Evidence for enhancement of anisotropy persistence in kinematic magnetohydrodynamic turbulent systems with finite-time correlations

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Using the field-theoretic renormalization group approach and the operator product expansion technique in the second order of the corresponding perturbative expansion, the influence of finite-time correlations of the turbulent velocity field on the scaling properties of the magnetic field correlation functions as well as on the anisotropy persistence deep inside the inertial range are investigated in the framework of the generalized Kazantsev-Kraichnan model of kinematic magnetohydrodynamic turbulence. Explicit two-loop expressions for the scaling exponents of the single-time two-point correlation functions of the magnetic field are derived and it is shown that the presence of the finite-time velocity correlations has a nontrivial impact on their inertial-range behavior and can lead, in general, to significantly more pronounced anomalous scaling of the magnetic field correlation functions in comparison to the rapid-change limit of the model, especially for the most interesting three-dimensional case. Moreover, by analyzing the asymptotic behavior of appropriate dimensionless ratios of the magnetic field correlation functions, it is also shown that the presence of finite-time correlations of the turbulent velocity field has a strong impact on the large-scale anisotropy persistence deep inside the inertial interval. Namely, it leads to a significant enhancement of the anisotropy persistence, again, especially in three spatial dimensions.

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I. INTRODUCTION

One of the most interesting consequences of the existence of anomalous scaling in developed turbulent environments [1–6], i.e., the existence of deviations from the simple scaling behavior predicted by the classical phenomenological Kolmogorov-Obukhov (KO) theory [7], is the evidence for the persistence of various types of symmetry breaking (such as large-scale anisotropy, which are usually related to the form of the energy pumping into strongly dissipative systems to maintain steady state) even deep inside the inertial interval, where, in accordance with the KO theory, one would expect full restoration of homogeneity and isotropy of fully developed turbulent systems in the statistical sense.

Using the field-theoretic renormalization group (RG) technique together with the operator product expansion (OPE) [8–12], significant successes were achieved especially in investigations of the anomalous scaling behavior of single-time two-point structures or correlation functions of scalar (temperature, concentration of impurities, etc.) or vector (e.g., weak magnetic) fields passively advected by given turbulent environments. In this respect, a systematic investigation of the influence of large- and small-scale uniaxial anisotropy [10,13–19], of spatial parity violation (helicity) [20], and of compressibility of various turbulent environments [21–23] on the scaling behavior of structure or correlation functions of scalar and vector fields was performed in the framework of the Gaussian rapid-change Kraichnan model of passively advected scalar fields [24] as well as in the framework of the analogous Kazantsev-Kraichnan model of kinematic magnetohydrodynamic (MHD) turbulence [25]. Here, it is worth mentioning that the anomalous scaling of various passively advected quantities even within simplified Gaussian models of developed turbulence is more strongly pronounced than the anomalous scaling of the turbulent velocity field in genuine models of fully developed turbulence (see, e.g., Refs. [4–6,13,14,26–42] and references cited therein).

However, very often the lowest first-order (one-loop) approximation is insufficient and at least second-order (two-loop) calculations are needed. For example, it is known that the anomalous dimensions of the leading composite operators that drive the anomalous scaling of the correlation functions of the magnetic field in the framework of the Kazantsev-Kraichnan model of kinematic MHD turbulence are completely the same as those in the Kraichnan model of passive scalar advection, although the set of the leading composite operators in this case is completely different [10,13]. But the two-loop calculations performed in Refs. [18,19] have shown that this equivalence is only an artifact of the one-loop level of approximation. Moreover, the same is also true when the corresponding non-Gaussian models driven by the stochastic Navier-Stokes equation are considered [43–45].

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A similar situation is true when some kinds of symmetry breaking are considered. For instance, if one wants to investigate the influence of spatial parity violation (the presence of helicity) on the properties of a turbulent system, then it is necessary to analyze it at least in the two-loop approximation since, due to the symmetry properties of the corresponding Feynman diagrams, the helicity effects are invisible at the one-loop level of approximation [20,23,46].

The same is true when one considers the finite-time correlations of the turbulent velocity field. In this case, in the one-loop approximation, the anomalous dimensions of the leading composite operators that drive the scaling behavior of the scalar and vector fields in the corresponding generalized Kraichnan and Kazantsev-Kraichnan models are completely the same and, moreover, do not depend at all on the parameters that control the presence of the finite-time velocity correlations in turbulent environments [40,47]. However, as recently shown in Ref. [48], where the two-loop corrections to the leading anomalous dimensions that drive the anomalous scaling in the framework of the generalized Kazantsev-Kraichnan model are calculated, these anomalous dimensions strongly depend on the presence of finite-time velocity correlations in the two-loop approximation and are significantly different in comparison to those obtained in the generalized Kraichnan model of passive scalar advection [49].

However, two important questions remain unanswered here. The first of them is related to the dependence of the critical exponents of the magnetic field correlation functions of the model on the parameter that control the presence of finite-time velocity correlations, i.e., how the presence of finite-time velocity correlations influences the inertial-range scaling behavior of the magnetic field in comparison to the rapid-change limit of the model with δ-time correlations of the turbulent velocity field. The second important but still open question is related to the problem of the influence of finite-time velocity correlations on the anisotropy persistence deep inside the inertial interval. We try to find answers to these two questions in the present paper. As we see, the strong dependence of the anomalous dimensions of the leading composite operators on the parameter that controls the amount of finite-time velocity correlations in the model also leads to the strong dependence of the critical exponents of the magnetic field correlation functions on this parameter, especially in the most interesting three-dimensional case. At the same time, the performed analysis shows that the anomalous scaling is, in general, more pronounced in the system with finite-time velocity correlations than in the rapid-change limit of the model. Even more interesting is the fact that the presence of finite-time velocity correlations in the model leads to a significant enhancement of the anisotropy persistence deep inside the inertial range.

From the phenomenological point of view, it is known that the synthetic Gaussian models of turbulent advection analyzed in the framework of the zero-mode technique allow one to establish a direct relation between the anomalous scaling and the statistical conservation laws (see, e.g., Ref. [29] as well as lectures by Gawedzki in Ref. [50]). Moreover, the existence of statistical conservation laws was also proven in Navier-Stokes turbulence [51]. These results allow investigation of not only the deviations from scaling predicted by KO theory but also the geometric structures formed in turbulent flows. Although, in what follows, our aim is to investigate the influence of finite-time correlations of the velocity field on the anomalous indices using the corresponding field-theoretic RG technique, nevertheless the obtained results also allow us to make some general conclusions about their influence on the formed geometric structures (regarding the connection between the language of the statistical conservation laws and the RG language, see Ref. [52]). First, the fact that the anomalous scaling of the magnetic field is stronger for finite-time velocity correlations means that one can also expect more pronounced intermittent geometric structures formed in the flow. Moreover, since the anisotropy persistence in the inertial range is supported by the finite-time correlations, one can also expect that the formed geometric structures will be more anisotropic in the statistical sense under the influence of finite-time correlations.

Note that although the passive advection of a weak magnetic field is considered in the framework of the Kazantsev-Karichnan model with Gaussian statistics of the velocity field, nevertheless, we suppose that the obtained results, at least at the qualitative level, correctly describe the scaling behavior of weak magnetic fields in electrically conductive turbulent environments, a complete analysis of which in the framework of the field-theoretic RG analysis of genuine MHD turbulence is still missing because of its extreme complexity. Therefore, we suppose that the obtained results not only are important for a fundamental theoretical understanding of the intermittency and anomalous scaling in turbulent systems with finite-time velocity correlations but can also be interesting from the phenomenological point of view for a potentially deeper understanding of plasma physics experiments (see, e.g., Refs. [53–60] and references cited therein) as well as MHD turbulent phenomena in astrophysics, e.g., such as the scaling properties of the solar wind (see, e.g., Refs. [61–72] as well as references cited therein).

For completeness, it is necessary to mention that the very presence of finite-time correlations of the velocity field violates the Galilean invariance of the studied model [26]. Therefore, the legitimate and natural question of the survival of finite-time correlations deep inside the inertial interval of various turbulent environments arises here. Although this question is beyond the scope of our analysis, it is worth mentioning that the first such investigation was performed recently in the framework of Navier-Stokes turbulence driven by a random force with finite-time correlations [73] in the lowest (one-loop) approximation. There, it was shown that, at the one-loop level of approximation of the corresponding field-theoretic model, it seems that finite-time correlations are completely suppressed in the inertial interval. However, this conclusion cannot be considered the ultimate one since higher-order corrections can change it.

The paper is organized as follows. In Sec. II, the model is defined, its field-theoretic formulation is given, and the already known basic facts are briefly discussed. In Sec. III, the influence of finite-time velocity correlations on the anomalous scaling of the magnetic field correlation functions is studied. In Sec. IV, the anisotropy persistence deep inside the inertial interval of the model is investigated. Obtained results are briefly reviewed and discussed in Sec. V.

II. SCALING IN THE KAZANTSEV-KRAICHNAN MODEL WITH FINITE-TIME CORRELATIONS

A. Kazantsev-Kraichnan model of kinematic magnetohydrodynamics with finite-time correlations of the velocity field

In this paper, we intend to study scaling properties of correlation functions of the magnetic field $\mathbf{b} \equiv \mathbf{b}(x)$, $x \equiv \{t, \mathbf{x}\}$ in the framework of the Kazantsev-Kraichnan model of kinematic MHD turbulence with the assumption of finite-time correlations of the turbulent velocity field and described by the following stochastic equation:

$$\partial_t \mathbf{b} = \nu_0 \Delta \mathbf{b} - (\mathbf{v}\cdot\partial)\mathbf{b} + (\mathbf{b}\cdot\partial)\mathbf{v} + \mathbf{f^b}. \tag{1}$$

Here, fluctuations of the magnetic field $\mathbf{b}$ are simulated by the Gaussian random noise $\mathbf{f^b} = \mathbf{f^b}(x)$ with zero mean and the correlation function in the form

$$D^b_{ij}(x_1;x_2) \equiv \langle f^b_i(x_1) f^b_j(x_2)\rangle = \delta(t_1 - t_2) C_{ij}(\mathbf{r}/L), \tag{2}$$

and the statistics of the turbulent velocity field $\mathbf{v}(x)$ of the electrically conductive environment is also supposed to be Gaussian with zero mean and with the correlator [40,47]

$$\begin{aligned} D^v_{ij}(x_1;x_2) &\equiv \langle v_i(x_1) v_j(x_2)\rangle \\ &= \int \frac{d\omega d\mathbf{k}}{(2\pi)^{d+1}} \frac{g_0 \nu_0^3 k^{4-d-2\varepsilon-\eta}}{\omega^2 + (u_0 \nu_0 k^{2-\eta})^2} \\ &\quad \times P_{ij}(\mathbf{k}) e^{-i[\omega(t_1-t_2) - \mathbf{k}\cdot(\mathbf{x_1}-\mathbf{x_2})]}, \end{aligned} \tag{3}$$

which simulates the presence of finite-time correlations of the velocity field.

In Eq. (1), $\partial_t \equiv \partial/\partial t$, $\partial_i \equiv \partial/\partial x_i$, $\Delta \equiv \partial^2$ denotes the Laplace operator, ν_0 is the magnetic diffusivity $\nu_0 = c^2/(4\pi\sigma_0)$, c is the speed of light, and σ_0 is the conductivity (in what follows, subscript 0 always denotes bare parameters of the unrenormalized theory). Moreover, due to the assumption of incompressibility, both vector fields $\mathbf{v}$ and $\mathbf{b}$ are divergent-free (solenoidal), i.e, $\partial\cdot\mathbf{v} = \partial\cdot\mathbf{b} = 0$. In Eq. (2), $\mathbf{r} = \mathbf{x_1} - \mathbf{x_2}$, L represents an integral scale related to the corresponding stirring, and C_{ij} is a tensor function finite in the limit $L \to \infty$ that must decrease rapidly for $|\mathbf{r}| \gg L$. Although, in what follows, the explicit form of the tensor function C_{ij} is not relevant, it is worth mentioning that large-scale anisotropy can be introduced to the system through the correlator, (2), e.g., by supposing that $\mathbf{f^b}$ has the form $(\mathbf{B}\cdot\partial)\mathbf{v}$, where $\mathbf{B}$ represents a constant large-scale (macroscopic) magnetic field (see, e.g., Ref. [13] for more details).

Finally, in Eq. (3), d represents the spatial dimension of the studied turbulent system, $P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j/k^2$ is the ordinary transverse projector, $\mathbf{k}$ is the wave vector (the momentum), $k = |\mathbf{k}|$, and $g_0 \nu_0^3$ is a positive amplitude factor, where the bare coupling constant g_0 (formally small parameter of the perturbation theory) is already extracted. The finite-time correlations of the velocity field are described by the parameter u_0 and by the additional exponent η in Eq. (3), which control the relation between the frequency ω and the wave number k in the form $\omega \simeq u_0 \nu_0 k^{2-\eta}$ [29,30,33,47,74,75]. At the same time, the second exponent ε controls the energy spectrum: $E(k) \sim k^{1-2\varepsilon}$. The value $\varepsilon = 4/3$ leads to the Kolmogorov "two-thirds law" for the spatial statistics of the velocity field or to the "five-thirds law" for the energy spectrum. Besides, $\eta = 4/3$ corresponds to the Kolmogorov frequency. In addition, the needed infrared (IR) regularization of the integral in Eq. (3) is reached by the cutoff from below $k = k_{\min} \equiv 1/L$, where L is an integral turbulent scale different, in general, from the scale L used in Eq. (2). However, this difference is not important in what follows.

Note that the model defined by Eqs. (1)–(3) is a generalization of the standard Kazantsev-Kraichnan rapid-change model [25] with δ-time correlations of the velocity field with the correlator in the form

$$\begin{aligned} D^v_{ij}(x_1;x_2) &= \delta(t_1 - t_2) g'_0 \nu_0 \\ &\quad \times \int \frac{d\mathbf{k}}{(2\pi)^d} P_{ij}(\mathbf{k}) k^{-d-2\varepsilon+\eta} e^{i\mathbf{k}\cdot(\mathbf{x_1}-\mathbf{x_2})}, \end{aligned} \tag{4}$$

obtained from (3) in the limit $u_0 \to \infty$ with $g'_0 \equiv g_0/u_0^2 =$ const.

On the other hand, in the limit $u_0 \to 0$ with $g''_0 \equiv g_0/u_0 =$ const., one obtains the second nontrivial special case of the general model, namely, the so-called quenched model with a time-independent (frozen) velocity field, with the velocity field correlator

$$D^v_{ij}(x_1;x_2) = \frac{g''_0 \nu_0^2}{2} \int \frac{d\mathbf{k}}{(2\pi)^d} P_{ij}(\mathbf{k}) k^{2-d-2\varepsilon} e^{i\mathbf{k}\cdot(\mathbf{x_1}-\mathbf{x_2})}. \tag{5}$$

B. Field-theoretic formulation and scaling regimes of the model

The stochastic model, (1)–(3), is equivalent to the corresponding field-theoretic model with an additional solenoidal auxiliary field $\mathbf{b}'$ and with the action functional (see, e.g., Ref. [40] for details)

$$\begin{aligned} S(\Phi) = &-\frac{1}{2}\int dx_1 dx_2\ v_i(x_1)\big[D^v_{ij}(x_1;x_2)\big]^{-1} v_j(x_2) \\ &+ \frac{1}{2}\int dx_1 dx_2\ b'_i(x_1) D^b_{ij}(x_1;x_2) b'_j(x_2) \\ &+ \int dx\ \mathbf{b}'\cdot[-\partial_t \mathbf{b} + \nu_0\Delta\mathbf{b} - (\mathbf{v}\cdot\partial)\mathbf{b} + (\mathbf{b}\cdot\partial)\mathbf{v}]. \end{aligned} \tag{6}$$

Here, $\Phi = \{\mathbf{v}, \mathbf{b}, \mathbf{b}'\}$, $dx = dt d^d\mathbf{x}$, and the correlators D^b_{ij} and D^v_{ij} are given in Eqs. (2) and (3), respectively. Moreover, the corresponding summations over all dummy indices are assumed.

As follows from the two-loop RG analysis of the field-theoretic model described by the action functional, (6), and performed in Ref. [48], the model is multiplicatively renormalizable and, depending on the values of the exponents ε and η, exhibits five IR stable fixed points that drive all possible asymptotic inertial-range scaling regimes of the model. Two of them are related to the rapid-change limit of the model—one trivial with zero fixed-point value of the coupling constant g' and the second one nontrivial with $g'_* > 0$ (note that, in what follows, the index "*" always denotes the fixed-point value of arbitrary quantity)—and two of them are related to the frozen limit of the model—again, one trivial with zero fixed-point value of the coupling constant g'' and the second one nontrivial with $g''_* > 0$.

The fifth fixed point, which is the most interesting one and plays a central role in what follows, describes scaling properties of the model with finite-time correlations of the velocity field with arbitrary finite fixed-point value of the parameter u. In this case, the fixed-point value of the coupling constant g depends explicitly on the chosen value of u_* and, in the two-loop approximation, is given as [48]

$$g_* \frac{S_d}{(2\pi)^d} = \frac{2du_*(1+u_*)}{d-1}\varepsilon + \frac{2du_*(d+u_*)}{(d-1)^2(d+2)(1+u_*)^2} \times {}_2F_1\left(1,1;2+\frac{d}{2};\frac{1}{(1+u_*)^2}\right)\varepsilon^2, \qquad (7)$$

where S_d denotes the surface area of the d-dimensional unit sphere

$$S_d \equiv \frac{2\pi^{d/2}}{\Gamma(d/2)} \qquad (8)$$

and ${}_2F_1(a_1, a_2; b; c)$ is the hypergeometric function. This fixed point is realized for $\varepsilon = \eta$ (a detailed analysis of its IR stability can be found in Ref. [48]).

C. Scaling behavior of the magnetic field correlation functions

The existence of an IR stable fixed point of the RG equations means that various correlation functions of the model have asymptotic scaling form deep inside the inertial range with well-defined critical exponents. In what follows, our aim is to investigate in detail the influence of the finite-time correlations of the velocity field on the IR scaling behavior of equal-time two-point correlation functions of the magnetic field defined as

$$B_{N-m,m}(r) \equiv \left\langle b_r^{N-m}(t,\mathbf{x}) b_r^m(t,\mathbf{x}')\right\rangle, \quad r = |\mathbf{x}-\mathbf{x}'|, \qquad (9)$$

where b_r denotes the component of the magnetic field $\mathbf{b}$ directed along the vector $\mathbf{r} = \mathbf{x} - \mathbf{x}'$ (see, e.g., Refs. [13,40]). It can be shown [13,40,48] that, using the RG analysis, their scaling behavior deep inside the inertial interval has the form

$$B_{N-m,m}(r) \simeq \nu_0^{-N/2}(r/l)^{-\gamma^*_{N-m}-\gamma^*_m} R_{N,m}(r/L), \qquad (10)$$

where γ^*_{N-m} and γ^*_m are the anomalous dimensions of the composite operators b_r^{N-m} and b_r^m (taken at the corresponding fixed-point values g_* and u_*) and the scaling functions $R_{N,m}(r/L)$ remain unknown in the framework of the standard RG analysis. However, their asymptotic behavior in the limit $r/L \to 0$ can be studied using the OPE technique [9].

In the framework of the OPE technique, the scaling functions $R_{N,m}(r/L)$ have the power form

$$R_{N,m}(r/L) = \sum_i C_{F_i}(r/L)(r/L)^{\Delta_{F_i}}, \quad r/L \to 0, \qquad (11)$$

where the summation is performed over all possible renormalized composite operators F_i allowed by the symmetry of the problem, Δ_{F_i} are their critical dimensions, and the corresponding coefficient functions $C_{F_i}(r/L)$ are regular in r/L. It is clear from representation (11) that scaling functions $R_{N,m}(r/L)$ can significantly change the IR asymptotic behavior of the correlation functions, (9), if there exist composite operators (usually called "dangerous") with negative critical dimensions that give singular contributions to the OPE, (11), in the limit $r/L \to 0$ (see, e.g., Ref. [13] for details). Moreover, if there exist more composite operators with negative critical dimensions, then the leading contribution to the expansion, (11), is given by the composite operators with the smallest critical dimensions. Note also that this behavior is commonly known as the anomalous scaling and is typical for fully developed turbulent systems.

In the fully isotropic case, i.e., in the case when even large-scale anisotropy is not present, the leading role in the scaling behavior of the magnetic field correlation functions is played by composite operators

$$F_N = (\mathbf{b}\cdot\mathbf{b})^{N/2}. \qquad (12)$$

Then the final asymptotic inertial-range behavior of the correlation functions, (9), has the form

$$B_{N-m,m}(r) \sim r^{-\gamma^*_{N-m}-\gamma^*_m+\gamma^*_N}, \qquad (13)$$

where γ^*_M for $M = N, m, N-m$ are the fixed-point values of the anomalous dimensions of the corresponding composite operators, (12).

On the other hand, in the more general case with the presence of uniaxial large-scale anisotropy given, e.g., by the unit vector $\mathbf{n} = \mathbf{B}/|\mathbf{B}|$, where $\mathbf{B}$ is the large-scale magnetic field discussed above, the leading role in the description of the scaling behavior of the model is played by the composite operators [13,17,19,48]

$$F_{N,p} = [\mathbf{n}\cdot\mathbf{b}]^p(\mathbf{b}\cdot\mathbf{b})^l, \quad N = 2l + p. \qquad (14)$$

Their fixed-point anomalous dimensions in the general case of the model with the presence of finite-time correlations of the velocity field were calculated in Ref. [48] up to the two-loop approximation and can be written as (we present their complete form here since they play the central role in our analysis)

$$\gamma^*_{N,p} = \gamma^{*(1)}_{N,p}\varepsilon + \gamma^{*(2)}_{N,p}\varepsilon^2 + O(\varepsilon^3), \qquad (15)$$

where

$$\gamma^{*(1)}_{N,p} = \frac{2N(N-1)-(N-p)(d+N+p-2)(d+1)}{2(d+2)(d-1)} \qquad (16)$$

represents the first-order approximation (the one-loop approximation) result and

$$\gamma^{*(2)}_{N,p} = -\frac{(d+u_*)[(d+1)k_1-2k_2]}{2(1+u_*)^3(d-1)^2(d+2)^2}\,{}_2F_1\left(1,1;2+\frac{d}{2};\frac{1}{(1+u_*)^2}\right) - \frac{2du_*^2}{(d+2)(d-1)^3}\frac{\Gamma\left(\frac{d}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)} \times \int_0^1 dx(1-x^2)^{\frac{d-1}{2}} x\left\{[(d+1)k_1-2k_2]X_1 + 2(dk_2-k_1)(x^2-1)X_2 + \frac{3}{d+4}[3((d+1)k_3-2k_4)X_3 + 2((d+2)k_4-3k_3)(x^2-1)X_4]\right\} \qquad (17)$$

is the second-order (the two-loop) contribution, where

$$k_1 = (N-p)(d+N+p-2), \tag{18}$$

$$k_2 = N(N-1), \tag{19}$$

$$k_3 = (N-2)(N-p)(d+N+p-2), \tag{20}$$

$$k_4 = N(N-1)(N-2), \tag{21}$$

$$X_1 = \frac{W_{11}(Y_1+Y_2)+W_{12}Y_3+W_{13}Y_4+W_{14}Y_5}{2u_*^3(u_*^2-1)[1+u_*^2+2u_*(2x^2-1)]}, \tag{22}$$

$$X_2 = \frac{W_{21}(Y_1+Y_2)+W_{22}Y_3+W_{23}Y_4+W_{24}Y_5}{2u_*^3(u_*-1)[1+u_*^2+2u_*(2x^2-1)]}, \tag{23}$$

$$X_3 = \frac{W_{31}(Y_1+Y_2)+W_{32}Y_3+W_{33}Y_6}{9u_*^2(u_*-1)^2}, \tag{24}$$

$$X_4 = \frac{-u_*(Y_1+Y_2)+Y_3+u_*^2Y_6}{3u_*^2(u_*-1)^2}, \tag{25}$$

$$W_{11} = 2(d-2)u_*^2(1+u_*)[2x^2(1+2u_*)+u_*^2-1], \tag{26}$$

$$W_{12} = (2+u_*)[2+5u_*+u_*^2-d(1+3u_*)] \times [1+u_*^2+2u_*(2x^2-1)], \tag{27}$$

$$W_{13} = 2(d-2)(1-u_*)(1+u_*)^2 \times [1+u_*^2+u_*(4x^2-1)], \tag{28}$$

$$W_{14} = 4(d-2)u_*^2(1+u_*)x, \tag{29}$$

$$W_{21} = 2u_*^2(1-u_*-x^2), \tag{30}$$

$$W_{22} = 1+u_*^2+2u_*(2x^2-1), \tag{31}$$

$$W_{23} = (u_*^2-1)[1+u_*^2+u_*(4x^2-1)], \tag{32}$$

$$W_{24} = -2u_*^2x, \tag{33}$$

$$W_{31} = u_*[-7-2u_*+d(4+u_*)-4x^2], \tag{34}$$

$$W_{32} = 5+4u_*-d(3+2u_*)+4x^2, \tag{35}$$

$$W_{33} = u_*^2(9-5d+4x^2), \tag{36}$$

and

$$Y_1 = \frac{\arctan\left(\frac{2-x}{\sqrt{2+2u_*-x^2}}\right)-\arctan\left(\frac{2+x}{\sqrt{2+2u_*-x^2}}\right)}{\sqrt{2+2u_*-x^2}}, \tag{37}$$

$$Y_2 = \frac{\arctan\left(\frac{1+u_*-x}{\sqrt{2+2u_*-x^2}}\right)-\arctan\left(\frac{1+u_*+x}{\sqrt{2+2u_*-x^2}}\right)}{\sqrt{2+2u_*-x^2}}, \tag{38}$$

$$Y_2 = \frac{\arctan\left(\frac{1+u_*-x}{\sqrt{(1+u_*)^2-x^2}}\right)-\arctan\left(\frac{1+u_*+x}{\sqrt{(1+u_*)^2-x^2}}\right)}{\sqrt{(1+u_*)^2-x^2}}, \tag{39}$$

$$Y_4 = \frac{\arctan\left(\frac{1-x}{\sqrt{1-x^2}}\right)-\arctan\left(\frac{1+x}{\sqrt{1-x^2}}\right)}{\sqrt{1-x^2}}, \tag{40}$$

$$Y_5 = \ln 2 - \ln(1+u_*), \tag{41}$$

$$Y_6 = \frac{\arctan\left(\frac{2-x}{\sqrt{4-x^2}}\right)-\arctan\left(\frac{2+x}{\sqrt{4-x^2}}\right)}{\sqrt{4-x^2}}. \tag{42}$$

As follows from Eqs. (15)–(42), in the anisotropic case, for a given value of N, there are several anomalous dimensions $\gamma_{N,p}$ for all possible values of p that can contribute to the final scaling expression for the magnetic field correlation functions. Of course, the leading one is that with the minimal fixed-point value. The analysis shows [48] that, at least up to the two-loop approximation, the following important hierarchy relations are valid among anomalous dimensions $\gamma_{N,p}^*$:

$$\gamma_{N,p}^* < \gamma_{N,p'}^*, \quad p < p', \tag{43}$$

$$\gamma_{N,0}^* < \gamma_{N',0}^*, \quad N > N', \tag{44}$$

$$\gamma_{N,1}^* < \gamma_{N',1}^*, \quad N > N', \tag{45}$$

where relation (44) holds for even values of N and N' and relation (45) is valid for odd values of N and N', respectively. This means that the scaling behavior of various statistical quantities deep inside the inertial interval must be driven by the anomalous dimensions $\gamma_{N,0}^*$ for even values of N and $\gamma_{N,1}^*$ for odd values of N, respectively. Note that this nontrivial fact is in accordance with Kolmogorov's local isotropy restoration hypothesis.

Note also that, as follows from Eq. (16), the scaling properties of the model do not depend at all on the presence of finite-time velocity correlations in the model in the one-loop approximation since the functions $\gamma_{N,p}^{*(1)}$ are independent of the parameter u_*. Thus, it is clear from this fact that it is necessary to investigate the model at least in the two-loop approximation to be able to study the influence of the finite-time velocity correlations on the scaling behavior of the model deep inside the inertial interval. We investigate this influence in detail in the next two sections.

III. INFLUENCE OF FINITE-TIME VELOCITY FIELD CORRELATIONS ON THE ANOMALOUS SCALING OF THE MAGNETIC FIELD CORRELATION FUNCTIONS

The existence of strict anisotropy hierarchy relations, (43)–(45), which are valid in the one-loop as well as in the two-loop approximation, leads to the definite prediction for the asymptotic inertial-range behavior of the correlation functions, (9), in the presence of large-scale uniaxial anisotropy. Its form depends on the values of N and m [13,17,19], namely,

$$B_{N-m,m}(r) \sim r^{\gamma_{N,0}^*-\gamma_{N-m,0}^*-\gamma_{m,0}^*} \tag{46}$$

for even values of N and m,

$$B_{N-m,m}(r) \sim r^{\gamma_{N,0}^*-\gamma_{N-m,1}^*-\gamma_{m,1}^*} \tag{47}$$

for an even value of N and an odd value of m, and

$$B_{N-m,m}(r) \sim r^{\gamma_{N,1}^*-\gamma_{N-m,0}^*-\gamma_{m,1}^*} \tag{48}$$

for odd values of N and m. The fourth possibility, with an odd value of N and an even value of m, is in fact contained in the last case.

Now, using the explicit expressions, (15)–(42), for the anomalous dimensions taken at the fixed point of the model, the final scaling asymptotic behavior of the correlation functions $B_{N-m,m}(r)$ in the two-loop approximation can be written in the general form

$$B_{N-m,m}(r) \sim r^{\zeta_{N,m}} = r^{\zeta^{(1)}_{N,m}\varepsilon + \zeta^{(2)}_{N,m}\varepsilon^2}, \tag{49}$$

where the well-known one-loop result $\zeta^{(1)}_{N,m}$ has the form [13,19]

$$\zeta^{(1)}_{N,m} = -\frac{m(N-m)}{d+2} \tag{50}$$

when N and m are simultaneously even or odd and

$$\zeta^{(1)}_{N,m} = -\frac{m(N-m)+d+1}{d+2} \tag{51}$$

for even values of N and odd values of m. On the other hand, the two-loop corrections $\zeta^{(2)}_{N,m}$ to the exponents $\zeta_{N,m}$ in Eq. (49) explicitly depend on the parameter u_*, which control the presence of finite-time velocity correlations and are given as

$$\zeta^{(2)}_{N,m} = -\frac{(d+u_*)D_1}{2(1+u_*)^3(d-1)(d+2)^2}\,{}_2F_1\left(1,1;2+\frac{d}{2};\frac{1}{(1+u_*)^2}\right) - \frac{2du_*^2}{(d+2)(d-1)^2}\frac{\Gamma\left(\frac{d}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)}$$
$$\times \int_0^1 dx(1-x^2)^{\frac{d-1}{2}} x\left\{D_1X_1 + D_2(x^2-1)X_2 + \frac{3}{d+4}[D_3X_3 + D_4(x^2-1)X_4]\right\}, \tag{52}$$

where functions X_i, $i = 1, \ldots, 4$, are defined in Eqs. (22)–(25) and

$$D_1 = 2m(N-m), \tag{53}$$

$$D_2 = 4m(N-m), \tag{54}$$

$$D_3 = 3m(N-m)(3N+2d-4), \tag{55}$$

$$D_4 = 6m(N-m)(N-4) \tag{56}$$

for even values of N and m,

$$D_1 = 2m(N-m), \tag{57}$$

$$D_2 = 4m(N-m), \tag{58}$$

$$D_3 = 3(N-m)[m(3N+2d-4)-d-1], \tag{59}$$

$$D_4 = 6(N-m)[m(N-4)+1] \tag{60}$$

for odd values of N and m, and

$$D_1 = 2[m(N-m)+d+1], \tag{61}$$

$$D_2 = 4[m(N-m)-1], \tag{62}$$

$$D_3 = 3[m(N-m)(3N+2d-4)+(N-4)(d+1)], \tag{63}$$

$$D_4 = 6(N-4)[m(N-m)-1] \tag{64}$$

for even N and odd m.

Thus, as follows from the explicit two-loop expressions for the exponents $\zeta_{N,m}$, the presence of the two-loop corrections leads to their dependence on the finite-time correlations of the velocity field (the dependence on the parameter u_* appears at the two-loop level of approximation) and, moreover, the form of the corresponding expressions for $\zeta_{N,m}$ for simultaneous even and odd values of N and m, which are the same in the one-loop approximation [see Eq. (50)], become different when two-loop corrections are taken into account [compare expressions (55) and (56) to the corresponding expressions (59) and (60)].

The explicit dependence of all scaling exponents $\zeta_{N,m}$ for $N = 2$, 3, 4, and 7 on the spatial dimension d and on the parameter u_* for the physically most important value $\varepsilon = 4/3$ in the two-loop approximation is shown in Figs. 1–7. As follows from all these figures, the amount of finite-time correlations of the turbulent velocity field significantly influences the scaling properties of the magnetic field correlation functions, i.e., significantly changes the corresponding scaling exponents, especially for spatial dimensions in the vicinity of the physically most relevant dimension $d = 3$. On the other hand, the role of the finite-time correlations rapidly decreases when d increases. It is important to stress once more here that all these scaling exponents are completely independent of the parameter u_* at the one-loop level of approximation. Therefore, at least two-loop calculations are required to be able to discuss this nontrivial question.

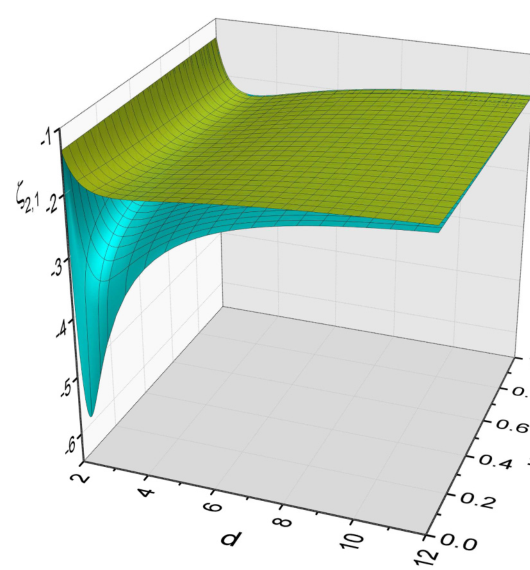

FIG. 1. Dependence of the scaling exponent $\zeta_{2,1}$ on the spatial dimension d as well as on the parameter u_* (the lower surface) and $w_* = 1/u_*$ (the upper surface) for $\varepsilon = 4/3$ in the two-loop approximation.

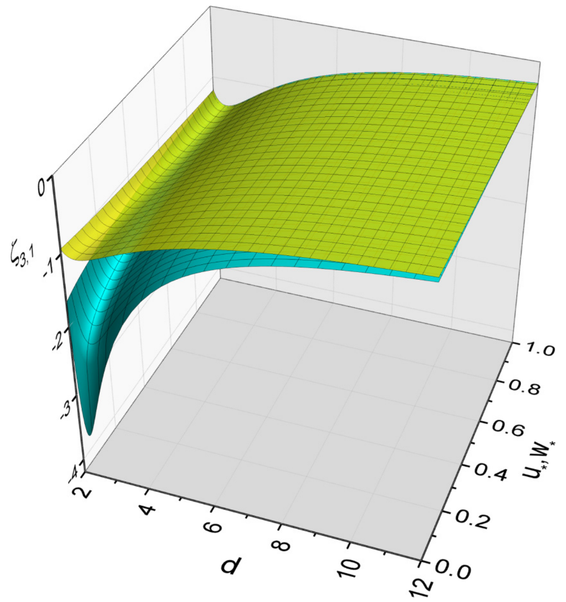

FIG. 2. Dependence of the scaling exponent $\zeta_{3,1}$ on the spatial dimension d as well as on the parameter u_* (the lower surface) and $w_* = 1/u_*$ (the upper surface) for $\varepsilon = 4/3$ in the two-loop approximation.

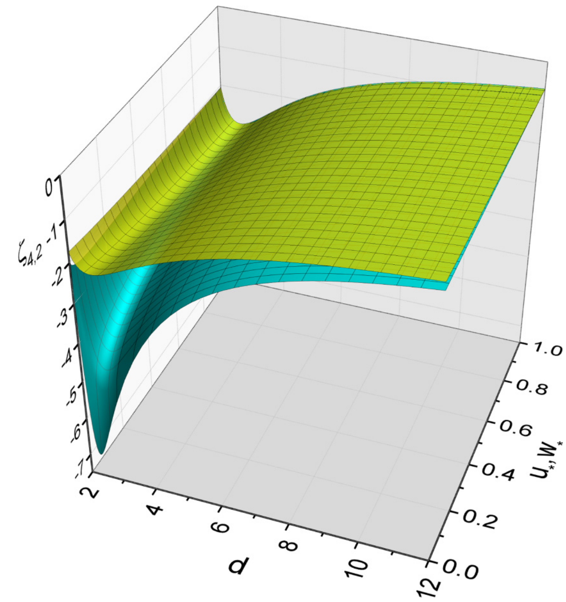

FIG. 4. Dependence of the scaling exponent $\zeta_{4,2}$ on the spatial dimension d as well as on the parameter u_* (the lower surface) and $w_* = 1/u_*$ (the upper surface) for $\varepsilon = 4/3$ in the two-loop approximation.

Moreover, looking at Figs. 1–7, the following general conclusions about the inertial-range scaling behavior of the single-time two-point correlation functions of the magnetic field, (9), can be drawn: (i) All scaling exponents $\zeta_{N,m}$ exhibit qualitatively very similar behavior as the functions of the spatial dimension d as well as the functions of the parameter u_*; (ii) in general, the scaling exponent values increase with increasing parameter u_* for all spatial dimensions $d \geqslant 3$, i.e., the most anomalous behavior of the magnetic field correlation functions is pronounced in the frozen limit of the model with $u_* \to 0$; (iii) the opposite behavior, in the framework of which the more anomalous behavior is observed in the rapid-change limit ($u_* \to \infty$ or $w_* \to 0$) than in the frozen limit of the model with $u_* \to 0$, can be observed only for some scaling exponents and only in the vicinity of the two-dimensional case (see Figs. 3, 6, and 7); and (iv) the scaling exponents $\zeta_{N,m}$ decrease as the functions of the spatial dimension d in the close vicinity of $d = 2$ but are increasing functions of d for

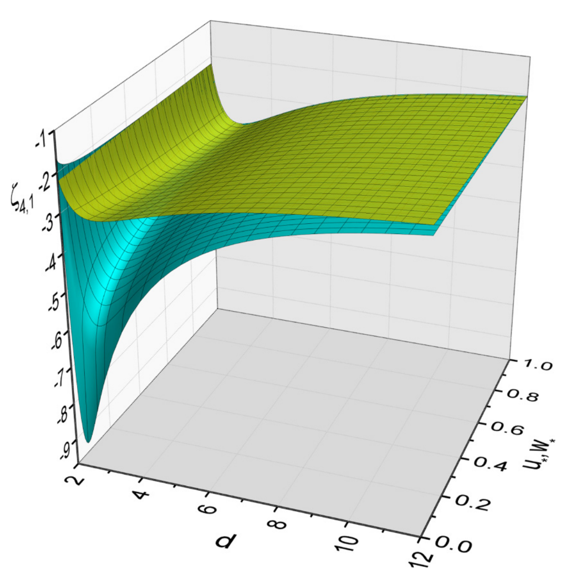

FIG. 3. Dependence of the scaling exponent $\zeta_{4,1}$ on the spatial dimension d as well as on the parameter u_* (the lower surface) and $w_* = 1/u_*$ (the upper surface) for $\varepsilon = 4/3$ in the two-loop approximation.

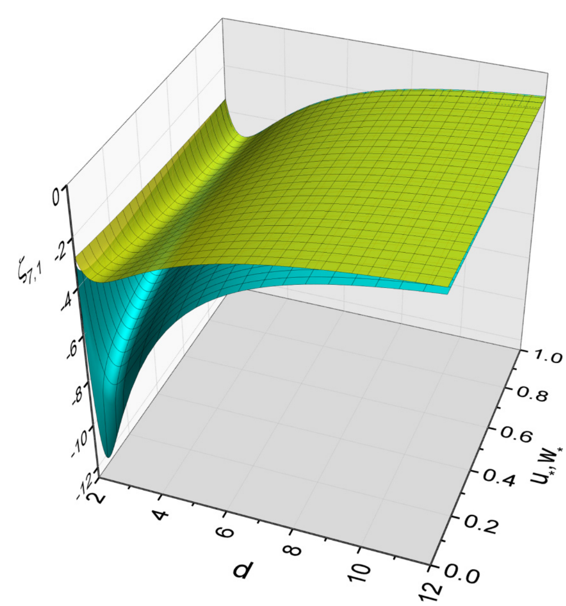

FIG. 5. Dependence of the scaling exponent $\zeta_{7,1}$ on the spatial dimension d as well as on the parameter u_* (the lower surface) and $w_* = 1/u_*$ (the upper surface) for $\varepsilon = 4/3$ in the two-loop approximation.

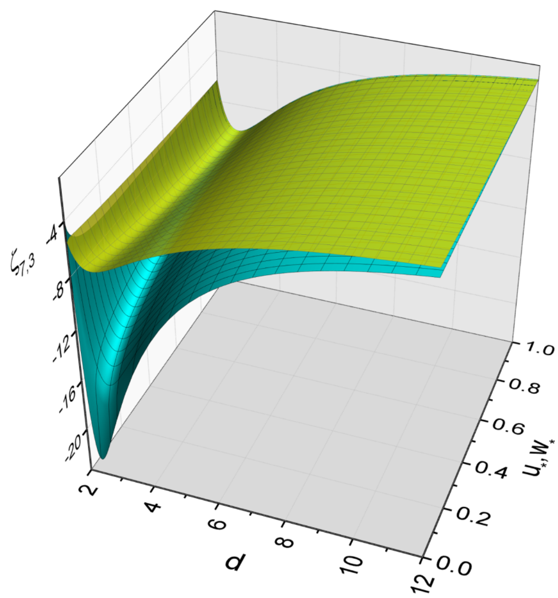

FIG. 6. Dependence of the scaling exponent $\zeta_{7,3}$ on the spatial dimension d as well as on the parameter u_* (the lower surface) and $w_* = 1/u_*$ (the upper surface) for $\varepsilon = 4/3$ in the two-loop approximation.

$d > 3$. The only exception to this rule can be observed for the exponent $\zeta_{2,1}$ in the vicinity of the rapid-change model limit (see Fig. 1).

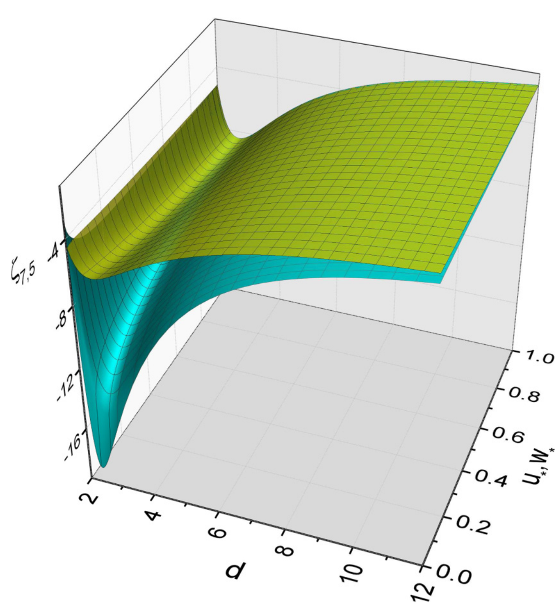

FIG. 7. Dependence of the scaling exponent $\zeta_{7,5}$ on the spatial dimension d as well as on the parameter u_* (the lower surface) and $w_* = 1/u_*$ (the upper surface) for $\varepsilon = 4/3$ in the two-loop approximation.

IV. INFLUENCE OF FINITE-TIME VELOCITY CORRELATIONS ON THE PERSISTENCE OF ANISOTROPY IN TURBULENT MHD SYSTEMS

As mentioned in Sec. I, the main aim of the present study is to investigate in detail the role of finite-time correlations of the turbulent velocity field for the persistence of large-scale anisotropy in the behavior of the magnetic field correlation functions deep inside the inertial interval. For this purpose, it is suitable to analyze the scaling behavior of dimensionless ratios of the single-time two-point correlation functions $B_{N-m,m}(r)$ studied in the previous section defined as follows:

$$\mathcal{R}_N \equiv \frac{B_{N-1,1}}{B_{1,1}^{N/2}} = \frac{\langle b_r^{N-1}(t,\mathbf{x}) b_r(t,\mathbf{x}')\rangle}{\langle b_r(t,\mathbf{x}) b_r(t,\mathbf{x}')\rangle^{N/2}}. \tag{65}$$

Then, using the asymptotic expressions for the corresponding correlation functions, (10), generalized to the anisotropic case together with the OPE representation of the scaling functions, (11), as well as using the anisotropy hierarchy relations, (43)–(45), for the fixed-point values of the anomalous dimensions of the leading composite operators of the model, the quantities $\mathcal{R}_N$ can be written as the explicit functions of the ratios r/l and r/L in the form [13,19]

$$\mathcal{R}_{2n} \propto \left(\frac{r}{l}\right)^{-\gamma^*_{2n-1,1}} \left(\frac{r}{L}\right)^{\gamma^*_{2n,0} - n\gamma^*_{2,0}} \tag{66}$$

for even values of $N = 2n$ and

$$\mathcal{R}_{2n+1} \propto \left(\frac{r}{l}\right)^{-\gamma^*_{2n,0}} \left(\frac{r}{L}\right)^{\gamma^*_{2n+1,1} - (n+1/2)\gamma^*_{2,0}} \tag{67}$$

for odd values of $N = 2n+1$, where various $\gamma^*_{x,y}$ are the fixed-point anomalous dimensions of the composite operators, (14), two-loop expressions of which are given in Eqs. (15)–(42).

A convenient method for quantitative estimation of the persistence of anisotropy deep inside the inertial interval is to investigate the dependence of functions (66) and (67) on the Pécklet number $\mathrm{Pe} \equiv (L/l)^{\varepsilon}$ obtained in the limit $r \to l$. In this case, one can write

$$\mathcal{R}_N \propto \mathrm{Pe}^{\xi_N}, \tag{68}$$

where

$$\xi_{2n} = [n\gamma^*_{2,0} - \gamma^*_{2n,0}]/\varepsilon, \tag{69}$$

$$\xi_{2n+1} = [(n+1/2)\gamma^*_{2,0} - \gamma^*_{2n+1,1}]/\varepsilon \tag{70}$$

for even and odd values of N, respectively. Finally, in the two-loop approximation, the exponents ξ_N can be written as

$$\xi_N = \xi_N^{(1)} + \xi_N^{(2)}\varepsilon + O(\varepsilon^2), \tag{71}$$

where the one-loop expressions are [13]

$$\xi_{2n}^{(1)} = \frac{2n(n-1)}{d+2}, \tag{72}$$

$$\xi_{2n+1}^{(1)} = \frac{4n^2-d-2}{2(d+2)}, \tag{73}$$

and the two-loop corrections are given as

$$\xi_N^{(2)} = \frac{(d+u_*)E_1}{2(1+u_*)^3(d-1)(d+2)^2}\,{}_2F_1\left(1,1;2+\frac{d}{2};\frac{1}{(1+u_*)^2}\right) + \frac{2du_*^2}{(d+2)(d-1)^2}\frac{\Gamma\left(\frac{d}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)} \times \int_0^1 dx(1-x^2)^{\frac{d-1}{2}}x\left\{E_1X_1+E_2(x^2-1)X_2+\frac{3}{d+4}\left[E_3X_3+E_4(x^2-1)X_4\right]\right\}, \tag{74}$$

where the functions X_i, $i=1,\dots,4$, are again defined in Eqs. (22)–(25) and

$$E_1 = 4n(n-1), \tag{75}$$

$$E_2 = 8n(n-1), \tag{76}$$

$$E_3 = 12n(n-1)(2n+d), \tag{77}$$

$$E_4 = 16n(n-1)(n-2) \tag{78}$$

for even values of $N=2n$ and

$$E_1 = 4n^2-d-2, \tag{79}$$

$$E_2 = 8n^2, \tag{80}$$

$$E_3 = 6n(2n-1)(2n+d+2), \tag{81}$$

$$E_4 = 8n(2n-1)(n-1) \tag{82}$$

for odd values of $N=2n+1$.

The explicit behavior of the two-loop exponents ξ_N up to $N=8$ as functions of the spatial dimension d and the parameters u_* and w_* is shown in Figs. 8–13 for $\varepsilon=4/3$. First, it is necessary to stress that these exponents do not depend at all on the parameter u_* (w_*) at the one-loop level of approximation [see Eqs. (72) and (73)], i.e., regardless of the value of u_* (w_*) these exponents are the same as in the Kazantsev-Kraichnan rapid-change model (the curves for $w_*=0$ in Figs. 8–13). This is a nontrivial fact that directly requires the two-loop analysis to be performed in order to be able to estimate, at least qualitatively, the influence of finite-time correlations of turbulent velocity fields on the studied statistical properties of correlation functions of the magnetic field deep inside the inertial interval.

As follows from all these figures, the presence of finite-time correlations of the velocity field almost always increases the value of the exponent ξ_N [the exception is the behavior of these exponents in the vicinity of two spatial dimensions, but only for large enough values of N (see Fig. 13)]. At the same time, the most pronounced enhancement of the values of the exponents ξ_N, i.e., the most pronounced enhancement of the anisotropy persistence in the inertial range, can be seen in the vicinity of the physically most interesting three-dimensional case. The only exception is observed for the smallest value of $N=3$, where the most pronounced persistence of the anisotropy in the inertial range is demonstrated in the two-dimensional case (see Fig. 8). The explicit behavior of

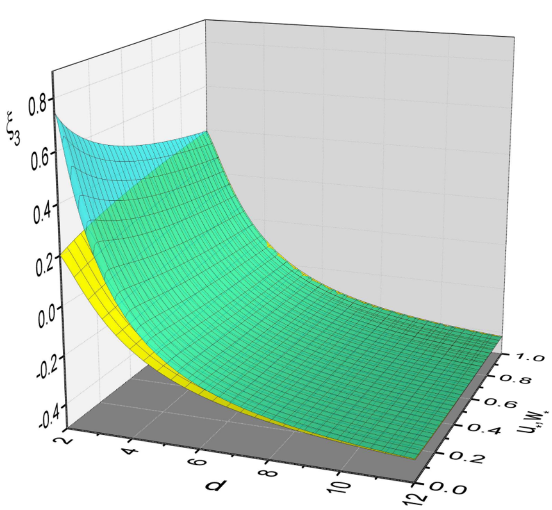

FIG. 8. Dependence of the total two-loop exponent ξ_3 on the spatial dimension d as well as on the parameter u_* (the upper surface) and $w_*=1/u_*$ (the lower surface) for $\varepsilon=4/3$.

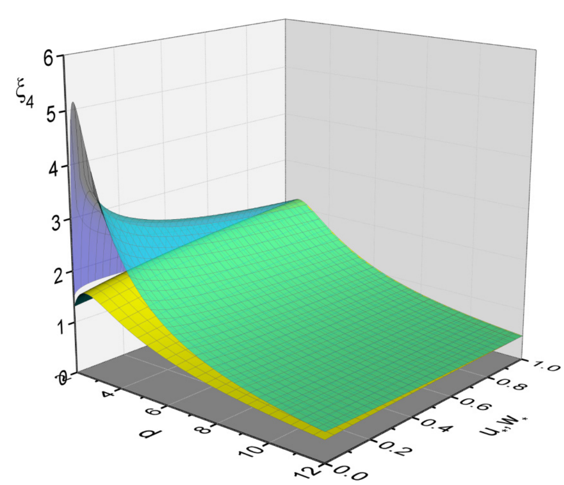

FIG. 9. Dependence of the total two-loop exponent ξ_4 on the spatial dimension d as well as on the parameter u_* (the upper surface) and $w_*=1/u_*$ (the lower surface) for $\varepsilon=4/3$.

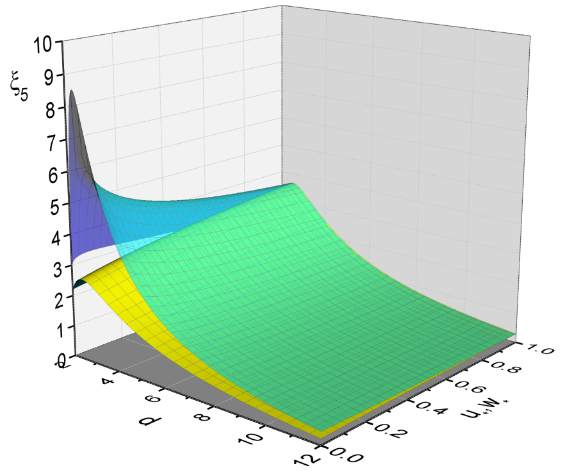

FIG. 10. Dependence of the total two-loop exponent ξ_5 on the spatial dimension d as well as on the parameter u_* (the upper surface) and $w_* = 1/u_*$ (the lower surface) for $\varepsilon = 4/3$.

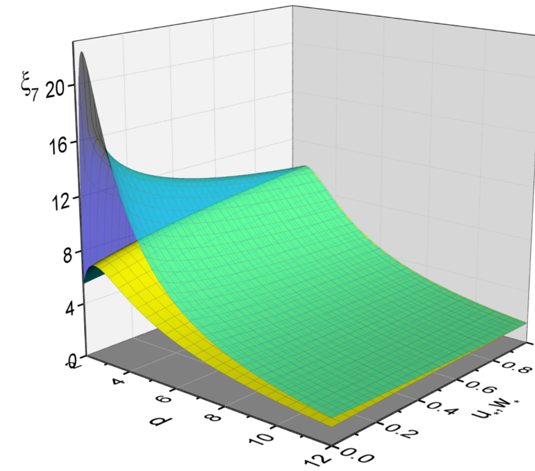

FIG. 12. Dependence of the total two-loop exponent ξ_7 on the spatial dimension d as well as on the parameter u_* (the upper surface) and $w_* = 1/u_*$ (the lower surface) for $\varepsilon = 4/3$.

the two-loop exponents ξ_N for $N = 3, \dots, 8$ directly for three spatial dimensions is shown in Figs. 14–16, where they are also compared to the corresponding one-loop results that do not depend on the parameter u_* (w_*). Here, it is also necessary to note that, from the persistence of the anisotropy point of view, the most important is the behavior of the exponents ξ_N for odd values of N since, as follows from symmetry considerations, they must be identically equal to 0 in the fully isotropic case (see Figs. 8, 10, and 12).

As already mentioned, with the exception of the case $N = 3$, the enhancement of the anisotropy persistence in the inertial interval caused by the presence of the finite-time correlations of the turbulent velocity field is most pronounced in the vicinity of $d = 3$. On the other hand, the influence of the finite-time correlations rapidly decreases with increasing value of the spatial dimension (see Figs. 8–13). At the same time, the dependence of the exponents ξ_N on the parameter u_* (w_*) disappears completely in the limit $d \to \infty$. Similarly, a significant suppression of the role of finite-time velocity correlations for the persistence of anisotropy deep inside the inertial interval can also be seen near two spatial dimensions for all $N \geqslant 4$. Moreover, it seems that for large enough values of N (see Fig. 13) the presence of finite-time correlations can even lead to less pronounced anisotropy persistence than in the case of the rapid-change model. Here, the behavior of the exponent ξ_3 is completely unique since, only in this case, the two-dimensional system demonstrates the most significant dependence on the parameter u_* (w_*) (see Fig. 8), i.e., the presence of finite-time correlations significantly increases the anisotropy persistence observed in the behavior of the function $\mathcal{R}_3$.

V. CONCLUSION

In this paper, using the field-theoretic RG analysis of the generalized Kazantsev-Kraichnan model in the second-order (two-loop) perturbative approximation, we have performed a detailed analysis of the influence of the finite-time

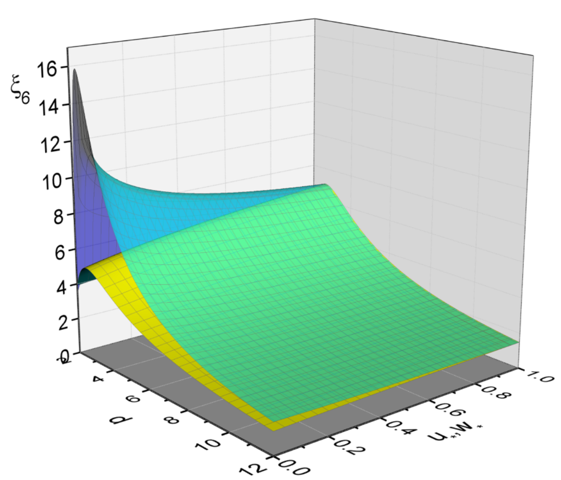

FIG. 11. Dependence of the total two-loop exponent ξ_6 on the spatial dimension d as well as on the parameter u_* (the upper surface) and $w_* = 1/u_*$ (the lower surface) for $\varepsilon = 4/3$.

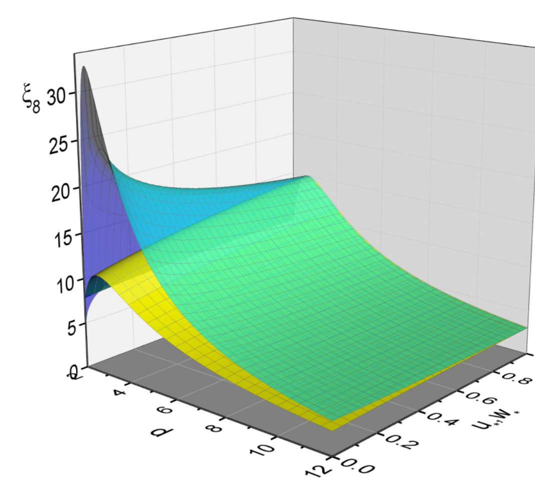

FIG. 13. Dependence of the total two-loop exponent ξ_8 on the spatial dimension d as well as on the parameter u_* (the upper surface) and $w_* = 1/u_*$ (the lower surface) for $\varepsilon = 4/3$.

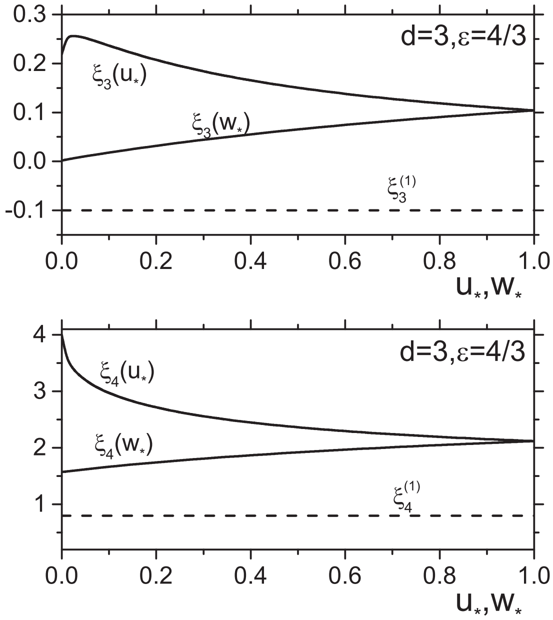

FIG. 14. Dependence of the total two-loop exponents ξ_3 and ξ_4 on the parameter u_* and $w_* = 1/u_*$ for $\varepsilon = 4/3$ and for the spatial dimension $d = 3$. The one-loop results are represented by the corresponding dashed lines.

correlations of the turbulent velocity field on the asymptotic scaling inertial-range behavior of the single-time two-point correlation functions $B_{N,N-m}(r)$ of the weak magnetic field defined in Eq. (9) as well as on the corresponding behavior of the dimensionless ratios of these correlation functions $\mathcal{R}_N$ defined in Eq. (65). It is shown for the first time that the

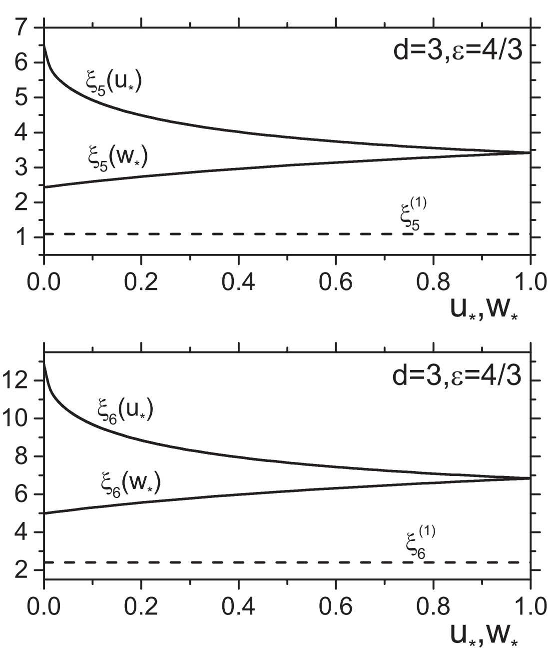

FIG. 15. Dependence of the total two-loop exponents ξ_5 and ξ_6 on the parameter u_* and $w_* = 1/u_*$ for $\varepsilon = 4/3$ and for the spatial dimension $d = 3$. The one-loop results are represented by the corresponding dashed lines.

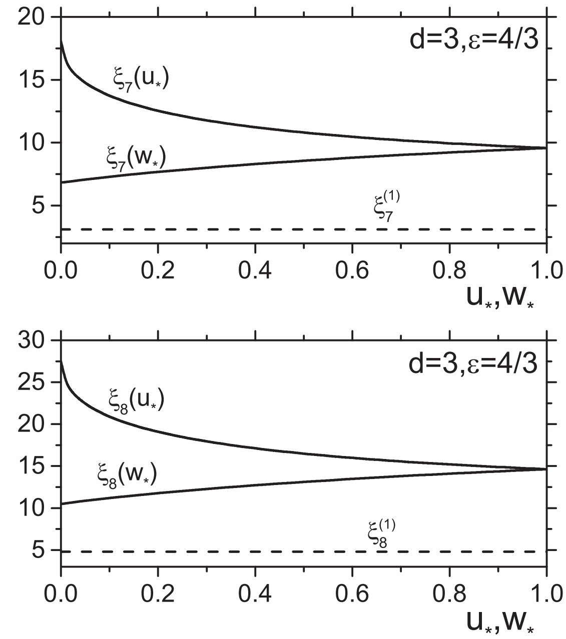

FIG. 16. Dependence of the total two-loop exponents ξ_7 and ξ_8 on the parameters u_* and $w_* = 1/u_*$ for $\varepsilon = 4/3$ and for the spatial dimension $d = 3$. The one-loop results are represented by the corresponding dashed lines.

finite-time correlations of the turbulent velocity fields can have a nontrivial and, at the same time, significant impact on the anomalous scaling of the magnetic field correlation functions as well as on the anisotropy persistence deep inside in the inertial range of developed turbulent environments. The two-loop explicit form of the corresponding critical exponents is found and it is demonstrated that the critical exponents of the magnetic field correlation functions become more negative when finite-time velocity correlations are present, i.e., the anomalous scaling becomes much more pronounced in the general case with finite-time velocity correlations than in the rapid-change limit of the studied model. On the other hand, it is also shown that the exponents, which characterize the behavior of the ratios $\mathcal{R}_N$ of the magnetic field correlation functions as functions of the Pécklet number, are increasing functions of the free parameter $w_* = 1/u_*$ of the model. This means that the persistence of the anisotropy is more pronounced in systems with finite-time correlations of the velocity field than in the rapid-change limit of the model with $w_* = 0$.

From the phenomenological point of view, the fact that the anomalous scaling of the magnetic field is more pronounced in the model with finite-time velocity correlations also means that the intermittency must be more visible in the formed geometric structures in this case than in the case of the rapid-change model. A similar conclusion must also be valid as for the anisotropy persistence in the inertial interval, namely, since the anisotropy persistence in the inertial range is enhanced by finite-time correlations, one can expect that the formed geometric structures will be more anisotropic in the statistical sense under the influence of finite-time correlations.

Let us emphasize that all studied effects of the presence of finite-time correlations of the velocity field in the model become visible starting from the performed two-loop analysis only since all studied quantities are independent of the parameter u_*, which directly describes the presence of finite-time velocity field correlations, at the one-loop level of approximation.

In conclusion, we suppose that a similar influence of the presence of finite-time correlations of the velocity field on the scaling properties of the magnetic field correlation functions must also be observed in more realistic but also more complex non-Gaussian models, e.g., in the framework of genuine kinematic MHD turbulence with the velocity field driven by the stochastic Navier-Stokes equation.

ACKNOWLEDGMENTS

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