Exploring the nonextensive thermodynamics of partially ionized gas in magnetic field

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Contrary to classical thermodynamics, which deals with systems in thermal equilibrium, partially ionized gases generally do not reach thermal equilibrium. Nonextensive statistical mechanics has helped extend classical thermodynamics to nonequilibrium ionized gas. However, the fundamental question on whether the statistics of non-Maxwellian electrons satisfy the laws of thermodynamics has not been resolved. Here, we verify the thermodynamic laws of reversible and adiabatic processes for a magnetically expanding ionized gas. Together with the experimental evidence of the non-Maxwellian electron distribution, the κ distribution, which measures the thermal equilibrium states, shows the Tsallis entropy to be nearly constant and the polytropic index to be close to adiabatic values along a divergent magnetic field. These results verify that the collisionless magnetic expansion of a nonequilibrium plasma is reversible and adiabatic, and an isentropic process is the origin of the high-energy tail of the energy distribution far downstream.

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I. INTRODUCTION

Thermodynamics is a fundamental science with strict physical laws for changes of state, and it deals with energy, heat, work, entropy, and spontaneity of processes [1–3]. The laws of thermodynamics are a universal theoretical system that indicates the transitivity of thermal equilibrium, energy conservation, irreversibility of thermal phenomena, and absolute zero point of a thermally equilibrated system. Importantly, thermodynamics defines the physical parameters of an ionized gas and is combined with local or nonlocal equilibrium theory to understand the complex physics of such a system [4,5].

Contrary to classical thermodynamics, which deals with systems in thermal equilibrium, ionized gases generally do not reach thermal equilibrium among all particle species or within each particle species in a volume [6–8]. In particular, non-Maxwellian electron distributions are observed in space plasmas [9–12], which are essentially collisionless systems. As a result, stationary states of ionized gases out of equilibrium are not readily understood through classical statistical descriptions of thermal equilibria.

Classical thermodynamics has evolved to include partially ionized gas systems that are not in equilibrium and to explain their physical properties. The field of nonextensive statistical mechanics pioneered by Tsallis generalizes the thermodynamic laws of nonequilibrium systems [13,14]. Under statistical mechanics, the non-Maxwellian distribution observed in a space plasma is valid for entropy and other relevant thermodynamic properties. Most importantly, the ability to independently observe temperature and entropy enables numerous analyses on phenomena such as plasma oscillation [15], turbulence [16], magnetic reconnection [17,18], and wave-particle interaction [19,20]. Nonextensive statistical mechanics is currently essential to our understanding of nonequilibrium astrophysical plasmas.

The laws of physics have been accepted as established and universally valid through rigorous empirical verification, and the laws of thermodynamics are being completed through measurement and observation of variables in a statistical context from the Joule paddle-wheel and Stirling engine to black holes [21]. In plasma thermodynamics, the magnetic nozzle is used to analyze astrophysical phenomena [22–38] because they share fundamental plasma physics such as non-Maxwellian electron energy distributions in collision-free conditions. However, although a magnetic nozzle plasma experiment well reproduces the space plasma environment, thermodynamic laws have only been verified under the assumption of classical thermal equilibrium.

In this study, we conduct the most rudimentary experiment on whether a magnetically expanding, nonequilibrium, collisionless plasma satisfies the laws of thermodynamics. Experimentally proving the reversibility and adiabaticity of nonequilibrium electron systems requires the following: (i) appearance of a non-Maxwellian electron energy distribution during the magnetic expansion, (ii) a minimized electric boundary in the magnetic nozzle, and (iii) a negligible electric field along the divergent magnetic field. In the modeled system, we minimize the trapped motion accompanied by the axial electric field, and we assume that the electrons only perform magnetic expansion.

II. EXPERIMENTAL SETUP

A. Magnetic nozzle device

Our experiment was performed with a filament plasma source and a grounded expansion chamber that was 0.66 m long and 0.6 m in diameter (Fig. 1), which maximized

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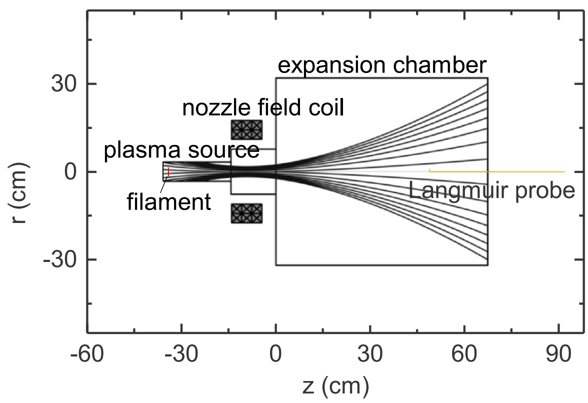

FIG. 1. Schematic of the magnetic nozzle device. The magnetic nozzle was driven by the filament plasma source and the divergent magnetic field. A single cylindrical Langmuir probe moved along the axis of the expansion region. The standard deviation of the magnetic field strength over a radius corresponding to the half-length of the probe tip (5.5 mm) was within 0.001 G in the axial measurement range.

the mean free path of electrons and minimized the electron potential energy. Using a mass flow controller with a flow rate of 3.5 sccm, argon gas was injected through a gas feeding port into the source region. The base pressure was maintained under 10^{-6} Torr, and the operating pressure was fixed at 0.38 mTorr to ensure the electrons were collision-free (an electron-neutral collisional mean free path of 1.3 m). A nozzle field coil, installed at the left end of the diffusion chamber, formed a convergent-divergent magnetic field configuration. A discharge voltage of 100 V was applied between thoriated hot filaments and the grounded chamber, resulting in emission and acceleration of thermionic electrons in the plasma source.

B. Single Langmuir probe diagnostics

The design of the probe follows [39] with a probe tip constructed from tungsten wire of radius 0.15 mm, which is larger than the minimum electron Larmor radius of 1.0 mm. The axial direction of the tip was oriented perpendicular to the direction of the divergent magnetic field to maximize the collection area. Then, the time-averaged electron energy probability function (*eepf*) was obtained by calculating the second derivative of the measured current-voltage characteristic. It is assumed that the beam-plasma interaction only occurred in a limited area of a few centimeters of the plasma source [40,41]. The plasma potential V_p was determined from the zero-crossing of the second derivative of the current-voltage curve. Each *eepf* was averaged over 110 shots to minimize error during the measurement; errors of less than 5% were achieved for V_p, electron density n_e, and electron temperature T.

III. RESULTS AND DISCUSSION

A. Magnetic expansion of plasma in the absence of an axial electric field

Thermodynamic laws were verified on the basis of nonextensive statistical properties established for a non-Maxwellian system, and the effect of electron thermodynamics on the electron energy distribution function was investigated. We conducted precise diagnostics of the spatially varying *eepf* along the diverging magnetic field under collisionless conditions. The radius of the probe was carefully chosen to ensure the reliability of the measurement of magnetized plasma [39]. The *eepfs* were measured at distances of 10–50 cm from the nozzle throat along the divergent magnetic field, and the kinetic electron temperature corresponding to the mean electron energy was determined from $T = 2/(3n_e)\int_0^\infty \varepsilon^{3/2} f(\varepsilon)\, d\varepsilon$, where ε is the electron kinetic energy and n_e is the electron density, i.e., $n_e = \int_0^\infty \varepsilon^{1/2} f(\varepsilon)\, d\varepsilon$. The measured plasma properties decreased along the axial magnetic field as with other magnetic nozzle devices except for the plasma potential V_p. The plasma potential was close to that of the grounded anode and had the characteristics of the potential of a magnetized dc plasma source [26,27,34], and the change in V_p over the axial measurement range was only 0.4 V [Fig. 2(a)]. The distribution was nearly Maxwellian near the nozzle throat [Fig. 2(b)] but noticeably non-Maxwellian near the region far downstream, where it showed a strong high-energy tail. Previously, the convex shape of the non-Maxwellian electron energy distribution was explained by nonlocal electron dynamics and adiabatic processes [38] based on the classical definition of electron temperature. The changes in *eepfs* found in this study show different evolution from that in the nonlocal approach for magnetic nozzle devices in which a double layer is observed. As indicated by the cooling ratio of electrons found in a similar experimental setup [26,34], the spatially averaged polytropic index $\overline{\gamma_e}$ is close to the adiabatic value (1.88 ± 0.75) [Fig. 4(c)]. However, the nonequilibrium state in the far-field region, as reflected by the strong high-energy tail, cannot yet be explained by classical thermodynamics or nonlocal kinetics with average kinetic energy applied to magnetic nozzles. Although all previous studies describe electron thermodynamics through average kinetic energy [24–26,28,33,34,38], the thermodynamic temperature defined by the classical framework does not coincide with the kinetic temperature if the stationary state of the system is not in thermal equilibrium; the logical consistency of the concept of temperature disappears [42,43]. Eventually, the zeroth law of thermodynamics that precedes the first and second laws is not satisfied only by the average kinetic energy.

In recent years, extensive research has been conducted on the magnetic nozzle, and a wide range of physical phenomena related to electrons has been reported, such as cooling [24–26,28,33,34,38], diamagnetism [22,35,36], and demagnetization [29]. Nonetheless, the results of these studies are still based on the classical definition of kinetic temperature (average electron temperature), which is only valid in classical thermodynamic analysis of a system in thermal equilibrium. Accordingly, regardless of the progress in magnetic nozzle physics, the problem regarding consistency with thermodynamic laws remains. The approach based solely on the density and average energy of electrons is significantly limited in that it cannot explain the appearance of nonequilibrium states as a fundamental phenomenon of magnetically expanding collisionless plasmas.

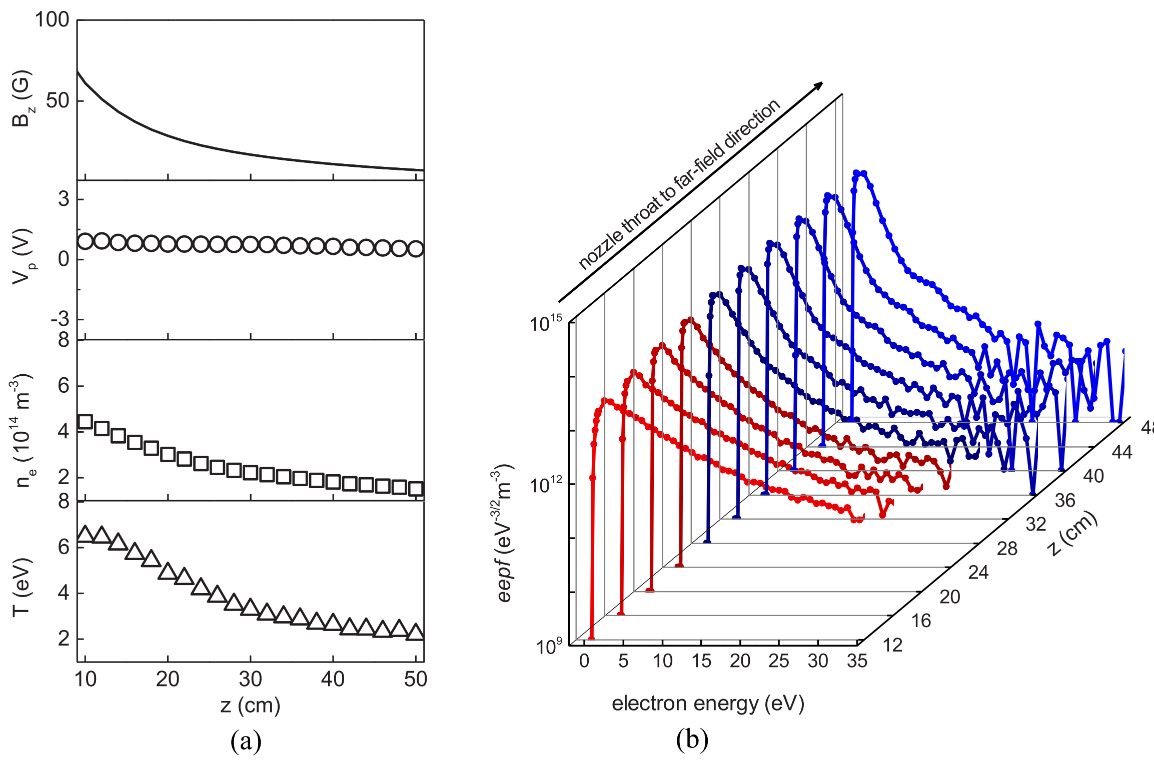

FIG. 2. Axial variation of plasma properties. (a) Axial profiles of magnetic field strength B_z and plasma parameters at distances of 10–50 cm from the nozzle throat. The measured V_p has a difference of about 0.9 V from the potential of the grounded chamber wall, and hence we expected most of the electrons to be freely escaping. (b) Axial evolution of *eepfs* at different axial positions (12–48 cm from the nozzle throat at intervals of 4 cm). The deviation from a Maxwellian energy distribution is greater approaching the far downstream region, where a heavy high-energy tail above 5 eV develops.

B. Kappa distribution and nonextensive thermodynamics

The non-Maxwellian *eepfs* observed in the magnetic nozzle show similar characteristics to those of the distribution function observed in the solar wind. Most of the solar wind electrons consist of the nearly isotropic core and halo electrons and are characterized by the κ distribution. In the κ distribution, the temperature has both thermodynamic and kinetic features that coincide with a non-Maxwellian space plasma; therefore, this non-Maxwellian electron system in a magnetic nozzle should be analyzed thermodynamically.

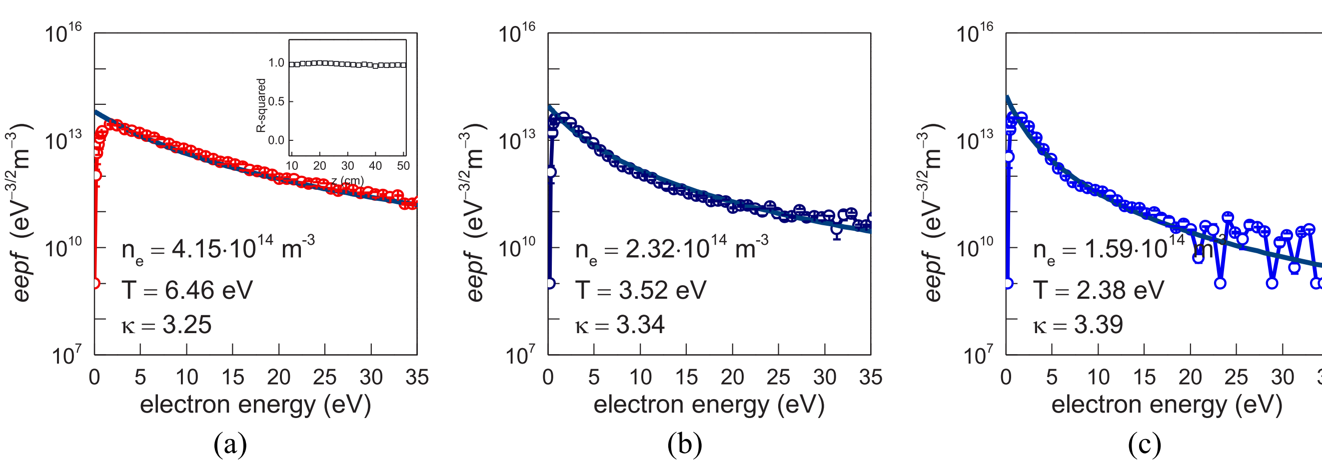

FIG. 3. Fitting of *eepfs*. *eepfs* were fitted to the κ distribution function of Eq. (1): (a) 12 cm, (b) 28 cm, and (c) 48 cm. The variables E_p and κ were determined using T and n_e obtained from the distribution function and fitted with the experimentally measured *eepfs* having an R-squared of 0.95 or more. The inset shows the R-squared values (the proportion of the variance of the fitted curve and the experimentally obtained *eepfs* in the range from 3 to 35 eV). The discrepancy at the low-energy range is due to the limitations of measurement corresponding to the distortion of the low-energy range of the *eepfs*, and it is assumed that the fitting by the κ distribution is consistent even for low-energy range not recorded.

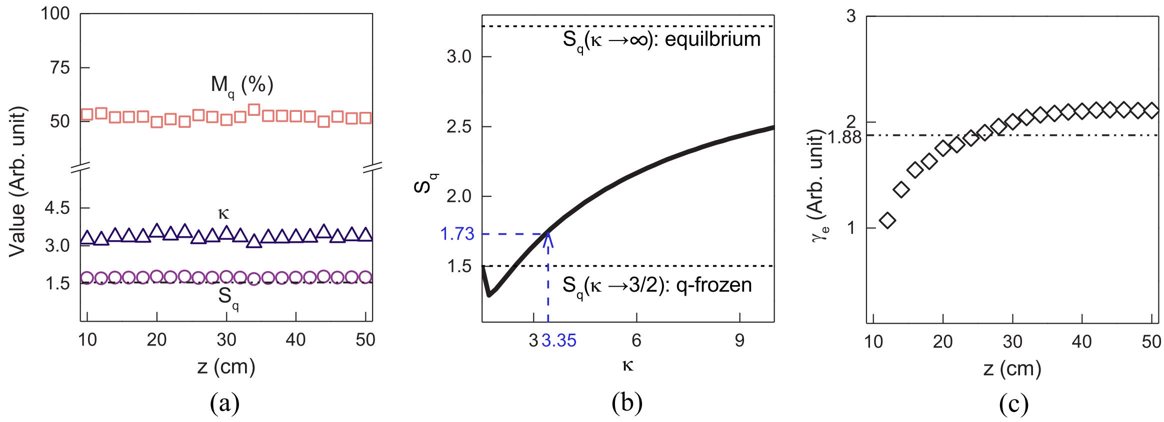

FIG. 4. Thermodynamic properties. (a) Axial variation of q-metastability M_q, kappa κ, and Tsallis entropy S_q. The dashed line indicates the statistical minimum of κ, 3/2. The κ obtained along the axial direction is nearly constant at 3.35 ± 0.05. (b) S_q plotted with respect to κ. Kappa has two extreme stationary states corresponding to q-frozen, $\kappa \to 3/2$ and equilibrium $\kappa \to \infty$. (c) When the axial electric field along the divergent magnetic field is removed, electrons undergo an adiabatic process (the spatially averaged polytropic index $\overline{\gamma_e}$ is 1.88 ± 0.75).

The κ distribution is a function of two independent parameters, temperature T and κ, defining the characteristic states. Under isotropic conditions, the distribution takes the form [14,19]

$$f_e(\varepsilon) = n_e \frac{2}{\sqrt{\pi}} \sqrt{\varepsilon} T^{-\frac{3}{2}} \left[\frac{\Gamma(\kappa+1)}{\left(\kappa - \frac{3}{2}\right)^{\frac{3}{2}} \Gamma\left(\kappa - \frac{1}{2}\right)} \right] \times \left(1 + \frac{\varepsilon}{\left(\kappa - \frac{3}{2}\right) T}\right)^{-\kappa-1}, \quad (1)$$

where ε and Γ denote the electron energy and the gamma (generalized factorial) function, respectively. In contrast to the thermodynamic temperature in classical Boltzmann-Gibbs statistical mechanics, which is valid only for a stationary system in thermal equilibrium, the electron temperature in the κ distribution enables use of relevant thermodynamic parameters of a nonequilibrium electron system. For the κ distribution, the most probable kinetic energy is defined as $E_p = T(\kappa - 3/2)/\kappa$, and the ratio E_p/T represents the degree to which a given κ distribution can be approximated by a Maxwellian distribution. We derive κ by fitting the recorded *eepfs* with the κ distribution [Figs. 3(a)–3(c)], and we identify the stationary states of the electron system along the divergent magnetic field.

One of the important roles of κ is to determine the nonequilibrium stationary states and to measure the "thermodynamic distance" from thermal equilibrium. Livadiotis proved that the q-exponential distribution is exactly the same as the κ distribution, and established the relation $\kappa = 1/(q-1)$, indicating the identity with the entropy index q that characterizes the Tsallis entropy S_q [42]. Accordingly, we examined the thermodynamic distance of each stationary state from equilibrium through the q-metastability $M_q = 4[(q-1)/(q+1)]$, where the equilibrium is described by the classical equilibrium limit $M_q = 0$ for $q \to 1$ and the q-frozen state $M_q = 1$ for $q \to 5/3$, which is the state 100% away from equilibrium. The calculated M_q (expressed as a percentage) for all axial positions is within $52 \pm 0.7\,\%$, implying invariance of the equilibrium state [Fig. 4(a)]. Quantifying the entropy enables discussion of the energy flow of the electrons in a magnetic nozzle. The nonextensive entropy S_q in terms of κ is given by [43]

$$S_q(\kappa) = \kappa - \kappa^{\frac{1}{\frac{2}{3}\kappa+1}} \left[\pi^{-\frac{3}{2}} \left(\kappa - \frac{3}{2}\right)^{\kappa - \frac{1}{2}} \frac{\Gamma(\kappa)}{\Gamma\left(\kappa - \frac{1}{2}\right)} \right]^{\frac{1}{\kappa + \frac{3}{2}}}. \quad (2)$$

The estimated nonextensive entropy appears nearly fixed at 1.73 ± 0.02 along the divergent magnetic field in Fig. 4(a), and the spatially averaged S_q (1.73) is outside the cavity (the nonmonotonic part of S_q, $\kappa < 2.45$) in Fig. 4(b), implying the absence of changes in κ through isentropic switching [42]. The calculated entropy, which does not change along the divergent magnetic field, indicates a reversible process. The information contained in the source region is preserved even after the thermodynamic process is completed along the divergent magnetic field (i.e., κ determined in the source region is maintained along with electron cooling in the divergent magnetic field). Finally, we conclude that the electrons in the nonequilibrium state undergo a reversible adiabatic process, and the formation of the non-Maxwellian distribution in the far field of the magnetic nozzle is understood within the laws of thermodynamics (as a natural consequence of a reversible adiabatic process).

IV. CONCLUSIONS

In summary, we found an answer to the fundamental question of whether collisionless, magnetically expanding, nonequilibrium electrons satisfy the laws of thermodynamics. Introducing the κ distribution into nonextensive statistical mechanics enables examination of the reversibility of electrons out of thermal equilibrium via nonextensive Tsallis entropy. This study validates thermodynamic laws for plasma along with adiabaticity. We have also found the clear origin of non-Maxwellian electron energy distributions far downstream of magnetically expanding plasma. The departure from a Maxwellian distribution in the far field of the magnetic nozzle is direct evidence of a reversible adiabatic process (an isentropic process). This study has found that the non-Maxwellian distribution is generated far downstream of the magnetically expanding plasma under collision-free conditions without wave interactions or additional heating mechanisms during the expansion. The final stationary equilibrium states in the region where the magnetic field is weak are subject to not only heating in the plasma generation region but also thermodynamic processes in the magnetic field. Although fluctuations in plasma properties were not observed in this study, wave-particle interaction should not be overlooked in space plasma. Therefore, in the future, a theoretical study should be carried out on the possibility of the emergence of the κ distribution in a laboratory plasma resulting from a self-consistent wave-particle interaction. Although the nonextensive statistical mechanics is considered as the optimal theory to elucidate the thermodynamic laws of space and laboratory plasma so far, it is considered desirable for the future development of plasma thermodynamics to adopt entropy from other perspectives in the interpretation of experimental results [20,44–51].

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The authors declare no competing interests.

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