Mechanism of subcritical avalanche propagation in three-dimensional disordered systems

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(Received 15 April 2021; accepted 25 May 2021; published 14 June 2021)

We present a numerical study on necessary conditions for the appearance of infinite avalanche below the critical point in disordered systems that evolve throughout metastable states. The representative of those systems is the nonequilibrium athermal random-field Ising model. We investigate the impact on propagation of infinite avalanche of both the interface of flipped spins at the avalanche's starting point and the number of independent islands of flipped spins in the system at the moment when the avalanche starts. To deduce what effects are originated due to finite system's size, and to distinguish them from the real necessary conditions for the appearance of the infinite avalanche, we examined lattices of different sizes as well as other key parameters for the avalanche propagation.

DOI: 10.1103/PhysRevE.103.062123

I. INTRODUCTION

Disordered systems that evolve through avalanchelike events are widely present all around us. In the vast variety of occasions, starting from social phenomena (e.g., financial market irregularities [1,2]) to the natural or experimentally induced phenomena (such as neuronal activities in the brain [3–6], earthquakes [7,8], response of the mechanically pressured wooden materials [9], behavior of disordered ferromagnets [10–13], thin striplike systems [14–17]), we notice avalanchelike relaxation to be the mechanism of system evolution. In the course of the above-mentioned researches, the need for adequate theoretical approaches appeared. Consequentially, several models were introduced and studied [18–28]. All these models tend to explain some of the observed phenomena, meaning that the described systems also evolve in an avalanchelike manner through the states determined by locally quenched disorders. The random-field Ising model (RFIM) turned out to be highly prominent among these models due to its simplicity and applicability [29–31].

Theoretical investigation of the RFIM turned out to be a rather difficult task. The perturbative renormalization group approach led to insufficiently precise predictions on the nontrivial critical behavior in three dimensions [32–34], while the nonperturbative methods offered some corrections [35,36]. Concomitantly, several studies on the equilibrium version of the model appeared raising (and answering) some key questions [37–40], whereas in Ref. [41] the authors studied the nonequilibrium response of the system to the varying temperature, and in Ref. [42] the authors offered a new approach on obtaining the critical disorders in nonequilibrium version of the model.

In this paper, we focus on the athermal nonequilibrium version of the RFIM, i.e., the version in which the thermal fluctuations are absent, and the system evolves throughout many avalanches traversing the metastable states that are not necessarily the global energy minima. Extensive numerical simulations of this version were performed, and the results on its critical behavior are presented in Refs. [43–46], and in Refs. [47–49] on the avalanche parameters' scaling laws for dimensions $D \geqslant 3$ for the systems situated at the hypercubic lattices. Additionally, the results for quadratic $D = 2$ lattices are presented in Refs. [50–52], the nonequilateral systems were investigated in Refs. [17,53–56], while the question of reconsideration of universality classes has been recently raised in Refs. [57–59].

Although avalanches represent the mechanism for evolution in variety of phenomena, many aspects of their expansion still remained veiled. The reason lies in the practical impossibility of tracking down the moment-by-moment avalanche propagation except for small avalanches, whose propagation is of less importance. What we are interested in here are the big avalanches that span the whole system and eventually cause the phase transition [60]. As these avalanches appear only under certain conditions, it is essential to know what these conditions are, which could reveal important details regarding the mechanism of some phenomena (such as earthquakes, snow avalanches, cracks in materials, avalanches in magnetic materials, etc.) [47,52,61].

In the present work, we investigate the mechanism of large avalanches formation. Such avalanches are extreme events, which overtake almost the whole system, meaning that they are infinite in the systems in the thermodynamic limit. However, they cannot appear before some necessary conditions are satisfied and that is why, for some values of parameters, such avalanches are absent. In other words, the interplay between various system's parameters determines whether such extreme events could be and will be present. The decisive outcome of that interplay turns out to be the archipelago of islands of flipped spins in the system whose number and size determine the moment when an extreme event can or cannot happen. To adequately investigate this phenomenon, one has to isolate each avalanche separately, no matter whether it is small or big.

In papers [62–64] are presented researches similar to our work just for a two-dimensional system. The authors studied the same athermal nonequilibrium RFIM and the impact of introducing an interface of flipped spins prior to the simulation start. Although the findings in these references, and references within, confront at some points, it became clear that the presence of the interface of flipped spins, as well as the presence of islands of flipped spins, influences to a large extent the system's response to the external driving.

The paper is organized as follows. In Sec. II are described the model and simulation details. In Sec. III are explained the subcritical avalanche evolution and the main factors that influence that evolution. We further show the impact of these factors, i.e., the islands of flipped spins in Sec. IV and the size of the interface neighboring the starting spin of an extreme avalanche in Sec. V. Finally, in Sec. VI we discuss the present results to the previous ones and offer possible future directions for the research on this topic, while in Sec. VII we conclude the paper.

II. MODEL AND SIMULATION DETAILS

The RFIM describes the system of N classical spins $S_i = \pm 1$ located at the sites of some underlying lattice. The system's Hamiltonian reads:

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} S_i S_j - H \sum_i S_i - \sum_i h_i S_i, \tag{1}$$

where $\langle \cdot \rangle$ denotes the summation over all distinct pairs of nearest neighbors, J represents the exchange coupling constant ($= 1$ in this paper), while H is the external driving magnetic field that is homogeneous and acts on each spin in the system. Local impurities, that are inevitably present in experimental samples, are modeled by the random magnetic field $\{h_i\}_{i=1}^N$ that at each site i acts on the associated spin S_i via interaction $-h_i S_i$. The values of this field are chosen independently at different sites from some zero mean distribution specified by its standard deviation R, called disorder, because this model parameter quantifies the amount of disorder in the system. Therefore, $\langle h_i h_j \rangle = R^2 \delta_{ij}$, where $\langle \ldots \rangle$ denotes averaging over all random-field configurations, and δ_{ij} is the Kronecker delta function. We pick the values h_i of the random field from the Gaussian distribution $\rho(h) = \frac{1}{\sqrt{2\pi}R} \exp(-\frac{h^2}{2R^2})$ and in this way facilitate the comparison between new and already known findings dominantly obtained in the past using the same distribution with remark that the type of distribution does not affect some of the model's responses as is evidenced in Ref. [65].

In the nonequilibrium version of the RFIM, considered here, the system evolves following the local dynamical rule according to which the spin S_i is stable provided that $S_i h_i^{\text{eff}} > 0$, where

$$h_i^{\text{eff}} = S + H + h_i, \tag{2}$$

is the effective magnetic field that acts on the spin S_i and $S = \sum_{\langle j \rangle} S_j$ is the sum of S_i's nearest-neighbor spins. Otherwise, S_i is unstable and will flip in the next moment of (discrete) simulation time which changes the value of effective magnetic field for S_i's nearest neighbors. Those that become unstable will flip in the next-next moment of time, and in this way an avalanche of spin flipping is created propagating as long as there are unstable spins in the system.

The spin stability is also affected by the change of external magnetic field H. We perform analysis at the rising part of magnetization curve setting initially all spins to $S_i = -1$ and increasing the external magnetic field from $H = -\infty$ to $H = +\infty$ in the adiabatic driving regime, i.e., at infinitely slow rate. This means that H is kept constant during any avalanche implying that at most one avalanche can be active at a time. The ongoing avalanche dies when there are no unstable spins in the system and then the H is increased exactly to the value that causes flipping of the least stable spin. This may trigger the next avalanche; otherwise, H is again increased so to flip the next least stable spin, and so on.

To reduce the simulation time, the increase of H between avalanches is not done in very small increments, but in a single step because all spins remain stable until H reaches the smallest value that flips only the least stable spin. Also, each simulation is started with so large negative value of H (mimicking $H = -\infty$) that the initial spin configuration is stable for the chosen configuration $\{h_i\}_{i=1}^N$ of the random magnetic field, and finished when all spins' values are $+1$. Our simulations were performed on the equilateral three-dimensional (3D) $L \times L \times L$ cubic lattices of linear size L with $N = L^3$ sites using periodic boundary conditions (for better resemblance with infinite systems). For each set of simulation parameters R, L the simulation is repeated for a number of different random-field configurations that was large enough for collecting reliable statistics.

The most time-consuming part of the simulation turns out to be finding the least stable spin once the avalanche is over. To reduce that time, we used the sorted list algorithm [44,66]. Briefly, this algorithm sorts all values of the random field in descending order. The first spin to flip in the system is, obviously, the first spin in the mentioned sorted array. After the avalanche that was created from that first spin, the rule for finding the least stable spin is the following: (i) create an array *nextPossible[S]* whose each element points to the spin that would flip first if the sum of its neighbors would be S (for three-dimensional cubic lattice S can range from -6 to 6); (ii) choose the spin from *nextPossible[S]* that has the largest value of $S + h_i$; (iii) set *nextPossible[S]* to the next spin in the sorted list; (iv) if the sum of nearest neighbors of the spin with the largest value of $S + h_i$ is S then it is the least stable spin, otherwise go back to the step (ii).

III. SUBCRITICAL AVALANCHE EVOLUTION

In the three-dimensional (3D) $L \times L \times L$ cubic lattice systems with disorders below the critical disorder $R_c = 2.16$ [43,45] avalanches that span the system along at least one of its spatial dimensions appear. We denote these avalanches as subcritical avalanches [67], and in this paper we focus on discovering the necessary conditions for their propagation. Properties of distributions of these avalanches were studied in Refs. [47,61].

In Fig. 1(a) is presented the system state just before the appearance of the spanning avalanche. The system's linear

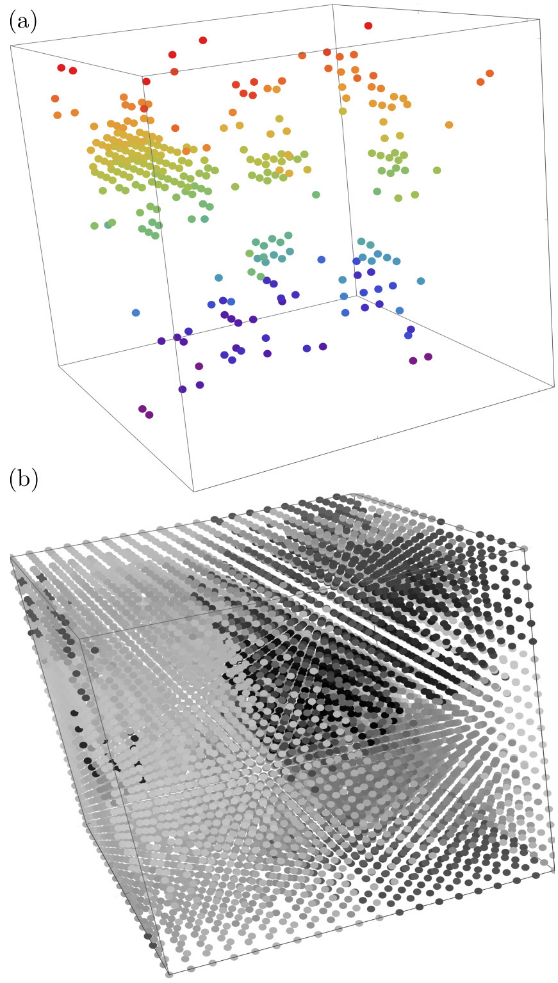

FIG. 1. (a) Spins flipped during the evolution of the system with 20^3 spins and disorder $R = 1.9$ before the spanning avalanche appears. (b) The evolution of the spanning avalanche triggered in the foregoing system immediately after the state shown in (a).

size is $L = 20$ and disorder $R = 1.9$. We see that there are some islands of flipped spins in the system before beginning of a spanning avalanche. In Sec. IV, we examine the importance of the presence of these islands for the propagation of the spanning avalanche. The islands lower the value of the external field needed to flip their neighboring spins since the sum of the nearest neighbors in expression (2) for h_i^{eff} becomes larger, as presented in Fig. 2 in the two-dimensional case. This is why the starting point of spanning avalanche usually is a spin that is in contact with some island of flipped spins. In the following text we call such an island as interface island, or shorter, interface. In Sec. V, we investigate the impact of the interface size on the appearance and propagation of the spanning avalanche. Note that in simulations of finite systems with very small disorders there would be no islands of flipped spins in the system before the appearance of spanning avalanche. This happens because the system is too small and the external field should grow to a large value to flip only the first spin in the system. However, once the first spin is flipped, it increases the value of the first term in expression (2) for h_i^{eff},

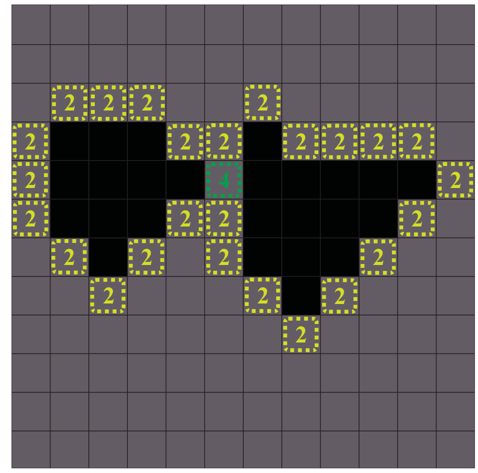

FIG. 2. Schematic depiction of two islands of flipped spins on a 2D square lattice (black squares) creating a strait between the islands making the spins in the strait less stable and easier to flip. The numbers in lighted squares represent a decrease of the barrier for avalanche passing caused by the reduced sum of the nearest-neighbor spins.

which, in combination with already large H, leads to flipping of all spins in the system.

For any finite system, there is a range of disorder from zero to some maximum value $R_{\max}^{(\text{ni})}(L)$ for which there are no islands of flipped spins prior to the advent of spanning avalanche. This value, depending on system size L, decreases to zero when $L \to \infty$ because for the systems in thermodynamic limit there would be infinitely many (islands of) flipped spins for any value of external field at any $R > 0$ (this is correct in the case of Gaussian and other unbounded distribution of random-field values, but not for bounded, e.g., uniform, distributions). As $R_{\max}^{(\text{ni})}(L) < 1$ for all values of L used in our simulations, in what follows we restrict our research to the values of disorder $R \geqslant 1$.

In Fig. 1 we illustrate a spanning avalanche emerging immediately after the system state shown in Fig, 1(a). Spins flipped during this spanning avalanche are shown in Fig. 1(b) by points in different shades of gray ranging from dark shades for early to light shades for lastly flipped spins. The avalanche did not evolve equally in all directions, suggesting that it is not that easy for the spanning avalanche to propagate as it might be expected. The reason is that, despite facilitating the spreading of avalanches by crawling of avalanche along the islands' rim, flipping the spins with a smaller number of unflipped neighbors and subsequently hopping between the islands, the spanning avalanche had to find passages between the islands propagating through the whole system. Hence, for $1 \leqslant R < R_c$, a suitable archipelago of islands of flipped spins has to be built by increasing H before a spanning avalanche emerges. For finite systems the spanning field H_{sp}, i.e., the value of H at which the spanning avalanche is triggered, depends on the chosen configuration of the random magnetic field $\{h_i\}_{i=1}^N$ and is, therefore, a random quantity. The distribution of H_{sp} is symmetric and unimodal [52,61], so its mean value is equal to the effective critical field $H_c^{\text{eff}}(R, L)$, i.e., the value of external field H at which the susceptibility $\chi = dM/dH$ attains its maximum for given disorder R and lattice size L.

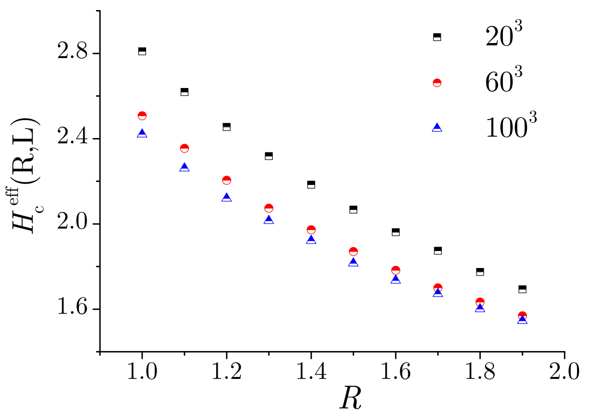

FIG. 3. The effective critical field $H_c^{\text{eff}}(R,L)$ versus disorder R for the system with 20^3, 60^3, and 100^3 spins, shown by black squares, red circles, and blue triangles, respectively.

With the increase of lattice size L, the width of distribution of spanning field tends to zero, while the effective critical field decreases, as shown in Fig. 3 and in Refs. [17,42,61], tending to the effective critical field $H_c^{\text{eff}}(R)$ in thermodynamic limit at which the susceptibility shows its maximum giving a jump of magnetization for $R < R_c$ due to infinite avalanche appearing at $H = H_c^{\text{eff}}(R)$ [17,42].

To investigate the impact of the number of islands of flipped spins on the spanning avalanches, we simulated three different systems with 20^3, 60^3, and 100^3 spins. Disorders for each system size ranged from $R = 1.0$ to $R = 1.9$ so that all spanning avalanches, appearing in this range of disorder, span the whole system, i.e., along all three spatial dimensions. For larger systems, reachable in simulations, the differences in the bellow presented quantities were very small, within the error bars.

IV. IMPACT OF THE ISLANDS OF FLIPPED SPINS

In this section, we want to see whether the islands of flipped spins play crucial role in the spanning process. To this end, for any chosen spanning avalanche (which was triggered, say, at its initial spin S_{i_0} by increasing the external field to $H = H_{\text{sp}}$ at the moment t_0), it would be useful to observe whether it would stay spanning under modified island conditions after being triggered at t_0 by setting the external magnetic filed to $H = H_{\text{sp}}$ and initial spin to $S_{i_0} = 1$. The modifications read:

(i) at t_0 the spins from all islands are set to -1 except the spins from the interface (consisting of (usually) one or more islands such that the starting spin of the spanning avalanche is the nearest neighbor of all islands from the interface [68]);

(ii) front-propagation flipping rule prescribes that a spin with $h_i^{\text{eff}} > 0$ for $t > t_0$ flips only if one of its neighbors was flipped at the previous moment of time.

Without modification (ii) of island flipping rule, the avalanche will stay spanning and the only difference in its evolution between the original and modified version would be the flipping in modified version of all island spins at the moment $t_0 + 1$ (because all of them became unstable at the moment t_0). On the other hand, this does not happen in the (fully) modified version due to modification (ii) of island flipping conditions. Indeed, although in this case $h_i^{\text{eff}} > 0$ for all spins from the islands, none of them will flip at the moment $t_0 + 1$ due to lack of neighbors flipped at the previous moment t_0. So, until it comes next to some of the spins from the islands' rim, the avalanche advances in the same way, both in original and (fully) modified version. These spins at the islands' rim might be (and likely will be) flipped (during the spanning avalanche) in the original version, but hardly likely in the modified version as their effective field is by (at least) a factor of 2 bigger in the modified version due to the negative value of their neighboring spins from the islands, see Fig 2. Because of such obstruction in the modified version to the crawling of avalanche along the island rims, the avalanche (that was spanning in the original version) often ceases to span the system in the modified version.

Here, one could also ask the question of whether the spanning is stopped not only by the (unflipped) spins at the rim of islands but with the aid of the spins from the first layer of spins inside the islands because these spins were flipped back to the value -1 and that changed their effective magnetic field. To test this, we introduce the next modification:

(iii) in addition to modifications (i) and (ii), the values of random magnetic field for all spins from islands are changed to the same value

$$h_i^*(S^*) = S^* - H_{\text{sp}}, \tag{3}$$

depending on the value of parameter S^*, which can take one of the values 4,2,0.

Thus, for $S^* = 4$, all spins from islands have the same (modified) value $h_i^*(4) = 4 - H_{\text{sp}}$ of their random field so, when any of the nearest neighbors of an island spin $S_{i'}$ is flipped, this spin will flip due to modification (ii) of island flipping conditions because the value of its (modified) effective magnetic field is $h_{i'}^{\text{eff}} \geqslant 0$. In a consequence, if the avalanche flips one spin from the island's rim it will flip all the spins inside this island for sure, facilitating the avalanche to become a spanning one.

In Fig. 4(a) we present the number of spanning avalanches per single run, N_{sp}, in the systems with 100^3 spins for various flipping conditions. When the evolution of the system is regular (i.e., with original flipping rules without any modification) then one spanning avalanche appears in each run for all disorders in question (i.e., $N_{\text{sp}} = 1$). However, this is not the case under any of the foregoing modifications. Even under the most relaxed flipping conditions, set upon islands by modifications (i), (ii), and (iii) with $S^* = 4$, we see that $N_{\text{sp}} < 1$, and that N_{sp} for $S^* = 2$ is less than for $S^* = 4$ due to more restrictive conditions, and even less for the most restrictive conditions when $S^* = 0$. Additionally, for modifications (i) and (ii) set upon the islands, the values of N_{sp} lie between the $S^* = 4$ and $S^* = 2$ data, enlightening altogether the role of spins from the islands' rim in preventing the avalanche from spanning propagation.

Furthermore, the data in Fig. 4(a) show that N_{sp} decreases with the increase of disorder. This happens due to larger number of islands in the systems with larger disorder, which creates more obstacles for the avalanche propagation. For higher disorders, the islands of spins are forming straits between them, filled with spins that can flip easier, because of

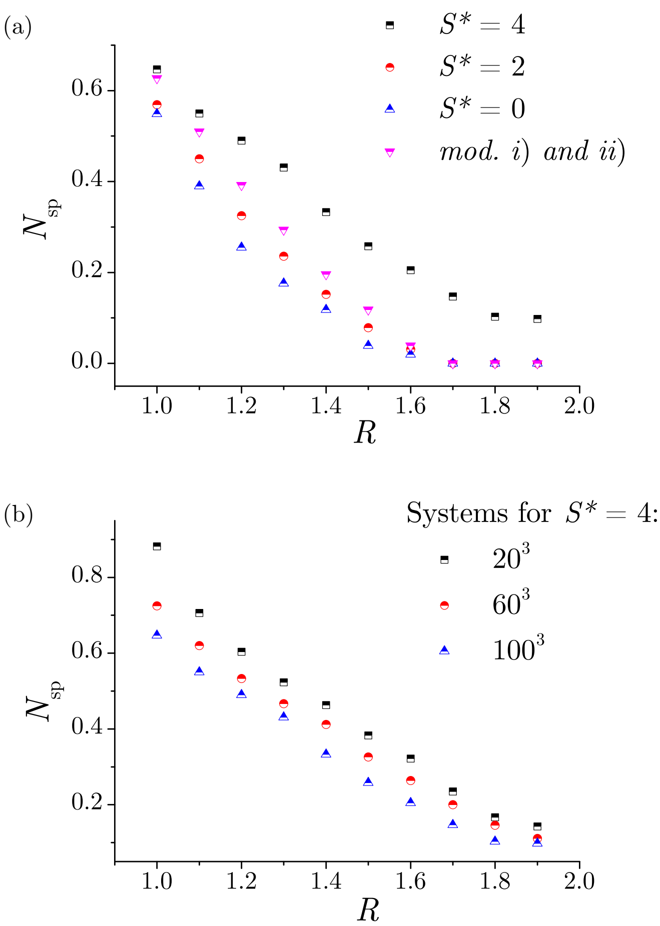

FIG. 4. (a) Number of spanning avalanches per single run, N_{sp}, averaged over different random field configurations, shown against disorder R for the system with 100^3 spins. Black squares represent N_{sp} under the most relaxed, i.e., $S^* = 4$, modifications imposed on islands for flipping a spin inside it when only one flipped neighbor is needed. Red circles represent the modifications with $S^* = 2$, so that the island spin flips if any two of its neighbors are flipped. Similarly, the blue triangles represent the $S^* = 0$ modifications that require any three flipped neighbors. Pink triangles represent N_{sp} under modifications (i) and (ii). b) N_{sp} vs R for the systems with 20^3, 60^3, and 100^3 spins, presented by black squares, red circles, and blue triangles, respectively, under the $S^* = 4$ modifications.

rather large sum of neighboring spins S, securing thus the way for an avalanche to span the system. However, at the same time, those islands act like traps when disrupted by the spin reversal according to modification (i), making it occasionally impossible for the avalanche to pass, see Fig. 2. That is why the lower external field is needed in ordinary conditions for a spanning avalanche to appear for larger disorders, see Fig. 3, while at the same time, there are less spanning avalanches per simulation for larger disorders when the islands are reversed, see Fig. 4, both being the consequence of the interplay between the shape of islands and distance between them.

Still, one could argue whether the spanning avalanche would appear always under the foregoing modifications if the disorder is low enough in which case the distance between islands is larger. Accordingly, we show in Fig. 4(b) the number of spanning avalanches per one simulation, N_{sp}, realized for

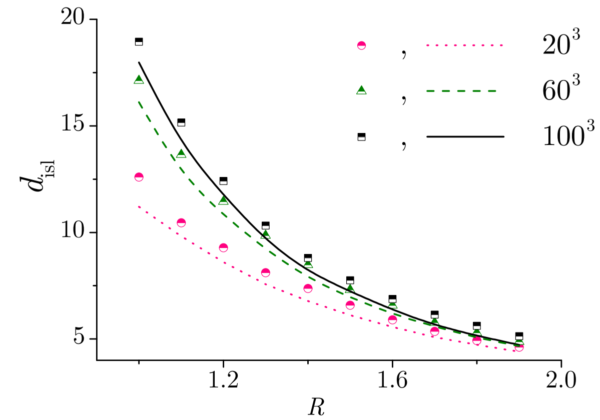

FIG. 5. The average distance between the islands, d_{isl}, versus disorder R for the systems with 20^3, 60^3, and 100^3 spins. Simulation data are shown by symbols, specified in legend, while the accompanying solid lines depict our prediction (7) approximating the average distance between the islands.

various system sizes under the most relaxed, i.e., $S^* = 4$, modifications for flipping the island spins. We see that N_{sp} decreases with the increase of the system size, suggesting that in the thermodynamic limit there would be zero spanning avalanches for any disorder. The only way for the spanning avalanche to appear in such a system would be if the external field is larger. However, we showed in Fig. 3 that the H_c decreases with the increase of the system size. Thus, it would be contradictory to claim that the H_c should be higher for the bigger system. Consequently, all of the foregoing indicates that the number and spatial distribution of the islands play a significant role in the spanning avalanche propagation.

Investigating the spatial distribution of islands further, we found that the average distance between the islands, d_{isl}, is reducing with the growing disorder R, as shown by symbols in Fig. 5. This distance is calculated as

$$d_{isl} = \sqrt[3]{N/N_{isl}}, \tag{4}$$

where N_{isl} is the average number of islands (i.e., the number of islands in a single run averaged over different random field configurations for a given value of disorder R) created before the advent of spanning avalanche. To predict this distance we approximately take that N_{isl} is given by the number of those spins that were flipped by the corresponding increase of external magnetic field and that had six down neighbors at the moment of their flipping. So,

$$N_{isl} \approx N p_{H_c^{\text{eff}}(R,L),R}(-6), \tag{5}$$

where

$$p_{H,R}(S) = \int_{-S-H}^{\infty} \rho(h)dh = \frac{1}{2}\left[1 + \text{erf}\left(\frac{S+H}{R\sqrt{2}}\right)\right] \tag{6}$$

is the flipping probability (at the external magnetic field H and disorder R of a spin having the sum of nearest-neighbor spins S) expressed in terms of the standard error function $\text{erf}(x) = 2/\sqrt{\pi}\int_0^x \exp(-t^2)dt$. Therefore, our prediction for

TABLE I. The values of effective critical fields, H_c^{eff}, average number of islands created in the system before the appearance of spanning avalanche, N_{isl}, and the average size of interface, V_{int}, for the system with 20^3 spins.

R	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
H_c^{eff}	2.81	2.62	2.45	2.32	2.18	2.07	1.96	1.87	1.77	1.69
N_{is}	4	7	10	15	20	28	39	52	67	81
V_{int}	3	4	5	6	9	12	14	16	17	18

the average distance between the islands reads

$$d_{\mathrm{isl}} \approx \frac{1}{\sqrt[3]{p_{H_c^{\mathrm{eff}}(R,L),R}(-6)}}. \tag{7}$$

Its results, presented by lines in Fig. 5, lie somewhat below the numerical data, i.e., points. There are two reasons why this occurs. One is that it may happen that one island could be created at some value of external field $H < H_c^{\mathrm{eff}}(R, L)$, and when the external field reaches $H = H_c^{\mathrm{eff}}(R, L)$ there is more than one spin in that island that satisfies the condition $H_c^{\mathrm{eff}}(R, L) + h_i \geqslant 6$, making thus the number $N p_{H_c^{\mathrm{eff}}(R,L),R}(-6)$ greater than the actual number of islands. Another reason is that islands can merge, and then the two (or more) spins that flipped with all neighbors' spins pointing down in (what used to be) separate islands, now give the single contribution to the number of islands. However, in the expression $N p_{H_c^{\mathrm{eff}}(R,L),R}(-6)$ they are still calculated as the two (or more) spins. These two reasons make the number $N p_{H_c^{\mathrm{eff}}(R,L),R}(-6)$ greater (or equal if the above-mentioned phenomena do not appear) than the actual number of islands, making the theoretical prediction curve (7) to lie a bit below the real numerical results. Despite this deficiency, we consider that our rough prediction (7) provides, to a reasonable extent, a qualitatively correct explanation of the dependence of d_{isl} on the disorder R.

The values of effective critical fields, H_c^{eff}, average number of islands created in the system before the appearance of spanning avalanche, N_{isl}, and the average size of interface, V_{int}, analyzed in the next section, are presented in Tables I, II, and III for the systems with 20^3, 60^3, and 100^3 spins, respectively. All mentioned quantities are averaged over different random field configurations for a given R.

V. IMPACT OF INTERFACE

Complementary to the previous section, in which we have analyzed the effects induced by modifications regarding only the islands of flipped spins, let us see now what happens when we modify only the interface and when we modify both islands and interface. Thus, in this section, after we let the system to evolve according to original rules until the external field reaches the value $H = H_{\mathrm{sp}}$ and flips the spin S_{i_0} that would start the spanning avalanche, we proceed with one of the following two mutually exclusive modification options:

(A) Islands remain intact; the original interface is replaced by an artificial interface, located next to i_0 and consisting of up spins; the remaining spins from the original interface (if any) are turned down and their random field changed to the same value

$$h_i^{**} = -H_{\mathrm{sp}}; \tag{8}$$

the front-propagation flipping rule, i.e., modification (ii) from the previous section, is adopted.

(B) Flip down the islands; enlarge the interface by flipping up a chosen number of randomly selected spins at its rim layer by layer not surrounding the spin S_{i_0}; and flip the spins throughout the ongoing avalanche triggered at S_{i_0} in agreement with:

(B1) either modification (ii) from Sec. IV,

(B2) or modification (iii) from Sec. IV.

With option (A), according to Eq. (8) at least three flipped neighbors are needed to flip some of the spins that originally were (but not anymore) included in the interface, meaning that their flipping is not completely prevented. Regarding the artificial interface, we set it to be either empty, or to contain one of the neighbors of the spin S_{i_0}, or to span some 2D 3×3, or 5×5, or 7×7 square next to S_{i_0}. Option (B) is same as the modification in Sec. IV with the addition of interface enlargement.

In Fig. 6, one can see that the number of spanning avalanches per single run, N_{sp}, is high even without the interface. Furthermore, with small interfaces, like 5×5 and 7×7, N_{sp} is practically equal to 1. This further indicates the importance of islands of flipped spins on the propagation of spanning avalanche for disorders $R < R_c$.

As seen from Fig. 6, after all islands are formed the spanning avalanches appear when there is a very small interface (or even without any interface with the probability greater than 0.5). However, in the previous section we saw that when the islands are reversed, the percentage of spanning avalanches under the modifications introduced there is significantly smaller, even though the averaged maximum interface size per disorder grows with the larger disorder (see Tables I, II, and III).

TABLE II. Same as in Table I, but for the system with 60^3 spins.

R	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
H_c^{eff}	2.51	2.35	2.20	2.07	1.97	1.87	1.78	1.70	1.63	1.57
N_{isl}	43	85	144	226	355	553	758	1122	1459	1868
V_{int}	13	16	17	18	20	22	25	28	30	34

TABLE III. Same as in Tables I and II, but for the system with 100^3 spins.

R	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
H_c^{eff}	2.42	2.26	2.12	2.01	1.92	1.81	1.73	1.67	1.60	1.54
N_{is}	147	287	523	909	1463	2146	3077	4308	5639	7394
V_{int}	14	18	19	21	22	26	27	29	32	37

When we applied the modification (B1) or (B2), we performed something like the process of crystallization by adding 10, or 100, or 1000 flipped spins to the existing interface, where the addition of 1000 spins was performed just in the case of the largest systems having 100^3 spins. In Fig. 7(a), one can see that even when the interface is enlarged by 1000 spins, and when the most relaxed flipping condition [i.e., (B2) with $S^* = 4$] were imposed, the avalanche started at S_{i_0} still does not span the system with 100^3 spins in each run.

In Fig. 7(b) we compare the systems with 20^3, 60^3, and 100^3 spins with their interface enlarged by 100 spins, and subjected to modifications (B1) or (B2). The number of spanning avalanches per single run, N_{sp}, decreases with the increasing system size under all analyzed scenarios, indicating that in the thermodynamic limit there will be no spanning avalanches even for the large interface sizes if the islands are reversed. However, the expansion of the interface improves the spanning ratio, see Fig. 7(c), where we compared two types of conditions of island's spins flipping and two sizes of interface in the system of size 100^3. Still, even for the most relaxed condition for flipping of the islands' spins, the N_{sp} is significantly lower than 1, especially for higher disorders.

For lower disorders, one could ask whether the observed behavior is a consequence of finite system size, as explained in Sec. III. That is why the results for larger disorders should be more reliable. Thus, when the islands are reversed, even larger interfaces cannot help the avalanche to overcome the effective fields of spins. Thus, islands remain unapproachable, and the system is trapped in some of the metastable states.

All this suggests that the presence of islands is a necessary condition for the avalanche to span the system. The external field will grow until the appropriate matrix for spanning avalanche propagation is formed, which obviously depends on the system's disorder.

VI. DISCUSSION

A lot of natural phenomena evolve through a number of separate or overlapping avalanches, which makes the theoretical and numerical study of the avalanche properties important. In this respect, it is of particular interest to monitor the evolution of a single avalanche because this can give clues for revealing the conditions for the appearance and progress of the extreme large events that are often catastrophic (earthquakes, snow avalanches, tsunami, etc.).

Nevertheless, from our results, it is clear that a system-spanning avalanche will not appear if there is no adequate setup for its propagation, i.e., a matrix of flipped islands. In the past, more attention was given to the study of the interface size and type impact on the avalanche propagation than of the islands' impact [63,64,69]. In Refs. [62,63] the authors were motivated by the fluid invasion experiments to investigate the RFIM with the interface set before the start of simulation and to allow flipping of only those spins that are in contact with the moving interface. The conclusions resulting from such approach could be significantly different from the ones derived for the systems where each unstable spin is allowed to flip. As we showed, when there are no flipped islands in the system, then the avalanche propagates much harder. These two types of systems and propagation conditions led to some differences even in the existence of the critical point in two-dimensional RFIM [50,62,64]. Thus, one should be very careful when deriving conclusions on the behavior of some model since the initial setup, the propagation conditions, or even the driving regime, nontrivially affect the system behavior.

On the other hand, although the presence of an interface of large size is, in general, not enough for the advent of a spanning avalanche if there are no suitably distributed islands of flipped spins, it still helps the avalanche to propagate further than it would in the case of smaller interface(s). In the extreme case, the interface size can be equal to the whole system's cross section (a whole cross-sectional line in the two-dimensional, or a whole cross-sectional plane in the three-dimensional case). Such interface could change the behavior of the system, especially for smaller disorders where it effectively lowers the dimension of the system due to propagation of avalanches layer by layer so that three-dimensional systems become effectively two-dimensional. The magnitude of the effects described in this paper cannot be seen in the simulations with the preset interface of the size of the whole system's cross section. This interface allows the avalanche to propagate far enough so that one could conclude that this is a system-spanning avalanche. However, this may be misleading since it might happen that the same interface would lead to the ordinary nonspanning avalanche if the system was much bigger. That could be the reason behind unexpected results in Refs. [69,70] or behind the disagreement between the results obtained in Refs. [64] and [50].

Although there were many papers regarding the behavior of the RFIM above the critical disorder, to the knowledge of the authors, to this date, there have been no attempts to elucidate the local conditions on the terrain explaining why the spanning avalanches do not appear for $R > R_c$ [71]. This paper might suggest further research regarding this topic. Imagine the system described via RFIM from this paper with all spins pointing down, and the external field H_0 is applied. We want to see how big would be the avalanche that H_0 would create. There would appear a spanning avalanche even for $R > R_c$ under such conditions. Thus, the islands represent the obstacles that prevent an avalanche from spanning for $R > R_c$. The crossover from ferromagnetic to paramagnetic

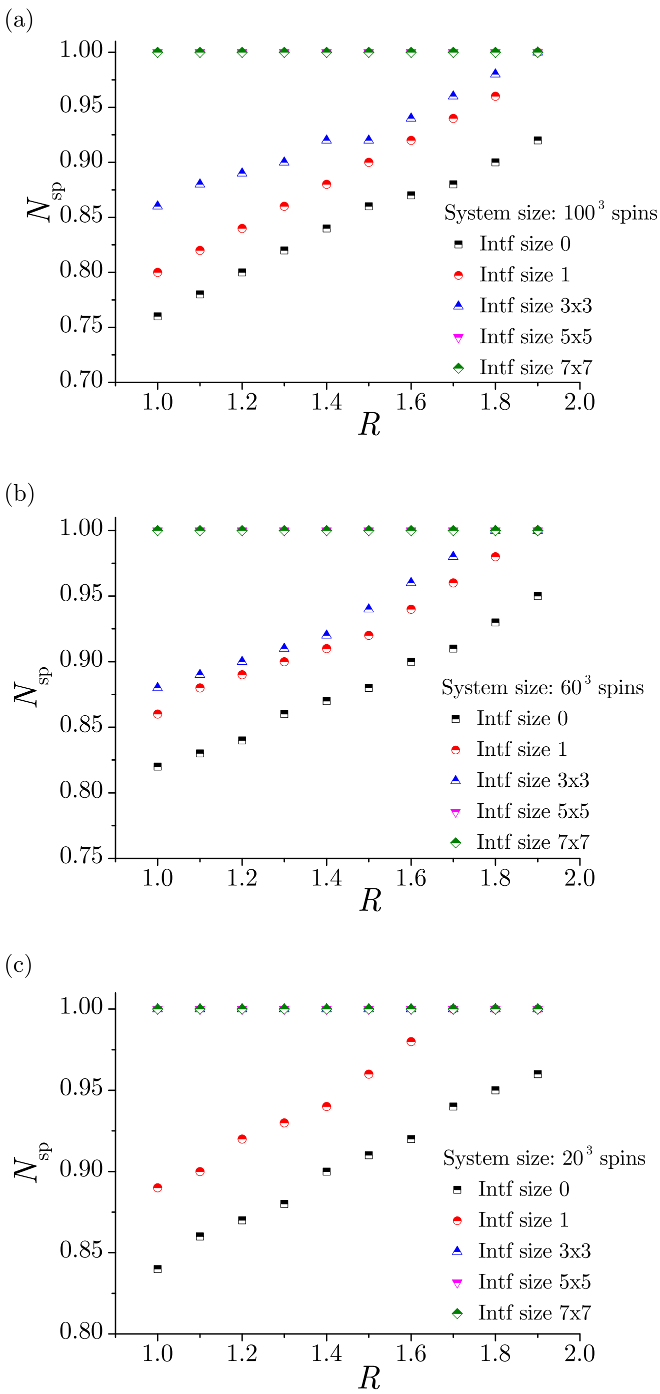

FIG. 6. Number of spanning avalanches per single run, N_{sp}, against disorder R for the model with adopted modification (A) at the systems having (a) 100^3 spins, (b) 60^3 spins, and (c) 20^3 spins. Black squares represent the case of artificial interface of size zero, red circles the case of artificial interface containing 1 spin, blue triangles the case of artificial interface containing 3×3 spins, pink triangles the case of artificial interface containing 5×5 spins, and green rhombuses the case of artificial interface containing 7×7 spins.

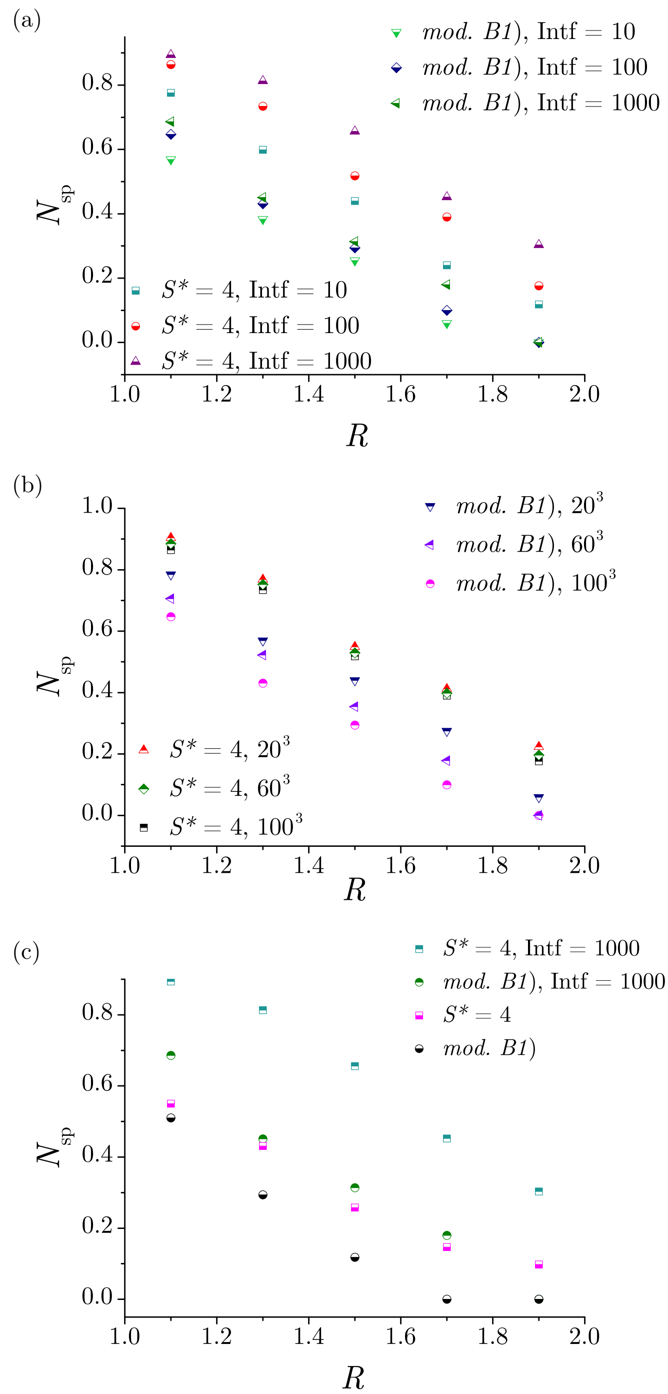

FIG. 7. (a) Number of spanning avalanches per single run, N_{sp}, against disorder R at the system having 100^3 spins. For adopted modification (B1) or (B2) with $S^* = 4$, light-green/light-blue squares represent the case of interface enlarged by ten spins, navy-blue/red circles of interface enlarged by 100 spins, olive-green/purple triangles of interface enlarged by 1000 spins, (b) N_{sp} vs R for the systems having 20^3, 60^3, and 100^3 spins, their interface enlarged by 100 spins, and adopted option (B1) or (B2) as specified in legend. (c) N_{sp} vs R for the system with 100^3 spins enlarged by 1000 spins and adopted option (B1) or (B2) specified in legend.

phase would then be when the islands start to prevent rather than help the propagation of avalanches.

The presented findings could also potentially lead to progress in other fields of research, contributing to a better understanding of real phenomena, such as earthquakes or financial shocks, and (hopefully) their prevention [1,8].

VII. CONCLUSION

In conclusion, we presented a study of the necessary conditions for the appearance of the extreme event, such as an infinite avalanche, in the cubic equilateral three-dimensional athermal nonequilibrium RFIM driven by the external magnetic field in the adiabatic regime. The value of the critical disorder R_c, above which there are only small avalanches in the system, was known before. However, that is a macroscopic quantity, averaged over all random fields in the system. What we found here are the local conditions that lead or do not lead to the extreme event. We showed that islands of flipped spins that create a matrix for avalanche propagation are the crucial factor in the existence of the spanning avalanche in the system. When the number of those islands is reduced, the avalanche cannot propagate at the same value of the external field at which it propagated when the islands were present, regardless of the size of the interface, i.e., the island of flipped spins neighboring the avalanche's starting spin. We have done that through the simulation of various conditions that the system finds itself in. We have artificially flipped back the islands that were flipped before the appearance of the spanning avalanche and set the condition for their flipping even less demanding than they were originally. This did not help, since under new circumstances, the avalanche (that was a spanning one under ordinary evolution rules) was not able to reach those sites of flipped islands at all, due to the more negative effective field of the spins that were neighbors to the islands of flipped spins.

Finally, the results from this work, and (hopefully) future results regarding the same topic, will help to obtain the closer insight to the creation and propagation of large avalanches. Once such knowledge is gathered, it might be possible to reach a big goal of finding the mechanism of controlling and preventing extreme events, especially in the real natural or social phenomena.

ACKNOWLEDGMENT

This work was supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia.

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