is the contribution to the permittivity from photoelectrons at $z > L$. Since the permittivity (6) is a linear function of the coordinate, Eq. (4) is reduced to the Airy differential equation

$$\frac{d^2E(\xi)}{d\xi^2} - \xi E(\xi) = 0, \quad (8)$$

where

$$\xi = \xi(z) = \left(\frac{\omega^2}{z_0 c^2}\right)^{1/3}(z - z_0), \quad (9)$$

and

$$z_0 = -\frac{L}{\Delta\varepsilon}. \quad (10)$$

The general solution of Eq. (8) is

$$E(\xi) = C_1\mathrm{Ai}(\xi) + C_2\mathrm{Bi}(\xi), \quad (11)$$

where $\mathrm{Ai}(\xi)$ and $\mathrm{Bi}(\xi)$ are the Airy functions and $C_1$ and $C_2$ are unknown coefficients. In the region $z > L$, where the permittivity is constant, the solution of Eq. (4) is described by an expression

$$E(z) = E(L)\exp\left[i\frac{\omega}{c}\sqrt{\varepsilon(\omega)}(z - L)\right], \quad (12)$$

with $\varepsilon(\omega) \equiv \varepsilon(\omega, L)$ and $\mathrm{Im}\sqrt{\varepsilon(\omega)} > 0$.

The unknown coefficients $C_1, C_2, E(L)$, and $E_r$ (the complex amplitude of the electric field of the reflected wave at the boundary $z = 0$) are found from the continuity conditions for the tangential components of the electric and magnetic fields at $z = 0$ and $z = L$. Taking into account the expressions (11) and (12) and the solution of Eq. (4), corresponding to the reflected wave in vacuum, for coefficients $C_1$ and $C_2$ we have

$$C_1 = -\frac{2E_0}{D}\left[-i\frac{\omega}{c}\mathrm{Bi}(\xi_L)\sqrt{\varepsilon(\omega)} + \xi'\mathrm{Bi}'(\xi_L)\right], \quad (13)$$

$$C_2 = \frac{2E_0}{D}\left[-i\frac{\omega}{c}\mathrm{Ai}(\xi_L)\sqrt{\varepsilon(\omega)} + \xi'\mathrm{Ai}'(\xi_L)\right], \quad (14)$$

where

$$D = \left[\mathrm{Bi}(\xi_0) - i\frac{c}{\omega}\xi'\mathrm{Bi}'(\xi_0)\right]\left[-i\frac{\omega}{c}\mathrm{Ai}(\xi_L)\sqrt{\varepsilon(\omega)} + \xi'\mathrm{Ai}'(\xi_L)\right] - \left[\mathrm{Ai}(\xi_0) - i\frac{c}{\omega}\xi'\mathrm{Ai}'(\xi_0)\right] \times \left[-i\frac{\omega}{c}\mathrm{Bi}(\xi_L)\sqrt{\varepsilon(\omega)} + \xi'\mathrm{Bi}'(\xi_L)\right], \quad (15)$$

and the notations $\xi' = d\xi(z)/dz$, $\xi(0) = \xi_0$, $\xi(L) = \xi_L$ are used. The sign $'$ in the functions $\mathrm{Ai}'$ and $\mathrm{Bi}'$ denotes differentiation by the argument of these functions.

Using the expressions (11), (13), and (14) and continuity of the electric field tangential component at the plasma boundary, we find the complex reflection coefficient $R = E_r/E_0$:

$$R = \frac{1}{D}\left\{\left[\mathrm{Bi}(\xi_0) + i\frac{c}{\omega}\xi'\mathrm{Bi}'(\xi_0)\right]\left[-i\frac{\omega}{c}\mathrm{Ai}(\xi_L)\sqrt{\varepsilon(\omega)} + \xi'\mathrm{Ai}'(\xi_L)\right] - \left[\mathrm{Ai}(\xi_0) + i\frac{c}{\omega}\xi'\mathrm{Ai}'(\xi_0)\right] \times \left[-i\frac{\omega}{c}\mathrm{Bi}(\xi_L)\sqrt{\varepsilon(\omega)} + \xi'\mathrm{Bi}'(\xi_L)\right]\right\}. \quad (16)$$

The absorption coefficient $A$, determining the fraction of incident wave energy transferred to the plasma, is found from the formula

$$A = 1 - |R|^2. \tag{17}$$

The expressions for the field strength and the absorption coefficient contain the Airy functions and their derivatives at the points $\xi_0$ and $\xi_L$. Taking into account the formulas (6), (9), and (10) we represent $\xi_0$ and $\xi_L$ as

$$\xi_0 = -\left(\frac{-\omega L}{c\Delta\varepsilon}\right)^{2/3}, \quad \xi_L = \xi_0\varepsilon(\omega). \tag{18}$$

The presence of parameters (18) allows us to consider several modes of field penetration into plasma with an inhomogeneous photoelectron density profile.

## III. HIGH-FREQUENCY SKIN EFFECT

In a rarefied plasma the photoelectron collision frequency with neutral atoms is comparatively low and it is easy to realize a mode of electromagnetic field penetration into the plasma when $\omega \gg \nu$, but the field frequency is noticeably less than the electron plasma frequency $\omega \ll \omega_L$. In this case the contribution from photoelectrons to the permittivity (7) in the linear approximation of $\nu/\omega$ is

$$\Delta\varepsilon \approx -\frac{\omega_L^2}{\omega^2}\left[1 - i\left(1 + \frac{\alpha}{3}\right)\frac{\nu}{\omega}\right], \quad \omega \gg \nu. \tag{19}$$

In the zeroth-order approximation by the small parameter $\nu/\omega$, taking into account the formula (19) and the inequality $\omega \ll \omega_L$ for $\xi_0$ and $\xi_L$, from (18) we have

$$\xi_0 \approx -\frac{\omega^2}{\omega_L^2}\left(\frac{\omega_L}{c}L\right)^{2/3}, \quad \xi_L \approx \left(\frac{\omega_L}{c}L\right)^{2/3}. \tag{20}$$

From (20) we can see that the values $-\xi_0$ and $\xi_L$ depend on the layer thickness of the variable photoelectron density. Below we consider three ranges of layer thicknesses at which different asymptotic expressions for the electric field in the plasma (11) and (12) and the absorption coefficient (17) are realized.

### A. Thin layer

When the incident frequency $\omega$ is much smaller than the plasma frequency $\omega_L$, then the inequality $-\xi_0 \ll \xi_L$ is fulfilled. If the layer thickness is less than $\delta$ (the depth of the skin layer in a constant density region), i.e., the conditions

$$L \ll \delta = \frac{c}{\omega_L} \ll \frac{c}{\omega} \tag{21}$$

are fulfilled, then from (20) and (21) it can be seen that $-\xi_0 \ll \xi_L \ll 1$. Using the Airy functions expansion for small argument values [see formulas (A1) and (A2)], for the complex amplitude of the electric field inside the layer with variable photoelectron density from (11) and (13)–(15) we have

$$E(z) = 2E_0\frac{(L-z)\gamma + \delta}{(L + ic/\omega)\gamma + \delta}, \tag{22}$$

where the notation $\gamma = 1 - (i/2)(1 + \alpha/3)\nu/\omega$ is used. In the region of constant photoelectron density $z > L$ from (12) and

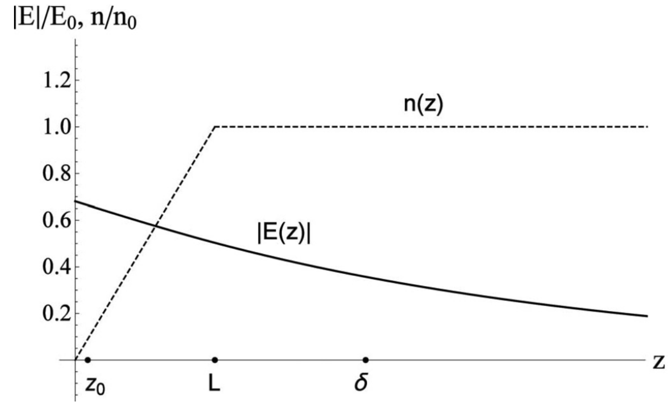


FIG. 1. Dependence of the field strength absolute value $E(z)$ inside the plasma on the distance to the plasma boundary in the case of a thin layer with variable density.

(22) for $E(z)$ we have the expression

$$E(z) = 2E_0\left[\left(L + i\frac{c}{\omega}\right)\frac{\gamma}{\delta} + 1\right]^{-1}\exp\left[-\gamma\frac{(z-L)}{\delta}\right]. \tag{23}$$

The numerically obtained under the conditions (21) dependence of $|E(z)|$ (the absolute value of the field strength) on the distance to the plasma boundary $z$ is shown in Fig. 1. The curve in Fig. 1 is plotted using the general expressions (11), (13), (14), and (15). Numerical calculations are performed for $\alpha = 4.8$, which corresponds to the plasma with average photoelectron energy $\epsilon_0 = 2.87$ eV, obtained by three-photon ionization of xenon atoms [14]. The remaining plasma parameters are assumed to be $\omega = 0.3\omega_L$, $\nu = 0.01\omega_L$, and $L = c/3\omega_L = \delta/3$. In Fig. 1 the characteristic scales that determine the electric field behavior inside the plasma are marked: the coordinate of the point $z_0$ at which the plasma density equals to the critical one; the layer thickness $L$ and the skin layer depth in a plasma with a constant photoelectron density $\delta$. From Fig. 1 and formulas (22) and (23) it follows that if the skin layer depth is greater than the thickness of layer with variable density, the field penetration features in plasma with inhomogeneous density profile are close to those realized in plasma with sharp boundary.

From (16) taking into account the Airy function expansion for small values of the argument [see formulas (A1) and (A2)], for the reflection coefficient we have

$$R = \frac{\omega - i\gamma\omega_L}{\omega + i\gamma\omega_L}. \tag{24}$$

Taking into account the interrelation between reflection coefficient (24) and absorption coefficient (17) and using the expression for $\gamma$, we obtain

$$A(\omega) = 2\frac{\nu}{\omega_L}\left(1 + \frac{\alpha}{3}\right). \tag{25}$$

The same expression for the absorption coefficient was obtained earlier in Ref. [18] under the assumption that the plasma photoelectron density changes abruptly, i.e., $L = 0$.

### B. Intermediate thickness layer

When $\omega \ll \omega_L$, it is possible that $-\xi_0 \ll 1 \ll \xi_L$. Such an interrelation between the parameters $-\xi_0$ and $\xi_L$ is realized for a variable density layer thickness satisfying the inequalities

$$\frac{c}{\omega_L} \ll L \ll \frac{\omega_L^3}{\omega^3}\frac{c}{\omega_L}. \tag{26}$$

The right-hand inequality (26) means that the distance from the plasma boundary to the point $z_0$ is small compared to $c/\omega = \lambda/2\pi$, where $\lambda$ is the wavelength. To calculate the coefficients $C_1$ (13) and $C_2$ (14), on the boundary $z = L$ we use asymptotic formulas for the Airy functions at large values of the argument [see formulas (A3) and (A4)] and at the boundary $z = 0$ we use approximate formulas for small values of the argument [see formulas (A1) and (A2)]. In that case, the coefficient $C_2$ is much smaller than $C_1$ and the summand $C_2\mathrm{Bi}(\xi)$ in the expression (11) is a small correction to $C_1\mathrm{Ai}(\xi)$. The correction value is smaller than $\xi_L^{-3/2} \ll 1$, for all values of $\xi$. Taking this into account, the complex amplitude of the electric field (11) inside the variable density layer $0 < z < L$ can be represented as

$$E(z) \approx -2iE_0 3^{1/3}\Gamma(1/3)\left(\frac{\omega z_0}{c}\right)^{1/3}\mathrm{Ai}\left[\left(\frac{\omega z_0}{c}\right)^{2/3}\left(\frac{z}{z_0}-1\right)\right] \approx -2iE_0 3^{1/3}\Gamma(1/3)\frac{\delta_L\omega}{c}\mathrm{Ai}\left(\frac{z-z_0}{\delta_L}\right), \tag{27}$$

where $\Gamma(x)$ is the Euler gamma function and $\delta_L = (Lc^2/\omega_L^2)^{1/3}$ is the effective skin layer depth in plasma with linearly varying photoelectron density profile. According to the left-hand inequality (26) in the considered case the field penetrates the plasma for a distance greater than the skin layer depth in a plasma with a sharp density profile. In the region of constant photoelectron density $z > L$, the amplitude of the field strength (12) is exponentially small

$$E(z) \approx -iE_0\frac{3^{1/3}\Gamma(1/3)}{\sqrt{\pi}}\left(\frac{\omega z_0}{c}\right)^{1/6}\left(\frac{z_0}{L-z_0}\right)^{1/4}\exp\left[-\gamma\frac{(z-L)}{\delta} - \frac{2}{3}\frac{\omega z_0}{c}\left(\frac{\gamma\omega_L}{\omega}\right)^3\right]. \tag{28}$$

The numerically obtained under the conditions (26) dependence of $|E(z)|$ on the distance to the plasma boundary $z$ is shown in Fig. 2. The curve in Fig. 2 is plotted using the general expressions (11)–(15). Numerical calculations are performed for the following plasma parameters: $\alpha = 4.8$, $\omega = 0.3\omega_L$, $\nu = 0.01\omega_L$, and $L = 10\,c/\omega_L$. In the same way as in Fig. 1, the characteristic scales which determine the electric field behavior inside the plasma are marked in Fig. 2. Figure 2 and formulas (27) and (28) show that the electric field becomes exponentially small at $z > \delta_L \gg z_0$, where $\delta_L \ll L$. From the expression (16) combined with (26) we obtain the reflection coefficient as

$$R \approx \frac{\Gamma(1/3) - i(3c/\omega z_0)^{1/3}\Gamma(2/3)}{\Gamma(1/3) + i(3c/\omega z_0)^{1/3}\Gamma(2/3)}. \tag{29}$$

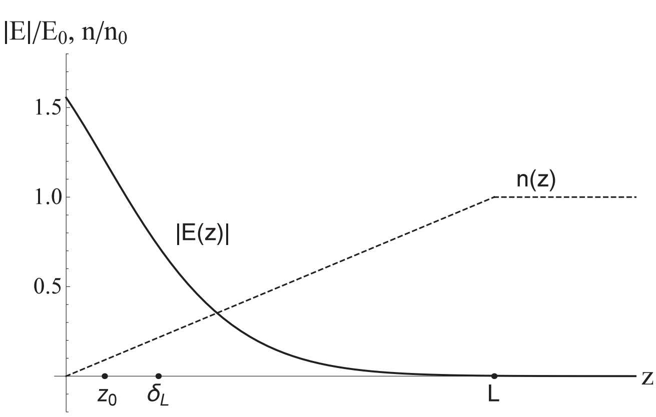


FIG. 2. Dependence of the field strength absolute value $E(z)$ inside the plasma on the distance to the plasma boundary in the case of an intermediate thickness layer.

It follows that the absorption coefficient (17) has the form

$$A(\omega) \approx 4\frac{\Gamma(1/3)}{3^{4/3}\Gamma(2/3)}\left(\frac{L\omega_L}{c}\right)^{1/3}\frac{\nu}{\omega_L}\left(1+\frac{\alpha}{3}\right). \tag{30}$$

A comparison of expressions (25) and (30) shows that an increase in the variable density layer thickness leads to a $(L\omega_L/c)^{1/3}$ times increase in the absorption coefficient. This increase is caused by the greater depth of the skin layer in a plasma with a smooth density profile.

### C. Thick layer

When $\omega \ll \omega_L$, as the thickness of the variable density layer increases, conditions are possible in which not only $-\xi_0$ but also $\xi_L$ is significantly greater than unity. Such conditions are realized if

$$L \gg \frac{\omega_L^3}{\omega^3}\frac{c}{\omega_L}, \tag{31}$$

when $z_0 \gg c/\omega$. Calculating the electric field inside the plasma, at $z = 0$ we use asymptotics of the Airy functions for large arguments (A5) and (A6), and at $z = L$ we use asymptotics of the Airy functions for large arguments (A3) and (A4). If the inequality (31) is satisfied, then the summand $C_2\mathrm{Bi}(\xi)$ in the expression (11) is a small correction to $C_1\mathrm{Ai}(\xi)$ for all values $\xi$ and can be omitted. Thus, using the expressions (13) and (15), for the electric field strength (11) inside the layer $0 < z < L$ we have

$$E(z) = 2E_0\sqrt{\pi}\left(\frac{\omega z_0}{c}\right)^{1/6}\exp\left[i\left(\frac{2}{3}\frac{\omega z_0}{c} - \frac{\pi}{4}\right)\right]\mathrm{Ai}\left(\frac{z-z_0}{\delta_L}\right). \tag{32}$$

Since when the inequality (31) is satisfied, the distance to the critical density point $z_0$ is much larger than $\delta_L$, it follows from

(32) that in the considered case the field penetrates the plasma at a depth of order $z_0 + \delta_L \approx z_0 \gg \delta_L$. At $z_0 - z \gg \delta_L$, i.e., at distances close to the plasma boundary, the expression (32) describes a weakly increasing oscillation of the field whose period increases as the critical density point is approached [see (33)]

$$E(z) \approx 2E_0\left(\frac{z_0}{z_0 - z}\right)^{1/4} \exp\left[i\left(\frac{2}{3}\frac{\omega z_0}{c} - \frac{\pi}{4}\right)\right] \sin\left[\frac{2}{3}\left(\frac{z_0 - z}{\delta_L}\right)^{3/2} + \frac{\pi}{4}\right]. \quad (33)$$

Up to the critical density point, the field is formed due to the interference of the incident and reflected waves. This is particularly clear at the plasma boundary, when $z = 0$ and the expression (33) takes a simpler form

$$E(0) \approx E_0 + E_0 \exp\left[i\left(\frac{4}{3}\frac{\omega z_0}{c} - \frac{\pi}{2}\right)\right]. \quad (34)$$

It follows from (32) that in the constant density region $z > L$ the field strength is negligibly small

$$E(z) = E_0\left(\frac{\omega}{\gamma\omega_L}\right)^{1/2} \exp\left[-\gamma\frac{(z - L)}{\delta} + i\left(\frac{2}{3}\frac{\omega z_0}{c} - \frac{\pi}{4}\right) - \frac{2}{3}\frac{\omega z_0}{c}\left(\gamma\frac{\omega_L}{\omega}\right)^3\right]. \quad (35)$$

The numerically obtained under the conditions (31) dependence of $|E(z)|$ on the distance to the plasma boundary $z$ is shown in Fig. 3. The curve in Fig. 2 is plotted using the general expressions (11)–(15). Numerical calculations are performed for the following plasma parameters: $\alpha = 4.8$, $\omega = 0.3\omega_L$, $\nu = 0.01\omega_L$, and $L = 800\, c/\omega_L$. Just as in Figs. 1 and 2, Fig. 3 marks the characteristic scales defining the electric field behavior inside the plasma. From Fig. 3 and the formulas (32)–(35), we see that as we approach the point $z_0$ the field oscillation amplitude increases slightly and its frequency decreases. At $z > z_0$ the field strength decreases at a distance of order $\delta_L$.

The reflection coefficient (16), when the inequality (31) is satisfied, is

$$R = \exp\left[i\left(\frac{4}{3}\frac{\omega z_0}{c} - \frac{\pi}{2}\right)\right]. \quad (36)$$

Thus for the absorption coefficient (17) we have

$$A(\omega) = 1 - \exp\left[-\frac{8}{3}\left(1 + \frac{\alpha}{3}\right)\frac{\nu L}{c}\frac{\omega^2}{\omega_L^2}\right]. \quad (37)$$

When $L \approx c\omega_L^2/\omega^3$, the expression (37) is stitched with the expression (30). If the thickness of the variable density layer is large $L \gg c\omega_L^2/\nu\omega^2$, then the field is almost completely absorbed in it.

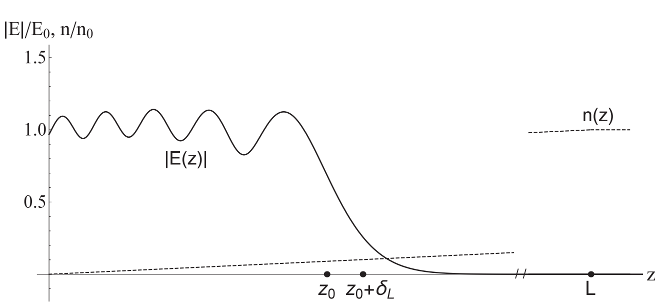


FIG. 3. Dependence of the field strength absolute value $E(z)$ inside the plasma on the distance to the plasma boundary in the case of a thick layer.

## IV. NORMAL SKIN EFFECT

Let us consider the case when collision frequency of electrons with neutral atoms satisfies the inequality $\nu \gg \omega$. Then the contribution from photoelectrons to the permittivity (7) in the linear approximation of $\omega/\nu$ is

$$\Delta\varepsilon \approx \frac{\omega_L^2}{\nu^2}\left[i\left(1 - \frac{\alpha}{3}\right)\frac{\nu}{\omega} + 2\frac{\alpha}{3} - 1 + i(\alpha - 1)\frac{\omega}{\nu}\right]. \quad (38)$$

We will focus on the case where, in addition to the inequality $\nu \gg \omega$, the inequalities

$$\frac{\omega_L^2}{\omega}\left(1 - \frac{\alpha}{3}\right)^2 \gg \nu\left(1 - \frac{\alpha}{3}\right) \gg \omega \quad (39)$$

are satisfied. The right-hand inequality (39) means that a photoionized plasma in which the parameter $\alpha$ is not only less than 3, but also $1 - \alpha/3 \gg \omega/\nu$, is considered. The case $\alpha > 3$ is not discussed in this communication. Then, when the inequalities (39) are satisfied, the permittivity can be represented in the form characteristic for the normal skin effect

$$\varepsilon(\omega) \approx \Delta\varepsilon \approx i\left(1 - \frac{\alpha}{3}\right)\frac{\omega_L^2}{\nu\omega}. \quad (40)$$

Taking (40) into account, for the values of $\xi$ at the variable density layer boundaries from (18) we have

$$\xi_0 \approx -\exp\left(\frac{i\pi}{3}\right)\left(\frac{\omega^2}{\omega_L^2}\frac{\nu L/c}{1 - \alpha/3}\right)^{2/3},$$

$$\xi_L \approx \xi_0\Delta\varepsilon \approx \exp\left(-\frac{i\pi}{6}\right)\left[\frac{(1 - \alpha/3)\omega}{\nu}\frac{L^2\omega_L^2}{c^2}\right]^{1/3}. \quad (41)$$

It follows from (41) that, as before, several field penetration modes are possible, depending on the thickness of the layer with a variable photoelectron density.

### A. Thin layer

When the left-hand inequality (39) is satisfied, then it follows from (40) and (41) that $|\xi_0| \ll |\xi_L|$. If also the width of the layer satisfies the condition

$$L \ll \delta_n = (c/\omega_L)\sqrt{2\nu/(1 - \alpha/3)\omega}, \quad (42)$$

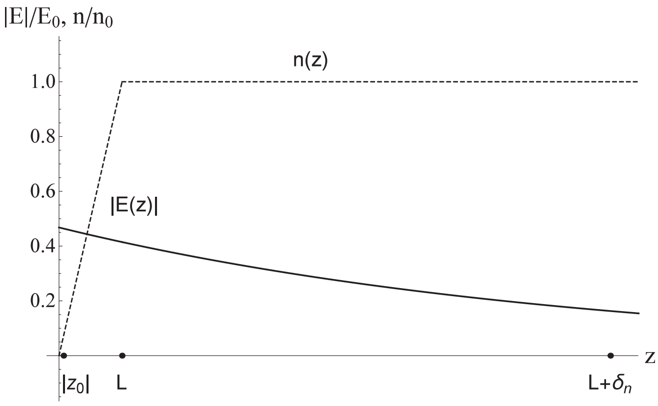


FIG. 4. Dependence of the field strength absolute value $E(z)$ inside the plasma on the distance to the plasma boundary in the case of a thin layer with a variable photoelectron density.

where $\delta_n$ is the skin layer depth in the normal skin effect mode, then the absolute values of the parameter $\xi$ at the variable density layer boundaries are small $|\xi_0| \ll |\xi_L| \ll 1$. Using the Airy functions expansion for small argument values [see formulas (A1) and (A2)], for the complex amplitude of the electric field inside the layer with a variable photoelectron density from (11) and (13)–(15) we have

$$E(z) = 2E_0\frac{(L-z)+(1+i)\delta_n/2}{(L+ic/\omega)+(1+i)\delta_n/2}. \tag{43}$$

From (12) and (43) we obtain $E(z)$ in the region $z > L$

$$E(z) = E_0\frac{(1+i)\delta_n}{(L+ic/\omega)+(1+i)\delta_n/2}\exp\left[(i-1)\frac{z-L}{\delta_n}\right]. \tag{44}$$

For a thin layer with a variable photoelectron density, the numerically obtained dependence of the absolute field strength $E(z)$ on the distance to the plasma boundary $z$ is shown in Fig. 4. The curve in Fig. 4 is plotted using the general expressions (11)–(15). Numerical calculations are performed for the following plasma parameters: $\alpha = 2$, corresponding to a plasma with an average photoelectron energy $\epsilon_0 \approx 4.5$ eV, produced by xenon ionization; incident field frequency $\omega = 0.05\omega_L$; photoelectron collision frequency $\nu = 0.5\omega_L$ and layer thickness $L = c/\omega_L$. The characteristic scales which determine the electric field behavior inside the plasma are marked in Fig. 4: The coordinate of the point $|z_0|$, the layer thickness $L$, and the skin layer depth in a plasma with a constant photoelectron density $\delta_n$. From Fig. 4 and formulas (43) and (44) it follows that, as for the high-frequency skin effect mode, when a skin layer depth greater than the variable density layer thickness, the field penetration features in plasma with an inhomogeneous density profile in the normal skin effect mode are close to those realized in plasma with a sharp boundary.

From (16) taking into account the Airy function expansion for small values of the argument, for the reflection coefficient we have

$$R = -\frac{\sqrt{\varepsilon(\omega)}-1}{\sqrt{\varepsilon(\omega)}+1}, \tag{45}$$

where $\sqrt{\varepsilon(\omega)} \approx (1+i)c/\omega\delta_n$. Taking into account the interrelation between reflection coefficient and absorption coefficient (17), we obtain

$$A(\omega) \approx \sqrt{\frac{8\nu\omega}{\omega_L^2}}\frac{1}{\sqrt{1-\alpha/3}}. \tag{46}$$

The expression (46) was obtained earlier in Ref. [18] assuming that the plasma photoelectron density changes abruptly, i.e., $L = 0$.

### B. Intermediate thickness layer

As the thickness of the variable density layer increases, conditions are possible when $|\xi_0| \ll 1$ and $|\xi_L| \gg 1$. Such absolute values of the variable $\xi$ at the inhomogeneous density layer boundaries correspond to a layer thickness that satisfies the inequalities

$$\delta_n \ll L \ll \frac{\omega_L^2}{\omega^2}\frac{c}{\nu}(1-\alpha/3), \tag{47}$$

where the right-hand inequality provides a small distance to the point $|z_0|$ compared to $c/\omega = \lambda/2\pi$. To calculate the electric field inside a layer with a variable photoelectron density on the boundary $z = L$ we use asymptotic formulas for the Airy functions at large values of the argument [see formulas (A3) and (A4)] and at the boundary $z = 0$ we use approximate formulas for small values of the argument [see formulas (A1) and (A2)]. When the inequalities (47) are satisfied, the coefficient $C_2$ (14) is much smaller than $C_1$ (13) and the term $C_2\mathrm{Bi}(\xi)$ in (11) is a negligibly small correction to $C_1\mathrm{Ai}(\xi)$. Taking this into account, the complex amplitude of the electric field (11) at $0 < z < L$ can be represented as

$$E(z) = 12^{1/3}e^{-i\pi/3}E_0\Gamma(1/3)\frac{\omega\delta_{nL}}{c}\mathrm{Ai}\left(2^{1/3}e^{-i\pi/6}\frac{z-z_0}{\delta_{nL}}\right), \tag{48}$$

where $\delta_{nL} = [Lc^2 2\nu/\omega_L^2\omega(1-\alpha/3)]^{1/3}$ is the effective skin layer depth in a plasma with smoothly varying photoelectron density profile, realized when (39) and (47) are satisfied. We note that $\delta_{nL} \gg |z_0|$. It follows from (48) that in the normal skin effect mode the electric field penetrates the plasma to distances $(L/\delta_n)^{1/3}$ times larger than in the case of a thin inhomogeneous layer. In the region of constant photoelectron density, when $z > L$, from (12) and (48) for the field strength we have

$$E(z) \approx E_0\frac{6^{1/3}\Gamma(1/3)}{\sqrt{2\pi}}\frac{\omega\delta_{nL}}{c}\left(\frac{\delta_{nL}}{2L}\right)^{1/4}\exp\left\{(i-1)\left[\frac{z-L}{\delta_n}+\frac{2}{3}\left(\frac{L}{\delta_{nL}}\right)^{3/2}\right]-i\frac{7\pi}{24}\right\}. \tag{49}$$

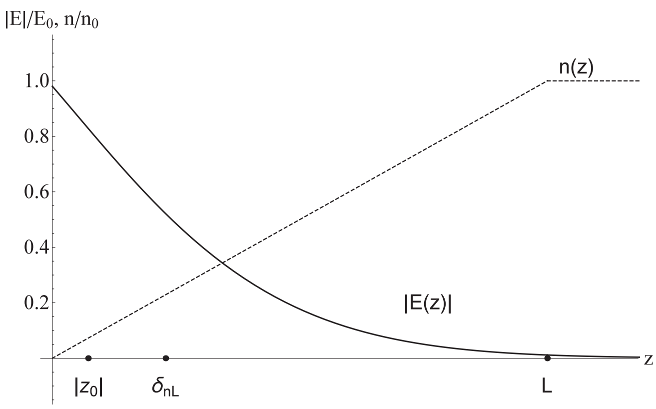


FIG. 5. Dependence of the field strength absolute value $E(z)$ inside the plasma on the distance to the plasma boundary in the case of an intermediate thickness layer.

For an intermediate thickness layer with a variable photoelectron density, the numerically obtained dependence of the absolute field strength $E(z)$ on the distance to the plasma boundary $z$ is shown in Fig. 5. The curve in Fig. 5 is plotted using the general expressions (11)–(15). Numerical calculations are performed for the following parameters: $\alpha = 2$, incident field frequency $\omega = 0.05\omega_L$, photoelectron collision frequency $\nu = 0.5\omega_L$, and layer thickness $L = 50c/\omega_L$. In Fig. 5 the coordinate of the point $|z_0|$, the thickness of the layer $L$ and the depth of the skin layer $\delta_{nL}$ are marked. Figure 5 and formulas (48) and (49) show that, as in the high-frequency skin effect regime, the electric field becomes exponentially small at $z > \delta_{nL} \gg |z_0|$, where $\delta_{nL} \ll L$.

From the expression (16) we find the reflection coefficient

$$R \approx \frac{\Gamma(1/3) - e^{i\pi/3}6^{1/3}\Gamma(2/3)c/\omega\delta_{nL}}{\Gamma(1/3) + e^{i\pi/3}6^{1/3}\Gamma(2/3)c/\omega\delta_{nL}}. \tag{50}$$

It follows that the absorption coefficient has the form

$$A(\omega) \approx \frac{2^{2/3}\Gamma(1/3)}{3^{1/3}\Gamma(2/3)}\frac{\omega\delta_{nL}}{c}. \tag{51}$$

A comparison of the expressions (46) and (51) shows that increasing of the variable density layer thickness leads to a relative increase of the absorption coefficient by $\approx \sqrt{L/\delta_{nL}}$ times.

### C. Thick layer

If the variable density layer is so thick that the conditions

$$L \gg \frac{\omega_L^2}{\omega^2}\frac{c}{\nu}(1 - \alpha/3), \tag{52}$$

or $|z_0| \gg c/\omega$ are satisfied, then the absolute values of the variable $\xi$ at the inhomogeneous layer boundaries $|\xi_0|, |\xi_L| \gg 1$. The electric field inside the plasma is found using asymptotic expressions for the Airy functions. When $z = 0$ we use the formulas (A5) and (A6), and when $z = L$ we use the formulas (A3) and (A4). Ignoring the small term $C_2\mathrm{Bi}(\xi)$ in the expression (11) and considering conditions (39) and (52), for complex amplitude of electric field strength inside layer $0 < z < L$ we find

$$E(z) = 2^{5/6}E_0\sqrt{\pi}\left(\frac{\omega\delta_{nL}}{c}\right)^{1/2}\exp\left[-\frac{1}{3}\left(\frac{\omega\delta_{nL}}{c}\right)^3 - i\frac{\pi}{6}\right]\mathrm{Ai}\left(2^{1/3}e^{-i\pi/6}\frac{z - z_0}{\delta_{nL}}\right). \tag{53}$$

If the inequality (52) is satisfied, then the distance to the point $|z_0| \gg \delta_{nL}$, and from (53) it follows that in this case the field penetrates the plasma to a depth of order $|z_0| + \delta_{nL} \approx |z_0| \gg \delta_{nL}$. It follows from (53) that in the region of constant photoelectron density $z > L$ the field strength is negligibly small

$$E(z) \approx 2^{-1/3}E_0\left(\frac{\omega\delta_{nL}}{c}\right)^{1/2}\left(\frac{\delta_{nL}}{2L}\right)^{1/4}\exp\left\{(i-1)\left[\frac{z - L}{\delta_n} + \frac{2}{3}\left(\frac{L}{\delta_{nL}}\right)^{3/2}\right] - \frac{1}{3}\left(\frac{\omega\delta_{nL}}{c}\right)^3 - i\frac{5\pi}{24}\right\}. \tag{54}$$

For a thick layer with a variable photoelectron density, the numerically obtained dependence of the absolute field strength $E(z)$ on the distance to the plasma boundary $z$ is shown in Fig. 6. The curve in Fig. 6 is plotted using the general expressions (11)–(15). Numerical calculations are performed for the following parameters: $\alpha = 2$; incident field frequency $\omega = 0.05\omega_L$; photoelectron collision frequency $\nu = 0.5\omega_L$ and layer thickness $L = 1000c/\omega_L$. From Fig. 6 and formulas (53) and (54) it follows that, as in the high-frequency skin effect regime, the electric field becomes exponentially small at $z - |z_0| > \delta_{nL}$.

The reflection coefficient (16), when the inequality (52) is satisfied, is

$$R = \exp\left[-\frac{4}{3}\frac{\nu L}{c(1 - \alpha/3)}\frac{\omega^2}{\omega_L^2} - i\frac{\pi}{2}\right]. \tag{55}$$

Thus for the absorption coefficient (17) we have

$$A(\omega) = 1 - \exp\left[-\frac{8}{3}\frac{\nu L}{c(1 - \alpha/3)}\frac{\omega^2}{\omega_L^2}\right]. \tag{56}$$

Taking (52) into account, we see that electric field is almost completely absorbed in the layer with a variable density of photoelectrons.

## V. CONCLUSION

Using the assumption of linear variation of photoelectron density in space, the features of probing radiation interaction with inhomogeneous photoionized plasma formed during multiphoton ionization of inert gas atoms are studied. The conditions under which the assumption of a sharp change in the photoelectron density can be used to describe

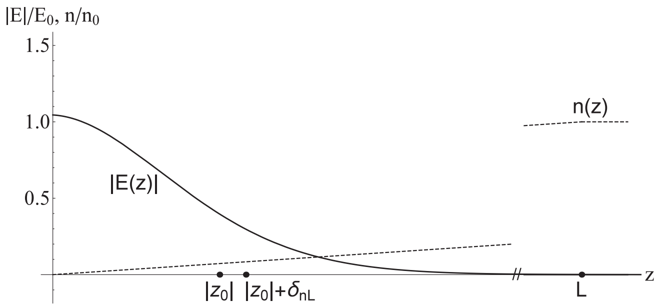


FIG. 6. Dependence of the field strength absolute value $E(z)$ inside the plasma on the distance to the plasma boundary in the case of a thick layer.

the interaction of short pulses with a nonequilibrium photoionized plasma are revealed. It is shown to what changes in the absorption and reflection of the probing radiation the blurring of a photoionized plasma boundary leads to. The obtained results form the basis for further study of possible modes of short electromagnetic pulses interaction with photoionized plasma of inert gases, including such modes when it is necessary to take into account the development of instabilities.

## ACKNOWLEDGMENTS

The reported study was funded by Russian Foundation for Basic Research (RFBR) Project No. 20-32-90158.

## APPENDIX

According to formulas 10.4.2 and 10.4.3 from Ref. [26], which give the expansion of the Airy functions for small values of the argument, we have approximate expressions:

$$\mathrm{Ai}(\xi) = \frac{1}{3^{2/3}\Gamma(2/3)} - \frac{\xi}{3^{1/3}\Gamma(1/3)}, \tag{A1}$$

$$\mathrm{Bi}(\xi) = \frac{1}{3^{1/6}\Gamma(2/3)} + \frac{3^{1/6}\xi}{\Gamma(1/3)}, \tag{A2}$$

where $\Gamma(x)$ is a Gamma function. For large values of the argument, using the known asymptotic expressions (see formulas 10.4.59, 10.4.63, 10.4.60, and 10.4.64 from Ref. [26]) we have for the Airy functions

$$\mathrm{Ai}(\xi) = \frac{1}{2\sqrt{\pi}\xi^{1/4}} \exp\left[-\frac{2}{3}\xi^{3/2}\right] \times \left(1 - \frac{5}{48}\frac{1}{\xi^{3/2}}\right), \quad |\arg \xi| < \pi, \tag{A3}$$

$$\mathrm{Bi}(\xi) = \frac{1}{\sqrt{\pi}\xi^{1/4}} \exp\left[\frac{2}{3}\xi^{3/2}\right] \times \left(1 + \frac{5}{48}\frac{1}{\xi^{3/2}}\right), \quad |\arg \xi| < \pi/3. \tag{A4}$$

$$\mathrm{Ai}(-\xi) = \frac{1}{\sqrt{\pi}\xi^{1/4}} \sin\left(\frac{2}{3}\xi^{3/2} + \frac{\pi}{4}\right), \quad |\arg \xi| < 2\pi/3. \tag{A5}$$

$$\mathrm{Bi}(-\xi) = \frac{1}{\sqrt{\pi}\xi^{1/4}} \cos\left(\frac{2}{3}\xi^{3/2} + \frac{\pi}{4}\right), \quad |\arg \xi| < 2\pi/3. \tag{A6}$$

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