Letter

Multiplicative noise can induce a velocity change of propagating dissipative solitons

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We investigate the influence of spatially homogeneous multiplicative noise on propagating dissipative solitons (DSs) of the cubic complex Ginzburg-Landau equation stabilized by nonlinear gradient terms. Here we focus on the nonlinear gradient terms, in particular on the influence of the Raman term and the delayed nonlinear gain. We show that a fairly small amount of multiplicative noise can lead to a change in the mean velocity for such systems. This effect is exclusively due to the presence of the stabilizing nonlinear gradient terms. For a range of parameters we find a velocity change proportional to the noise intensity for the Raman term and for delayed nonlinear gain. We note that the dissipative soliton decreases the modulus of its velocity when only one type of nonlinear gradient is present. We present a straightforward mean field analysis to capture this simple scaling law. At sufficiently high noise strength the nonlinear gradient stabilized DSs collapse.

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Stable localized solutions and their interactions in dissipative driven systems have become the focus of interest due to experimental investigations in pattern-forming systems such as binary fluid mixtures near convective onset [1–3], surface reactions [4–6], and nonlinear optics [7,8], including recent observations of exploding solitons [9] and bio-inspired systems [10]. All the stable localized solutions in these systems fulfill a balance between the driving force and dissipation and are therefore frequently called dissipative solitons [11].

One of the evolution equations which turned out to be very useful to model dissipative solitons is the cubic-quintic complex Ginzburg-Landau (CGL) equation, which is the envelope equation associated with a weakly inverted bifurcation to traveling waves [12]. Following the pioneering work by Thual and Fauve [13], dissipative solitons and their interaction associated with the cubic-quintic CGL equations have been investigated in detail (compare, for example, Refs. [14–16]). As for nonlinear gradient terms, they have been shown to change the speed of propagating dissipative solitons (DSs) deterministically [17,18].

A few years ago a new type of DS, dissipative solitons in the cubic CGL equation stabilized exclusively by nonlinear gradient terms, nonlinear gradient stabilized (NLGS) DSs, and in the absence of a stabilizing quintic term was elucidated by Facão and Carvalho [19]. This class of DSs was further investigated in Refs. [20–23]; it was shown that oscillatory DSs stably exist [20] and that three out of four nonlinear gradient terms familiar from nonlinear optics can generate such DSs [21,23]. In addition a mechanical model was proposed [22].

Noise, a phenomenon extensively studied in physics and chemistry [24] and, more recently, in biology [25], is ubiquitous in nature. While in most studies the effect of noise added to a deterministic equation is addressed, multiplicative noise for which the stochastic force multiplies a function of the stochastic variables has also attracted a considerable amount of attention over the years. First, this interest was driven by experiments on spatially homogeneous systems such as electronic circuits [26] and optical systems, namely, the dye laser [27]. This led to theoretical investigations of these zero-dimensional problems [28,29].

For spatially extended pattern-forming systems early experimental work focused on the effect of spatially homogeneous multiplicative noise on the onset of pattern formation in electroconvection in nematic liquid crystals. It was shown that the onset of spatial patterns could be postponed by a substantial amount by superposing noise on the driving voltage and that relaxation rates showed a strong linear dependence on the noise strength [30].

Noise effects on DSs have so far been studied for the cubic-quintic CGL equation. Weak additive noise can lead to the partial annihilation of counterpropagating pulses [31], a phenomenon that was observed before experimentally near the onset of binary fluid convection [3] and for surface reactions [4,5]. For single DSs weak noise was shown to induce explosions via various routes [32], while multiplicative noise can lead to the collapse of dissipative solitons [33].

Here we demonstrate that multiplicative noise can change the speed of this class of DSs, while no such effects are obtained for polynomial nonlinearities, for example, for the cubic-quintic CGL equation. For the Raman effect as well as for delayed nonlinear gain we find for the cubic CGL equation with these two nonlinear gradient terms a simple scaling law: the velocity change in the NLGS DSs is proportional to the intensity of the multiplicative noise. These results are also derived using a mean field model.

To motivate our equations we consider a wave packet moving inside a nonlinear fiber around a frequency ω_0 and wave number k_0. $W = \mathrm{Re}[\Psi(X, T)e^{ik_0x - i\omega_0 t}]$ is a wave moving to the right, where Ψ is the envelope modulating the wave

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packet, which depends on the slow space X and the slow time T.

Making a formal expansion around $k_0 : k - k_0 = (\frac{\partial k}{\partial \omega})(\omega - \omega_0) + \frac{1}{2}(\frac{\partial^2 k}{\partial \omega^2})(\omega - \omega_0)^2 + \cdots$, where $\frac{\partial k}{\partial \omega}$ evaluated at ω_0 corresponds to $1/v_g$ and $\frac{\partial^2 k}{\partial \omega^2} \equiv \beta_2$ is the group velocity dispersion ($\beta_2 < 0$, anomalous dispersion), and setting $k - k_0 = K$ and $\omega - \omega_0 = \Omega$, we obtain a dispersion relation for the envelope $\Psi : K = \frac{1}{v_g}\Omega + \frac{1}{2}\beta_2\,\Omega^2 + \cdots$. The above-mentioned dispersion relation for the envelope Ψ along with the Kerr effect, fiber losses ($\delta < 0$), nonlinear gain of energy $\epsilon > 0$, and the spectral filtering $\beta > 0$ results in the cubic CGL equation [left side of Eq. (1)].

For short pulses and a wide spectrum we should add to the above equation nonlinear gradient terms: self-steepening S_r and a delayed Raman response R_r [right side of Eq. (1)]:

$$i\left[\Psi_X + \frac{1}{v_g}\Psi_T\right] - \frac{1}{2}\beta_2\,\Psi_{TT} + |\Psi|^2\Psi - i\delta\Psi - i\epsilon|\Psi|^2\Psi$$
$$- i\beta\Psi_{TT} = R_r\Psi(|\Psi|^2)_T - iS_r(\Psi|\Psi|^2)_T. \tag{1}$$

To obtain spatially homogeneous multiplicative noise in an amplitude or envelope equation one inspects the physical origin of the prefactor of the term linear in the amplitude. For example, in the case of thermal convection in fluids this is the distance of the applied temperature difference from its critical value for convective onset, and for electroconvection in nematic liquid crystals it is the difference in the applied voltage compared to the critical voltage [30]. Then one can superpose spatially homogeneous noise on this quantity experimentally in a controlled way. One can apply this type of noise most easily to experimental systems showing spatiotemporal pattern formation. This applies to the two systems just mentioned as well as to concentration noise applied to the catalytic oxidation of CO under ultrahigh vacuum conditions [34]. In the case of nonlinear optics the analog was discussed, for example, in Refs. [27–29]. After eliminating adiabatically the fast variable in favor of the slowest variable, one obtains an amplitude equation containing a noise term multiplying the slow variable, which is the subharmonic wave amplitude for subharmonic generation and the Stokes field for Raman scattering [28]. Typically, in nonlinear optics the source of multiplicative noise is fluctuations of the pump field [27–29].

The stochastic cubic CGL equation with nonlinear gradient terms and spatially homogeneous multiplicative noise we investigate here reads, in the moving frame, $v_g + v_0$, in which the DSs are at rest,

$$\begin{aligned} A_t - v_0 A_x = {} & \mu A + (\beta_r + i\beta_i)|A|^2 A \\ & - i(R_r + iR_i)A\,(|A|^2)_x \\ & - (S_r + iS_i)\,(A|A|^2)_x \\ & + (D_r + iD_i)A_{xx} + A\eta\,\xi, \end{aligned} \tag{2}$$

where v_0 is the speed that the pulses acquire due to the nonlinear gradient terms [19–21]. $A(x,t)$ is a complex field, β_r is positive, R_r accounts for the delayed Raman response, and S_r accounts for self-steepening; R_i stands for a delayed nonlinear gain, and S_i stands for the dispersion of the nonlinear gain. For the case in which only the Raman effect is present we showed in Ref. [22] that $v_0 \sim 1/R_r$. The stochastic force $\xi(t)$ denotes white noise with the properties $\langle\xi\rangle = 0$, and $\langle\xi(t)\,\xi(t')\rangle = \delta(t - t')$. That means we consider multiplicative noise, which is real and homogeneous in space.

The parameter values we keep fixed in all runs are $\mu = -0.012$, $\beta_i = 1.0$, and $D_i = 0.5$. Positive values of D_i correspond to the regime of anomalous linear dispersion and are necessary to obtain stable NLGS DSs in the present case. We also note that the chosen value of μ is only weakly subcritical. The other parameters came in two groups. For studies of the Raman effect we used $R_r = 0.2$, $\beta_r = 0.3$, and $D_r = 0.6$, and for the investigations of the influence of the delayed nonlinear gain $R_i = 1.0$, $\beta_r = 0.2$, and $D_r = 0.3$.

We numerically solved Eq. (2) by implementing a fourth order Runge-Kutta algorithm, together with a pseudospectral split-step scheme. All derivatives were computed in Fourier space, while the nonlinear terms were solved in physical space.

The numerical code used $N = 625$ Fourier modes on a periodic domain of length $L = 50$; thus, our grid spacing is $dx = 0.08$. This number was enough to resolve even the smallest scales, developed from the solutions of Eq. (2). Finally, the applied time step was $dt = 0.005$, as this value ensured the numerical stability of our code.

All simulations were performed from an initial state, generated by a noiseless solution of Eq. (2) after a total time of 1250. This long simulation time was used to ensure a stationary solution as a starting point to study the stochastic behavior of Eq. (2). Posterior simulations with multiplicative noise were performed during a total time $T_{\mathrm{MAX}} = 1250$. This time allowed us to measure the effects of noise on the stationary state while keeping the total computation time relatively low, granting us the execution of several thousand numerical simulations.

To get a first qualitative overview of the phenomena triggered by spatially homogeneous multiplicative noise we show in Fig. 1 x-t plots of DS peaks generated by 100 realizations each for $\eta = 0.03$ for a box size $L = 50$ and a run time $T = 1000$. In Fig. 1(a) $R_r = 0.2$, $\beta_r = 0.3$, $D_r = 0.6$, and $v_0 = 0.904$; in Fig. 1(b) $R_i = 1.0$, $\beta_r = 0.2$, and $D_r = 0.3$. We note that for delayed nonlinear gain R_i the dispersion of the trajectories is significantly larger than for the Raman effect R_r.

In Fig. 2 we present x-t plots of $|A|$ for three values of the multiplicative noise strength η, $\eta = 0.01, 0.03$, and 0.05, for $R_r = 0.2$, $\beta_r = 0.3$, $D_r = 0.6$, and $v_0 = 0.904$. The box size is $L = 50$, and the run time plotted is $T = 350$. Inspecting Fig. 2, we see immediately that there is a noise-induced velocity change which grows more strongly than linear as a function of the noise strength η.

In the inset in Fig. 3 the time series for the area of the pulse, $I(t) = \int |A(x,t)|dx$, is plotted for noise strength $\eta = 0.03$, reflecting the influence of the multiplicative noise. In Fig. 3 we plot the Fourier spectrum for the area of the pulse for noise strength $\eta = 0.03$, clearly revealing a broad spectrum due to the effects of the multiplicative noise containing many Fourier components.

In the top panel of Fig. 4 we plot the velocity change $\Delta v'$ as a function of the noise strength η for 100 realizations for

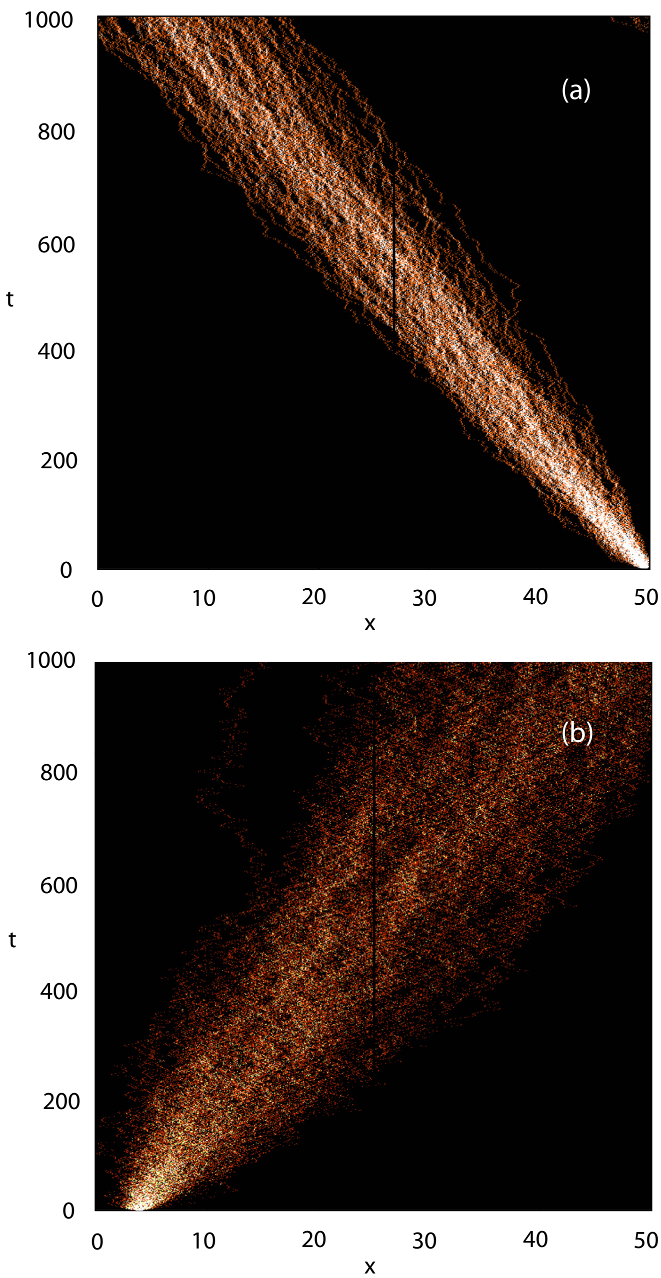

FIG. 1. x-t plots for the DS peaks of 100 realizations for $\eta = 0.03$, a box size $L = 50$, and a run time $T = 1000$. In (a) $R_r = 0.2$, $\beta_r = 0.3$, $D_r = 0.6$, and $v_0 = 0.904$, and in (b) $R_i = 1.0$, $\beta_r = 0.2$, and $D_r = 0.3$. The color indicates the peak of the pulse.

each value of η in the range $\eta = 0 \to \eta = 0.05$ for the Raman effect ($R_r = 0.2$). The black dots denote the average value. In the bottom panel of Fig. 4 the velocity change $|\Delta v'|$ is shown as a function of noise strength η for the case of delayed nonlinear gain, $R_i = 1.0$; the other parameter values are $\beta_r = 0.2$ and $D_r = 0.3$. As for the case of the Raman effect, we obtain an effective exponent γ with the value $\gamma = 2.068$ when an exponential fit is made using the black dots for the average value. For self-steepening ($S_r \neq 0$) we find no net effect on the effective velocity. This is due to the fact that only the boundary values enter when averaging deterministically in this case.

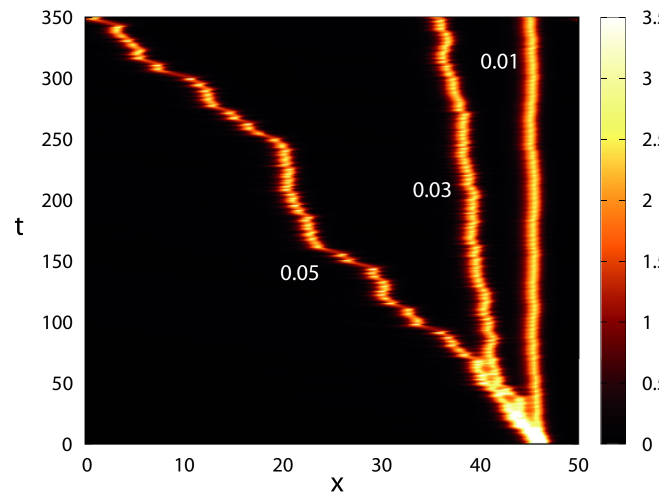

FIG. 2. x-t plots of $|A|$ for three values of the multiplicative noise strength η: $\eta = 0.01, 0.03$, and 0.05. The parameters are $R_r = 0.2$, $\beta_r = 0, 3$, $D_r = 0.6$, and $v_0 = 0.904$. The colors indicate the values of the modulus of the pulse.

Our mean field analysis proceeds as follows. The cubic CGL equation with delayed Raman response R_r reads

$$A_t = v_0 A_x + \mu A + (\beta_r + i\beta_i)|A|^2 A - iR_r(|A|^2)_x A + (D_r + iD_i)A_{xx}. \tag{3}$$

Introducing spatially homogeneous multiplicative noise into Eq. (3), we get

$$\tilde{A}_t = v_0 \tilde{A}_x + [\mu + \eta\,\xi(t)]\tilde{A} + (\beta_r + i\beta_i)|\tilde{A}|^2\tilde{A} - iR_r(|\tilde{A}|^2)_x\tilde{A} + (D_r + iD_i)\tilde{A}_{xx}. \tag{4}$$

The stochastic force $\xi(t)$ is a white noise obeying $\langle\xi\rangle = 0$ and $\langle\xi(t)\,\xi(t')\rangle = \delta(t - t')$.

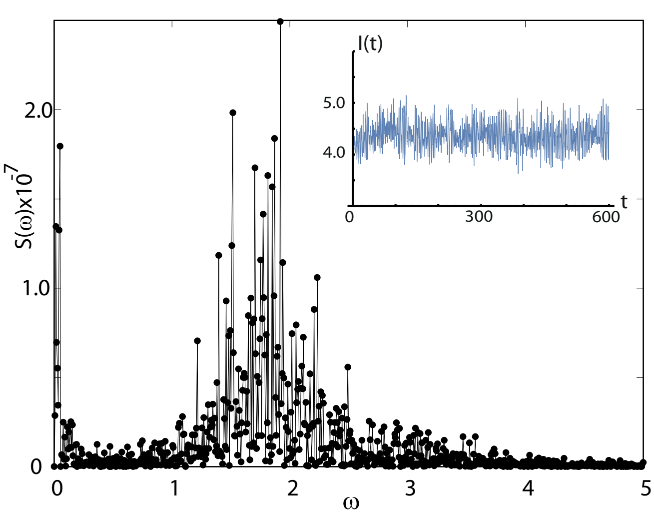

FIG. 3. Inset: The time series for the area of the pulse is plotted for $\eta = 0.03$. The Fourier spectrum for the area of the pulse is plotted as a function of the angular frequency ω for $\eta = 0.03$. The other parameters are $R_r = 0.2$, $\beta_r = 0.3$, and $D_r = 0.6$.

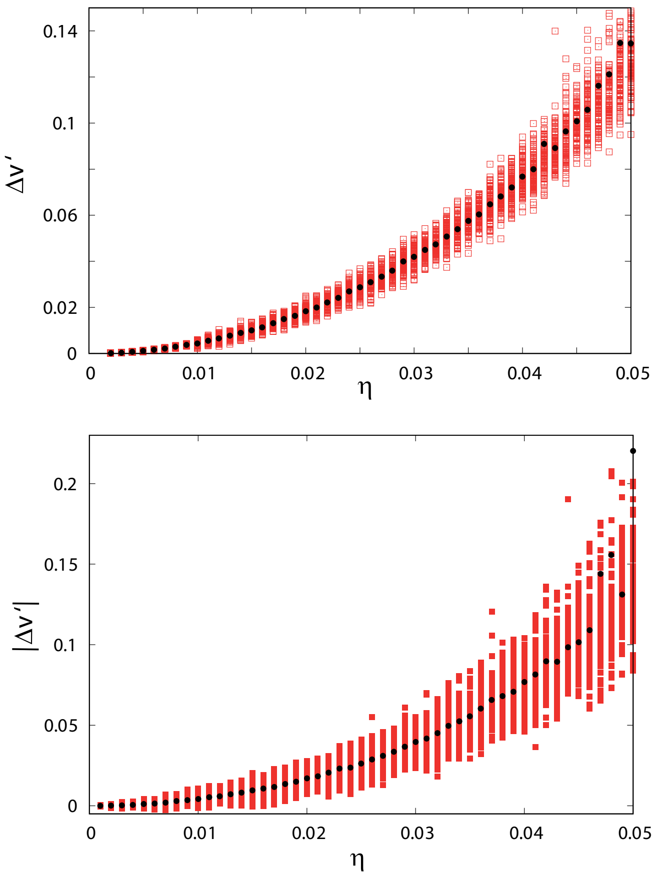

FIG. 4. The velocity change $\Delta v'$ is shown as a function of the noise strength η for 100 realizations for each value of η in the range $\eta = 0 \rightarrow \eta = 0.05$ for the Raman effect with parameters $R_r = 0.2$, $\beta_r = 0.3$, and $D_r = 0.6$ (top) and for delayed nonlinear gain with $R_i = 1.0$, $\beta_r = 0.2$, and $D_r = 0.3$ (bottom). The black dots denote the average value and give rise to exponent $\gamma = 2.066$ (for Raman) and $\gamma = 2.068$ (for delayed nonlinear gain) when they are fitted to $\langle|\Delta v'|\rangle \sim \eta^{\gamma}$.

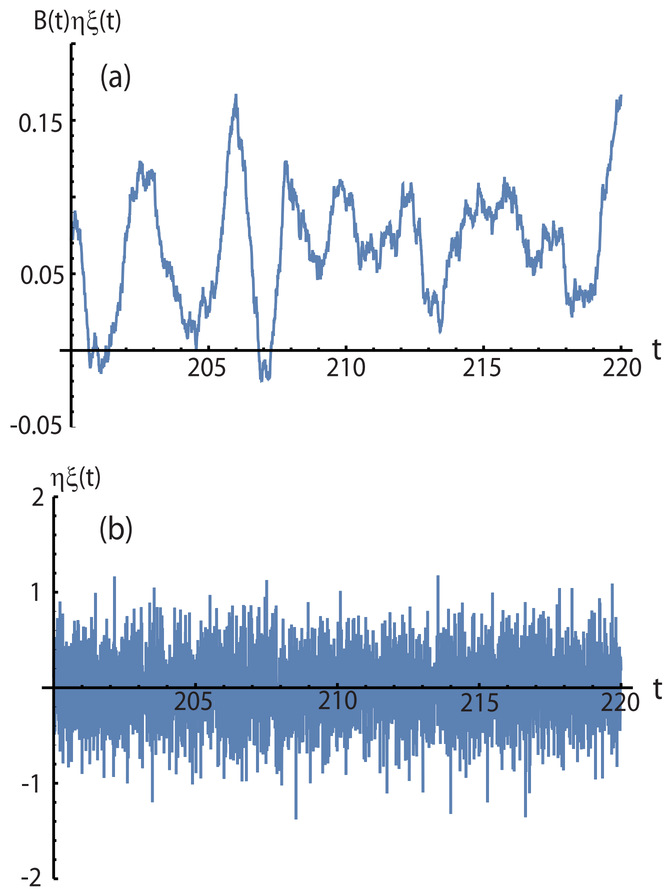

FIG. 5. The quantities (a) $B(t)\eta\xi(t)$ and (b) $\eta\xi(t)$ are plotted as a function of t in the asymptotic regime, demonstrating that the timescale of the noise is much shorter than the timescale associated with $B(t)$.

To study the dynamics of the resulting stochastic pulse we make the following approximate ansatz:

$$\tilde{A} = A(x + v't, t)[1 + B(t)\eta\,\xi(t)], \tag{5}$$

where v' is the small velocity of the stochastic pulse and $B(t)$ is a function varying slowly in time. To check the validity of this ansatz, we plot in Fig. 5(a) $B(t)\eta\xi(t)$ and in Fig. 5(b) $\eta\xi(t)$: the timescale of the noise is much shorter than that of $B(t)$. From Eq. (5) we obtain $B(t)\eta\,\xi(t) = (\int \tilde{A}\,dx)/(\int A\,dx) - 1$. In addition we checked numerically the $\langle B^2\eta^2\xi^2\rangle \rightarrow$ const and that the average size of the stochastic pulses $\langle \int \tilde{A}\,dx\rangle$ increases linearly with the noise strength η.

We introduce the following change in variables in order to study the pulse in its moving frame: $X = x + v't$, $T = t$. Then Eq. (4) takes the form

$$\begin{aligned} A_T = (v_0 - v')A_X + \left(\mu + \eta\,\xi(t) - \frac{[B\,\eta\,\xi(t)]_T}{[1 + B\eta\,\xi(t)]}\right)A \\ + (\beta_r + i\beta_i)|A|^2 A\,[1 + B\,\eta\,\xi(t)]^2 \\ - iR_r(|A|^2)_X A\,[1 + B\,\eta\,\xi(t)]^2 + (D_r + iD_i)A_{XX}. \end{aligned} \tag{6}$$

However, what we solve numerically is the discrete equation. Thus, $A \rightarrow A_i$, and $\eta\,\xi(t) \rightarrow \eta\,\zeta/\sqrt{dt}$, where ζ is a random number obeying a distribution $N(0, 1)$. In a mean field spirit we seek an "average equation" containing an "average velocity." Therefore, we have $\langle(1 + \frac{B\,\eta\,\zeta}{\sqrt{dt}})^2\rangle \rightarrow 1 + \frac{\langle B^2\rangle\eta^2}{dt}$.

By comparing Eqs. (3) and (6) we see that $R_r \rightarrow R_r(1 + \frac{\langle B^2\rangle\eta^2}{dt})$. Then $v_0 - \langle v'\rangle \sim \frac{1}{R_r}(1 + \frac{\langle B^2\rangle\eta^2}{dt})^{-1} \sim \frac{1}{R_r}(1 - \frac{\langle B^2\rangle\eta^2}{dt} + \cdots)$, and we obtain $\langle v'\rangle = v_0(\frac{\langle B^2\rangle\eta^2}{dt} + \cdots) > 0$ because $v_0 > 0$ since $v_0 \sim \frac{1}{R_r}$. Thus, for $R_r > 0$ and $R_i = 0$ we see that the homogeneous multiplicative noise decreases the velocity of the pulses, as shown in Fig. 1(a). The variance increases with noise strength, as the distributions in Fig. 6 show. Each histogram was generated using 500 realizations.

For $R_r = 0$ and $R_i > 0$ we replace $-iR_r$ by R_i in Eq. (4). The following procedure is analogous, and we get $R_i \rightarrow R_i(1 + \frac{\langle B^2\rangle\eta^2}{dt})$ and obtain the same scaling law $\langle v'\rangle = v_0(\frac{\langle B^2\rangle\eta^2}{dt} + \cdots) < 0$ because $v_0 < 0$ since $v_0 \sim -\frac{1}{R_i}$. Thus, for $R_r = 0$ and $R_i > 0$ we see that the homogeneous multiplicative noise also decreases the modulus of the velocity of the pulses, as shown in Fig. 1(b).

In conclusion, we have investigated the influence of spatially homogeneous multiplicative noise on DSs in the cubic CGL equation stabilized by nonlinear gradient terms. The

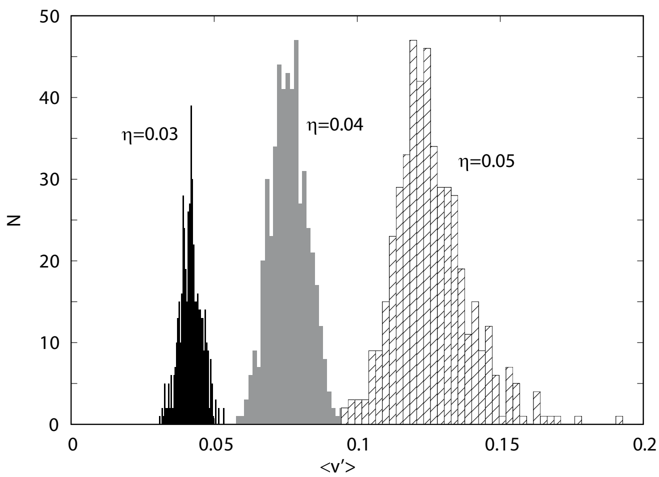

FIG. 6. Histograms showing the variance as a function of $\langle v' \rangle$ for three noise strengths: $\eta = 0.03$, $\eta = 0.04$, and $\eta = 0.05$. We clearly see that the variance increases as a function of noise strength.

main result in this Letter is that the dissipative soliton due to homogeneous multiplicative noise decreases the modulus of its velocity when only one type of nonlinear gradient is present. We demonstrated that this velocity change is proportional to the noise intensity of the multiplicative noise for both parameters: the Raman effect and delayed nonlinear gain. To elucidate this scaling law we presented a simple mean field analysis.

Comparing this result with the DSs familiar from the cubic-quintic CGL equation without nonlinear gradient terms, we note that such a velocity shift does not arise in this case, showing that nonlinear gradient terms play a key role in this effect that is unknown from other nonlinear evolution equations, allowing for the stable existence of DSs. We note that for large enough noise strength of the multiplicative noise the DSs collapse. The present study opens the door for several other areas of investigation. Perhaps the most clear-cut candidate to study the effects predicted here experimentally is from nonlinear optics, where, frequently, Raman-type effects and delayed nonlinear gain play an important role. Candidates for other experimental systems are easily controllable chemical reactions for which solitonlike structures and their collisions have been observed [4,5]. This expectation is based on two features: (a) for nearly all weakly inverted bifurcations to traveling waves in these systems nonlinear gradient terms in the associated envelope equations [12] arise, and (b) how to superpose multiplicative noise on surface reactions experimentally has been shown [35].

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