

Skewness of local logarithmic exports

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(Received 7 January 2021; accepted 5 March 2021; published 29 March 2021)

The distributions of trade values and relationships among countries and product categories reflect how countries select their trade partners and design export portfolios. Here we consider the exporter-importer network and the exporter-product network with links weighted by the logarithm of the corresponding export values each year from 1962 to 2018, and study how the weights of the outgoing links from each country are distributed. Such local logarithmic export distributions by destinations and products are found to follow approximately the Gaussian distribution across exporters and time, implying random assignment of export values on a logarithmic scale. However, a nonzero skewness is identified, changing from positive to negative as exporters have more partner importers and more product categories in their portfolios. Seeking the origin, we analyze how local exports depend on the out-degree of the exporter and the in-degrees of destinations or products and formulate their quantitative and measurable relation incorporating randomness, which uncovers the fundamental nature of the export strategies of individual countries.

DOI: [10.1103/PhysRevE.103.032314](https://doi.org/10.1103/PhysRevE.103.032314)

I. INTRODUCTION

Individual countries, though equipped with different resources for production and put in different environments, commonly fine tune the strategies on how to allocate resources, invest capital, and make international trade partnership towards economic development [1–6]. Such efforts are universal and thus can be imprinted in the patterns of domestic production, consumption, and international trade. Especially in international trade are reflected the decisions on what products, to what countries, and how much each country exports [7]. The empirical and analytical study of the trade value distributions, along with the analysis of the mechanism generating the observed characteristics [8,9], may illuminate the underlying principles of global economic systems. Computational methods and models can be useful for this paper, which are increasingly applied to various socioeconomic systems [10,11].

The network approach has been instrumental to quantify various natures of international trade, revealing its far-from-random connection among countries [12–14], and the broadness of the distributions of trade values by countries [15–17] or products [18–20]. These studies mostly aimed at identifying typical representative characteristics, consequently neglecting local variations which may be essential for understanding the principles governing the trade strategies of individual countries. In this light, here we investigate how the export values of a country are distributed by its destinations (partner importers) and product categories. We compile the international trade datasets from two sources [21,22],

covering different time periods, to obtain the annual export value for each pair of an exporter and an importer country, and for each pair of an exporter country and a product category. Using the result, we construct the exporter-importer (EI) network and the exporter-product (EP) network each year, where links connect exporter countries to their partner importers or to their exported product categories, and are weighted by the logarithm of the corresponding annual export values. Then the distribution of the weights of the outgoing links from each country represents how the values of the exports from the country are distributed by destinations or products, which we analyze in the present paper.

We find that such local logarithmic export distributions measured for individual exporters follow approximately the Gaussian distribution, implying the possibility of the first two moments to fully characterize the whole distributions. However, the distributions exhibit significant asymmetry and significant nonzero skewness. Moreover, the skewness shows a systematic variation—decreases with the out-degree of the exporter, the number of partner importers, or the number of the exported products. In seeking the origin, we first find that the average export of a country per destination or product increases with its out-degree, meaning that the whole distribution is pushed relatively to the right (left) for a hub (nonhub) exporter and thus may give rise to a thicker left (right) tail. From the network perspective, this finding characterizes the influence of a source node on the average weight of its outgoing links. Individual link weights are found to be dependent also on the in-degrees of the target nodes, the number of connected exporters of destination or product in the EI or the EP network. Yet the dependence is accompanied by large fluctuations. We derive the formula decomposing a local export into a deterministic term dependent on the in-degree of the

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target and a random noise, which is well defined empirically and leads us to disentangle the skewness of local exports in terms of the third-order joint cumulants of the in-degrees of the targets and random noises. These results therefore uncover the mechanisms generating the skewed distribution of local exports but also provide a quantitative framework to analyze the architecture of weighted complex networks.

The paper is organized as follows. In Sec. II, we introduce briefly the datasets and present the empirical findings on the skewness of the local logarithmic export distributions. We study in Sec. III the distributions of the in-degrees of destinations and products receiving links from each exporter and compare the skewness of such local in-degrees and local exports. In Sec. IV, we show how the in-degrees of destinations or products and random noises together generate the skewness of local exports. We discuss our findings and future works in Sec. V.

II. LOCAL EXPORT DISTRIBUTIONS

A. Datasets and two trade networks

We compile the National Bureau of Economic Research–United Nations (NBER-UN) data [21] and the World Trade Organization (WTO) data [22] to obtain the annual export value $V_{cpc'}(t)$ in US dollars of a product category p exported by a country c to another country c' in the period $1962 \leq t \leq 2018$. The number of countries exporting or importing at least one product ranges between 154 and 190 and the total number of exported products fluctuates between 686 and 1458 in the studied period (Appendix A).

Each country exports selected products to selected destination countries in different amounts, which constitutes its export portfolio. To represent the whole collection of such trade relationships, here we consider two kinds of weighted networks, the EI network and the EP network each year as follows (Appendix A). Nodes in the EI network are countries exporting or importing at least one product in given year t . A link is assigned from a country c to another country c' if the aggregated export from c to c' , $V_{cc'}(t) = \sum_p V_{cpc'}(t)$, is positive. The (binary) adjacency matrix $\mathbf{A}^{(\text{EI})} = (A_{cc'})$ is given by $A_{cc'} = \theta(V_{cc'})$ with $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x \leq 0$. The EP network is a bipartite network consisting of two types of nodes, countries, and product categories, and they are connected if $V_{cp}(t) = \sum_{c'} V_{cpc'}(t) > 0$. Therefore the adjacency matrix $\mathbf{A}^{(\text{EP})} = (A_{cp})$ is given by $A_{cp} = \theta(V_{cp})$. The links in the EI and the EP network are then weighted by the base-10 logarithm of the corresponding export values as

$$\begin{aligned} v_{cc'}(t) &= \log_{10} [V_{cc'}(t)], \\ v_{cp}(t) &= \log_{10} [V_{cp}(t)]. \end{aligned} \quad (1)$$

The link-weight matrix, or the weighted adjacency matrix, is given by $\mathbf{v}^{(\text{EI})} = (v_{cc'})$ and $\mathbf{v}^{(\text{EP})} = (v_{cp})$, respectively. We take the logarithm given that export values span several orders of magnitude [14,15,17,23]. As we will show, the distributions of v 's are close to the Gaussian distribution and we will investigate their characteristics. On the other hand, the distribution of the raw values of export V 's is close to power laws [15,17].

B. Distribution of local logarithmic exports

How a country c divides its export among different destinations (c') or product categories (p) can be represented by the distributions of its logarithmic export values:

$$\begin{aligned} P_c^{(\text{EI})}(v) &= \frac{1}{q_c^{(\text{EI})}} \sum_{c' \in I_c^{(\text{EI})}} \delta(v_{cc'} - v), \\ P_c^{(\text{EP})}(v) &= \frac{1}{q_c^{(\text{EP})}} \sum_{p \in I_c^{(\text{EP})}} \delta(v_{cp} - v), \end{aligned} \quad (2)$$

where $I_c^{(\text{EI})} = \{c' | A_{cc'} > 0\}$ and $I_c^{(\text{EP})} = \{p | A_{cp} > 0\}$ are the sets of the partner importers and of the exported products of c . The out-degrees $q_c^{(\text{EI})}$ and $q_c^{(\text{EP})}$ are the sizes of those sets equal to the number of outgoing links from c :

$$\begin{aligned} q_c^{(\text{EI})} &= \sum_{c'} A_{cc'} = \sum_{c' \in I_c^{(\text{EI})}} 1, \\ q_c^{(\text{EP})} &= \sum_p A_{cp} = \sum_{p \in I_c^{(\text{EP})}} 1. \end{aligned} \quad (3)$$

$q_c^{(\text{EI})}$ and $q_c^{(\text{EP})}$ can be considered as a measure of the diversity of country c in the trade partnership and inventory. $P_c^{(\text{EI})}(v)$ and $P_c^{(\text{EP})}(v)$ correspond to egocentric link-weight distributions in the EI and the EP network and we call them *local* (logarithmic) export distributions in that they are measured only for the outgoing links of c .

The local export distributions $P_c(v)$ commonly take a bell shape while the mean $\langle v \rangle_c^{(\text{EI})}$ or $\langle v \rangle_c^{(\text{EP})}$ is different from exporter to exporter [Figs. 1(a)–1(d)]. We present the results obtained for year 1980 as an example in most figure plots, but all our statements are valid throughout the studied period unless stated otherwise. The standard deviation $\sigma_{v;c}^{(\text{EI})}$ or $\sigma_{v;c}^{(\text{EP})}$ does not vary with c as significantly as the mean does. Therefore no significant correlation is present between $\sigma_{v;c}$'s and $\langle v \rangle_c$'s, which is contrasted to their well-known scaling law or Taylor's law ubiquitously observed in various complex systems [24–26]. We will soon see the nonuniversal aspects of the statistics of local exports, depending on exporters and thus complicating the relationship among the moments.

Let us introduce the standardized variable \tilde{v} such that

$$v_{cv} = \langle v \rangle_c^{(\text{EG})} + \sigma_{v;c}^{(\text{EG})} \tilde{v}_{cv} \quad \text{with } (v, G) = \begin{cases} (c', I) \text{ or} \\ (p, P) \end{cases} \quad (4)$$

where we used v and G to avoid repeating almost the same expressions such that v represents an importer c' or a product p , and G represents I or P . Then the standardized distributions $\tilde{P}_c(\tilde{v})$ for different c 's appear to be collapsing onto the Gaussian function of zero mean and unit variance $\tilde{P}^{(\text{Gaussian})}(\tilde{v}) = \frac{e^{-\frac{\tilde{v}^2}{2}}}{\sqrt{2\pi}}$ [Figs. 1(e) and 1(f)]. If the data collapse were perfect, $\{\tilde{v}_{cv} | v \in I_c\}$ would be considered as Gaussian random variables satisfying $\langle \tilde{v} \rangle_c = 0$, $\langle \tilde{v}^2 \rangle_c = 1$, $\langle \tilde{v}^n \rangle_c = 0$ for $n \geq 3$ with $\langle \tilde{v}^n \rangle_c = \frac{1}{q_c} \sum_{v \in I_c} \tilde{v}_{cv}^n$. We would think that every country divides its export among destinations and products randomly on a logarithmic scale universally [8,9] and that the mean and the standard deviation fully characterize the statistics of the local logarithmic export values.

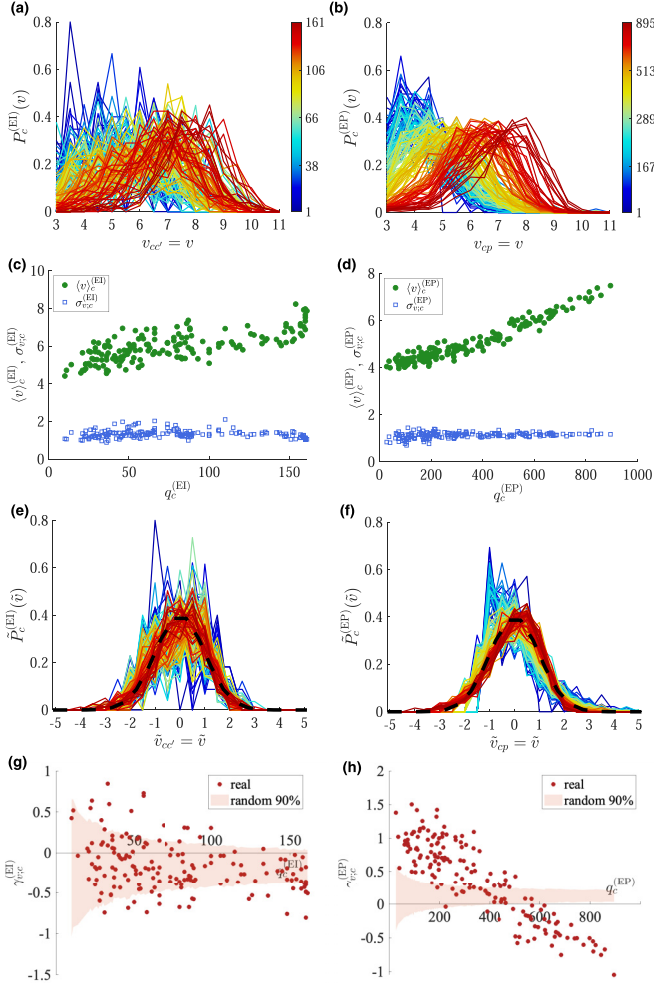


FIG. 1. Statistics of the logarithm of local exports. (a), (b) Distribution of the logarithm of the exports from each country c (a) to different destinations and (b) of different product categories in year 1980. Color varies with the out-degree of c . (c), (d) The mean and standard deviation of the logarithmic exports from country c (c) to different destinations and (d) of different products vs the out-degree of c . (e), (f) Distribution of the standardized logarithmic exports from country c (e) to different destinations and (f) of different products. The dashed lines in (e) and (f) are $\tilde{P}^{(\text{Gaussian})}(\tilde{v}) = \frac{e^{-\tilde{v}^2/2}}{\sqrt{2\pi}}$. (g), (h) The skewness of the logarithmic exports from c (g) to different destinations and (h) of different products vs the out-degree of c . Shades in (g) and (f) represent the middle 90% range of the random-version skewness.

C. Skewness

The data collapse is not perfect, however. To quantify the deviation, we measure the skewness of local logarithmic exports for each exporter c as

$$\gamma_{v;c} = \frac{\frac{1}{q_c} \sum_{v \in I_c} (v_{cv} - \langle v \rangle_c)^3}{\sigma_{v;c}^3} = \langle \tilde{v}^3 \rangle_c, \quad (5)$$

where we dropped the superscript (EI) and (EP) but they are assumed implicitly. The results are in Figs. 1(g) and 1(h). Skewness represents the normalized asymmetry of a distribution such that the skewness is positive (negative) if the

right (left) tail is thicker. The observed $\gamma_{v;c}^{(\text{EP})}$'s are significant for most exporters c , and $\gamma_{v;c}^{(\text{EI})}$'s seem partly so. To check the significance, we shuffle weights—logarithmic export values—among links to obtain a thousand realizations of weight-shuffled EI and EP networks and measure their skewness. In case of year 1980 presented in Figs. 1(g) and 1(h), 140 among 161 (= 87%) exporters with out-degrees not smaller than 10 find their true skewness $\gamma_{v;c}^{(\text{EP})}$ outside the middle 90% range of the skewness from randomized networks [Fig. 1(h)]; 42 (= 26%) exporters have true skewness $\gamma_{v;c}^{(\text{EI})}$ outside the middle 90% range of the random-version skewness [Fig. 1(g)]. See Appendix B for more details.

The skewness of local logarithmic exports tends to decrease, from positive to negative, with the out-degree of the exporter c , more strongly for $\gamma_{v;c}^{(\text{EP})}$ than for $\gamma_{v;c}^{(\text{EI})}$ [Figs. 1(g) and 1(h)]. Such correlation is robustly observed in the considered period, except for the weak correlation between $\gamma_{v;c}^{(\text{EI})}$ and $q_c^{(\text{EI})}$ in the middle 1980s and 1990s (Appendix C).

When the distribution of a finite-ranged variable is much closer to its lower (upper) bound, a thicker right (left) tail can be developed, yielding a positive (negative) skewness. This can explain the observed negative correlation between $\gamma_{v;c}$ and q_c : $P_c(v)$ moves to the right on the v axis if c has a large out-degree, as captured by the growth of the mean logarithmic export $\langle v \rangle_c$ with q_c [Figs. 1(a)–1(d)]. Empirically $\gamma_{v;c} = \langle \tilde{v}^3 \rangle_c$ is negatively correlated with $\langle v \rangle_c$ as expected (Appendix C). It is noteworthy that the export value of a country per destination or product grows with the out-degree of the exporter, as it implies a superlinear growth of the whole export of a country $V_c = \sum_{c',p} V_{cc'p}$ with the out-degree $q_c^{(\text{EI})}$ and $q_c^{(\text{EP})}$. More developed or richer countries may have many partners and export products [13,15,16,23] and therefore our results point out different skewness of the export portfolio between developed and developing countries.

We have just argued that the skewness of local exports $\gamma_{v;c}$ is influenced by the exporter c via the c -dependent mean $\langle v \rangle_c$. Individual local exports $v_{cc'}$'s and v_{cp} 's must be dependent also on the importer country c' and the product category p [5]. Exporters have different sets of partners and export products, possibly depending on the exporters' out-degrees, which might be influencing the skewness of local export distributions. In the next sections, we study how the standardized logarithmic exports \tilde{v}_{cv} vary with v , which will deepen our understanding of the statistics of local exports and of the export strategies of countries in international trade.

III. LOCAL IN-DEGREE DISTRIBUTIONS

What products and to what countries a country exports and how large its value is may depend on the domestic industrial landscape of the exporter itself but also on the characteristics of the partner importers and product categories such as the economic development of and the social and political relationship with the partner countries, and the technology level of products, and so on. Among them, how many countries export to a country and how many countries export a product category can be used as important information when an exporter country constructs its product portfolio, as they represent the *popularity* in international trade, reflecting the

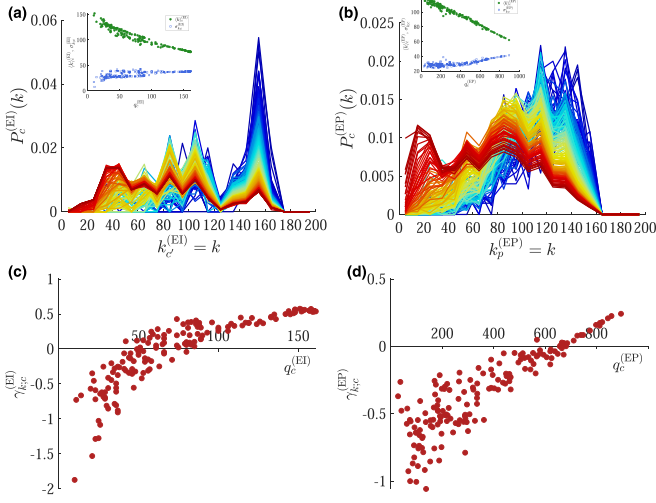


FIG. 2. Statistics of local in-degrees in the EI and the EP network. The in-degree of a product or a country node means the number of exporter countries exporting the product or to the country. (a), (b) Distribution of the in-degrees of (a) the country nodes in the EI network and (b) the product nodes in the EP network receiving links from each exporter country c in year 1980. Color varies with the out-degree of c . Inset: The mean and standard deviation of local in-degrees vs the out-degree of the exporter. (c), (d) The skewness of the in-degrees of (c) the country in the EI network and (d) the product nodes in the EP network receiving links from c vs the out-degree of the exporter c .

collective evaluation by countries [5]. They correspond to the in-degrees of a country c' and of product p in the EI and the EP network, defined as

$$k_v = \sum_c A_{cv} = \sum_{c \in \mathcal{O}_v} 1, \quad (6)$$

with v being c' or p , and $\mathcal{O}_{c'/p}$ the set of nodes (countries) sending outgoing links to c' or p . We take the in-degree as the representative characteristic of an importer or a product and investigate how the local logarithmic export value to or of a destination or product relates to the in-degree of the latter. To this end, we examine in this section the distributions of the in-degrees of the partner importers and the exported products of given exporter, which we call local in-degree distributions, and their correlations, if any, with local export values.

A. Local in-degree distribution and its moments

Like the local export distributions, the local in-degree distributions $P_c^{(\text{EI})}(k) = \frac{1}{q_c^{(\text{EI})}} \sum_{c' \in I_c^{(\text{EI})}} \delta(k_{c'}^{(\text{EI})} - k)$ and $P_c^{(\text{EP})}(k) = \frac{1}{q_c^{(\text{EP})}} \sum_{p \in I_c^{(\text{EP})}} \delta(k_p^{(\text{EP})} - k)$ show a large variation with exporters c , related to the variation of the mean local in-degree $\langle k \rangle_c$ [Figs. 2(a) and 2(b)]. Differently from the mean local logarithmic export, $\langle k \rangle_c$ decreases with the out-degree of the exporter. The standard deviations $\sigma_{k;c}^{(\text{EI})}$ and $\sigma_{k;c}^{(\text{EP})}$ increase with q_c . Such behaviors of the first two moments and the whole distributions imply that hub exporters select unpopular destinations or products as well as popular ones while nonhub exporters tend to select popular partners and products only. In the context of complex networks, hub exporters and popular

destinations or products have links to and from one another, forming a core, while nonhub exporters and unpopular destinations or products tend to have links only to and from the nodes in the core, viewed as periphery, which is characteristic of the core-periphery structure [27].

The skewness of the local in-degree distribution $\gamma_{k;c} = \frac{1}{q_c} \sum_{v \in I_c} (k_v - \langle k \rangle_c)^3 / \sigma_{k;c}^3$ increases with the out-degree of the exporter as shown in Figs. 2(c) and 2(d). See Appendix C as well for its robustness against time. As we discussed for the correlation of the skewness of local export and the out-degree of the exporter, the growth of $\gamma_{k;c}$ with q_c may be caused by the shift of local in-degree distributions in the k axis to the left as the out-degree of the exporter increases. Also the distributions are narrower for nonhub exporters, explaining the larger absolute values of $\gamma_{k;c}$ for nonhub exporters than for hub ones given that cubic fluctuations are normalized by the standard deviations in computing the skewness.

B. Correlation of local exports and in-degrees at the aggregate level

At the aggregate level, the local export values of destinations or products and their in-degrees are correlated. The Pearson correlation coefficient between the aggregated standardized values $\{\tilde{v}_{cv} | c, v \in I_c\}$ and $\{\tilde{k}_{cv} | c, v \in I_c\}$ is 0.51 and 0.29 for EI and EP, respectively, in year 1980. Here we introduced the standardized local in-degrees \tilde{k} for exporter c defined by

$$k_v = \langle k \rangle_c + \sigma_{k;c} \tilde{k}_{cv}. \quad (7)$$

The joint distributions of the standardized local in-degrees and export in Figs. 3(a) and 3(b) also support their positive correlations.

If the local export value to or of a destination or product is strongly correlated with the in-degree of the latter, then one can expect

$$\tilde{v}_{cv} \simeq \tilde{k}_{cv}. \quad (8)$$

Then the two skewnesses should be identical or at least similar $\gamma_{v;c} = \langle \tilde{v}^3 \rangle_c \simeq \langle \tilde{k}^3 \rangle_c = \gamma_{k;c}$. In Figs. 3(c) and 3(d), however, one can see that they are quite different and some of them have even different signs. Moreover, $\gamma_{v;c}^{(\text{EP})}$ shows a significant negative correlation with $\gamma_{k;c}^{(\text{EP})}$. Such discrepancy between the two skewnesses shows that Eq. (8) needs correction. In the next section, we present an improved version of Eq. (8) which relates the two skewnesses correctly and leads us to understand at a deeper level the skewness of local logarithmic exports.

IV. ROLES OF IN-DEGREES AND FLUCTUATIONS IN LOCAL EXPORTS

A. Decomposing local exports

The standardized local exports to or of destinations or products and in-degrees of destinations or products are shown as examples for two selected countries in Figs. 4(a) and 4(b). The slope of the best fitting line of the data points $\{(\tilde{k}_{cv}, \tilde{v}_{cv}) | v \in I_c\}$ corresponds to their Pearson correlation coefficient, which differs from exporter to exporter. Also we find that the data points fluctuate considerably deviating from the

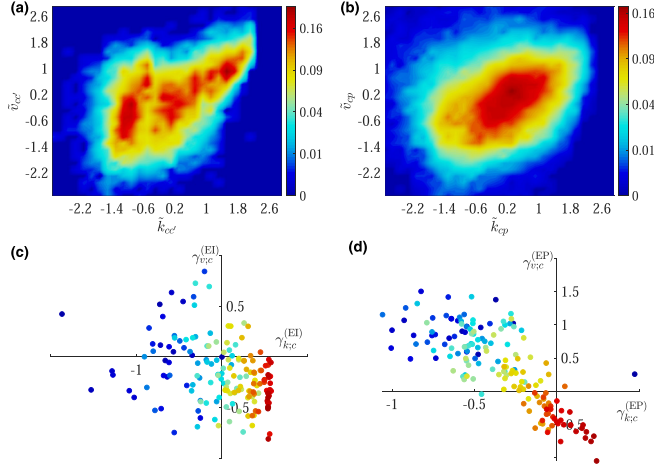


FIG. 3. Correlation between local in-degrees and local logarithmic exports and their skewness. (a) The joint probability density function (PDF) $P^{(EI)}(\tilde{k}, \tilde{v}) = \sum_{c,c'} A_{cc'} \delta(\tilde{k}_{cc'} - \tilde{k}) \delta(\tilde{v}_{cv} - \tilde{v}) / \sum_{c,c'} A_{cc'}$ of standardized local in-degrees and local logarithmic exports in the EI network. The orientation of the red (high density) region to the upper right indicates a positive correlation. (b) The joint PDF $P^{(EP)}(\tilde{k}, \tilde{v}) = \sum_{c,p} A_{cp} \delta(\tilde{k}_{cp} - \tilde{k}) \delta(\tilde{v}_{cp} - \tilde{v}) / \sum_{c,p} A_{cp}$. (c) Scatter plot of the skewness of local logarithmic exports to different destinations $\gamma_{v;c}^{(EI)}$ vs the skewness $\gamma_{k;c}^{(EI)}$ of local in-degrees in the EI network. Color varies with the out-degree of c as in Fig. 2. (d) Scatter plot of the skewness of local logarithmic exports of different products $\gamma_{v;c}^{(EP)}$ vs the skewness $\gamma_{k;c}^{(EP)}$ of local in-degrees in the EP network.

fitting line. Such fluctuation is different in magnitude and sign depending on the target—destination or product—for a given exporter. We take the sum of all the factors contributing to local exports except for the in-degrees as a *random noise* and represent the local export to or of a destination or product as

$$\tilde{v}_{cv} = \rho_c \tilde{k}_{cv} + \sqrt{1 - \rho_c^2} \xi_{cv}, \quad (9)$$

where $\rho_c^{(EI)}$ and $\rho_c^{(EP)}$ are the Pearson correlation coefficients between local exports and in-degrees given by

$$\rho_c = \langle \tilde{k} \tilde{v} \rangle_c = \frac{1}{q_c} \sum_{v \in I_c} \tilde{k}_v \tilde{v}_{cv}, \quad (10)$$

and $\xi_{cc'}$ and ξ_{cp} are random noises of zero mean and unit strength satisfying

$$\begin{aligned} \langle \xi \rangle_c &= \frac{1}{q_c} \sum_{v \in I_c} \xi_{cv} = 0, \\ \langle \tilde{k} \xi \rangle_c &= \frac{1}{q_c} \sum_{v \in I_c} \tilde{k}_v \xi_{cv} = 0, \\ \langle \xi^2 \rangle_c &= \frac{1}{q_c} \sum_{v \in I_c} \xi_{cv}^2 = 1. \end{aligned} \quad (11)$$

The relations in Eq. (11) are derived by using Eq. (10), $\langle \tilde{k} \rangle_c = \langle \tilde{v} \rangle_c = 0$, and $\langle \tilde{k}^2 \rangle_c = \langle \tilde{v}^2 \rangle_c = 1$. The graphical meaning and measurement of ρ 's and ξ 's are presented in Fig. 4(a). All the quantities in Eq. (9) can be measured empirically, which advances greatly our paper as will be presented in the remaining part. We expect that the statistics of random noise

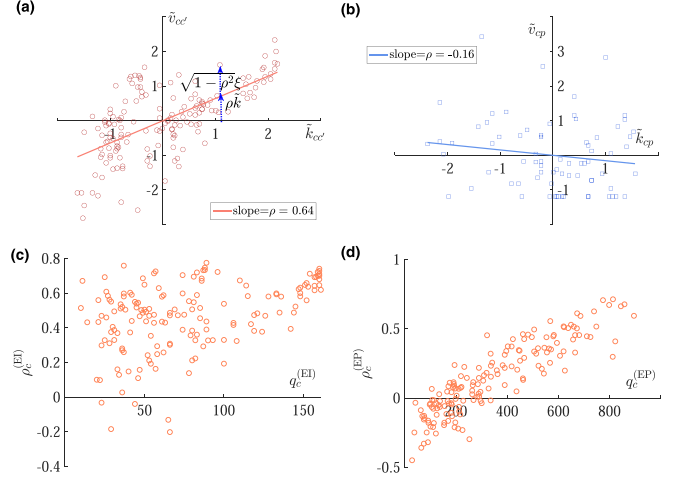


FIG. 4. Formulating local exports as functions of local in-degrees for given exporter. (a) Standardized logarithmic exports $\{\tilde{v}_{cc'} | c' \in I_c\}$ to different destinations c' 's from a selected country c vs their standardized in-degrees $\{\tilde{k}_{cc'} | c' \in I_c\}$ in 1980. The slope ρ_c of the fitting line is equal to their Pearson correlation coefficient. The deviation of $\tilde{v}_{cc'}$ from the prediction by the in-degree, $\rho_c \tilde{k}_{cc'}$, is set to $\sqrt{1 - \rho_c^2} \xi_{cc'}$, by which the random noise $\xi_{cc'}$ is defined. (b) $\{\tilde{v}_{cp}\}$ vs $\{\tilde{k}_{cp}\}$ for a selected country c in 1980. (c) Pearson correlation coefficients $\rho_c^{(EI)}$ between the logarithmic exports to different destinations from an exporter c and their in-degrees vs the out-degree of c . (d) Pearson correlation coefficients $\rho_c^{(EP)}$ between the logarithmic exports of different products from an exporter c and their in-degrees vs the out-degree of c .

ξ_{cv} encodes the relevant factors mainly stemming from the exporter c while the standardized in-degree \tilde{k} reflects those from the destination or product. They are independent of each other in that their covariance $\langle \tilde{k} \xi \rangle_c$ vanishes, but, as we will show below, they have nonzero higher-order joint cumulants.

From the dependence of ρ_c on the out-degree q_c , more significant in the EP than the EI network [Figs. 4(c) and 4(d)], we see that nonhub exporters determine their export values to or of destinations or products mainly by referring to their own circumstances encoded by the noise term while the local exports from hub exporters depend strongly on the popularity of destinations or products.

B. Joint third-order cumulants of in-degree and random noise

Using Eq. (9), one can evaluate the skewness of local exports as

$$\begin{aligned} \gamma_{v;c} &= \rho_c^3 \langle \tilde{k}^3 \rangle_c + 3\rho_c^2 \sqrt{1 - \rho_c^2} \langle \tilde{k}^2 \xi \rangle_c \\ &\quad + 3(1 - \rho_c^2) \rho_c \langle \tilde{k} \xi^2 \rangle_c + (1 - \rho_c^2)^{3/2} \langle \xi^3 \rangle_c, \end{aligned} \quad (12)$$

where four third-order cumulants, $\langle \tilde{k}^3 \rangle_c$, $\langle \tilde{k}^2 \xi \rangle_c$, $\langle \tilde{k} \xi^2 \rangle_c$, and $\langle \xi^3 \rangle_c$, can be measured empirically and capture asymmetry in the distribution of \tilde{k} 's and ξ 's. The four cumulants therefore represent four different modes to generate asymmetry in the local export distribution. We have seen that $\gamma_{k;c} = \langle \tilde{k}^3 \rangle_c$ depends on q_c in the opposite way to how $\langle \tilde{v}^3 \rangle_c$ does, which we now understand can happen if ρ_c is sufficiently small as $\langle \tilde{k}^3 \rangle_c$ is weighted by ρ_c^3 in Eq. (12). \tilde{k} and ξ have zero covariance $\langle \tilde{k} \xi \rangle_c = 0$ but we find empirically that they have

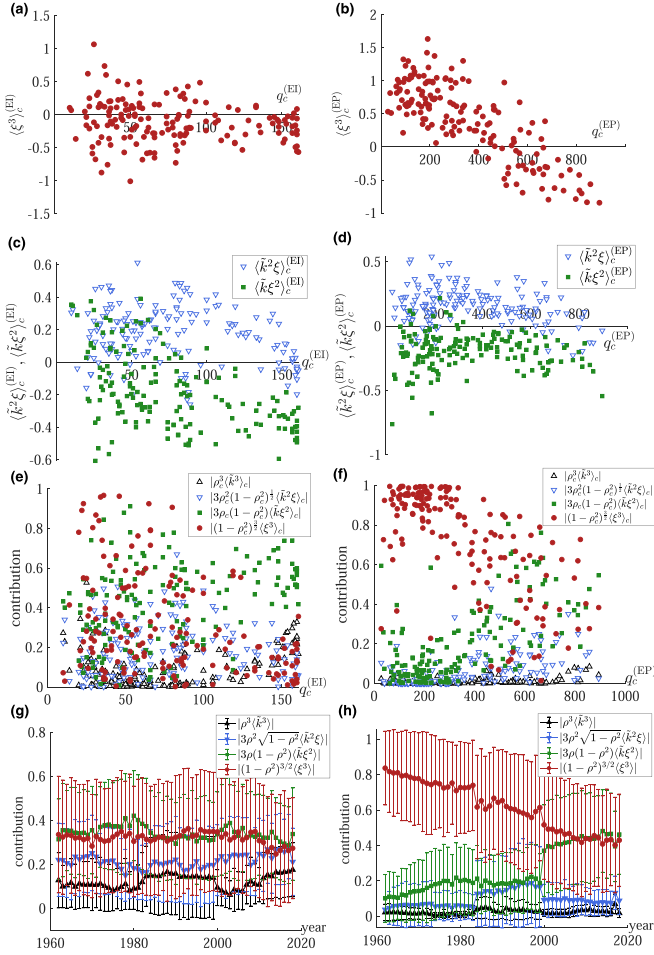


FIG. 5. Third-order cumulants of local in-degrees and random noises and their relative magnitudes. (a)–(f) are obtained from the data of year 1980. (a) The skewness of random noises $\langle \xi^3 \rangle_c^{(EI)}$ in the local exports from each exporter c to different destinations vs the out-degree $q_c^{(EI)}$. (b) The skewness of random noises $\langle \xi^3 \rangle_c^{(EP)}$ in the local exports from exporter c of different products vs the out-degree $q_c^{(EP)}$. (c) Third-order joint cumulants $\langle \tilde{k}^2 \xi \rangle_c^{(EI)}$ and $\langle \tilde{k} \xi^2 \rangle_c^{(EI)}$ vs $q_c^{(EI)}$. (d) $\langle \tilde{k}^2 \xi \rangle_c^{(EP)}$ and $\langle \tilde{k} \xi^2 \rangle_c^{(EP)}$ vs $q_c^{(EP)}$. (e) Relative magnitudes of the four third-order cumulants of local in-degrees and random noises in the EI network. (f) The same plot as (e) for the EP network. (g) The averages of the relative magnitudes of the third-order cumulants in the EI networks as functions of time. Error bars represent the standard deviations. (h) The same plot as (g) for the EP networks.

nonzero third-order joint cumulants $\langle \tilde{k}^2 \xi \rangle_c$ and $\langle \tilde{k} \xi^2 \rangle_c$ (Fig. 5 and Appendix D). In principle, $\langle \tilde{k}^2 \xi \rangle_c$ is positive (negative) when the data points of (\tilde{k}, \tilde{v}) tend to be located above (below) the fitting line $\tilde{v}^{(\text{fit})} = \rho \tilde{k}$ when \tilde{k}^2 is large. If (\tilde{k}, \tilde{v}) tends to spread more (less) as \tilde{k} increases, the joint cumulant $\langle \tilde{k}^2 \xi \rangle_c$ will be positive (negative).

The skewness of the random noise ξ is expected to make a considerable contribution to the whole skewness of local export, for the magnitudes and the q_c dependence of $\langle \xi^3 \rangle_c$ are empirically close to those of $\langle \tilde{v}^3 \rangle_c$ [Figs. 5(a) and 5(b)]. It suggests that the statistics of the random noise ξ , the nature of which mainly depends on the exporter itself, may govern the statistics of local exports. However,

the joint cumulants $\langle \tilde{k}^2 \xi \rangle_c$ and $\langle \tilde{k} \xi^2 \rangle_c$ are not negligible [Figs. 5(c) and 5(d)] but even make bigger contributions to the whole skewness $\langle \tilde{v}^3 \rangle_c$ for some exporters c [Figs. 5(e) and 5(f)]. We measure the contributions of the four third-order cumulants to the skewness of local exports by the ratio of the absolute values of the four terms in Eq. (12), $|\rho^3 \langle \tilde{k}^3 \rangle_c|$, $|3\rho_c^2 \sqrt{1 - \rho_c^2} \langle \tilde{k}^2 \xi \rangle_c|$, $|3(1 - \rho_c^2) \rho_c \langle \tilde{k} \xi^2 \rangle_c|$, $|(1 - \rho_c^2)^{3/2} \langle \xi^3 \rangle_c|$ to their sum $|\rho_c^3 \langle \tilde{k}^3 \rangle_c| + |3\rho_c^2 \sqrt{1 - \rho_c^2} \langle \tilde{k}^2 \xi \rangle_c| + |3(1 - \rho_c^2) \rho_c \langle \tilde{k} \xi^2 \rangle_c| + |(1 - \rho_c^2)^{3/2} \langle \xi^3 \rangle_c|$. Therefore the sum of those four contributions is 1 (100%) for each exporter. As expected, the contribution of the skewness of in-degrees is not significant, $9.0 \pm 9.0\%$ to $\gamma_{v;c}^{(EI)}$ and $1.0 \pm 1.7\%$ to $\gamma_{v;c}^{(EP)}$ in year 1980 [Figs. 5(e) and 5(f)]. On the other hand, the contribution of the skewness of the random noise $\langle \xi^3 \rangle_c$ is 31 ± 25 and $73 \pm 26\%$ to $\gamma_{v;c}^{(EI)}$ and $\gamma_{v;c}^{(EP)}$, respectively. Interestingly, the contribution of a joint third-order cumulant $\langle \tilde{k} \xi^2 \rangle_c$ makes a large contribution, 42 ± 20 and $20 \pm 19\%$ to $\gamma_{v;c}^{(EI)}$ and $\gamma_{v;c}^{(EP)}$, respectively, in year 1980. Across time [Figs. 5(g) and 5(h)], $\langle \xi^3 \rangle_c$ and $\langle \tilde{k} \xi^2 \rangle_c$ contribute about 32 and 34%, respectively, to $\gamma_{v;c}^{(EI)}$. For $\gamma_{v;c}^{(EP)}$, time variations are stronger than for $\gamma_{v;c}^{(EI)}$; $\langle \xi^3 \rangle_c$ makes the leading contribution 40–84% and the next-leading contribution is made by $\langle \tilde{k} \xi^2 \rangle_c$, ranging from 10 to 48%.

The empirically measured third-order cumulants allow us to better understand the skewness of local exports. $\langle \xi^3 \rangle_c$ and $\langle \tilde{k} \xi^2 \rangle_c$ together contribute 66 ± 4 and $88 \pm 5\%$ to $\gamma_{v;c}^{(EI)}$ and $\gamma_{v;c}^{(EP)}$, respectively. They tend to decrease with the out-degree of the exporter and are thus responsible for the out-degree dependence of $\gamma_{v;c}$. As such, it is mainly via the two cumulants $\langle \tilde{k} \xi^2 \rangle_c$ and $\langle \xi^3 \rangle_c$ that an exporter and its partners or products affect the skewness of their local logarithmic exports.

Moreover, the analysis based on Eq. (12) elucidates the nature of the export strategies of individual countries. As shown in Figs. 5(c) and 5(d), $\langle \tilde{k} \xi^2 \rangle_c$ is negative for most exporters. In 1980, 126 among 161 countries have $\langle \tilde{k} \xi^2 \rangle_c^{(EI)} \leq 0$ and 152 countries have $\langle \tilde{k} \xi^2 \rangle_c^{(EP)} \leq 0$. Such negative $\langle \tilde{k} \xi^2 \rangle_c$ is observed for many countries throughout the considered period (Appendix D), and means that the spread of local exports is weaker for larger \tilde{k} . As such, a country determines its export value to a popular destination or of a popular product by more referring to its in-degree than to or of an unpopular destination or product, for the latter of which the circumstances of the exporter itself can be important. Also we find that $\langle \tilde{k}^2 \xi \rangle_c$ is positive for many countries, which implies that the local export to or of quite a popular or quite an unpopular destination or product tends to be larger than the average. See Appendix D for graphical examples. All these results illuminate how the export values of an exporter country are allocated to destinations or products and, combined with the different connection patterns between hub and nonhub exporters, illuminate the universal and the nonuniversal aspects of the strategies of countries in international trade.

V. SUMMARY AND DISCUSSION

We have investigated how a country divides its export value over destinations and products by analyzing the local logarithmic export distributions. They are close to the Gaussian distribution, but exhibit significant skewness. We studied the

origin and the implications of such skewness for the export strategies of countries in international trade. The local export per destination or product is not constant but large for hub exporters and small for nonhub exporters, which contributes to the variation of the skewness of local exports with exporters. We presented the formula relating the local export value to or of a destination or product to the in-degree of the latter and a random noise, which are measurable empirically and thereby allow us to decipher the origin of the skewness of local exports and to understand quantitatively the export strategies of countries. We showed that the statistics of the random noise and its third-order correlation with in-degrees are responsible for the skewness of local exports and that the local exports to or of more popular destinations or products are governed more by their in-degrees than the export to or of less popular ones are.

Such quantitative dependence of local exports on the degrees of exporters, importers, and products and the involved randomness are not only of theoretical importance but also can be utilized as the analysis framework that helps countries evaluate and revise their trade portfolios. For instance, a country should compare its export distributions with those of countries having similar out-degrees not to draw any misleading conclusions from the comparison, given our discovery of the strong correlation between the out-degree and the statistics of local exports. Whether the export value of a specific product category is typical or atypical in the export portfolio of a country should be assessed by considering the in-degree of the product and the out-degree of the exporter country in the manner presented in this paper.

Our paper aimed at understanding the connection topology and the local link-weight distributions in weighted networks, here the world trade networks, and therefore our methods and results can be applied to other weighted complex networks including natural, informational, social, and artificial networks. The triangular relationships involving exporters, importers, and products can be represented by hypergraphs [28], and generalizing our analysis for them is in progress.

The analysis of the empirical probability distributions is demanded frequently across disciplines in this big data era, and our methods and results provide a relevant framework. While many of the distributions appear to take the Gaussian form, partly due to the central limit theorem, our paper suggests that their asymmetry or skewness may be significant and contain valuable information on the underlying principles of the considered systems.

ACKNOWLEDGMENTS

We thank Su Do Yi for valuable comments. This work was supported by a National Research Foundation of Korea grant funded by the Korean Government (Grant No. 2019R1A2C1003486) and a Korea Institute for Advanced Study individual grant (Grant No. CG079901).

APPENDIX A: DATASETS AND NETWORKS

In this paper, the NBER-UN dataset in the period $1962 \leq t \leq 1999$ [21] and the WTO dataset for $2000 \leq t \leq 2018$ [22] are compiled to obtain the set of export values $\{V_{cc'p}(t) | c, c' \in \mathcal{C}(t), p \in P(t)\}$ for $1962 \leq t \leq 2018$. Here $\mathcal{C}(t)$ is the set of

countries exporting or importing at least one product category and $P(t)$ is the set of trade product categories exported by at least one country in year t . To classify trade products, the four-digit standard international trade classification (SITC) [29] is used in Ref. [21]. The WTO datasets [22] are based on the harmonized system [30] as the product classification scheme, and we also select its four-digit version to make

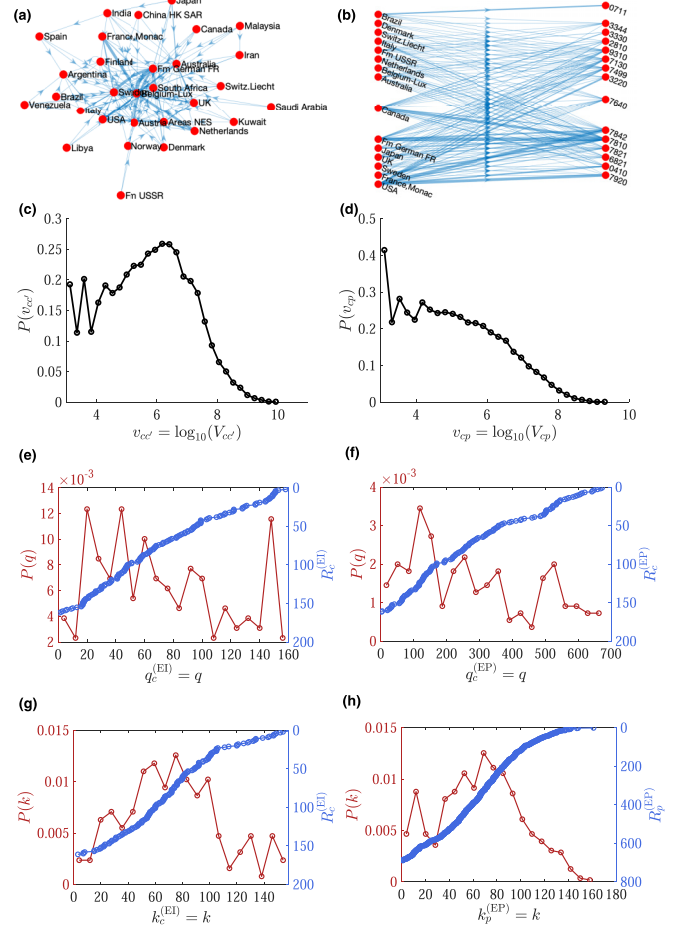


FIG. 6. Trade networks in 1970 and their global degree and link-weight distributions. (a) A subnetwork of the EI network in 1970, consisting of 30 countries having the largest (out + in)-degrees connected by 141 links of standardized local weight (logarithmic export value) larger than 1, i.e., $\tilde{v} > 1$. (b) A subnetwork of the EP network of 15 countries having the largest out-degrees and 15 products having the largest in-degrees connected by 133 links of $\tilde{v} > 1$. (c) Global distribution of the logarithm of exports $P^{(EI)}(v) = \frac{\sum_{c,c'} A_{cc'} \delta(v_{cc'} - v)}{\sum_{c,c'} A_{cc'}}$ in the EI network with $A_{cc'}$ the adjacency matrix. (d) Global distribution of the logarithm of exports $P^{(EP)}(v)$ in the EP network. (e) Global out-degree distribution $P^{(EI)}(q) = \frac{\sum_c \delta(q_c^{(EI)} - q)}{\sum_c 1}$ and the plot of the rank $R_c^{(EI)}$ vs the out-degree $q_c^{(EI)}$ of each exporter c in the EI network. (f) Global out-degree distribution $P^{(EP)}(q)$ and the plot of the rank $R_c^{(EP)}$ vs the out-degree $q_c^{(EP)}$ of exporter c in the EP network. (g) Global in-degree distribution $P^{(EI)}(k) = \frac{\sum_c \delta(k_c^{(EI)} - k)}{\sum_c 1}$ and the plot of the rank $R_c^{(EI)}$ vs the in-degree $k_c^{(EI)}$ of importer c in the EI network. (h) Global in-degree distribution $P^{(EP)}(k) = \frac{\sum_p \delta(k_p^{(EP)} - k)}{\sum_p 1}$ and the plot of the rank $R_p^{(EP)}$ vs the in-degree $k_p^{(EP)}$ of product p in the EP network.

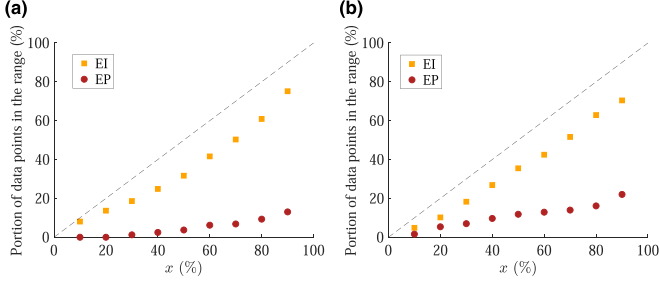


FIG. 7. Significance of skewness of local logarithmic exports. (a) The fraction of exporters the true skewness of which remains in the middle $x(\%)$ range of the random-version skewness in 1980. Squares and circles are for the EI and the EP network. (b) The same plot as (a) in year 2010.

the total number of product categories similar to that of the four-digit SITC. These two schemes of product classification can be mapped to each other [31].

We remark that the datasets in the period 1984 to 1999 contain some virtual product categories inserted to fill some gaps in the total values of trade across different levels of classification [21]. The NBER-UN datasets record trade over 1000 US dollars. Yet these differences depending on data source and time period do not affect the main results of the present paper.

In Fig. 6, parts of the EI and the EP network in year 1970 are shown along with their statistical features.

APPENDIX B: SIGNIFICANCE OF SKEWNESS AND ITS CORRELATION WITH OUT-DEGREE

To check the significance of the skewness of local logarithmic exports, we compare the true skewness with the values of the skewness from $\mathcal{N} = 1000$ realizations of weight-shuffled EI and EP networks. The out-degrees of each exporter

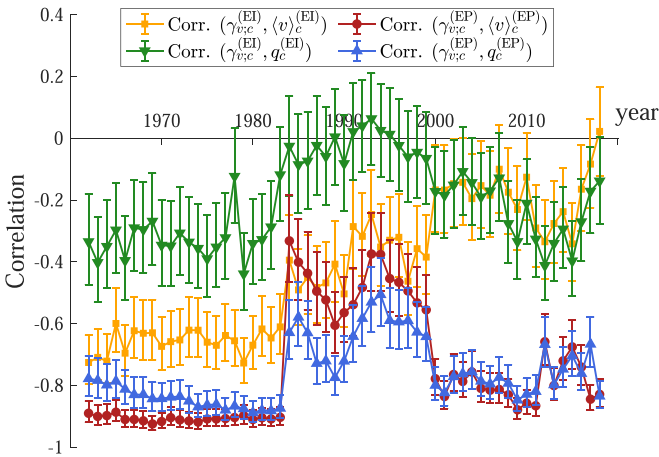


FIG. 8. Time-dependent correlation of the skewness with the average of local logarithmic exports and with the out-degree of the exporter. The Pearson correlation coefficient between the skewness of local logarithmic exports $\gamma_{v;c}^{(EI,EP)}$ and the out-degree $q_c^{(EI,EP)}$ of the exporter or the average local logarithmic export $\langle v \rangle_c^{(EI,EP)}$ as a function of time t .

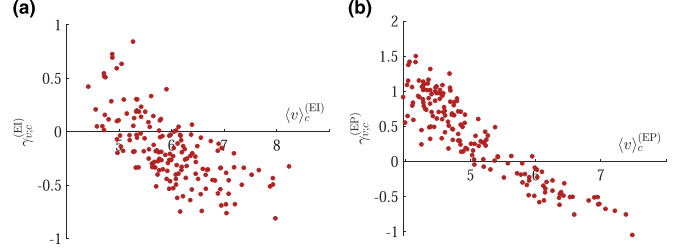


FIG. 9. Plot of the skewness vs the average of local logarithmic exports. (a) Plot of the skewness of local logarithmic exports $\gamma_{v;c}^{(EI)}$ vs the average of local logarithmic exports $\langle v \rangle_c^{(EI)}$ in the EI network in year 1980. (b) The same plot as (a) in the EP network.

$q_c^{(EI)}$ and $q_c^{(EP)}$ do not change by this randomization. Then, for each exporter c we are given \mathcal{N} values of the random-version skewness $\{\gamma_{v;c}^{(EI;random)(\ell)} | \ell = 1, 2, \dots, \mathcal{N}\}$ in the EI network and $\{\gamma_{v;c}^{(EP;random)(\ell)} | \ell = 1, 2, \dots, \mathcal{N}\}$ in the EP network, and we check whether the true skewness $\gamma_{v;c}^{(EI)}$ and $\gamma_{v;c}^{(EP)}$ belong to the middle $x(\%)$ range of those random-version skewness values. Repeating this for all exporters, we obtain the fraction $y(\%)$ of exporters the true skewnesses of which are included in their respective middle $x(\%)$ ranges of the random-version skewness. If y as a function of x remains close to x , then one can say that true skewness is not distinguished from the random-version ones and therefore the true skewness is not a significant one. On the other hand, if y is far smaller than x , then it means that the true skewness tends to be outside the range expected in the randomized networks, demonstrating the significance of the observed skewness in a sense that it is hardly expected in randomized networks.

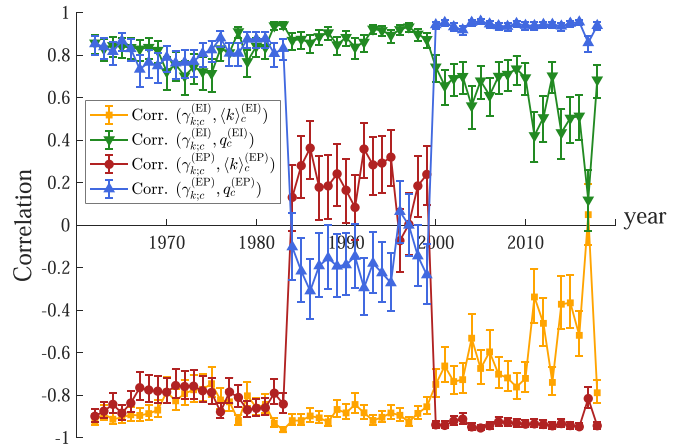


FIG. 10. Correlation of skewness of local in-degrees with the out-degree of the exporter and the average local in-degree. The Pearson correlation coefficient between the skewness of local in-degrees $\gamma_{k;c}^{(EI,EP)}$ and the out-degree $q_c^{(EI,EP)}$ of the exporter or the average local in-degree $\langle k \rangle_c^{(EI,EP)}$ as a function of time t . Correlations are observed robustly in the studied period, except for the correlations of $\gamma_{k;c}^{(EP)}$ with $q_c^{(EP)}$ and with $\langle k \rangle_c^{(EP)}$ which change sign in the period 1984–1999 presumably owing to the virtual product categories added artificially (see Appendix A).

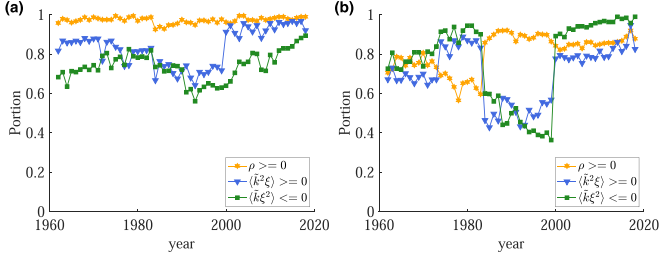


FIG. 11. Robustness against time of the sign of the correlation coefficient and third-order cumulants. (a) The portions of exporters showing a non-negative correlation coefficient between local logarithmic exports and local in-degrees ($\rho_c^{(EI)} \geq 0$), a non-negative cumulant $\langle \tilde{k}^2 \xi \rangle_c^{(EI)} \geq 0$, and another nonpositive cumulant $\langle \tilde{k} \xi^2 \rangle_c^{(EI)} \leq 0$ of local in-degrees and random noises are shown, respectively, for the EI network as functions of time. (b) The same plots as (a) for the EP network.

We found that the latter is the case. In Fig. 7, we present the plot of y versus x for the EI and the EP network in year 1980 and 2010. y is far below the line $y = x$ in case of the EP network. Also for the EI network, y is smaller than x though the difference is not so impressive as for the EP network. These results suggest that the empirically observed skewness of local logarithmic exports is significant.

APPENDIX C: SIGNIFICANCE OF CORRELATIONS

Here we consider the significance of the correlations between the skewness of local logarithmic exports and other properties. First, we measure the correlation of the skewness with the out-degree and with the average of local logarithmic exports of the exporter and present the results as functions of time in Fig. 8. The scatter plots of the skewness versus the average local logarithmic exports in 1980 are given in Fig. 9, demonstrating their negative correlations. The skewness of local in-degrees is also correlated with the out-degree and with the average of local in-degrees of the exporter, which is measured for every year and presented in Fig. 10.

APPENDIX D: APPENDIX D: SIGNS OF THE THIRD-ORDER CUMULANTS

Our paper shows that the signs of the correlation coefficient ρ_c and of two third-order cumulants $\langle \tilde{k}^2 \xi \rangle_c$ and $\langle \tilde{k} \xi^2 \rangle_c$ appearing in Eq. (12) are not randomly distributed for different exporters but seem fixed; the portions of the exporter countries having $\rho_c \geq 0$, $\langle \tilde{k}^2 \xi \rangle_c \geq 0$, and $\langle \tilde{k} \xi^2 \rangle_c \leq 0$, respectively, are larger than expected on a random basis, respectively, as shown in Fig. 11. The signs of the four third-order cumulants,

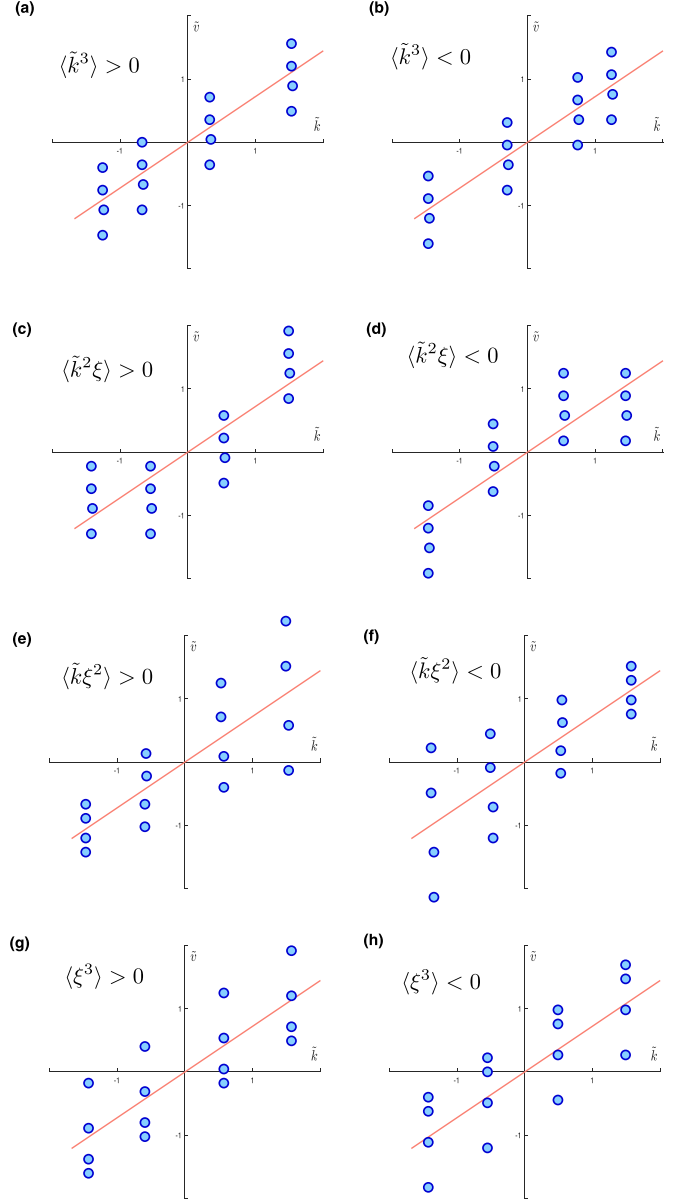


FIG. 12. Schematic distribution of $\{(\tilde{k}, \tilde{v})\}$ corresponding to positive or negative third-order cumulants. Each panel shows an example of the distribution of the data points of standardized local exports and in-degrees $\{(\tilde{k}, \tilde{v})\}$ exhibiting a given positive or negative cumulant. The solid line represents $\tilde{v} = \rho \tilde{k}$ with $\rho = 1$ for simplicity. Therefore the deviation of the data points from the line represents the random noise ξ . Note that $\langle \tilde{k} \rangle = \langle \xi \rangle = 0$ and $\langle \tilde{k}^2 \rangle = \langle \xi^2 \rangle = 1$.

$\langle \tilde{k}^3 \rangle_c$, $\langle \tilde{k}^2 \xi \rangle_c$, $\langle \tilde{k} \xi^2 \rangle_c$, and $\langle \xi^3 \rangle_c$, in Eq. (12) have implications for the behavior of the standardized local export as a function of the standardized local in-degree as shown in Fig. 12.

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