Editors' Suggestion

# Applicability of the absence of equilibrium in quantum system fully coupled to several fermionic and bosonic heat baths

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(Received 17 September 2020; accepted 4 January 2021; published 29 January 2021)

The time evolution of an occupation number is studied for a fermionic or bosonic oscillator linearly fully coupled to several fermionic and bosonic heat baths. The influence of the characteristics of thermal reservoirs of different statistics on the nonstationary population probability is analyzed at large times. Applications of the absence of equilibrium in such systems for creating a dynamic (nonstationary) memory storage are discussed.

DOI: 10.1103/PhysRevE.103.012137

## I. INTRODUCTION

Quantum systems are never completely isolated, and they interact with a large number of degrees of freedom of the surrounding environment. The coupling of a quantum system to a heat bath usually induces its evolution toward asymptotic equilibrium imposed by the complexity of the heat bath(s). In practice, a quantum system is often coupled to a few reservoirs [1–10]. In Refs. [11,12], it has been illustrated that the system linearly fully coupled to several baths of different statistics (fermionic and bosonic) might never reach a stationary asymptotic limit. This absence of equilibrium at large times can be used in some applications, e.g., in communication lines, quantum computers, and other modern quantum devices. In the present paper, we study the time evolution of occupation numbers of fermionic (two-level system) and bosonic oscillators embedded in fermionic and bosonic heat baths. A system fully coupled to two heat baths with the same or a different quantum nature is described here using the non-Markovian master-equation and quantum Langevin approaches [12], and taking into consideration the Ohmic dissipation with Lorentzian cutoffs [13–17]. The full coupling contains the resonant (the rotating wave approximation) and nonresonant terms [17]. The environmental effects on a quantum system could keep this system in a certain state or provide it with some specific properties.

## II. MODEL

### A. Hamiltonian

The Hamiltonian of the total system (the quantum system plus several heat baths "λ," $\lambda = 1, \dots, N_b$) is written as [12]

$$H = H_c + \sum_{\lambda=1}^{N_b} H_\lambda + \sum_{\lambda=1}^{N_b} H_{c,\lambda}, \tag{1}$$

where

$$H_c = \hbar\omega a^\dagger a \tag{2}$$

is the Hamiltonian of the isolated system being either a fermionic (two-level system) or bosonic oscillator with frequency $\omega$, and

$$H_\lambda = \sum_i \hbar\omega_{\lambda,i} c^\dagger_{\lambda,i} c_{\lambda,i}$$

are the Hamiltonians of the thermal baths. When we write down the creation/annihilation operators $a^+/a$ ($c^\dagger_{\lambda,i}/c_{\lambda,i}$), we mean the creation/annihilation operators of the transition with the corresponding energy $\hbar\omega$ ($\hbar\omega_{\lambda,i}$). So, each fermionic transition operator $a^+$ or $c^\dagger_{\lambda,i}$ is the product of operators of the creation and annihilation of a fermion in the excited and ground states, respectively. There is only the conversion of excitation quanta from the fermionic system to the bosonic ones or *vice versa* in our formalism. The value of $N_b$ is the number of heat baths. Each heat bath "λ" is modeled by the assembly of independent fermionic or bosonic oscillators labeled in both cases by "$i$" with frequencies $\omega_{\lambda,i}$. For the FC coupling between the system and heat baths, the interaction Hamiltonians $H_{c,\lambda}$ are

$$H_{c,\lambda} = \sum_i \alpha_{\lambda,i}(a^\dagger + a)(c^\dagger_{\lambda,i} + c_{\lambda,i}). \tag{3}$$

The real constants $\alpha_{\lambda,i}$ determine the coupling strengths. The interaction Hamiltonian (3) is linear in the system and bath operators. It has important consequences on the dynamics of the system by altering the effective collective potential and by allowing energy to be exchanged with the thermal reservoirs, thereby allowing the system to attain some equilibrium with the heat baths.

Here, the system and heat baths have fermionic or bosonic statistics. So, the creation and annihilation operators of the

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system and heat baths satisfy the commutation or anticommutation relations

$$aa^\dagger - \varepsilon_a a^\dagger a = 1, \quad a^\dagger a^\dagger - \varepsilon_a a^\dagger a^\dagger = aa - \varepsilon_a aa = 0,$$
$$c_{\lambda,i}c^\dagger_{\lambda,i} - \varepsilon_\lambda c^\dagger_{\lambda,i}c_{\lambda,i} = 1, \quad c^\dagger_{\lambda,i}c^\dagger_{\lambda,i} - \varepsilon_\lambda c^\dagger_{\lambda,i}c^\dagger_{\lambda,i} = c_{\lambda,i}c_{\lambda,i} - \varepsilon_\lambda c_{\lambda,i}c_{\lambda,i} = 0, \tag{4}$$

where $\varepsilon_a$ and $\varepsilon_\lambda$ are equal to 1 (−1) for the bosonic (fermionic) system and bosonic (fermionic) heat baths, respectively.

### B. Master equation for the occupation number of a quantum system

Employing the Hamiltonian (1) for the fermionic and bosonic systems, we deduce the equations of motion for the occupation number,

$$\begin{aligned}\frac{da^\dagger(t)a(t)}{dt} &= \frac{i}{\hbar}\sum_{\lambda,i}\alpha_{\lambda,i}[a(t) - a^\dagger(t)][c^\dagger_{\lambda,i}(t) + c_{\lambda,i}(t)]\\ &= \frac{i}{\hbar}\sum_{\lambda,i}\alpha_{\lambda,i}[c^\dagger_{\lambda,i}(t)a(t) - a^\dagger(t)c_{\lambda,i}(t)\\ &\quad + a(t)c_{\lambda,i}(t) - a^\dagger(t)c^\dagger_{\lambda,i}(t)]. \end{aligned}\tag{5}$$

For the operators $c^\dagger_{\lambda,i}(t)a(t)$ and $a(t)c_{\lambda,i}$ in Eq. (5), we derive the following equations:

$$\begin{aligned}\frac{dc^\dagger_{\lambda,i}a}{dt} &= i(\omega_{\lambda,i} - \omega)c^\dagger_{\lambda,i}a + \frac{i}{\hbar}\alpha_{\lambda,i}[a^\dagger a + aa][1 - (1-\varepsilon_\lambda)c^\dagger_{\lambda,i}c_{\lambda,i}]\\ &\quad - \frac{i}{\hbar}\sum_{\lambda',i'}\alpha_{\lambda',i'}[c^\dagger_{\lambda,i}c^\dagger_{\lambda',i'} + c^\dagger_{\lambda,i}c_{\lambda',i'}][1 - (1-\varepsilon_a)a^+a], \end{aligned}\tag{6}$$

$$\begin{aligned}\frac{dac_{\lambda,i}}{dt} &= -i(\omega_{\lambda,i} + \omega)ac_{\lambda,i} - \frac{i}{\hbar}\alpha_{\lambda,i}[a^\dagger a + aa][1 - (1-\varepsilon_\lambda)c^\dagger_{\lambda,i}c_{\lambda,i}]\\ &\quad - \frac{i}{\hbar}\sum_{\lambda',i'}\alpha_{\lambda',i'}[c_{\lambda,i}c^\dagger_{\lambda',i'} + c_{\lambda,i}c_{\lambda',i'}][1 - (1-\varepsilon_a)a^+a]. \end{aligned}\tag{7}$$

Substituting the formal solutions

$$\begin{aligned}c^\dagger_{\lambda,i}(t)a(t) &= e^{i(\omega_{\lambda,i}-\omega)t}c^\dagger_{\lambda,i}(0)a(0) + \frac{i}{\hbar}\alpha_{\lambda,i}\int_0^t d\tau\, e^{i(\omega_{\lambda,i}-\omega)[t-\tau]}[a^\dagger(\tau)a(\tau) + a(\tau)a(\tau)][1 - (1-\varepsilon_\lambda)c^\dagger_{\lambda,i}(\tau)c_{\lambda,i}(\tau)]\\ &\quad - \frac{i}{\hbar}\sum_{\lambda',i'}\alpha_{\lambda',i'}\int_0^t d\tau\, e^{i(\omega_{\lambda,i}-\omega)[t-\tau]}[c^\dagger_{\lambda,i}(\tau)c^\dagger_{\lambda',i'}(\tau) + c^\dagger_{\lambda,i}(\tau)c_{\lambda',i'}(\tau)][1 - (1-\varepsilon_a)a^+(\tau)a(\tau)],\\ a(t)c_{\lambda,i}(t) &= e^{-i(\omega_{\lambda,i}+\omega)t}a(0)c_{\lambda,i}(0) - \frac{i}{\hbar}\alpha_{\lambda,i}\int_0^t d\tau\, e^{-i(\omega_{\lambda,i}+\omega)[t-\tau]}[a^\dagger(\tau)a(\tau) + a(\tau)a(\tau)][1 - (1-\varepsilon_\lambda)c^\dagger_{\lambda,i}(\tau)c_{\lambda,i}(\tau)]\\ &\quad - \frac{i}{\hbar}\sum_{\lambda',i'}\alpha_{\lambda',i'}\int_0^t d\tau\, e^{-i(\omega_{\lambda,i}+\omega)[t-\tau]}[c_{\lambda,i}(\tau)c^\dagger_{\lambda',i'}(\tau) + c_{\lambda,i}(\tau)c_{\lambda',i'}(\tau)][1 - (1-\varepsilon_a)a^+(\tau)a(\tau)] \end{aligned}\tag{8}$$

of Eqs. (6) and (7) [also the solutions of the operators $a^\dagger(t)c_{\lambda,i}(t)$ and $a^\dagger(t)c^\dagger_{\lambda,i}(t)$] in Eq. (5) and taking the initial conditions $\langle c^\dagger_{\lambda,i}(0)a(0)\rangle = \langle a(0)c_{\lambda,i}(0)\rangle = \langle a^\dagger(0)c_{\lambda,i}(0)\rangle = \langle a^\dagger(0)c^\dagger_{\lambda,i}(0)\rangle = 0$ (the symbol $\langle\cdots\rangle$ denotes the averaging over the whole system of heat baths and oscillator), and assuming that $\langle aa\rangle = \langle a^\dagger a^\dagger\rangle = \langle c^\dagger_{\lambda,i}c^\dagger_{\lambda',i'}\rangle = \langle c_{\lambda',i'}c_{\lambda,i}\rangle = 0$, $\langle c^\dagger_{\lambda,i}c_{\lambda',i'}\rangle = \langle c^\dagger_{\lambda,i}c_{\lambda,i}\rangle = n_{\lambda,i}\delta_{\lambda,\lambda'}\delta_{i,i'}$ (the heat baths consist of independent oscillators), and $\langle a^\dagger ac^\dagger_{\lambda,i}c_{\lambda,i}\rangle = n_a n_{\lambda,i}$ (the mean-field approximation), we obtain the master-equation for the occupation number $n_a = \langle a^\dagger a\rangle$ of the oscillator ($a = f$ and $a = b$ for fermionic and bosonic systems, respectively) [12]:

$$\begin{aligned}\frac{dn_a(t)}{dt} &= \sum_{\lambda,i}\int_0^t ds\{W^-_{\lambda,i}(t-s)[\bar{n}_a(s)n_{\lambda,i}(s) - n_a(s)\bar{n}_{\lambda,i}(s)]\\ &\quad + W^+_{\lambda,i}(t-s)[\bar{n}_a(s)\bar{n}_{\lambda,i}(s) - n_a(s)n_{\lambda,i}(s)]\}, \end{aligned}\tag{9}$$

where

$$\begin{aligned}W^-_{\lambda,i} &= \frac{2\alpha^2_{\lambda,i}}{\hbar^2}\cos([\omega - \omega_{\lambda,i}][t-s]),\\ W^+_{\lambda,i} &= \frac{2\alpha^2_{\lambda,i}}{\hbar^2}\cos([\omega + \omega_{\lambda,i}][t-s]). \end{aligned}\tag{10}$$

Here, $\bar{n}_a(t) = 1 + \varepsilon_a\langle a^\dagger a\rangle$ and $\bar{n}_{\lambda,i}(t) = 1 + \varepsilon_\lambda\langle c^\dagger_{\lambda,i}c_{\lambda,i}\rangle$. One can rewrite Eq. (9) as

$$\begin{aligned}\frac{dn_a}{dt} &= \int_0^t d\tau\{W_+(t-\tau)\bar{n}_a(\tau) - W_-(t-\tau)n_a(\tau)\}\\ &= \int_0^t d\tau\{W_+(t-\tau) - W(t-\tau)n_a(\tau)\}, \end{aligned}\tag{11}$$

where

$$\begin{aligned} W_+ &= \sum_\lambda W_+^{(\lambda)} \\ &= \sum_{\lambda,i} [W_{\lambda,i}^-(t-\tau)n_{\lambda,i}(\tau) + W_{\lambda,i}^+(t-\tau)\bar{n}_{\lambda,i}(\tau)], \\ W_- &= \sum_\lambda W_-^{(\lambda)} \\ &= \sum_{\lambda,i} [W_{\lambda,i}^-(t-\tau)\bar{n}_{\lambda,i}(\tau) \\ &\quad + W_{\lambda,i}^+(t-\tau)n_{\lambda,i}(\tau)]. \end{aligned} \tag{12}$$

Here, $W = W_- - \varepsilon_a W_+$. The coefficient $W_+$ ($W_-$) defines the rate of occupation (leaving) of the state "$a$" in the open quantum system. The ratio between $W_+$ and $W_-$ characterizes the rate of equilibrium. The occupation number reaches the equilibrium value if the ratio of $W_+$ and $W_-$ has asymptotic at $t \to \infty$.

As shown in Refs. [11,18], for the fermionic ($a = f$) or bosonic ($a = b$) oscillator (with the renormalized frequency $\Omega$) linearly fully coupled to $N = N_f + N_b = N_{\bar{a}} + N_a$ heat baths with different statistics ($N_{\bar{a}}$ Fermi and $N_a$ Bose baths or *vice versa*), the master Eqs. (9) or (11) can be mapped to a simple diffusion equation,

$$\frac{dn_a(t)}{dt} = -2\lambda(t)n_a(t) + 2D(t), \tag{13}$$

provided that

$$\begin{aligned} W - 2\varepsilon_a \sum_{\lambda=1}^{N_{\bar{a}}} W_+^{(\lambda)} &= 2\dot{\lambda}(t) - 4\lambda(t)\lambda(t), \\ \sum_{\lambda=1}^{N_{\bar{a}}} W_+^{(\lambda)} + \sum_{\lambda=N_{\bar{a}}+1}^{N} W_+^{(\lambda)} &= 2\dot{D}(t) - 4\lambda(t)D(t), \end{aligned} \tag{14}$$

$$\lambda(t) = p\lambda_{\bar{a}}(t) + (1-p)\lambda_a(t) - 2\varepsilon_a \sum_{\lambda=1}^{N_{\bar{a}}} D_{\bar{a}_\lambda}(t), \tag{15}$$

and

$$D(t) = \sum_{\lambda=1}^{N_{\bar{a}}} D_{\bar{a}_\lambda}(t) + \sum_{\lambda=N_{\bar{a}}+1}^{N} D_{a_\lambda}(t). \tag{16}$$

Here, we have introduced the time-dependent friction $\lambda(t)$ and diffusion $D(t)$ coefficients (see Appendix). If $\bar{a} = f$ ($\bar{a} = b$) and $\bar{a}_\lambda = f_\lambda$ ($\bar{a}_\lambda = b_\lambda$), then $a = b$ ($a = f$) and $a_\lambda = b_\lambda$ ($a_\lambda = f_\lambda$), respectively. The value of $p$ is defined as $p = \sum_{\lambda=1}^{N_{\bar{a}}} \alpha_\lambda / \sum_{\lambda=1}^{N} \alpha_\lambda$, where $\alpha_\lambda$ is the coupling strength between the system and heat bath labeled by $\lambda$ ($\lambda = 1, \dots, N$). The time-dependent friction $\lambda_f(t)$ [$\lambda_b(t)$] and partial diffusion $D_{f_\lambda}(t)$ [$D_{b_\lambda}(t)$] coefficients for the fermionic (bosonic) system coupled with $N$ fermionic (bosonic) heat baths are given in Appendix. In the case of the non-Markovian dynamics, the baths affect the system and *vice versa*.

Using $D_{f_\lambda}(t)$, $D_{b_\lambda}(t)$, $\lambda_f(t)$, and $\lambda_b(t)$ from Eqs. (A10), (A11), and the solution

$$n_a(t) = e^{-2\int_0^t d\tau\lambda(\tau)} \left\{ n_a(0) + 2\int_0^t d\tau D(\tau) e^{2\int_0^\tau d\tau'\lambda(\tau')} \right\} \tag{17}$$

of Eq. (13), one can calculate the time-dependent occupation number of the quantum system.

### C. Asymptotic occupation number

Because the friction coefficient $\lambda_b(t)$ does not converge to a stationary value at $t \to +\infty$ (Fig. 1) [8,19], an asymptotic stationary value of occupation number $n_a$ in Eq. (13) can be reached if the condition

$$\frac{1}{p} \sum_{\lambda=1}^{N_{\bar{a}}} I_{\bar{a}_\lambda}(\infty) = \frac{\frac{1}{1-p} \sum_{\lambda=N_{\bar{a}}+1}^{N} I_{a_\lambda}(\infty)}{1 + \frac{2\varepsilon_a}{1-p} \sum_{\lambda=N_{\bar{a}}+1}^{N} I_{a_\lambda}(\infty)} \tag{18}$$

is satisfied [11,12]. In other cases, the occupation number remains oscillating at large time (Fig. 2) because the friction $\lambda_b(t)$ and, correspondingly, diffusion coefficient oscillate as a function of time (Fig. 1) [8,19]. To obtain Eq. (18), the relation $D_{a_\lambda} = \lambda_a I_{a_\lambda}$ is used at large time ($\Omega t \gg 1$).

The physical problem discussed here is considerably simplified when the $N$ baths have the same quantum nature. Then, the asymptotic occupation number is always stationary (Fig. 2) and is given by

$$\begin{aligned} n_a(\infty) &= \lim_{t\to\infty} \frac{D(t)}{\lambda(t)} = \lim_{t\to\infty} \frac{\sum_{\lambda=1}^{N} D_{a_\lambda}(t)}{\lambda_a(t)} \\ &= \sum_{\lambda=1}^{N} I_{a_\lambda}(\infty) = I_a(\infty) \end{aligned} \tag{19}$$

in the case when all reservoirs and system oscillators have the same quantum nature ($p = 0$, $N_{\bar{a}} = 0$, $N_a = N$), or

$$n_a(\infty) = \frac{\sum_{\lambda=1}^{N_{\bar{a}}} I_{\bar{a}_\lambda}(\infty)}{1 - 2\varepsilon_a \sum_{\lambda=1}^{N_{\bar{a}}} I_{\bar{a}_\lambda}(\infty)} = \frac{I_{\bar{a}}(\infty)}{1 - 2\varepsilon_a I_{\bar{a}}(\infty)} \tag{20}$$

in the case when all reservoirs have the same quantum nature ($\bar{a} = b$ or $f$), which differs from the case of the system oscillator ($a = f$ or $b$) ($1 - p = 0$, $N_{\bar{a}} = N$, $N_a = 0$). Equations (19) and (20) generalize the equations given in Ref. [18] for a single bath. If the baths have the same temperatures, then the asymptotic occupation number differs in general from the Fermi-Dirac or Bose-Einstein occupation number. Only in the Markovian weak-coupling limit and in the case of the same temperature $T_\lambda = T$ of all baths, Eqs. (19) and (20) are reduced to the usual Bose-Einstein and Fermi-Dirac thermal distributions $n_a(\infty) = [\exp(\hbar\omega/kT) - \varepsilon_a]^{-1}$ and the system has a thermal equilibrium.

## III. CALCULATED RESULTS FOR A FERMIONIC OR BOSONIC OSCILLATOR COUPLED WITH FERMIONIC AND BOSONIC BATHS IN THE CASE OF OHMIC DISSIPATION WITH LORENTZIAN CUTOFFS

In all of the figures in this paper presented for the fermionic or bosonic oscillator with two baths of the same or different statistics, we set $\gamma_1/\Omega = 10$, $\gamma_2/\Omega = 15$, $\alpha_1 = 0.1$, $\alpha_2 = 0.05$, $g_0 = \alpha_1 + \alpha_2 = 0.15$, $kT_1/(\hbar\Omega) = 1$, and $kT_2/(\hbar\Omega) = 0.1$. The values of $\gamma_{1,2}/\Omega$ are taken to hold the conditions $\gamma_{1,2} \gg \Omega$: the non-Markovian quantum Langevin approach can be applied when the system is slow in comparison to the relaxation times of the heat baths. The occupation numbers and the diffusion and friction coefficients depend on the

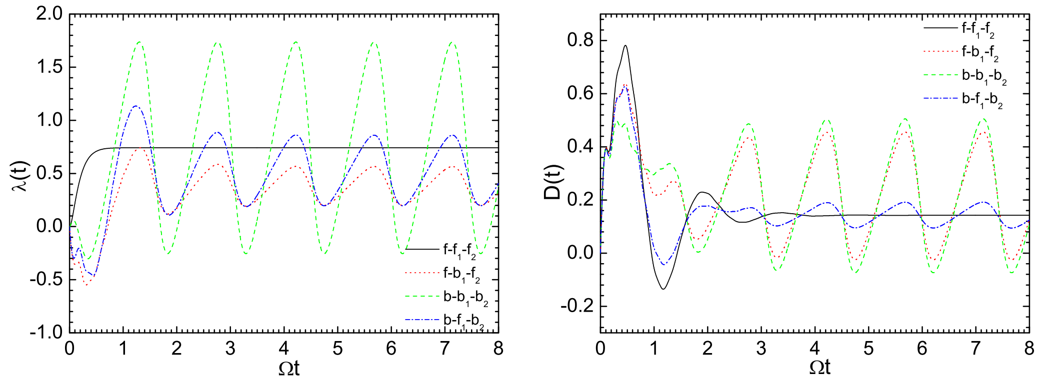


FIG. 1. The calculated dependencies of the friction and diffusion coefficients on time $t$ for the fermionic-fermionic-fermionic ($f$-$f_1$-$f_2$, solid line), bosonic-bosonic-bosonic ($b$-$b_1$-$b_2$, dashed line), mixed fermionic-bosonic-fermionic ($f$-$b_1$-$f_2$, dotted line), and bosonic-fermionic-bosonic ($b$-$f_1$-$b_2$, dash-dotted line) systems.

values of oscillator frequency $\omega$, coupling strengths $\alpha_1$, $\alpha_2$, inverse memory times $\gamma_{1,2}$, and heat bath temperatures $T_{1,2}$ (see Appendix). A zero chemical potential is assumed here. The values of $\alpha_1$ and $\alpha_2$ are chosen to have the realistic values of friction coefficients, which are known from the microscopic calculations. Indeed, these coupling strengths provide almost the same friction coefficient for relative motion of two nuclei as in Ref. [20]. As an example of a bosonic system, the atomic or nuclear molecular state can be considered. The bound or quasibound particle (an electron in the trap or a nucleon in the isomeric state) can be taken as an example of a fermionic system. The electromagnetic and temperature fields or the phonon bath can be treated as the bosonic baths. Free electrons and inclusion in the compound can act as the fermionic baths.

For the fermionic-fermionic-fermionic ($f$-$f_1$-$f_2$), bosonic-bosonic-bosonic ($b$-$b_1$-$b_2$), mixed fermionic-bosonic-fermionic ($f$-$b_1$-$f_2$), and bosonic-fermionic-bosonic ($b$-$f_1$-$b_2$) systems, the time-dependent friction and diffusion coefficients are shown in Fig. 1. The diffusion and friction coefficients are equal to zero at initial time. As seen, the time dependencies of these coefficients are not the same for the different systems. For the $f$-$f_1$-$f_2$ system, the friction and diffusion coefficients reach their asymptotic values relatively quickly (the transient time for the friction is quite short, $\Omega t \leqslant 0.5$), whereas in the case of $b$-$b_1$-$b_2$ and mixed $f$-$b_1$-$f_2$, $b$-$f_1$-$b_2$ systems, they oscillate with the same period of oscillations. The amplitudes of oscillations for the system with two bosonic baths are larger than those for the systems with one bosonic bath. For the $b$-$b_1$-$b_2$ system, the friction and diffusion coefficients oscillate in the phase and, as a result, the occupation number has an asymptotic limit (Fig. 2). In contrast, for the mixed systems $f$-$b_1$-$f_2$ and $b$-$f_1$-$b_2$, the occupation number oscillates around a certain average value at large times, so it has no asymptotic limit. For both systems, the periods of oscillations are the same. The occupation number for the fermionic oscillator oscillates with a larger amplitude than the one for the bosonic oscillator (Fig. 2). The absolute value of oscillations depends mainly on the coupling constants. The times to reach the asymptotic oscillations are almost the same for these systems.

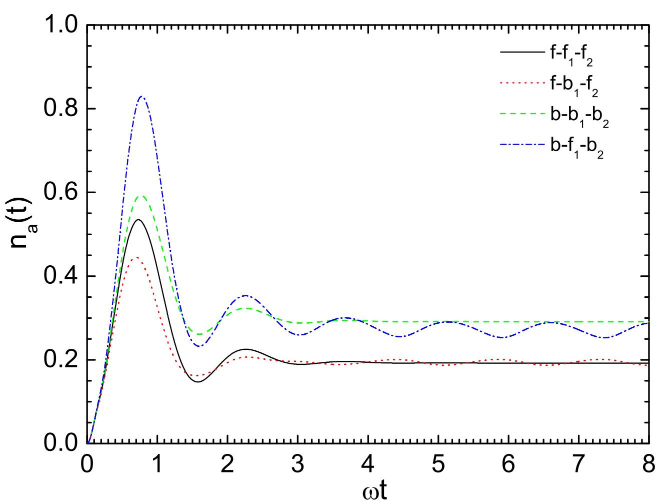


FIG. 2. For the fermionic-fermionic-fermionic ($f$-$f_1$-$f_2$, solid line), bosonic-bosonic-bosonic ($b$-$b_1$-$b_2$, dashed line), mixed fermionic-bosonic-fermionic ($f$-$b_1$-$f_2$, dotted line), and bosonic-fermionic-bosonic ($b$-$f_1$-$b_2$, dash-dotted line) systems, the calculated dependencies of the average occupation numbers on time $t$. The plots correspond to the initially unoccupied $n_a(t=0)=0$ system state.

In the case when a fermionic bath coexists with a bosonic bath, at large times the influence of the thermostats is minimal and reversible—it takes energy from the system and gives the same amount of energy back. As a result, the population of the excited state(s) decreases and then increases on the same level independent of the environment. As shown in Fig. 3, the period of oscillations of $n_a(t)$ at large $t$ depends on the frequency of the oscillator and, accordingly, carries information about the system. At $\Omega/\gamma_1 > 0.1$, the frequency of asymptotic oscillations is proportional to the oscillator frequency. Since the asymptotic oscillations of the occupation number depend on the oscillator frequency, this gives a new opportunity to control these oscillations by changing the oscillator frequency. For example, in this way one can control the amplification or attenuation of signal transmission. Since the asymptotic oscillations are independent of the medium, one can unambiguously judge the population of the excited state of a two-level system, which, for example, is important in quantum computers. In this case, it is necessary to ensure a sufficient degree of metastability of the excited states of the quantum register. These states must have a sufficiently large lifetime that determines their relaxation to the ground state

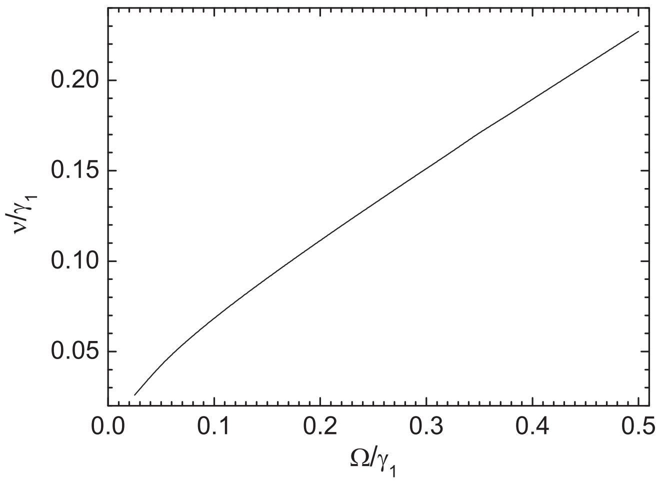


FIG. 3. For the mixed fermionic-bosonic-fermionic ($f$-$b_1$-$f_2$) and bosonic-fermionic-bosonic ($b$-$f_1$-$b_2$) systems, the calculated dependence of the frequency of oscillations of $n_a(t)$ at large $t$ on the oscillator renormalized frequency $\Omega$. For two systems, the results of calculations coincide.

due to dissipative processes. Such a system with nonstationary asymptotics can be used as a dynamic (nonstationary) memory system because the information about some properties of the system [population of excitation state(s) and frequency] is preserved at large times. So, we suggest to store information by using nonstationary memory systems. This idea can be effective, because such systems will be stable under external conditions.

## IV. CONCLUSIONS

In conclusion, for a bosonic or fermionic oscillator fully coupled with mixed bosonic-fermionic heat baths, the absence of the equilibrium asymptotic of the occupation number was predicted. At large times, the period of oscillations of the occupation number depends on the frequency of the oscillator and, accordingly, carries information about the system. It is an example of nonstationary (dynamic) memory storage. Each frequency corresponds to a certain state and can lead to the control of these states for recording data in quantum computers and increasing channels and speeds of communication. As shown, this behavior is also expected for other systems (not necessarily an oscillator fully coupled with several fermionic and bosonic heat baths) in which the asymptotic occupation number is a periodic function of time.

## ACKNOWLEDGMENTS

G.G.A. and N.V.A. were supported by Ministry of Science and Higher Education of the Russian Federation (Moscow, Contract No. 075-10-2020-117). V.V.S. acknowledges the Alexander von Humboldt—Stiftung (Bonn). D.L. thanks the CNRS for finacial support through the 80Prime program. This work was partly supported by the IN2P3(France)-JINR(Dubna) Cooperation Programme and DFG (Bonn, Grant No. Le439/16).

## APPENDIX: EXPLICIT EXPRESSIONS FOR FRICTION AND DIFFUSION COEFFICIENTS OF A FERMIONIC (BOSONIC) OSCILLATOR WITH SEVERAL FERMIONIC (BOSONIC) HEAT BATHS

Let us consider the case when all $N$ heat baths and system oscillators with frequency $\omega$ are either all bosonic or all fermionic. For these systems, the details of the procedure for obtaining the occupation number of a system are given in Ref. [8]. Here, we directly write the final expression for the time dependence of the occupation number:

$$n_a(t) = n_a(0)|A(t)|^2 + [1 + \varepsilon_a n_a(0)]|B(t)|^2 + I_a(t), \quad \text{(A1)}$$

where $I_a(t) = \sum_\lambda I_{a_\lambda}(t)$ and

$$I_{a_\lambda}(t) = \frac{\alpha_\lambda \gamma_\lambda^2}{\pi}\int_0^\infty dw \frac{w}{\gamma_\lambda^2 + w^2}\big[n^{(\lambda)}(w)|M(w,t)|^2 + \big[1+\varepsilon_\lambda n^{(\lambda)}(w)\big]|N(w,t)|^2\big], \quad \text{(A2)}$$

$$\begin{aligned}
A(t) &= \frac{1}{2}\sum_{k=1}^{N_0}\xi_k e^{s_k t}(s_k - s_0)\left\{2s_k - i[\Omega+\omega] - 2is_k\sum_{\lambda=1}^{N}\frac{\alpha_\lambda\gamma_\lambda}{s_k+\gamma_\lambda}\right\}\prod_{\mu=1}^{N}(s_k+\gamma_\mu)\\
&= i\sum_{\lambda=1}^{N}\alpha_\lambda\gamma_\lambda^2\sum_{k=1}^{N_0}\xi_k e^{s_k t}(s_k - s_0)\frac{s_k - i\omega}{s_k + i\omega}\prod_{\mu=1,\mu\neq\lambda}^{N}(s_k+\gamma_\mu),\\
B(t) &= \frac{i}{2}\sum_{k=1}^{N_0}\xi_k e^{s_k t}(s_k - s_0)\left\{\Omega - \omega + 2s_k\sum_{\lambda=1}^{N}\frac{\alpha_\lambda\gamma_\lambda}{s_k+\gamma_\lambda}\right\}\prod_{\mu=1}^{N}(s_k+\gamma_\mu)\\
&= i\sum_{\lambda=1}^{N}\alpha_\lambda\gamma_\lambda^2\sum_{k=1}^{N_0}\xi_k e^{s_k t}(s_0 - s_k)\prod_{\mu=1,\mu\neq\lambda}^{N}(s_k+\gamma_\mu),\\
N(w,t) &= \sum_{k=0}^{N_0}\xi_k e^{s_k t}(is_k - \omega)\prod_{\mu=1}^{N}(s_k+\gamma_\mu),\\
M(w,t) &= -\sum_{k=0}^{N_0}\xi_k e^{s_k t}(is_k + \omega)\prod_{\mu=1}^{N}(s_k+\gamma_\mu),
\end{aligned} \quad \text{(A3)}$$

where

$$\xi_k = \prod_{i=0,\ i\neq k}^{N_0} \frac{1}{s_k - s_i} \tag{A4}$$

with $s_0 = -iw$ and the roots $s_k$, $k = 1, \dots, N_0$, of the $N_0 = N + 2$ order polynomial:

$$\left[s^2 + \omega^2 - 2\omega \sum_{\lambda=1}^{N} \frac{\alpha_\lambda \gamma_\lambda^2}{s + \gamma_\lambda}\right] \prod_{\mu=1}^{N} (s + \gamma_\mu) = 0. \tag{A5}$$

Here,

$$\Omega = \omega - 2\sum_{\lambda=1}^{N} \alpha_\lambda \gamma_\lambda \tag{A6}$$

is the renormalized frequency and $\varepsilon_\lambda$ is equal to 1 ($-1$) for the bosonic (fermionic) heat bath "λ."

In Eq. (A2), $n^{(\lambda)}(w) = \{\exp[\hbar w/(kT_\lambda)] - \varepsilon_\lambda\}^{-1}$ is equilibrium Fermi-Dirac (Bose-Einstein) distribution of the fermionic (bosonic) heat bath "λ." The $T_\lambda$ is the initial thermodynamic temperature of the corresponding heat bath. Here, we introduce the spectral density $\rho_\lambda(w)$ of the heat-bath excitations, which allows us to replace the sum over $i$ by integral over the frequency $w$: $\sum_i \cdots \to \int_0^\infty dw\, \rho_\lambda(w) \cdots$. For all baths, we consider the following spectral function [13]:

$$\frac{\alpha_{\lambda,i}^2}{\hbar^2 w_{\lambda,i}} \to \frac{\rho_\lambda(w)\alpha_{\lambda,w}^2}{\hbar^2 w} = \frac{1}{\pi}\alpha_\lambda \frac{\gamma_\lambda^2}{\gamma_\lambda^2 + w^2}, \tag{A7}$$

where the memory time $\gamma_\lambda^{-1}$ of dissipation is inverse to the bandwidth of the heat-bath excitations, which are coupled to the collective system. This is the Ohmic dissipation with the Lorentzian cutoff (Drude dissipation). The relaxation time of the heat bath should be much less than the characteristic collective time. The similarity of expressions for the occupation numbers for fermionic and bosonic systems results from the similarity of the equations of motion for creation and annihilation operators [19,21].

Making a derivative of Eq. (A1) in $t$ and simple but tedious algebra, we derive the following differential equation for the occupation number:

$$\frac{dn_a(t)}{dt} = -2\lambda_a(t)n_a(t) + 2D_a(t), \tag{A8}$$

where

$$\lambda_a(t) = -\frac{1}{2}\frac{d}{dt}\ln[|A(t)| + \varepsilon_a|B(t)|^2] \tag{A9}$$

and

$$\begin{aligned} D_a(t) &= \sum_{\lambda=1}^{N} D_{a_\lambda}(t) = \lambda_a(t)[|B(t)|^2 + I_a(t)] \\ &\quad + \frac{1}{2}\frac{d}{dt}[|B(t)|^2 + I_a(t)], \\ D_{a_\lambda}(t) &= \lambda_a(t)[J_\lambda(t) + I_{a_\lambda}(t)] \\ &\quad + \frac{1}{2}\frac{d}{dt}[J_\lambda(t) + I_{a_\lambda}(t)] \end{aligned} \tag{A10}$$

are the time-dependent friction and diffusion coefficients, respectively. The following decomposition $|B(t)|^2 = \sum_\lambda J_\lambda(t)$ is used in Eq. (A10). Here, $\lambda_a(t=0) = D_a(t=0) = 0$. Therefore, we have obtained the equation for $n_a(t)$, which is local in time. In the case of constant transport coefficients, this equation describes the Markovian dynamics, i.e., the evolution of $n_a(t)$ is independent of the past. In Eq. (A8), the transport coefficients explicitly depend on time, and the non-Markovian effects are taken into consideration through this time dependence [8]. The non-Markovian feature of Eq. (A8) is well seen at $D_a = 0$. In this case, $n_a(t) \sim \exp(-2\int_0^t \lambda_a(t)dt)$, i.e., the occupation number depends on the time dependence of $\lambda_a$. Because $A(\infty) = B(\infty) = 0$ [8], the appropriate asymptotic equilibrium distribution

$$n_a(\infty) = \frac{D_a(\infty)}{\lambda_a(\infty)} = \sum_\lambda I_{a_\lambda}(\infty) \tag{A11}$$

is achieved [see Eqs. (A1) and (A8)]. Using Eqs. (A1) and (A2), the asymptotic values of $|M(w,t)|^2$ and $|N(w,t)|^2$ are found. With these values, we obtain from (A2)

$$\begin{aligned} I_{a_\lambda}(t\to\infty) &= \frac{\alpha_\lambda\gamma_\lambda^2}{\pi}\int_0^\infty dw \frac{w}{\gamma_\lambda^2 + w^2}\{[\omega + w]^2 n^{(\lambda)}(w) \\ &\quad + [\omega - w]^2[1 + \varepsilon_\lambda n^{(\lambda)}(w)]\} \\ &\quad \times \frac{\prod_{\mu=1}^{N_b}(\gamma_\mu^2 + w^2)}{\prod_{k=1}^{N_0}(s_k^2 + w^2)}. \end{aligned} \tag{A12}$$

The specific quantum nature of the baths enters into the diffusion coefficient through the appearance of occupation probabilities. The asymptotic diffusion and friction coefficients are related by the well-known fluctuation-dissipation relations connecting diffusion and damping constants. Fulfillment of the fluctuation-dissipation relations means that we have correctly defined the dissipative kernels in the non-Markovian equations of motion. In the Markovian limit (weak couplings and high temperatures), the asymptotic occupation number is

$$n_a(\infty) = \frac{1}{g_0}\sum_\lambda \alpha_\lambda n^{(\lambda)}(\omega),$$

where $g_0 = \sum_\lambda \alpha_\lambda$.

[1] E. Stefanescu and W. Scheid, Physica A 374, 203 (2007); E. Stefanescu, W. Scheid, and A. Sandulescu, Ann. Phys. 323, 1168 (2008); E. Stefanescu, Prog. Quantum Electron. 34, 349 (2010).

[2] L. Lamata, D. R. Leibrandt, I. L. Chuang, J. I. Cirac, M. D. Lukin, V. Vuletić, and S. F. Yelin, Phys. Rev. Lett. 107, 030501 (2011).

[3] G.-D. Lin and L.-M. Duan, New J. Phys. 13, 075015 (2011).

[4] A. Nunnenkamp, J. Koch, and S. M. Girvin, New J. Phys. **13**, 095008 (2011).
[5] A. Majumdar, D. Englund, M. Bajcsy, and J. Vucković, Phys. Rev. A **85**, 033802 (2012).
[6] S. D. Bennett, N. Y. Yao, J. Otterbach, P. Zoller, P. Rabl, and M. D. Lukin, Phys. Rev. Lett. **110**, 156402 (2013).
[7] M. Chen and J. Q. You, Phys. Rev. A **87**, 052108 (2013); C.-K. Chan, G.-D. Lin, S. F. Yelin, and M. D. Lukin, *ibid.* **89**, 042117 (2014).
[8] A. A. Hovhannisyan, V. V. Sargsyan, G. G. Adamian, N. V. Antonenko, and D. Lacroix, Phys. Rev. E **97**, 032134 (2018).
[9] M. Mwalaba, I. Sinayskiy, and F. Petruccione, Phys. Rev. A **99**, 052102 (2019).
[10] D. Lacroix, V. V. Sargsyan, G. G. Adamian, N. V. Antonenko, and A. A. Hovhannisyan, Phys. Rev. A **102**, 022209 (2020).
[11] A. A. Hovhannisyan, V. V. Sargsyan, G. G. Adamian, N. V. Antonenko, and D. Lacroix, Physica A **545**, 123653 (2020).
[12] A. A. Hovhannisyan, V. V. Sargsyan, G. G. Adamian, N. V. Antonenko, and D. Lacroix, Phys. Rev. E **101**, 062115 (2020).
[13] K. Lindenberg and B. J. West, *The Nonequilibrium Statistical Mechanics of Open and Closed Systems* (VCH, New York, 1990); Phys. Rev. A **30**, 568 (1984).
[14] F. Haake and R. Reibold, Phys. Rev. A **32**, 2462 (1985).
[15] V. V. Dodonov and V. I. Man'ko, *Density Matrices and Wigner Functions of Quasiclassical Quantum Systems*, Proceedings of the Lebedev Physics Institute of Sciences Vol. 167, edited by A. A. Komar (Nova Science, Commack, NY, 1987).
[16] A. Isar, A. Sandulescu, H. Scutaru, E. Stefanescu, and W. Scheid, Int. J. Mod. Phys. E **3**, 635 (1994).
[17] Th. M. Nieuwenhuizen and A. E. Allahverdyan, Phys. Rev. E **66**, 036102 (2002).
[18] V. V. Sargsyan, A. A. Hovhannisyan, G. G. Adamian, N. V. Antonenko, and D. Lacroix, Physica A **505**, 666 (2018).
[19] V. V. Sargsyan, D. Lacroix, G. G. Adamian, and N. V. Antonenko, Phys. Rev. A **95**, 032119 (2017).
[20] K. Washiyama and D. Lacroix, Phys. Rev. C **78**, 024610 (2008); K. Washiyama, D. Lacroix, and S. Ayik, *ibid.* **79**, 024609 (2009).
[21] V. V. Sargsyan, G. G. Adamian, N. V. Antonenko, and D. Lacroix, Phys. Rev. A **90**, 022123 (2014); **96**, 012114 (2017).