Universal two-level quantum Otto machine under a squeezed reservoir

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We study an Otto heat machine whose working substance is a single two-level system interacting with a cold thermal reservoir and with a squeezed hot thermal reservoir. By adjusting the squeezing or the adiabaticity parameter (the probability of transition) we show that our two-level system can function as a universal heat machine, either producing net work by consuming heat or consuming work that is used to cool or heat environments. Using our model we study the performance of these machine in the finite-time regime of the isentropic strokes, which is a regime that contributes to make them useful from a practical point of view.

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I. INTRODUCTION

Classical heat machines convert thermal resources into work and vice versa. For example, the heat engine draws heat from a hot reservoir, uses part of that heat to perform mechanical work, and discards the rest in a cold reservoir. The refrigerator, on the other hand, uses mechanical work to remove heat from a cold reservoir and discards it in a hot one. A third kind of heat machine, the heather, uses the mechanical work to heat one, usually the cold, or both reservoirs; see Fig. 1. The cyclic heat machines are the paradigm for these comparative studies, whose efficiency η for heat engines and coefficient of performances $\mathcal{C}$ for refrigerators are related by

$$\mathcal{C} = \frac{1}{\eta} - 1. \tag{1}$$

Equation (1) answers the following question: Given a cyclic heat machine operating reversibly, if work W is extracted with efficiency η, what is the performance coefficient $\mathcal{C}$ if that same heat machine operates in a reverse cycle consuming work, $-W$? The heat machine efficiency and performance coefficient are bounded by the Carnot efficiency and performance coefficient, which, for thermal reservoirs, are only attained in quasistatic or reversible cycles. It is currently a subject of intense study to compare heat machines running on purely classic resources with heat machines running on some kind of genuinely quantum resource, as, for example, coherence [1–8], entanglement [9–13], as well as exploring the finite dimension of Hilbert space [14–16].

A heat machine whose working substance is a quantum system is often called a quantum heat machine. Potential technological applications of quantum heat machines ranges from heat transport in nanodevices [17,18] to biological process control [19,20], among others [21]. One kind of quantum heat machine widely addressed by researchers in the field of quantum thermodynamics is the quantum Otto heat machine (QOHM) [15,22–29]. The QOHM consists of two isochoric strokes, one with the working substance coupled to the cold thermal reservoir and the other coupled to the hot thermal reservoir, and two isentropic strokes, in which the working substance is disconnected from the thermal reservoirs and evolves unitarily. In the past few years, nonthermal reservoirs have also been used in the theoretical and experimental study of QOHM; for instance, squeezed thermal reservoirs [30–35] and reservoirs at apparent negative temperature [36]. These unconventional quantum engines have drawn attention due to the promising gains in engine efficiency and power.

In this paper we study a minimal model of QOHM which is universal [16,37] in the sense that it can works either as a heat engine or a refrigerator or, yet, a heater (see Fig. 1), depending on the control parameter. Our model consists of a two-level system (TLS) driven by an external laser source and interacting with a cold thermal reservoir and with a squeezed hot thermal reservoir, which will be assumed as a free resource [33]. The relation between η and $\mathcal{C}$ [Eq. (1)] will be generalized to include the squeezing parameter, which will be our parameter of control in building these types of heat machines. Using our model, we are able to study both the efficiency and performance of this TLS machine at finite-time regime of the isentropic strokes, which contributes to making them useful from the point of view of applicability.

This paper is organized as follows. In Sec. II we present our model for a universal QOHM, which consists of a TLS as the working substance under a cold thermal and a squeezed hot thermal reservoir. In Sec. III we present the results of our calculation for the heats exchanged with the reservoirs and the work done or performed by the QOHM and the corresponding efficiency η and performance coefficient $\mathcal{C}$. In Sec. IV we generalize the relation between η and $\mathcal{C}$ given by Eq. (1) to include both the squeezed thermal reservoir and finite-time isentropic strokes. Finally, in Sec. V we present our conclusions.

II. UNIVERSAL QOHM

We are going to consider the TLS implementation of a four-stroke quantum Otto cycle. The four-stroke to our QOHM are the following:

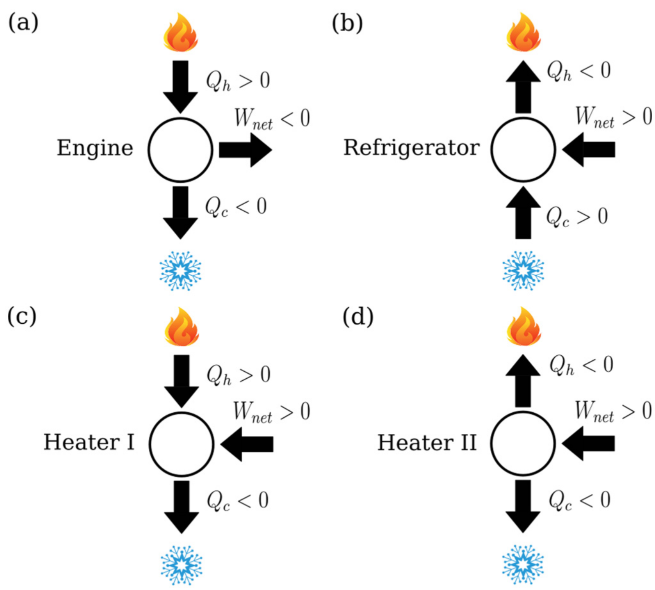

FIG. 1. The four types of heat machines: (a) a heat engine, which draws heat from the hot reservoir and dumps it in the cold one, performing work; (b) a refrigerator, on which work is performed and used to draw heat from the cold reservoir and pump it to the hot one; (c) a heater that uses the work done to pump heat from the hot to the cold reservoir; and (d) a second type of heat machine that uses the work done to heat two environments at once.

(i) Cooling stroke. In this first step, the TLS is weakly coupled to the cold thermal reservoir until thermalized, when it can be described by the Gibbs state $\rho_1^G = e^{-\beta_c H_c}/\mathrm{Tr}(e^{-\beta_c H_c})$, where H_c is the Hamiltonian and $\beta_c = 1/k_B T_c$, where k_B is the Boltzmann constant and T_c is the reservoir temperature. The TLS Hamiltonian remains unchanged during the thermalization process and has the form $H_c = \frac{1}{2}\hbar\omega_c\sigma_x$, with $\hbar$, ω_c, and σ_x being the reduced Planck constant, the angular frequency, and the x Pauli matrix, respectively.

(ii) Expansion stroke. In this stage the TLS evolves unitarily from the state ρ_1^G (at time $t = 0$) to $\rho_2 = U\rho_1^G U^\dagger$(at time $t = \tau$), where U is the unitary operator accounting for the external driven of the TLS Hamiltonian, which varies from $H_c = \frac{1}{2}\hbar\omega_c\sigma_x$ to $H_h = \frac{1}{2}\hbar\omega_h\sigma_y$, with ω_h being an angular frequency higher than ω_c (corresponding to the energy gap expansion). For our purpose, it is not necessary to specify the unitary operator U.

(iii) Heating stroke. This is the stage where the TLS is weakly coupled to the hot squeezed thermal reservoir until reaching the stead state $\rho_3^S = S\rho_3^G S^\dagger$. The reservoir squeezing changes the thermalized state $\rho_3^G = e^{-\beta_h H_h}/\mathrm{Tr}(e^{-\beta_h H_h})$ according to operator $S = (\mu|-_y\rangle\langle +_y| + \nu|+_y\rangle\langle -_y|)/\sqrt{\mu^2+\nu^2}$, where $\mu = \cosh r$ and $\nu = \sinh r$ [38]. The state $|-_y\rangle$ ($|+_y\rangle$) is the ground (excited) state of the TLS at this stage and r is the *squeezing parameter*. As in the cooling stroke, here the TLS Hamiltonian $H_h = \frac{1}{2}\hbar\omega_h\sigma_y$ also remains unchanged.

(iv) Compression stroke. This stage is accomplished by reversing the expansion protocol (ii), such that the TLS Hamiltonian is changed from $H_h = \frac{1}{2}\hbar\omega_h\sigma_y$ to $H_c = \frac{1}{2}\hbar\omega_c\sigma_x$, corresponding to the energy gap compression, making the TLS state to evolve unitarily from to ρ_3^S to $\rho_4 = U^\dagger\rho_3^S U$.

The quantities we are interested in is the efficiency η to the heat engine as well as the coefficient of performance $\mathcal{C}$ to the refrigerator. The engine efficiency is given by $\eta = -W_{\rm net}/Q_h$, where Q_h is the average heat absorbed from the hot reservoir and $W_{\rm net}$ is the average net work extracted from the engine, while the coefficient of performance, on the other hand, is given by $\mathcal{C} = Q_c/W_{\rm net}$, where Q_c is the average heat extracted from the cold thermal reservoir by performing an average net work $W_{\rm net}$ on the TLS. In order to determine both the efficiency and the coefficient of performance we resort to the first law of thermodynamics, together with work and heat definitions, as follows. According to the first law of thermodynamics, the change in the internal energy of a given system during a thermodynamic process can be decomposed into work W and heat Q. In quantum thermodynamics the first law is written as $\Delta E = Q + W$, where ΔE is the average change in the system internal energy, which is given by $E = \mathrm{Tr}(\rho H)$. Heat and work averages, in turn, are $Q = \int dt\,\mathrm{Tr}[(d\rho/dt)H]$ and $W = \int dt\,\mathrm{Tr}[\rho(dH/dt)]$ [39,40], respectively. Therefore, according to the first law, from the quantum Otto cycle described in (i)–(iv), we can promptly see that $W = 0$ and $\Delta E = Q$ in the heating and cooling strokes, and $Q = 0$ and $\Delta E = W$ in the expansion and compression strokes. This considerably simplifies the calculations, as compared to other cyclic machines, such as Carnot or Stirling, where work and heat are simultaneously exchanged.

Aiming at possible applications in nuclear magnetic resonance [27,36], we use here the following parameters in our numerical calculations: $\omega_c = 2\pi$ kHz, $\omega_h = 3.5\omega_c$, $\beta_c = 1/(10\text{ peV})$, and $\beta_h = 0.7\beta_c$.

III. RESULTS

With the information provided in (i)–(iv) strokes and definitions of work and heat as given in Sec. II, we can obtain the average heats exchanged with the cold and hot reservoirs as well as the average net work:

$$Q_c = -\frac{1}{2}\hbar\omega_c(\tanh\theta_c - \zeta\tanh\theta_h) - \hbar\xi\zeta\omega_c\tanh\theta_h, \quad (2)$$

$$Q_h = \frac{1}{2}\hbar\omega_h(\tanh\theta_c - \zeta\tanh\theta_h) - \hbar\xi\omega_h\tanh\theta_c, \quad (3)$$

and

$$W_{\rm net} = -\frac{1}{2}\hbar(\omega_h - \omega_c)(\tanh\theta_c - \zeta\tanh\theta_h) + \hbar\xi(\omega_h\tanh\theta_c + \zeta\omega_c\tanh\theta_h), \quad (4)$$

where $\theta_{c(h)} = \frac{1}{2}\beta_{c(h)}\hbar\omega_{c(h)}$, $\zeta = 1/(\mu^2+\nu^2)^2$, and $\xi = |\langle\pm_y|U|\mp_x\rangle|^2 = |\langle\pm_x|U^\dagger|\mp_y\rangle|^2$. The parameter ξ, which gives the probability of transition between the two levels of the TLS, is the *adiabaticity parameter* [27,36]. This parameter allows us to study the QOHM efficiency and performance coefficient in any time regime. In fact, this so-called adiabaticity parameter is the transition probability induced by the unitary evolution U, and the faster the unitary process the greater ξ. When $\xi = 0$, the process is called quasistatic and occurs at null power. Finite-time processes, on the other

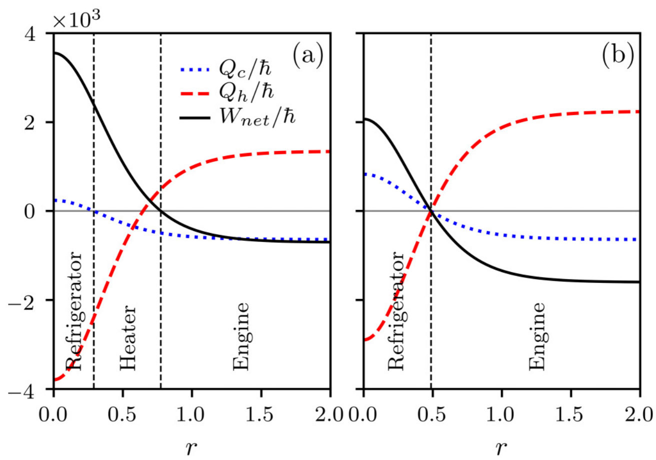

FIG. 2. Curves Q_c (dotted blue line), Q_h (dashed red line), and W_{net} (solid black line) versus the squeezing parameter r. These curves highlight the universality of our model, since depending on the value of the control parameter r the four types of machines shown in Fig. 1 can be engineered. The parameters used here are (a) $\xi = 0.2$, $\omega_c = 2\pi$ kHz, $\omega_h = 3.5\omega_c$, $\beta_c = 1/(10\text{ peV})$, $\beta_h = 0.7\beta_c$ and (b) $\xi = 0$, $\omega_c = 2\pi$ kHz, $\omega_h = 3.5\omega_c$, $\beta_c = 1/(10\text{ peV})$, $\beta_h = 0.7\beta_c$. Dotted black lines delimit the regions of the different types of machine shown in Fig. 1.

hand, occur at nonnull power, and for instantaneous process $U = I$, with I being the identity. As we shall see, the Otto efficiency and performance coefficient occurs to $\xi = 0$, corresponding to a machine operating at null power. As we are not attaching any secondary system to exchange work with our TLS machine, either in the case of heat engines or the in the case of refrigerators, we can think about this simplified model as a proof or concept [27], allowing us to impose theoretically maximum constraints on their efficiency or performance coefficient. It is worthwhile to mention that, from Eq. (3), if we consider a quasistatic process ($\xi = 0$) in the limit of high squeezing ($r \to \infty$), which implies $\zeta \to 0$, then $Q_h > 0$ irrespective of the bath temperatures. If we then let $T_c > T_h$ in this high squeezing limit, then this would imply a heat flow from cold to hot, apparently in contradiction with the second law of thermodynamics. However, it is to be noted that the squeezing operation takes the reservoir out of thermodynamics equilibrium, changing its excitation number to $n_{\text{th}} \to N = n_{\text{th}}(\cosh^2 r + \sinh^2 r) + \sinh^2 r$ [38] and, as a consequence, changing the so-called effective or apparent temperature, defined according to $N = (e^{-\hbar\omega_h/k_B T_{\text{eff}}} - 1)^{-1}$, of the squeezed reservoir. Hence, in the limit of high squeezing $r \to \infty$, $N \to \infty$, such that even letting $T_c > T_h$ the effective temperature of the reservoir will be $T_{\text{eff}} > T_c$, which can be seen as an increase in its effective temperature as well as a change in its ergotropy, such that part of the energy exchanged with the squeezed thermal bath is actually work. Because of the nonequilibrium nature of the squeezed reservoir, no second law violation occurs, as shown in Refs. [41,42].

According to our convention, $Q > 0$ ($Q < 0$) means heat energy flowing into (out of) the engine, while $W_{\text{net}} < 0$ ($W_{\text{net}} > 0$) means useful energy flowing out of (into) the engine; see Fig. 1. In Fig. 2 we show all the three relevant quantities Q_c (dotted blue line), Q_h (dashed red line), and W_{net} (solid black line) versus the squeezing parameter r for $\xi = 0$ [Fig. 2(a)] and $\xi = 0.2$ [Fig. 2(b)]. From Fig. 2(a) the universality of our TLS machine should be apparent. For example, if we want to build a heat engine, for which $Q_c < 0$, $Q_h > 0$, and $W_{\text{net}} < 0$, then we should choose $r \gtrsim 0.77$; if we want to build a refrigerator, for which $Q_h < 0$, $Q_c > 0$, and $W_{\text{net}} > 0$, then our control parameter should be $r \lesssim 0.29$; heater machines of types I and II, on the other hand, lie in region $0.29 \lesssim r \lesssim 0.77$, thus corresponding to the four types of heat machines as shown in Fig. 1. Also, note from Fig. 2(b) that to the quasistatic case $\xi = 0$ there are only two types of heat machines, and depending on the value of r, the machine switches directly from engine to refrigerator and vice versa.

In the following sections we will study the two main types of machines whose application has been highlighted in the most varied contexts, which are the engine and the refrigerator, by considering the squeezing as the control parameter.

A. Two-level heat engine

Following the definition of efficiency, which is $\eta = -W_{\text{net}}/Q_h$, we find to our model:

$$\eta = 1 - \frac{\omega_c}{\omega_h}\mathcal{R}, \tag{5}$$

where

$$\mathcal{R} = \frac{1 + 2\xi\mathcal{F}}{1 - 2\xi\mathcal{G}}, \tag{6}$$

with

$$\mathcal{F} = \frac{\zeta\tanh\theta_h}{\tanh\theta_c - \zeta\tanh\theta_h} \tag{7}$$

and

$$\mathcal{G} = \frac{\tanh\theta_c}{\tanh\theta_c - \zeta\tanh\theta_h}. \tag{8}$$

Note, from Eq. (6), that $\mathcal{R} = 1$ when $\xi = 0$, which corresponds to the quasistatic case, and $\eta = 1 - \frac{\omega_c}{\omega_h} \equiv \eta_{\text{Otto}}$.

In terms of the adiabaticity parameter ξ the condition to extract work is $W_{\text{net}} < 0$, such that

$$\xi < \frac{(\omega_h - \omega_c)(\tanh\theta_c - \zeta\tanh\theta_h)}{2(\omega_h\tanh\theta_c + \zeta\omega_c\tanh\theta_h)}, \tag{9}$$

which implies $\zeta < \tanh\theta_c/\tanh\theta_h$ (since $\xi \geqslant 0$). As we can confirm both analytically and numerically, this condition results in heat absorption from the squeezed hot thermal reservoir, $Q_h > 0$, and heat loss to the cold thermal reservoir, $Q_c < 0$, which characterizes the heat engine. In Fig. 3 we show the efficiency η versus the squeezing parameter for quasistatic regime $\xi = 0$ (solid black line), which gives the Otto efficiency, and for finite-time regime $\xi = 0.1$ (dashed blue line) and $\xi = 0.2$ (dotted red line). Figure 3 shows that although the engine efficiency can be enhanced by the squeezed reservoir, the Otto efficiency is never achieved for processes occurring in finite-time regimes. However, for a treatment of the engine efficiency when elaborated optimization procedure is carried out, see Ref. [34].

B. Two-level refrigerator

According to Eq. (1), a good engine is a poor refrigerator and vice versa. This leads us to the conclusion that, as we have seen in the previous section, since the squeezing parameter enhances the engine efficiency, squeezed reservoirs should not enhance the coefficient of performance. As we shall see, our results confirm that this is true. The refrigerator $\mathcal{C}$ is defined by the ratio $\mathcal{C} = Q_c/W_{\rm net}$, meaning that the goal is to extract as much heat as possible from the cold reservoir by doing a minimum of work. From Eqs. (2) and (4) we obtain

$$\mathcal{C} = \frac{\omega_c(\tanh\theta_c - \zeta\tanh\theta_h) - \xi\zeta\omega_c\tanh\theta_h}{(\omega_h-\omega_c)(\tanh\theta_c - \zeta\tanh\theta_h) + \xi(\omega_h\tanh\theta_c + \zeta\omega_c\tanh\theta_h)}, \tag{10}$$

or, after a little algebra,

$$\mathcal{C} = \frac{\mathcal{R}\mathcal{C}_{\rm Otto}}{1+\mathcal{C}_{\rm Otto}(1-\mathcal{R})}, \tag{11}$$

where $\mathcal{R}$ was defined in Eqs. (6)–(8) and

$$\mathcal{C}_{\rm Otto} = \frac{\omega_c}{\omega_h - \omega_c} \tag{12}$$

is the ideal $\mathcal{C}$ obtained from quasistatic processes, i.e., by letting $\xi = 0$ ($\mathcal{R} = 1$).

Recalling that Eq. (11) makes sense only for $Q_c > 0$ and $W_{\rm net} > 0$, the following constraint must be obeyed:

$$\xi < \frac{1}{2}\left(1 - \frac{\tanh\theta_c}{\zeta\tanh\theta_h}\right). \tag{13}$$

By imposing the constraint Eq. (13), we can numerically verify that the highest value of $\mathcal{C}$ is equal to the ideal Otto machine $\mathcal{C}_{\rm Otto} = \omega_c/(\omega_h - \omega_c)$, which occurs to the quasistatic process $\xi = 0$ or $\mathcal{R} = 1$. In Fig. 4 we show the $\mathcal{C}$ for the ideal Otto refrigerator $\xi = 0$ (solid black line), as well as for two other finite-time parameters $\xi = 0.1$ (dashed blue line) and $\xi = 0.2$ (dotted red line). As we can see, for $\xi > 0$ all curves in Fig. 4 start below the ideal Otto performance coefficient $\mathcal{C}_{\rm Otto} = 0.4$ and decrease further as the squeezing parameter increases. As expected, the performance coefficient behavior is contrary to the efficiency behavior shown in Fig. 3.

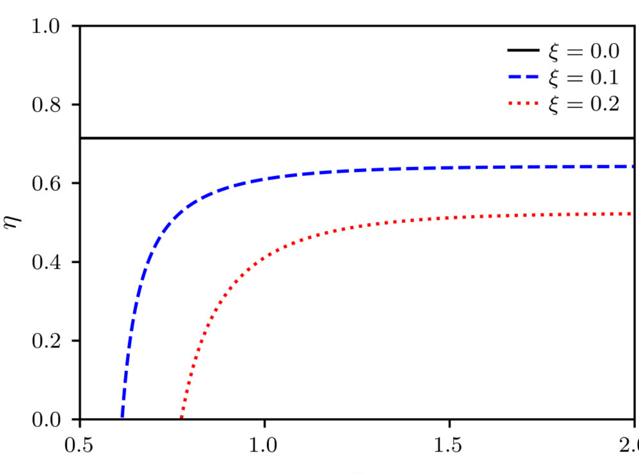

FIG. 3. Efficiency η to the TLS engine versus the squeezing parameter r considering quasistatic processes $\xi = 0$, resulting in the Otto efficiency $\eta_{\rm Otto} = 1 - \omega_c/\omega_h$ (horizontal solid black line) and the finite-time regime $\xi = 0.1$ (dashed blue line) and $\xi = 0.2$ (dotted red line). We also use the parameters $\omega_c = 2\pi$ kHz, $\omega_h = 3.5\omega_c$, $\beta_c = 1/(10\text{ peV})$, and $\beta_h = 0.7\beta_c$.

IV. η-$\mathcal{C}$ RELATION FOR SQUEEZED RESERVOIRS

In this section we will generalize Eq. (1) to take into account processes occurring under squeezed reservoirs. To this end, we need to eliminate $W_{\rm net}$ from both $\eta = -W_{\rm net}/Q_h$ and $\mathcal{C} = Q_c/W_{\rm net}$ using Eqs. (2)–(4). For $\xi > 0$ it should be noted, see Fig. 2(a), that in different regions in which the heat machine operates either as refrigerator or engine, there is no $W_{\rm net}$ such that its absolute value coincides in both regions, thus implying that it is not legitimate to consider $W_{\rm net}^{(\rm engine)} = -W_{\rm net}^{(\rm fridge)}$ when eliminating $W_{\rm net}$ from $\eta = -W_{\rm net}/Q_h$ and $\mathcal{C} = Q_c/W_{\rm net}$. As a consequence, unlike what happens in the quasistatic case shown in Fig. 2(b), for $\xi \neq 0$ there will not always be a balance between the work $W_{\rm net}$ that can be extracted from a universal TLS engine having η efficiency, and the $\mathcal{C}$ performance that would be obtained if that same work $W_{\rm net}$ were supplied to a fridge machine, at least for the parameters considered here. Nonetheless, note that when $\xi = 0$ [Fig. 2(b)] the different regions that delimit the different types of thermal machines do not overlap, regardless of the parameters used. For example, the work is positive in the region that limits the operation of the refrigerator but negative in the region that limits the operation of the heat engine, as

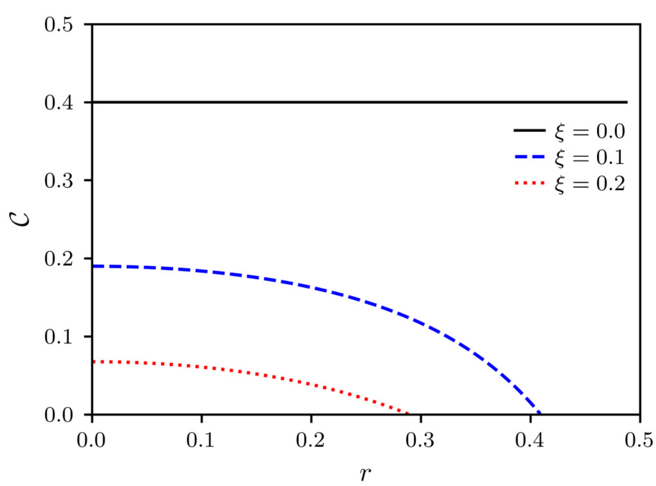

FIG. 4. Performance $\mathcal{C}$ versus the parameter r of the squeezed reservoir for a (a) quasistatic process $\xi = 0$ (solid black line) and finite-time processes (b) $\xi = 0.1$ (dashed blue line) and (c) $\xi = 0.2$ (dotted red line). The other parameters used here are $\omega_c = 2\pi$ kHz, $\omega_h = 3.5\omega_c$, $\beta_c = 1/(10\text{ peV})$, and $\beta_h = 0.7\beta_c$.

shown in Fig. 2(b). If we then restrict our analysis to the case $\xi = 0$, then we see that as defined by Eq. (6), $\mathcal{R} = 1$, and therefore, using Eq. (5), Eq. (11), and Eq. (12), the η-$\mathcal{C}$ relation follows:

$$\mathcal{C}_{\text{Otto}} = \frac{1}{\eta_{\text{Otto}}} - 1, \tag{14}$$

which is exactly the same as Eq. (1) and therefore the squeezing parameter does not modify the relation between η and $\mathcal{C}$ in the quasistatic limit, which is an interesting result.

V. CONCLUSION

We have proposed a universal QOHM based on a two-level system as the working substance that operates under two reservoirs: a cold thermal reservoir and a squeezed hot thermal reservoir. For universal QOHM we mean the possibility of changing the parameters of control, such as the squeezing r and the adiabaticity ξ parameters, to make the machine work as a thermal engine, a refrigerator, or as a heater. We also showed that the squeezing parameter, although useful to improve the efficiency of an engine, always leads to a worsening of the performance coefficient of a refrigerator. This is in contrast to the result of Ref. [31], where the authors considered a harmonic oscillator as the work substance and the cold reservoir as the squeezed one. Finally, we have demonstrated that the usual relation between η and $\mathcal{C}$, Eq. (1), remains unchanged for a heat machine working at null power under two reservoirs: a cold thermal reservoir and a hot squeezed thermal reservoir.

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