Formation of microwave frequency-chirped solitons of self-induced transparency under conditions of cyclotron resonance absorption

Naum S. Ginzburg,* Lev A. Yurovskiy, Alexander S. Sergeev, Irina V. Zotova, and Andrey M. Malkin
Institute of Applied Physics of RAS, 46 Ul'yanov Street, Nizhny Novgorod, 603950, Russia

(Received 24 May 2021; accepted 8 September 2021; published 30 September 2021)

We study the formation of solitons of microwave self-induced transparency (M/W-SIT) which occurs under cyclotron resonance interaction of an electromagnetic pulse with an initially rectilinear magnetized electron beam. Taking into account the relativistic dependence of the gyrofrequency on the particle energy for electromagnetic wave propagating with a phase velocity different from the speed of light (i.e., far from the autoresonance conditions), such a beam can be considered as a medium of nonisochronous unexcited oscillators. Thus, similar to passing light pulses in the two-level medium, for sufficiently large amplitude and duration the incident electromagnetic pulse decomposes into one or several solitons. We find analytically the generalized solution for the M/W-SIT soliton with amplitude and duration determined, besides the soliton velocity, by the frequency self-shift parameter. The feasibility and stability of the obtained solutions are confirmed in numerical simulations of a semibounded problem describing propagation and nonlinear interaction of an incident electromagnetic pulse.

DOI: 10.1103/PhysRevE.104.034218

I. INTRODUCTION

The self-induced transparency (SIT) effect, well-known in optics, manifests itself in the propagation of a short (in the scale of relaxation times) light pulse through a resonant noninverted medium that normally absorbs the light but becomes transparent when the pulse energy exceeds some threshold value [1–4]. In this case, the incident pulse transforms into one or more of the so-called SIT solitons, which propagate practically without change in their shapes. Under some conditions, the formation of a SIT soliton can be accompanied by nonlinear compression of the incident light pulse.

As shown in [5–7], similar effects can be observed in microwave electronics for the case of resonant interaction of an electromagnetic (EM) pulse with an initially rectilinear electron beam guided by a homogeneous magnetic field. Taking into account the relativistic dependence of the gyrofrequency on the electron energy, such a beam can be considered a resonant passive medium comprising nonisochronous nonexcited cyclotron oscillators. Thus, similarly to optics, an incident electromagnetic pulse with sufficiently large amplitude and duration decomposes into one or several solitons, i.e., the microwave self-induced transparency (M/W-SIT) effect appears. It should be noted that we consider interaction with the fast wave under the condition of the normal Doppler effect. This is in contrast with Refs. [8,9], which consider interaction of an initially rectilinear electron beam with a slow wave under the anomalous Doppler effect. In the latter case, unlike SIT effects, the beam of electrons moving with a superlight velocity can amplify and generate radiation.

In [5–7], we have found, for a particular case, the simplified analytical solution for a M/W-SIT soliton with its amplitude and duration depending only on velocity, similarly to single-parametric optical SIT soliton solutions obtained in the pioneering papers of Refs. [1–4]. At the same time, direct simulations of an EM pulse cyclotron resonance interaction with a nonexcited electron beam show that the found solution is not universally applicable to the transformation of the incident pulse into a soliton for all possible values of initial amplitudes, durations, and carrier frequencies. Actually, another important parameter associated with the shift of the soliton center frequency from the cyclotron resonance may be of significance for a description of soliton formation. In this sense, M/W-SIT solitons resemble nonlinear Schrödinger solitons [10–13] for a nonabsorbing medium with reactive nonlinearity.

The paper is organized as follows. In Sec. II, we formulate the basic nonstationary model of the interaction of an incident microwave pulse with an initially rectilinear magnetized electron beam propagating in a waveguide. In Sec. III, we obtain a generalized form of a M/W-SIT solitonlike solution that depends on two parameters, namely the soliton velocity and the frequency self-shift. In Sec. IV, the feasibility and stability of the obtained solitonlike solutions are confirmed based on numerical simulations within the framework of an averaged semibounded problem describing the transformation of an incident microwave pulse. In Sec. V, we present a brief conclusion about the obtained results, and we estimate the possibility of experimental observation of M/W-SIT soliton formation considering the nonlinear transformation of the microwave superradiant pulse.

II. BASIC MODEL

We consider the microwave SIT effect under interaction of radiation with a copropagated initially rectilinear electron beam [Fig. 1(a)] guided by a homogeneous magnetic field $\vec{H} = \vec{z}_0 H_0$. The electric field of an electromagnetic wave can

*ginzburg@ipfran.ru

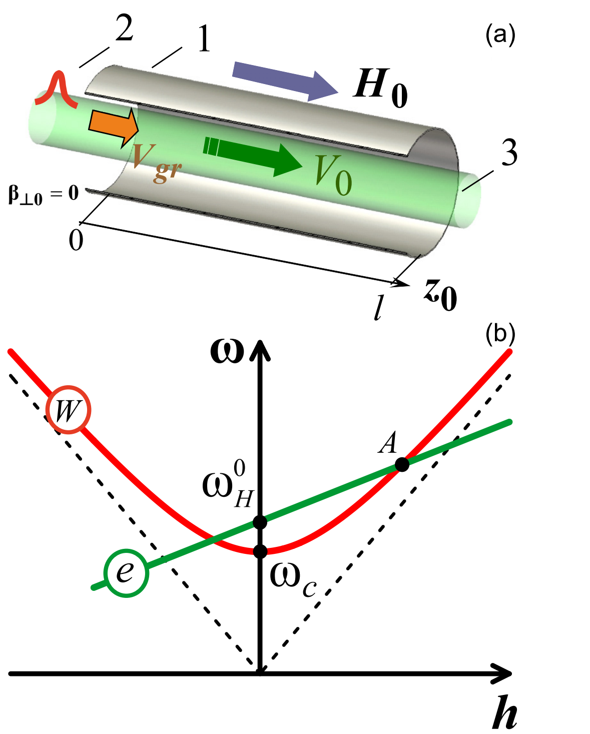

FIG. 1. (a) Scheme of the formation of M/W-SIT solitons in a waveguide (1) under cyclotron resonance interaction of an incident electromagnetic pulse (2) with an initially rectilinear (at $z = 0$) electron beam (3). (b) Dispersion diagram of a waveguide mode W ($\omega = \sqrt{c^2h^2 + \omega_c^2}$) and the beam line e ($\omega - hV_{||0} = \omega_H^0$); A is the resonant point for interaction with a copropagating electron beam.

be presented as

$$\vec{E} = \mathrm{Re}\{\vec{E}(\vec{r}_\perp)A(z,t)\exp(i\omega_r t - ih_r z)\}, \tag{1}$$

where $A(z,t)$ is the slow-varying wave amplitude, $\vec{E}(\vec{r}_\perp)$ is the transverse structure of a TE_{mn} waveguide mode, the frequency of the exact cyclotron resonance [Fig. 1(b)]

$$\omega_r - h_r V_{||0} = \omega_H^0 \tag{2}$$

is chosen as the reference one, $V_{||0} = \beta_{||0}c$ is the initial translational velocity of electrons, $\omega_H^0 = eH_0/m_e c\gamma_0$ is the unperturbed gyrofrequency, $\gamma_0 = (1-\beta_{||0}^2)^{-1/2}$, and $h_r = h(\omega_r)$ is the axial wave number at the frequency ω_r. Note that in the process of cyclotron absorption, an electromagnetic wave can increase the transverse momentum of electrons $p_\perp$, which is assumed to be zero at the entrance to the interaction space $z = 0$. Such a beam can be considered as an analog of a resonant medium consisting of nonexcited (noninverted) cyclotron oscillators, while rotating beams [which are usually used in gyroresonant devices, namely cyclotron-resonance masers (CRMs)] form active inverted media [14–16].

The systems of equations describing the electron-wave interaction under the indicated conditions can be obtained from the well-known equations of the theory of cyclotron-resonance masers [15,16], in which we must set $p_{\perp 0} = 0$. Thus, the CRM equations are reduced to the set of two equations for the normalized wave amplitude $\tilde{A}$ and the complex transverse momentum $\tilde{p}_+$:

$$\frac{\partial \tilde{A}}{\partial z} + \frac{1}{V_{\rm gr}}\frac{\partial \tilde{A}}{\partial t} = -G\frac{\omega_r}{c}\frac{\tilde{p}_+}{\hat{p}_{||}},$$
$$\frac{\partial \tilde{p}_+}{\partial z} + \frac{1}{V_{||0}}\frac{\partial \tilde{p}_+}{\partial t} + i\tilde{p}_+\frac{\omega_r}{c}\frac{\mu|\tilde{p}_+|^2}{\hat{p}_{||}} = \frac{\omega_r}{c}\frac{\tilde{A}}{\hat{p}_{||}}, \tag{3}$$

where the beam of initially nonexcited cyclotron oscillators is described by a single equation of motion. Note that for active rotating beams, the averaging over initial cyclotron rotating phases is needed (i.e., an electron beam is described by a set of motion equations for electron fractions that differ in the gyrophase at the system entrance). In Eqs. (3), we use following dimensionless variables and parameters:

$$\tilde{A} = \frac{eAJ_{m-1}(\nu_n r_b/r_w)}{m_e c\omega_r}\frac{1-\beta_{\rm ph}^{-1}\beta_{||0}}{2\beta_{||0}^2\gamma_0\sqrt{1-\beta_{\rm ph}^2}},$$
$$\tilde{p}_+ = \frac{p_x + ip_y}{m_e c\gamma_0\beta_{||0}}e^{-i\omega_r t + ih_r z},$$

where $\mu = \beta_{||0}(1-\beta_{\rm ph}^{-2})/2(1-\beta_{\rm ph}^{-1}\beta_{||0})$ is the bunching parameter (the parameter of the nonisochronism [16]), $b = \beta_{||0}/2\beta_{\rm ph}(1-\beta_{||0}/\beta_{\rm ph})$ is the recoil parameter characterizing the change in the translational momentum of electrons $\hat{p}_{||} = p_{||}/mV_{||0}\gamma_0 = 1 - b|\hat{p}_+|^2$, $V_{\rm gr} = \beta_{\rm gr}c$ and $V_{\rm ph} = \beta_{\rm ph}c$ are the group and the phase velocities of a resonant TE_{mn} wave at a frequency ω_r,

$$G = \frac{eI_b}{m_e c^3}\frac{2\mu\beta_{\rm ph}\left(1-\beta_{\rm ph}^{-1}\beta_{||0}\right)^2}{\gamma_0\beta_{||0}^3}\frac{J_{m-1}^2(\nu_n r_b/r_w)}{J_m^2(\nu_n)\left(\nu_n^2 - m^2\right)},$$

I_b is the current of a tubular beam with an injection radius r_b, r_w is the waveguide radius, ν_n is the nth root of the equation $dJ_m(x)/dx = 0$, and $J_m(x)$ is the Bessel function.

It should be noted that for development of microwave self-induced transparency, the fundamental factor is the nonisochronism of cyclotron oscillators, which is caused by the relativistic dependence of the gyrofrequency on the electron's energy [14–16]. However, under interaction with traveling waves, it can be partially compensated by the recoil effects arising in the process of wave radiation or absorption. For the phase velocity of the wave equal to the speed of light (autoresonance [17]), full compensation occurs, and the electrons behave like linear oscillators [18]. Thus, the considered M/W-SIT effects develop only outside the autoresonance regime, i.e., with the nonzero parameter μ. Actually, under the assumption of the low current density, the condition $\sqrt{Gb}/\mu \ll 1$ should be satisfied. It allows us to reduce Eqs. (3) to a simpler form:

$$\frac{\partial a}{\partial Z} - \frac{\partial a}{\partial \tau} = -p, \quad \frac{\partial p}{\partial Z} + ip|p|^2 = a, \tag{4}$$

where $p = \tilde{p}_+\mu^{1/2}G^{-1/4}$, $a = \tilde{A}\mu^{1/2}G^{-3/4}$, $\tau = \sqrt{G}\omega_r(t - z/V_{||0})(\beta_{||}^{-1} - \beta_{\rm gr}^{-1})^{-1}$, and $Z = \sqrt{G}\omega_r z/c$ are used as new independent variables. Here, we assume that the wave group velocity is larger than the translational velocity of electrons ($\beta_{\rm gr} > \beta_{||}$). This situation is typical for cyclotron interaction with copropagating nonrelativistic electron beams in waveguides and for the gyrofrequency exceeding the cutoff frequency of an operating waveguide mode. Note also that the above-mentioned anamalous Doppler effect [8,9] is described by equations similar to Eqs. (4), accurate to the sign in the right part of the motion equation.

For solution of the semibounded problem for propagation of an incident pulse, Eqs. (4) should be supplemented with boundary conditions at the input cross-section $Z = 0$ of the interaction space. According to the above explanation, we assume that the electron beam is injected into the system without the initial orbital velocity:

$$p|_{Z=0} = 0. \tag{5}$$

The incident pulse at the system's entrance is given in the form

$$a|_{Z=0} = a_{\text{in}} \sin^2(\pi\tau/T) e^{-i\delta\tau}, \tag{6}$$

where $\delta = \beta_{||0} G^{-1/2}(\omega_{\text{in}} - \omega_r)/\omega_{\text{in}}$ is the detuning of the pulse carrier frequency ω_{in} from the reference frequency ω_r; a_{in} and T are the amplitude and the duration of the pulse, respectively. Equations (4) with boundary conditions (5) and (6) are used in Sec. IV for simulations of stability of solitonlike solutions which are obtained in Sec. III for the case of an unbounded medium formed by a nonexcited electron beam.

To conclude this section, we note that equations similar to Eqs. (4) can be used (up to notation) for a description of different physical situations, including interaction with a relativistic electron beam moving in a nonmagnetized background plasma, or transverse (across a static magnetic field) propagation of an incident electromagnetic pulse through the magnetized cold plasma [5].

III. GENERALIZED SOLITONLIKE SOLUTION FOR MICROWAVE SELF-INDUCED TRANSPARENCY IN AN UNBOUNDED SYSTEM

As mentioned above, the simplest form of solitonlike solutions for microwave self-induced transparency was derived in [5], where some assumptions were used under which the amplitude and duration of the M/W-SIT soliton depended only on its velocity. In this section, we obtain a more general form of a soliton solution, in which the dependence on the detuning of the soliton carrier frequency from the reference one is taken into consideration.

Further, for convenience, we represent the complex field amplitude and the transverse momentum in the form $a(Z, \tau) = \hat{a}e^{i\Phi}$ and $p(Z, \tau) = \hat{p}e^{i\Psi}$, where $\hat{a} = |a(Z, \tau)|$, $\hat{p} = |p(Z, \tau)|$. Thus, Eqs. (4) can be rewritten for real and imaginary parts:

$$\frac{\partial \hat{a}}{\partial Z} - \frac{\partial \hat{a}}{\partial \tau} = -\hat{p}\cos\chi, \quad \frac{\partial \hat{p}}{\partial Z} = \hat{a}\cos\chi, \tag{7}$$

$$\frac{\partial \Phi}{\partial Z} - \frac{\partial \Phi}{\partial \tau} = \frac{\hat{p}}{\hat{a}}\sin\chi, \quad \frac{\partial \Psi}{\partial Z} = \frac{\hat{a}}{\hat{p}}\sin\chi - \hat{p}^2, \tag{8}$$

where $\chi = \Phi - \Psi$ is the phase difference. Then, we will search for the solutions of Eqs. (7) and (8) in the form of the stationary wave:

$$\hat{a} = \hat{a}(\xi), \quad \hat{p} = \hat{p}(\xi), \quad \chi = \chi(\xi), \tag{9}$$

assuming that the absolute values of the field amplitude $\hat{a}$ and the transverse momentum $\hat{p}$ as well as the phase difference χ depend only on the variable $\xi = Z + U\tau$, where U is the normalized soliton velocity. It is important to note that taking into account the condition $(\partial/\partial\tau - U\partial/\partial Z)\chi(\xi) = 0$, the phase terms Φ and Ψ depend not only on ξ but also on τ:

$$\Phi = \varphi(\xi) + \Omega\tau, \quad \Psi = \psi(\xi) + \Omega\tau, \tag{10}$$

where Ω is the frequency self-shift. In view of the indicated assumptions, we can represent Eqs. (7) and (8) as

$$\frac{d\hat{a}}{d\xi} = \frac{\hat{p}}{U-1}\cos\chi, \quad \frac{d\hat{p}}{d\xi} = \hat{a}\cos\chi, \tag{11}$$

$$\frac{d\chi}{d\xi} = -\sin\chi\left(\frac{1}{U-1}\frac{\hat{p}}{\hat{a}} + \frac{\hat{a}}{\hat{p}}\right) + \hat{p}^2 + \frac{\Omega}{U-1}. \tag{12}$$

Taking into account that for an unbounded medium we should set $\hat{a}, \hat{p}(\infty) = 0$, from Eq. (11) we have the integral of motion:

$$\hat{p} = s\hat{a}, \tag{13}$$

where $s = \sqrt{U-1}$ characterizes the difference between the soliton velocity and the unperturbed (i.e., without an electron beam) group velocity of radiation V_{gr} in a waveguide. It should be noted that this relation can be considered as a direct consequence of the energy conservation law. As a result, Eqs. (11) and (12) reduce to the following form:

$$\frac{d\hat{p}}{d\xi} = \frac{\hat{p}}{s}\cos\chi, \quad \frac{d\chi}{d\xi} = -\frac{2}{s}\sin\chi + \hat{p}^2 + \frac{\Omega}{s^2}. \tag{14}$$

Equations (14) have stationary states:

$$\hat{p}_0 = 0, \quad \sin\chi_0 = \frac{\Omega}{2s}, \tag{15}$$

where the second relation describes the equilibrium point of the saddle type (except for the case of $\cos\chi_0 = 0$). The soliton solution corresponds to the separatrix passing through the indicated points:

$$\hat{p}^2(\chi) = \frac{4}{s}(\sin\chi - \sin\chi_0). \tag{16}$$

Using relation (16), we can integrate Eqs. (14) for the phase difference χ, yielding

$$\chi(\xi) = 2\arctan\left(\frac{tg(\chi_0/2) + \exp[2s^{-1}\xi\cos\chi_0]}{1 + tg(\chi_0/2)\exp[2s^{-1}\xi\cos\chi_0]}\right). \tag{17}$$

A combination of relation (16) and solution (17) as well as the integral of motion (13) results in the solitonlike solution for the absolute value of the field amplitude:

$$\hat{a}(\xi) = \sqrt{\frac{4}{s^3}\left(\frac{1 - \sin^2\chi_0}{\sin\chi_0 + \cosh[2s^{-1}\xi\cos\chi_0]}\right)}. \tag{18}$$

Note that formula (18) reduces to the simplified form obtained in [5] for the special case $\sin\chi_0 = 0$:

$$\hat{a}(\xi) = 2s^{-3/2}[\text{sech}(2s^{-1}\xi)]^{1/2}. \tag{19}$$

We should emphasize that soliton solutions (18) and (19) have a meaning only for $U > 1$ when $s = \sqrt{U-1}$ is the real value. In coordinates (z, t), it corresponds to the fact that the soliton velocity V_s satisfies the inequality

$$V_{||} < V_s < V_{\mathrm{gr}}. \tag{20}$$

Thus, in waveguides, M/W-SIT solitons propagate faster than electrons but slower than the incident pulse in the absence of an electron beam.

For the normalized soliton energy, we get from solution (18)

$$W = \int_{-\infty}^{+\infty} \hat{a}^2(\tau')d\tau' = \frac{4}{s^2}\left(\frac{\pi}{2} - \chi_0\right),$$
$$\chi_0 = \arcsin\left(\frac{\Omega}{2s}\right) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right). \tag{21}$$

We also introduce the instantaneous self-shift of the soliton carrier frequency, which determines the frequency chirp:

$$\Omega_\Sigma(\xi) = \frac{\partial \Phi}{\partial \tau} = U\frac{d\varphi(\xi)}{d\xi} + \Omega$$
$$= \Omega\left(1 + \frac{U}{s^2}\right) - \frac{U}{s}\sin\left(\chi(\xi)\right). \tag{22}$$

According to relation (22), the instantaneous frequency self-shift varies over the soliton profile; it reaches its minimum value at the maximum of the amplitude (soliton center),

$$\Omega_\Sigma|_{\xi=0} = \Omega\left(1 + \frac{U}{s^2}\right) - \frac{U}{s}, \tag{23}$$

and it tends asymptotically to a constant at the soliton edges,

$$\Omega_\Sigma \xrightarrow{\xi\to\pm\infty} \Omega\left(1 + \frac{U}{2s^2}\right). \tag{24}$$

At the same time, for a chosen soliton velocity, formula (15) yields that a soliton exists in the following range of Ω:

$$-2s < \Omega < 2s. \tag{25}$$

Profiles of M/W-SIT solitons given by solution (18) and corresponding frequency chirps depend on two parameters: soliton velocity U and frequency self-shift Ω (Fig. 2). A decrease in the soliton velocity U leads to a decrease in the full width at half-maximum (FWHM) duration of the soliton while its peak amplitude and the energy W increase [see formula (21)]. The most intensive solitons with the shortest duration are realized for negative values of the frequency self-shift Ω, which correspond to the case when the soliton carrier frequency is lower than the reference frequency.

Figure 2 provides a simple interpretation of the M/W-SIT effect, which confirms its analogy with the optical case. The dependence of the transverse momentum $\hat{p}(\xi)$ shows that the transverse oscillations of electrons are excited by absorbing electromagnetic energy at the leading front of an incident pulse, and then electrons return to the nonexcited state by reradiating this energy to the field at the pulse trailing front. As a result, due to the reemission of the absorbed radiation, the EM pulse transforms to the soliton, which travels with anomalously low energy loss and unchanged shape. Note that an increase in the soliton amplitude and electron transverse momentum is replaced by their decrease as the phase difference $\chi(\xi)$ passes through the point $\pi/2$. Correspondingly, the faster the phase difference $\chi(\xi)$ changes, the faster the energy is transferred from the EM wave to the electrons in the leading and trailing edges of the M/W-SIT soliton.

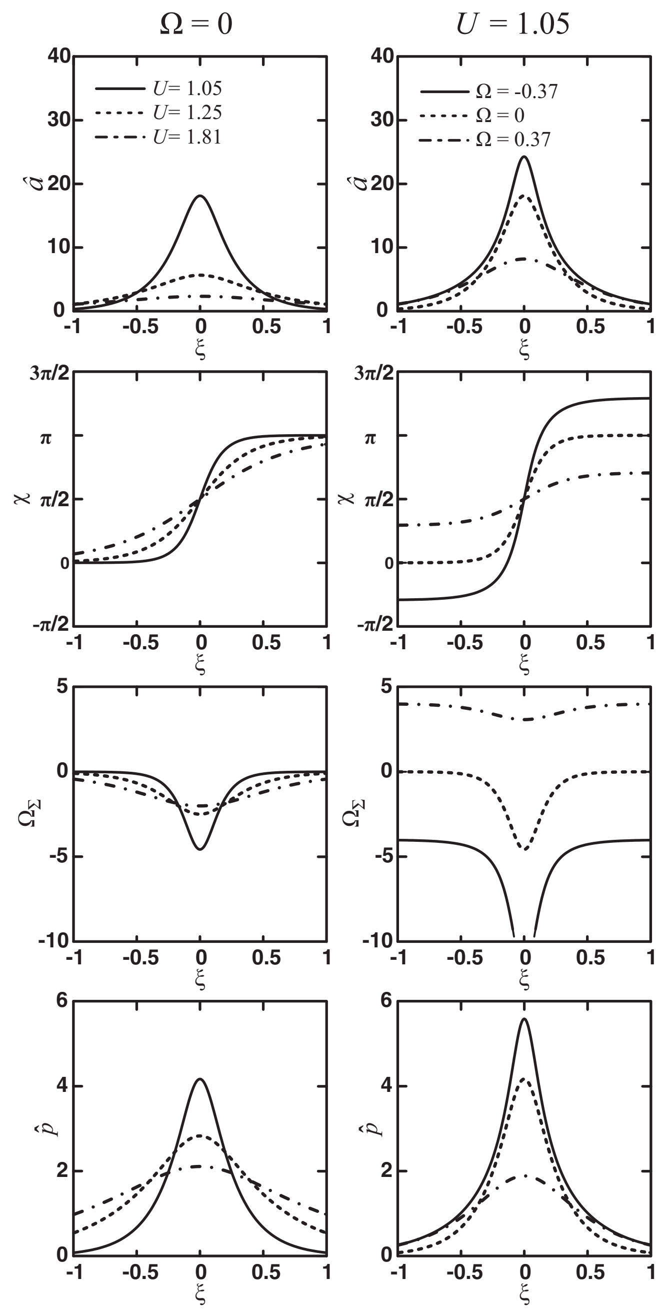

FIG. 2. Soliton profile $\hat{a}(\xi)$, phase difference $\chi(\xi)$, frequency chirp $\Omega_\Sigma(\xi)$, and amplitude of transverse momentum $\hat{p}(\xi)$ for different values of the soliton velocity U and soliton frequency self-shift parameter Ω.

IV. DEMONSTRATION OF THE STABILITY OF SOLITON SOLUTIONS BASED ON SIMULATIONS OF A SEMIBOUNDED PROBLEM

The results of numerical simulations of Eqs. (4) with the boundary conditions (5) and (6) presented in Figs. 3 and 4 confirm the analogy with self-induced transparency effects

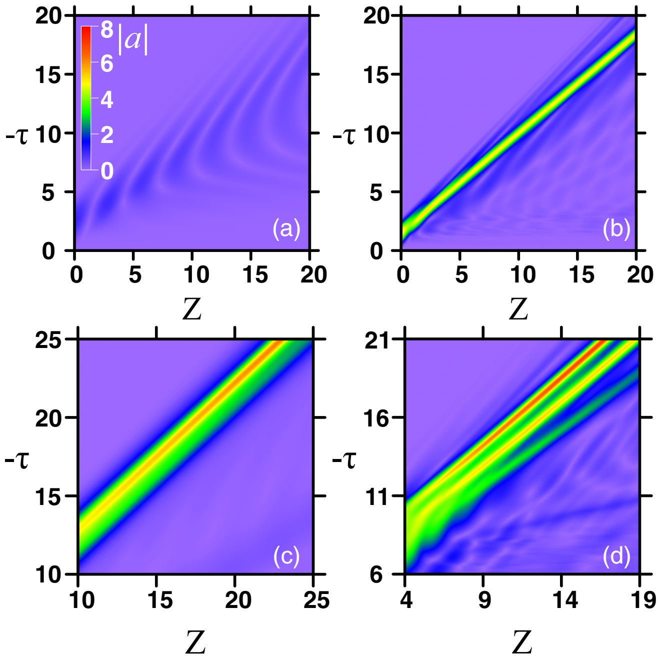

FIG. 3. Typical regimes of propagation of an electromagnetic pulse in an initially rectilinear electron beam under cyclotron resonance condition: (a) cyclotron resonance absorption pulse ($a_{\rm in} = 1$, $T = 5$, $\delta = 0$); (b) microwave self-induced transparency with the formation of a M/W-SIT soliton ($a_{\rm in} = 5$, $T = 3$, $\delta = 0$); (c) soliton formation with compression of the incident EM pulse ($a_{\rm in} = 5$, $T = 6$, $\delta = 3$); (d) decomposition of the incident pulse into several solitons ($a_{\rm in} = 5$, $T = 10$, $\delta = 0$).

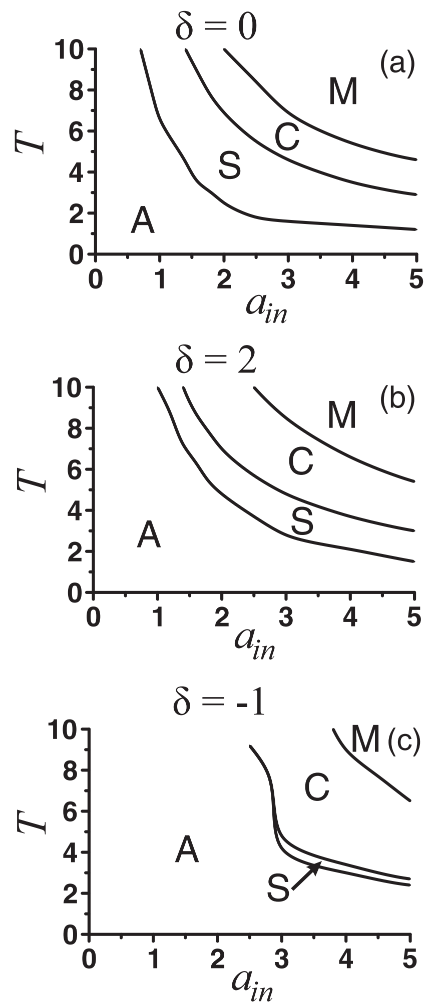

FIG. 4. Zones of different interaction regimes on the plane "amplitude $a_{\rm in}$ - duration T of the incident pulse" for different detuning parameters δ: *A*, cyclotron resonance absorption; *S*, self-induced transparency; *C*, compression of the incident pulse in the process of SIT soliton formation; *M*, multisoliton regime.

in optics. When the amplitude and duration of an incident electromagnetic pulse are small, the cyclotron absorption (A-zones in Fig. 4) is observed [Fig. 3(a)]. This process is accompanied by a quasiperiodic energy exchange between the electromagnetic pulse and electrons that is similar to optical Rabi oscillations of the population inversion [4]. As the energy of the initial microwave pulse increases, the self-induced transparency effect occurs when the incident pulse transforms into a M/W-SIT soliton (S-zones in Fig. 4), and after that it propagates practically without damping [Fig. 3(b)]. The amplitude of the formed M/W-SIT soliton can exceed the amplitude of the incident signal [Fig. 3(c)], i.e., the regime of nonlinear compression takes place (C-zones in Fig. 4). Rather long and high-power incident signals decompose into several solitons [Fig. 3(d)] with different amplitudes, durations, velocities, and frequency chirps (M-zones in Fig. 4).

In Fig. 4, zones of different interaction regimes are presented on the plane of the parameters "amplitude $a_{\rm in}$ - duration T" of the incident pulse depending on the parameter of the resonance detuning δ. For positive δ, when the carrier frequency of the incident pulse is higher than the reference frequency, the structure of interaction zones is similar to the case of the exact cyclotron resonance $\delta = 0$ [Fig. 4(a)]. However, with an increase in δ, the interaction zones shift towards a higher energy of the incident pulses [Fig. 4(b)]. For negative δ, compression of the input signal occurs practically in the entire zone of formation of single solitons [Fig. 4(c)].

Obviously, the analytical soliton solutions obtained in Sec. III are asymptotic. Nevertheless, these solutions can be approximately compared with the results of solving the semibounded problem (4)–(6) for a sufficiently large length of the interaction space (cyclotron absorber). Such a comparison was carried out for various parameters of the incident pulses entering at the input of the cyclotron absorber with a length of $L = 50$. To reconstruct the soliton shape from the results of numerical simulations, the peak values of the transverse momentum of electrons $p_{\rm max}$ and the soliton amplitude $a_{\rm max}$ were determined, which makes it possible to find the soliton velocity U based on the integral of motion (13):

$$U = 1 + (p_{\rm max}/a_{\rm max})^2. \tag{26}$$

Simultaneously, the frequency self-shift Ω is determined from Eq. (16) taking into account Eq. (15):

$$\Omega = s^2 p_{\rm max}^2/2 - 2s. \tag{27}$$

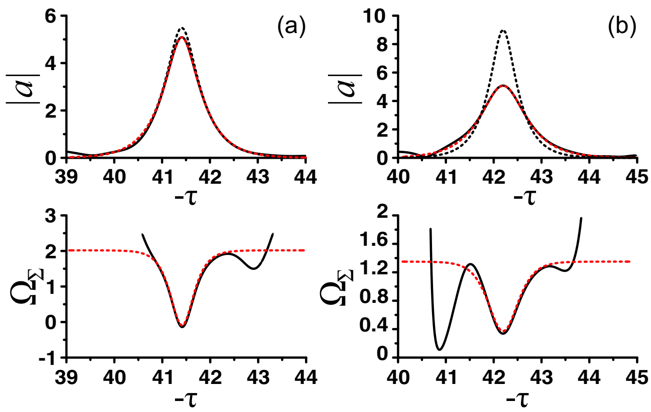

FIG. 5. Comparison of M/W-SIT solitons profiles (top) and frequency chirps (bottom) found in numerical simulations of the semiboundary problem (black solid curves) with the generalized analytical solutions (18) and (22) (red dashed curves). Black dashed curves correspond to the simplified soliton solutions found in [5] for a particular case $\chi_0 = 0$. Parameters of the input pulse in simulations: (a) $a_{\rm in} = 5$, $T = 3$, $\delta = -1$; (b) $a_{\rm in} = 5$, $T = 3$, $\delta = 1$.

Note that each pair of parameters U, Ω corresponds to a single analytical solution (18).

Field profiles and frequency chirps found based on numerical simulations of the semibounded problem are shown in Fig. 5. According to relations (26) and (27), for the initial pulse parameters $a_{\rm in} = 5$, $T = 3$, $\delta = -1$, the formed soliton has the normalized velocity $U = 1.26$ and the soliton frequency self-shift $\Omega = 0.15$ which corresponds to $\sin\chi_0 \approx 0$. Thus, the analytical solution (19) obtained previously in [5] [dotted line in Fig. 5(a)] is practically coincident with the numerical results. At the same time, for the initial pulse with $a_{\rm in} = 5$, $T = 3$, $\delta = 1$, parameters of the analytical solution $U = 1.13$ and $\Omega = 0.5$ correspond to $\sin\chi_0 \approx 0.7$. In this case, the numerically found soliton profile is also approximated well by formula (18), but it is significantly different from the profile given by the simplified solution (19).

V. CONCLUSION

We have shown that M/W-SIT solitons arising in cyclotron resonance interaction of an incident electromagnetic pulse with an initially rectilinear electron beam have a number of common features with optical SIT solitons and, at the same time, are governed by two parameters resembling the nonlinear Schrödinger solitons.

To conclude, we estimate the possibility of experimental observation of the formation of M/W-SIT solitons considering nonlinear transformation of the microwave superradiant (SR) pulse at the central frequency of 90 GHz with the following parameters [19]: a duration of 0.7 ns, a peak power of 160 MW, and a transverse structure of the TE_{11} mode of a cylindrical waveguide with a radius of 6 mm. Under the assumption that such a pulse interacts with an initially rectilinear near-axis electron beam with an energy of 100 keV and a current of 100 A guided by a magnetic field of $H_0 = 15$ kOe, the normalized parameters correspond to the case of soliton formation presented in Fig. 3(b). Thus, the initial SR pulse is transformed into a soliton over a length of the interaction region of 30 cm. An increase in the interaction length leads to compression of the formed M/W-SIT soliton. In particular, for an interaction length of 2 m, the formed soliton will have a peak power of 530 MW and a FWHM duration of 0.1 ns. Note that the length of the compression region can be significantly reduced (to 20–30 cm) in the case of interaction with a counterpropagating electron beam, when the input pulse enters from the collector end of the system. The soliton profile in this case is also described analytically by a formula similar to (18) up to the replacement $U \to -U$.

The data that support the findings of this study are available from the corresponding author upon reasonable request.

ACKNOWLEDGMENTS

This work was supported by Russian Science Foundation under Project No. 19-72-10119. The authors would like to thank N. M. Ryskin for useful discussion.

[1] S. L. McCall and E. L. Hahn, Self-induced transparency, Phys. Rev. **183**, 457 (1969).

[2] G. L. Lamb, Analytical descriptions of ultrashort optical pulse propagation in a resonant medium, Rev. Mod. Phys. **43**, 99 (1971).

[3] H. M. Gibbs and R. E. Slasher, Optical pulse compression by focusing in a resonant absorber, Appl. Phys. Lett. **18**, 505 (1971).

[4] L. Allen and J. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975).

[5] N. S. Ginzburg, A. S. Sergeev, and I. V. Zotova, Self-Induced Transparency and Electromagnetic Pulse Compression in a Plasma or an Electron Beam under Cyclotron Resonance Conditions, Phys. Rev. Lett. **105**, 265001 (2010).

[6] I. V. Zotova, N. S. Ginzburg, A. S. Sergeev, E. R. Kocharovskaya, and V. Yu. Zaslavsky, Conversion of Electromagnetic Wave Into a Periodic Train of Solitons Under Cyclotron Resonance Interaction with Backward Beam of Unexcited Oscillators, Phys. Rev. Lett. **113**, 143901 (2014).

[7] N. S. Ginzburg, I. V. Zotova, and A. S. Sergeev, Self-induced transparency, compression, and stopping of electromagnetic pulses interacting with beams of unexcited classical oscillators, JETP **113**, 772 (2011).

[8] N. S. Ginzburg, Nonlinear theory of electromagnetic wave generation and amplification based on the anomolous Doppler effect, Radiophys. Quantum Electron. **22**, 323 (1979).

[9] G. S. Nusinovich, M. Korol, and E. Jerby, Theory of the anomalous Doppler cyclotron-resonance-maser amplifier with tapered parameters, Phys. Rev. E **59**, 2311 (1999).

[10] V. E. Zakharov and A. B. Shabat, Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media, Zh. Eksp. Teor. Fiz. **61**, 118 (1971) [JETP **34**, 62 (1972)].

[11] G. P. Agrawal and Y. S. Kivshar, *Optical Solitons: From Fibers to Photonic Crystals* (Academic, New York, 2003).

[12] A. G. Alka, R. Gupta, C. N. Kumar, and T. S. Raju, Chirped femtosecond solitons and double-kink solitons in the cubic-quintic nonlinear Schrödinger equation with self-steepening and self-frequency shift, Phys. Rev. A **84**, 063830 (2011).

[13] H. Triki, K. Porsezian, A. Choudhuri, and P. T. Dinda, Chirped solitary pulses for a nonic nonlinear Schrödinger equation on a continuous-wave background, Phys. Rev. A **93**, 063810 (2016).

[14] A. V. Gaponov, M. I. Petelin, and V. K. Yulpatov, The induced radiation of excited classical oscillators and its use in high-frequency electronics, Radiophys. Quantum Electron. **10**, 794 (1967).

[15] G. S. Nusinovich, *Introduction to the Physics of Gyrotrons* (Johns Hopkins University Press, Baltimore, 2004).

[16] V. L. Bratman, N. S. Ginzburg, G. S. Nusinovich, M. I. Petelin, and P. S. Strelkov, Relativistic gyrotrons and cyclotron autoresonance masers, Int. J. Electron. **51**, 541 (1981).

[17] A. A. Kolomensky and A. N. Lebedev, Resonance effects associated with particle motion in a plane electromagnetic wave, J. Expt. Theor. Phys. (U.S.S.R.) **44**, 261 (1963) [JETP **17**, 179 (1963)].

[18] A. V. Gaponov, Instability of a system of excited oscillators with respect to electromagnetic perturbations, J. Expt. Theor. Phys. (U.S.S.R.) **39**, 326 (1960) [JETP **12**, 232 (1961)].

[19] N. S. Ginzburg, V. Yu. Zaslavsky, A. M. Malkin, A. S. Sergeev, I. V. Zotova, K. A. Sharypov, S. A. Shunailov, V. G. Shpak, M. R. Ul'masculov, and M. I. Yalandin, Generation of intense spatially coherent superradiant pulses in strongly oversized 2D periodical surface-wave structure, Appl. Phys. Lett. **117**, 183505 (2020).