Rotating ion beam effects on temperature gradient instability in completely ionized plasmas

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The aim of this paper is to investigate the effects of a rotating ion beam on the temperature gradient instability (TGI) in completely ionized plasmas. The interplay of the temperature and density gradients provides the basis for experiencing an unstable inhomogeneous plasma medium due to TGI taken under consideration. The density and temperature gradients are considered perpendicular to the magnetic field where a nonrelativistic rotating ion beam such as O^+ is present. By implementing the kinetic theory together with a zeroth-order approximation of geometrical optics, the dielectric permittivity tensor of the inhomogeneous plasma is obtained where by a suitable linear eikonal equation, the growth rate of the TGI in the collisional regime is calculated in the presence of a rotating ion beam. In such a configuration an unstable condition is experienced in regions with opposite electron density and temperature gradients, where it is destabilized by the temperature and plasma density gradients and the frequent electron collisions. As a consequence, the results reveal that the TGI can be damped or modified through interaction with the rotating ion beam depending on the characteristics of the ion beam, namely, velocity and density.

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I. INTRODUCTION

Drift wave instabilities highlight themselves in various physical phenomena, e.g., inhomogeneous plasmas [1–3], dusty plasmas [4], stochastic coronal heating [5], solar wind and the planetary magnetospheres [6,7], anomalous transport [8], and anomalous ion heating in laboratory confined plasmas [9], earth's magnetopause [10], and the ionosphere [11]. The earth's ionosphere is spatially inhomogeneous with respect to temperature and density [12]. This may be due to the photoelectron ionization production effect [13], effects of the inner magnetosphere on the ionosphere [14], chemical reactions between the ionized species and the neutral compositions, and dissipating planetary waves [15]. The density and temperature gradients create diamagnetic drifts of electrons and ions in opposite directions, which cause charge separation leading to an electric field **E** perturbation perpendicular to the background magnetic field **B**. These fields provide $(\mathbf{E} \times \mathbf{B})/B^2$ drifts that enhance the perturbation growth triggering instability. The perturbation propagates perpendicular to the magnetic field and the gradient [16–18] where the gradient drift frequency provokes ionospheric irregularities. The ionospheric irregularities are plasma density fluctuations that have been reported by several radar observations [19,20] and have been analyzed linearly [21] and nonlinearly [22] in the *F* region. The effects of various factors such as density and temperature gradients, electric fields, collisions, and anisotropies in plasma temperature on these irregularities have been studied in various situations, where the efficiency of the temperature gradient has been highlighted [23,24]. Observations of joint measurements obtained by the Millstone Hill incoherent scatter radar and the Super Dual Auroral Radar Network [25] have detected decameter-scale irregularities excited on the equatorward wall of midlatitude ionosphere trough in a region that density and temperature gradients are in opposite directions. Thus, it has been concluded that temperature gradient instability (TGI) is an effective factor in the generation of the irregularities, which fits well with the irregularity generation via TGI proposed by Hudson and Kelley [26]. The TGI is a type of collisional drift wave instability whose free energy arises mainly from the opposed temperature and density gradients perpendicular to the magnetic field. In addition, collisions have been shown to exert a destabilizing effect on the TGI [27].

Although the density and temperature gradients and collisions play an effective role in the TGI, they are not the only ones. For instance, TGI as the source of ionospheric irregularity can be damped by a conducting layer of the *E* region ionosphere that shorts out the electrostatic fields [25]. In fact, the charged particle beams would also affect the inhomogeneous plasma [28]. The characteristics of the wave generated by the interaction of a beam with an inhomogeneous plasma depend on the inhomogeneity parameters together with the consistent species of the plasma and beam [29,30]. Thus, the electron and ion beams would possess information regarding both the ionosphere and the magnetosphere [31]. The injection of charged particle beams is a very probable phenomenon in space. Regarding earth, they may enter the ionosphere due to satellites and space shuttle engines [32,33], which eventually provides a condition for water ion beams to possess a rotating behavior as the downstream distance from the shuttle is increased [34] (see also [35]). Natural

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processes also provide charged particle beams such as substorm injections and solar wind penetration in the earth's atmosphere where they interact with the plasma [21]. The charged particle beam-plasma interaction has already been studied in homogeneous and inhomogeneous media [36–38]. The free energy in the relative streaming of the two species can give rise to a variety of instabilities [39,40] and consequent plasma heating [41]. In particular, the perpendicular component of the velocity relative to the magnetic field provides a source of free energy to drive instabilities [42,43]. The cyclotron instability regarding a rotating electron beam in a cold magnetoactive plasma medium was investigated by Shokri and Khorashadizadeh [44] assuming the plasma to be homogeneous.

In the present study, the long-wavelength TGI driven by density and temperature gradients perpendicular to the magnetic field in the presence of a rotating ion beam such as O^+ is studied in a completely ionized plasmas. The beam is nonrelativistic and propagating parallel to the magnetic field. Using the kinetic theory, with the assumption of weak spatial inhomogeneities, the dielectric permittivity tensor is obtained by the zeroth-order approximation of geometrical optics, which is suitable for cases when the wavelength is smaller than the characteristic length of the inhomogeneity. By presenting a suitable linear eikonal equation for the gradient drift frequency $\omega \leqslant \omega_{d\alpha} \sim k_\perp v_{T\alpha}^2/\Omega_\alpha L_0$, where $\omega_{d\alpha}$ and L_0 are the drift frequency of particle type α ($\alpha = e, i$ for electrons and ions, respectively) and the characteristic length of inhomogeneity, respectively, the growth rate of the instability could be derived in the collisional regime $\omega \ll \nu_e$ and $|\omega + i\nu_\alpha| \gg k_z v_{T\alpha}$, where ν_α is the effective collisional frequency of a particle of type α. The perpendicular and parallel wave vectors with respect to the magnetic field are represented by $k_\perp$ and k_z, respectively. The thermal velocity is represented by $v_{T\alpha}$, while the inhomogeneous cyclotron frequency of particle of type α is shown by Ω_α. As a result, an unstable configuration is found in the region where the electron density and temperature gradients are in opposite directions. This implies the necessity of the temperature gradient for instability. The role of physical parameters such as plasma density and temperature inhomogeneity scale lengths together with electron collisions and ion beam parameters such as velocity and density on the instability growth rate is investigated.

The paper is organized as follows. The model and equations together with the appropriate assumptions are presented in Sec. II. The suitable eikonal equation and the corresponding dielectric permittivity tensor of the plasma system which enables estimating the growth rate of TGI in the presence of a rotating ion beam are presented in Sec. III, which are then discussed in Sec. IV. A summary is presented in Sec. V.

II. MODEL AND BASIC EQUATIONS

Consider a collisional magnetoactive plasma medium which is weakly inhomogeneous. The external magnetic field $\mathbf{B}_0$ and the gradients in plasma parameters (density and temperature) are considered to be along the $\mathbf{z}$ and $\mathbf{x}$ directions, respectively, as shown in Fig. 1. In the context of the present study the medium possesses low plasma kinetic pressure compared to the magnetic pressure, where the inhomogeneity of the magnetic field is neglected, and the low-frequency drift oscillation ($\omega \leqslant \omega_{d\alpha} \ll \Omega_\alpha$) is longitudinal with a high degree of accuracy. Furthermore, in order to investigate the drift oscillations and consequently consider the guiding center drift of particles, it must be assumed that $\nu_\alpha \ll \Omega_\alpha$. The wavelength λ considered in this medium is significantly smaller than the characteristic dimensions L_0 of the inhomogeneity ($\lambda/L_0 \ll 1$) of the medium. In such a condition, the zeroth-order approximation of geometrical optics implies that the polarization of the wave is almost plane, which, as in the case of a homogeneous medium, enables obtaining the dielectric permittivity tensor of inhomogeneous plasma. In this approximation, all local parameters are set constant with their local values and their local constant gradient direction and length scale.

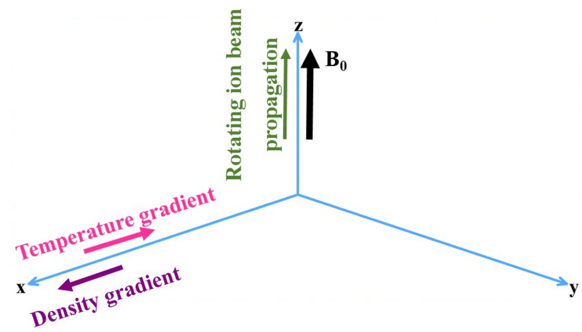

FIG. 1. Schematic of the inhomogeneous magnetoactive plasma system in the presence of a rotating ion beam.

It is supposed that the plasma is completely ionized, and therefore collisions between charged particles dominate. So the Landau kinetic equation for charged particles of type α with momentum $\mathbf{p}_\alpha$ and charge e_α is implemented to obtain the dielectric permittivity tensor of collisional inhomogeneous magnetoactive plasma [45]

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{r}} + e_\alpha \left\{ \mathbf{E} + \frac{1}{c}[\mathbf{v}, \mathbf{B}] \right\} \cdot \frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha} = \Sigma_\beta \left(\frac{\partial f_\alpha}{\partial t} \right)_{\text{col}}^{\alpha\beta}, \tag{1}$$

where $\mathbf{v}$, $\mathbf{E}$, and $\mathbf{B}$ are the velocity, the electric field, and the magnetic field at the position of the particle, respectively, t and $\mathbf{r}$ denote time and place coordinates, respectively, and c is the light speed in vacuum. Furthermore, the right-hand side relation denotes the variation of distribution function f_α due to the Coulomb collisions of charged particles of type α with charged particles of type β, which is defined as

$$\left(\frac{\partial f_\alpha}{\partial t} \right)_{\text{col}}^{\alpha\beta} = 2\pi L e_\alpha^2 \Sigma_\beta e_\beta^2 \frac{\partial}{\partial p_{\alpha i}} \int d\mathbf{p}_\beta \frac{u^2 \delta_{ij - u_i u_j}}{u^3} \times \left[f_\beta(\mathbf{p}_\beta) \frac{\partial f_\alpha(\mathbf{p}_\alpha)}{\partial p_{\alpha j}} - f_\alpha(\mathbf{p}_\alpha) \frac{\partial f_\beta(\mathbf{p}_\beta)}{\partial p_{\beta j}} \right], \tag{2}$$

with the notations L for the Coulomb logarithm and u for the relative velocity of particles of types α and β.

In order to calculate the dielectric permittivity tensor of the plasma under consideration, it is necessary to determine the corresponding equilibrium distribution function $f_{0\alpha}$ first. As the Larmor frequency of particles is high compared to the characteristic collision frequencies, the equilibrium distribution function of this inhomogeneous plasma can be calculated in the collisionless approximation. So, by neglecting the term associated with the collisions and using the cylindrical coordinates ($v_x = v_\perp \cos\phi$, $v_y = v_\perp \sin\phi$, and v_z, where v_x, v_y, and v_z are the velocities in the **x**, **y**, and **z** directions, respectively) for the stationary state where $\mathbf{E}_0 = 0$, Eq. (1) reduces to

$$v_\perp \cos\phi \frac{\partial f_{0\alpha}(\mathcal{E}_\alpha, C_\alpha)}{\partial x} - \Omega_\alpha \frac{\partial f_{0\alpha}(\mathcal{E}_\alpha, C_\alpha)}{\partial \phi} = 0, \tag{3}$$

where $C_\alpha = v_\perp \sin\phi + \Omega_\alpha x$, and by using the small parameter $\lambda_{L\alpha}/L_0 \ll 1$ ($\lambda_{L\alpha} = v_{T\alpha}/\Omega_\alpha$ is the Larmor radius of the particles of type α), the distribution function $f_{0\alpha}(\mathcal{E}_\alpha, C_\alpha) = f_{0\alpha}[\mathcal{E}_\alpha, v_y + \Omega_\alpha x]$ for plasma species α with the mass m_α and energy $\mathcal{E}_\alpha = m_\alpha v^2/2$ can be expanded as

$$f_{0\alpha}(\mathcal{E}_\alpha, C_\alpha) = \left(1 + \frac{v_\perp \sin\phi}{\Omega_\alpha}\frac{\partial}{\partial x}\right) F_\alpha(\mathcal{E}_\alpha, x), \tag{4}$$

where $F_\alpha(\mathcal{E}_\alpha, x)$ is an arbitrary function chosen to be the local Maxwellian distribution function with inhomogeneous density $N_\alpha(x)$ and temperature $T_\alpha(x)$,

$$F_\alpha(\mathcal{E}_\alpha, x) = \frac{N_\alpha(x)}{[2\pi m_\alpha T_\alpha(x)]^{3/2}} \exp\left(-\frac{\mathcal{E}_\alpha}{T_\alpha(x)}\right). \tag{5}$$

To derive the dielectric permittivity tensor, a linear small perturbation of local equilibrium must be considered, which, due to the plasma inhomogeneity being in the **x** direction, is considered in the form of $\delta f_\alpha = \delta f_\alpha(x) \exp(-i\omega t + ik_y y + ik_z z)$. While k_y is the wave vector component in the **y** direction, y and z are place coordinates in the **y** and **z** directions, respectively. By considering $C_\alpha = v_\perp \sin\phi + \Omega_\alpha x$, and therefore $\partial f_{0\alpha}/\partial p_{\alpha j} = v_j \partial f_{0\alpha}/\partial \mathcal{E}_\alpha + (\delta_{yj}/m_\alpha \Omega_\alpha)\partial f_{0\alpha}/\partial x$, substituting $f_\alpha = f_{0\alpha} + \delta f_\alpha$ in Eq. (1) represents

$$(\omega - k_z v_z)\delta f_e = \frac{e\Phi}{T_e}\left(k_z v_z - \frac{k_y v_{Te}^2}{\Omega_e} a_e\right) f_{0e} + i\left[\left(\frac{\partial f_e}{\partial t}\right)_{\text{col}}^{ee} + \left(\frac{\partial f_e}{\partial t}\right)_{\text{col}}^{ei}\right] \tag{6}$$

for electrons and

$$(\omega - k_z v_z)\delta f_i = \frac{e\Phi}{T_i}\left(k_z v_z - \frac{k_y v_{Ti}^2}{\Omega_i} a_i\right) f_{0i} + i\left(\frac{\partial f_i}{\partial t}\right)_{\text{col}}^{ii} \tag{7}$$

for ions, where $a_\alpha = \partial \ln N_\alpha/\partial x + \partial \ln T_\alpha/\partial x(-3/2 + v^2/2v_{T\alpha}^2)$ and the potential Φ satisfies $\mathbf{E} = -i\mathbf{k}\Phi$; **k** is the wave vector. Furthermore, Eq. (2) is used to calculate the terms associated with the collision contribution. Since the ion thermal velocity compared with the electron thermal velocity is very low, the ion-electron collision integral $(\partial f_i/\partial t)_{\text{col}}^{ie}$ is neglected. In addition, the terms associated with the perturbation of the ion distribution function δf_i are neglected in electron-ion collisions; then

$$\left(\frac{\partial f_e}{\partial t}\right)_{\text{col}}^{ei} = 2\pi e^2 e_i^2 L N_i \frac{\partial}{\partial p_i} \frac{u^2 \delta_{ij - u_i u_j}}{u^3} \frac{\partial \delta f_e}{\partial p_j}. \tag{8}$$

The consideration is restricted to the case of the frequent electron-electron collisions $\nu_i \ll \omega, k_z v_{Te} \ll \nu_e$, so the the terms associated with the collisions are the main terms in Eq. (6) and the Chapman-Enskog method is implemented. To simplify, the function F_e is introduced by

$$\delta f_e = -\frac{k_y v_{Te}^2}{\omega \Omega_e} \frac{e}{T_e} a_e \Phi f_{0e} + F_e \tag{9}$$

for electrons. Under these assumptions, by substituting Eq. (9) in Eq. (6), δf_e can be extracted by implementing the Chapman-Enskog procedure, i.e., by expanding F_e into a series of Sonin-Laguerre polynomials. Following the standard procedure, the conductivity tensor components σ_{ij} are calculated by an equation of the induced current $\mathbf{j}_i = \Sigma_\alpha e_\alpha \int d\mathbf{p}\, \mathbf{v}_i \delta f_\alpha = \sigma_{ij}\mathbf{E}_j$. Then the equation $\varepsilon_{ij} = \delta_{ij} + (4\pi i/\omega)\sigma_{ij}$ is used to obtain the dielectric permittivity tensor contribution due to the electrons, $\delta\varepsilon_e(\omega, \mathbf{k}, x)$ for $\nu_{\text{eff}}\omega \ll k_z^2 v_{Te}^2$,

$$\delta\varepsilon_e(\omega, \mathbf{k}, x) = \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left\{1 + i1.44 \frac{\omega \nu_{\text{eff}}}{k_z^2 v_{Te}^2}\left(1 - \frac{k_y v_{Te}^2}{\omega \Omega_e} \frac{\partial \ln N T_e^{-0.56}}{\partial x}\right)\right\}, \tag{10}$$

in which ν_{eff} is the effective collisions frequency of electrons. Furthermore, the function F_i is introduced by

$$\delta f_i = \frac{e_i}{T_i}\Phi f_{0i} + F_i \tag{11}$$

for ions. By substituting Eq. (11) in Eq. (7) and implementing successive approximations for $k_z v_{Ti}, \nu_i \ll \omega$, the dielectric permittivity tensor contribution due to the ions, $\delta\varepsilon_i(\omega, \mathbf{k}, x)$, is obtained as

$$\begin{aligned}\delta\varepsilon_i(\omega, \mathbf{k}, x) = \frac{\omega_{pi}^2}{k^2 v_{Ti}^2}\Bigg\{&\frac{k_y v_{Ti}^2}{\omega \Omega_i}\frac{\partial \ln N}{\partial x} + \frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2}\left(1 - \frac{k_y v_{Ti}^2}{\omega \Omega_i}\frac{\partial \ln N T_i}{\partial x}\right) + \frac{i\nu_i v_{Ti}^4}{10\omega}\Bigg[\left(16\frac{k_z^4}{\omega^4} + 28\frac{k_z^2 k_\perp^2}{\omega^2 \Omega_i^2} + 7\frac{k_\perp^4}{\Omega_i^4}\right)\left(1 - \frac{k_y v_{Ti}^2}{\omega \Omega_i}\frac{\partial \ln N}{\partial x}\right)\\ &- \left(24\frac{k_z^4}{\omega^4} + \frac{33}{2}\frac{k_z^2 k_\perp^2}{\omega^2 \Omega_i^2} - \frac{3}{4}\frac{k_\perp^4}{\Omega_i^4}\right)\frac{k_y v_{Ti}^2}{\omega \Omega_i}\frac{\partial \ln T_i}{\partial x}\Bigg]\Bigg\},\end{aligned} \tag{12}$$

where $\omega_{p\alpha}$ represents the Langmuir frequency of particle species α ($\alpha = e, i$).

Since the weakly inhomogeneous plasma under consideration is in the presence of a monoenergetic rotating ion beam, the eikonal equation for longitudinal plasma oscillations taking into account the correction caused by the rotating ion beam $\delta\varepsilon_{\text{beam}}(\omega, \mathbf{k})$ is given by

$$\varepsilon(\omega, \mathbf{k}, x) = 1 + \delta\varepsilon_e(\omega, \mathbf{k}, x) + \delta\varepsilon_i(\omega, \mathbf{k}, x) + \delta\varepsilon_{\text{beam}}(\omega, \mathbf{k}) = 0, \quad (13)$$

where

$$\delta\varepsilon_{\text{beam}}(\omega, \mathbf{k}) = \frac{k_i k_j}{k^2}\,\delta\varepsilon_{ij}(\omega, \mathbf{k}), \quad (14)$$

where $\delta\varepsilon_{ij}(\omega, \mathbf{k})$ is the dielectric permittivity tensor components of the rotating ion beam with a nonrelativistic parallel ($u_\parallel$) and perpendicular ($u_\perp$) velocity component with respect to the external magnetic field described by the distribution function with average longitudinal $p_{\parallel 0}$ and transverse $p_{\perp 0}$ momenta with respect to the direction of the magnetic field B_0,

$$f_{0b} = \frac{N_{0b}}{2\pi p_{\perp 0}}\delta(p_\perp - p_{\perp 0})\delta(p_\parallel - p_{\parallel 0}), \quad (15)$$

where N_{0b} is the total number of beam particles whose thermal spread has been neglected. Due to the smallness of the spatial size of the ion beam in comparison with the scale length of the inhomogeneity, the ion beam is considered to be spatially homogeneous. It should be stated that when the plasma density greatly exceeds the ion beam density, neutralization-induced charges and currents are observed in the plasma by injected ion beams enabling the assumption $\omega_b^2 \ll \omega_{pe}^2$, where ω_{pe} and ω_b are the Langmuir frequencies of the plasma and the beam, respectively. Now, in order to neglect the equilibrium azimuthal motion of the beam and the plasma components, it is assumed that $u_\perp^2 \ll c^2$.

The distribution function for the rotating ion beam together with Vlasov's equation regarding a perpendicular wave vector ($k_\perp$) along the x axis gives the dielectric permittivity of this system as [44,45]

$$\begin{aligned}
\delta\varepsilon_{xx} &= \Sigma_n\left\{\frac{2}{z}n^2 J_n(z)J_n'(z)P_n + n^2 J_n^2(z)Q_n\right\}, \quad \delta\varepsilon_{yy} = \Sigma_n\left\{\frac{1}{z}\left[z^2 J_n'^2(z)\right]' P_n + z^2 J_n'^2(z)Q_n\right\},\\
\delta\varepsilon_{zz} &= -\frac{\omega_b^2}{\omega^2} + \Sigma_n\left\{\left[\frac{2k_z u_\parallel}{\omega - k_z u_\parallel}J_n^2(z) + \frac{u_\parallel^2}{u_\perp^2}2zJ_n(z)J_n'(z)\right]P_n + \frac{u_\parallel^2}{u_\perp^2}z^2 J_n^2(z)Q_n\right\},\\
\delta\varepsilon_{xy} &= -\delta\varepsilon_{yx} = -i\Sigma_n\left\{\frac{n}{z}[zJ_n(z)J_n'(z)]'P_n + nzJ_n(z)J_n'(z)Q_n\right\},\\
\delta\varepsilon_{yz} &= -\delta\varepsilon_{zy} = i\Sigma_n\left\{\left(\frac{\Omega_i k_z z J_n(z)J_n'(z)}{k_\perp(\omega - k_z u_\parallel)} + \frac{u_\parallel}{u_\perp}[zJ_n(z)J_n'(z)]'\right)P_n + \frac{u_\parallel}{u_\perp}z^2 J_n(z)J_n'(z)Q_n\right\},\\
\delta\varepsilon_{xz} &= \delta\varepsilon_{zx} = \Sigma_n\left\{\left[\frac{n\Omega_i k_z}{k_\perp(\omega - k_z u_\parallel)}J_n^2(z) + \frac{u_\parallel}{u_\perp}2nJ_n(z)J_n'(z)\right]P_n + \frac{u_\parallel}{u_\perp}nzJ_n^2(z)Q_n\right\},
\end{aligned} \quad (16)$$

where $J_n(z)$ and $J_n'(z)$ are the Bessel function of the first kind and its derivation with respect to the argument z, respectively. Here

$$\begin{aligned}
P_n &\equiv \frac{\omega_b^2(\omega - k_z u_\parallel)}{\omega^2(\omega - k_z u_\parallel - n\Omega_i)},\\
Q_n &\equiv \frac{\omega_b^2\Omega_i^2(\omega^2 - k_z^2 c^2)}{\omega^2 c^2 k_\perp^2(\omega - k_z u_\parallel - n\Omega_i)^2},\\
z &= \frac{k_\perp u_\perp}{\Omega_i}.
\end{aligned} \quad (17)$$

Equations (10)–(17) provide a foundation for investigating the effects of a rotating ion beam on the TGI, which will be discussed in the following section.

III. TGI IN THE PRESENCE OF A ROTATING ION BEAM

Drift waves are prominent features of inhomogeneous plasmas, where in the context of the present study it is the long-wavelength gradient drift waves ($\omega \leqslant \omega_{d\alpha} < \Omega_\alpha$) in an inhomogeneous plasma that are bound to attract attention. Our inhomogeneous plasma consists of electrons and ions that host a rotating ion beam. The inhomogeneity includes temperature and density gradients that lead to diamagnetic drifts of electrons (v_{de}) and ions (v_{di}) heading in opposite directions, causing charge separation. The result would be the creation of an electric field $\mathbf{E}$ with a perturbation perpendicular to the magnetic field. Thus, in such a plasma medium, the wave is produced perpendicular to the magnetic field with the gradient drift frequency ($\omega_{d\alpha} = \mathbf{k}\cdot\mathbf{v}_{d\alpha} \sim k_\perp v_{T\alpha}^2/\Omega_\alpha L_0$) that propagates parallel to the electrons drift [16]. The magnetic field is taken in the z direction, while the gradients in density and temperature are assumed to be in the $\mathbf{x}$ direction. Thus, the wave vector would be perpendicular $k_\perp$, which is along the $\mathbf{y}$ direction in the direction of v_{de}. In order to satisfy the term $|\omega/k_z v_{Te}| \ll 1$, a parallel component ($k_z$) of the wave vector in the z direction is considered so that $|k_z/k_\perp| \ll 1$. The analysis is confined to the low-frequency waves under the conditions $\omega \ll \Omega_i \ll \omega_{pi}$, with a long wavelength $|k_\perp v_{T\alpha}/\Omega_\alpha| \ll 1$. By taking $k_x = 0$, Eq. (14) reduces to

$$\delta\varepsilon_{\text{beam}}(\omega, \mathbf{k}) = \frac{k_y^2}{k^2}\delta\varepsilon_{yy} + \frac{k_z^2}{k^2}\delta\varepsilon_{zz} + \frac{k_z k_y}{k^2}(\delta\varepsilon_{yz} + \delta\varepsilon_{zy}), \quad (18)$$

while the components of the dielectric permittivity tensor of the rotating ion beam can be evaluated from Eq. (16) by considering the asymptotic form of the Bessel function for small arguments ($z \ll 1$) [46],

$$J_n(z) = \frac{1}{\Gamma(n+1)}\left(\frac{z}{2}\right)^n, \tag{19}$$

where $\Gamma(n+1)$ represents the Gamma function. Due to the limit $u_\perp \ll c$, only the fundamental harmonic $n = 0, \pm 1$ is accounted. Using Eq. (16) for the perpendicular wave number $k_\perp$ along the y axis, the essential components of the dielectric permittivity tensor caused by the nonrelativistic rotating ion beam are obtained

$$\begin{aligned}
\delta\varepsilon_{yy} &= \frac{\omega_b^2}{\omega^2}\left\{\frac{\Omega_i(\omega - k_z u_\parallel)}{(\omega - k_z u_\parallel)^2 - \Omega_i^2} + \frac{u_\perp^2}{2}\left(\frac{\omega^2}{c^2} - k_z^2\right)\frac{(\omega - k_z u_\parallel)^2 + \Omega_i^2}{\left[(\omega - k_z u_\parallel)^2 - \Omega_i^2\right]^2}\right\},\\
\delta\varepsilon_{zz} &= \frac{\omega_b^2}{\omega^2}\left\{\left(\frac{k_y u_\perp}{\Omega_i}\right)^2\left(\frac{k_z u_\parallel}{\omega - k_z u_\parallel} + \frac{u_\parallel^2}{u_\perp^2}\right)\frac{(\omega - k_z u_\parallel)^2}{(\omega - k_z u_\parallel)^2 - \Omega_i^2}\right.\\
&\quad \left. + u_\parallel^2\left(\frac{\omega^2}{c^2} - k_z^2\right)\left[\frac{1}{(\omega - k_z u_\parallel)^2} + \left(\frac{k_y u_\perp}{\Omega_i}\right)^2\frac{(\omega - k_z u_\parallel)^2 + \Omega_i^2}{2[(\omega - k_z u_\parallel)^2 - \Omega_i^2]^2}\right] + \frac{u_\parallel^2}{u_\perp^2}\frac{2k_\perp u_\perp}{\Omega_i} + \frac{2k_z u_\parallel}{\omega - k_z u_\parallel} - 1\right\},\\
\delta\varepsilon_{yz} &= \delta\varepsilon_{zy} = \frac{\omega_b^2}{\omega^2}\frac{\Omega_i(\omega - k_z u_\parallel)}{(\omega - k_z u_\parallel)^2 - \Omega_i^2}\frac{k_\perp u_\parallel}{\Omega_i}\left\{\frac{k_z u_\perp}{(\omega - k_z u_\parallel)}\frac{u_\perp}{2u_\parallel} + \frac{2u_\perp^2\left(\frac{\omega^2}{c^2} - k_z^2\right)}{(\omega - k_z u_\parallel)^2 - \Omega_i^2} + 1\right\}.
\end{aligned} \tag{20}$$

We assume that $|\omega - k_z u_\parallel|/\Omega_i \ll 1$ and $|\omega| \ll |k_z c|$. By considering $T_e = T_i$, $\partial \ln N_e/\partial x = \partial \ln N_i/\partial x$, and $k_\perp$ along the y axis and substituting Eqs. (10), (12), and (18) into Eq. (13), the eikonal equation is obtained as

$$\begin{aligned}
\varepsilon(\omega, k, x) = 1 &+ \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left\{1 + \frac{k_\perp v_{Ti}^2}{\omega\Omega_i}\frac{\partial \ln N}{\partial x} + \frac{k_y^2 v_{Ti}^2}{\Omega_i^2}\left(1 - \frac{k_\perp v_{Ti}^2}{\omega\Omega_i}\frac{\partial \ln NT_i}{\partial x}\right)\right\} + \frac{\omega_b^2}{\omega^2}\frac{k_y^2}{k^2}\left\{\frac{-\omega + k_z u_\parallel}{\Omega_i} - \frac{k_z^2}{k_y^2}\left[\frac{3}{2}\frac{k_\perp^2 u_\perp^2}{2\Omega_i^2} + \frac{k_z^2 u_\parallel^2}{(\omega - k_z u_\parallel)^2}\right.\right.\\
&\left.\left. - \frac{u_\parallel^2}{u_\perp^2}\frac{2k_\perp u_\perp}{\Omega_i} - \frac{2k_z u_\parallel}{\omega - k_z u_\parallel} + 1\right]\right\} + i1.44\frac{\omega\nu_{\text{eff}}}{k_z^2 v_{Te}^2}\frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left\{1 - \frac{1}{L_N}\frac{k_\perp v_{Te}^2}{\omega\Omega_e}(1 - 0.56\eta_e)\right\} = 0,
\end{aligned} \tag{21}$$

where $\eta_e = L_N/L_{Te}$ is the relative parameters of the electron, while $L_{Te} = (\partial \ln T_e/\partial x)^{-1}$ and $L_N = (\partial \ln N/\partial x)^{-1}$ are the characteristic lengths of the temperature and density inhomogeneity of the electron, respectively. In addition, the dissipation due to the ion-ion collisions has been neglected. Assuming $k_y^2 v_{Ti}^2/\Omega_i^2 \ll 1$ reduces the eikonal Eq. (21) as

$$\begin{aligned}
\varepsilon(\omega, k, x) = 1 &+ \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left\{1 + \frac{k_\perp v_{Ti}^2}{\omega\Omega_i}\frac{\partial \ln N}{\partial x}\right\} + \frac{\omega_b^2}{\omega^2}\frac{k_y^2}{k^2}\left\{\frac{-\omega + k_z u_\parallel}{\Omega_i} - \frac{k_z^2}{k_y^2}\left[\frac{3}{2}\frac{k_\perp^2 u_\perp^2}{2\Omega_i^2} + \frac{k_z^2 u_\parallel^2}{(\omega - k_z u_\parallel)^2}\right.\right.\\
&\left.\left. - \frac{u_\parallel^2}{u_\perp^2}\frac{2k_\perp u_\perp}{\Omega_i} - \frac{2k_z u_\parallel}{\omega - k_z u_\parallel} + 1\right]\right\} + i1.44\frac{\omega\nu_{\text{eff}}}{k_z^2 v_{Te}^2}\frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left\{1 - \frac{1}{L_N}\frac{k_\perp v_{Te}^2}{\omega\Omega_e}(1 - 0.56\eta_e)\right\} = 0.
\end{aligned} \tag{22}$$

It should be noted that we have $\omega = \omega_r + i\gamma$, where $\gamma \ll \omega_r$. In addition, $\omega_r = \text{Re}\{\omega\}$ is the oscillation frequency and $\gamma = \text{Im}\{\omega\}$ is the growth rate of the wave with $\gamma > 0$ corresponding to the instability. Under such conditions and $\omega_b^2 \ll \omega_{pe}^2$, by calculating the root of the real part of the dielectric permittivity (22) without the contribution of the beam, the wave frequency is obtained as

$$\omega_r = -\frac{\frac{k_\perp v_{Ti}^2}{\Omega_i}\frac{\partial \ln N}{\partial x}}{1 + k^2\lambda_{De}^2} \sim \frac{k_\perp v_{Te}^2}{\Omega_e}\frac{\partial \ln N}{\partial x} = \omega_{Ne}, \tag{23}$$

where $\lambda_{De} = v_{Te}/\omega_{pe}$ is the Debye wavelength of the electron, with $k^2\lambda_{De}^2 \ll 1$, and $\omega_{Ne} = (k_\perp v_{Te}^2/\Omega_e)(\partial \ln N/\partial x) \sim (k_\perp v_{Te}^2/\Omega_e)1/L_N$ is the density gradient contribution in the gradient drift frequency of electrons. The instability growth rate is defined as

$$\gamma = \frac{-\text{Im}\varepsilon(\omega_r, k, x)}{\partial\, \text{Re}\varepsilon(\omega, k, x)/\partial\omega|_{\omega=\omega_r}}, \tag{24}$$

where $\text{Re}\varepsilon(\omega, k, x)$ and $\text{Im}\varepsilon(\omega, k, x)$ are the real and imaginary parts of Eq. (22), respectively. By substituting Eqs. (22) and (23) into Eq. (24), the growth rate of the instability for the plasma system of interest in the present study is obtained as

$$\gamma = -\frac{0.81\frac{\omega^2\nu_{\text{eff}}}{k_z^2 v_{Te}^2}\eta_e}{1 + A}, \tag{25}$$

where

$$A = \frac{\omega_b^2}{\omega_{pe}^2}\frac{k_\perp^2 v_{Te}^2}{\omega^2}\left\{\frac{\omega - 2k_z u_\parallel}{\Omega_i} + 2\frac{k_z^2}{k_y^2}\left(\frac{3}{4}\frac{k_\perp^2 u_\perp^2}{\Omega_i^2} + \frac{k_z^2 u_\parallel^2(2\omega - k_z u_\parallel)}{(\omega - k_z u_\parallel)^3} - \frac{u_\parallel^2}{u_\perp^2}\frac{2k_\perp u_\perp}{\Omega_i} - \frac{k_z u_\parallel(3\omega - 2k_z u_\parallel)}{(\omega - k_z u_\parallel)^2} + 1\right)\right\}. \tag{26}$$

TABLE I. Plasma parameters used for computing the wave frequency and the growth rate of TGI.

Particle	$\omega_{p\alpha}$ (rad/s)	$\Omega_{c\alpha}$ (rad/s)	$v_{T\alpha}$ (m/s)	T_α (K)	L_T^{-1} (km^{-1})	L_N^{-1} (km^{-1})
electron	2.33×10^7	8.81×10^6	1.38×10^5	1250	1/5	1/5
ion	1.36×10^5	301.5	803.05	1250	0	1/5

It could be readily noticed that $\eta_e = 0$ corresponds to $\gamma = 0$ and without the presence of the beam ($\omega_b = 0$), the instability ($\gamma > 0$) occurs for $\eta_e < 0$, which is known as TGI. In fact, the TGI is experienced in regions with opposite electron density and temperature gradients, while the temperature gradient is necessary to excite the instability. Indeed, when the plasma is perturbed, a charge accumulates at the interface between hot and cool regions and consequently the polarization electrostatic fields is formed, directed from the high- to low-density regions of the perturbed plasma. The polarization field of the perturbation will grow as a consequence of the diamagnetic drifts of the opposed density and temperature gradients. These fields, in combination with the ambient magnetic field, cause $\mathbf{E} \times \mathbf{B}$ flows that further enhance the perturbation.

As one can notice from Eq. (25), the growth rate of the instability also develops by the frequent electrons collisions. It thus follows that the electron collisions not only do not stabilize the TGI, but they also destabilize it.

In addition, it can now be understood that the presence of the beam can stabilize or destabilize the TGI that depends on the magnitude and sign of A [Eq. (26)]. Equation (25) yields straightforwardly that in the presence of the ion beam, the instability ($\eta_e < 0$) remains unstable if $1 + A > 0$ and it can be stabilized ($A > 0$) or destabilized ($A < 0$) by the ion beam. So the conditions in which the ion beam acts as a stabilizing effect can be defined by a stabilization switching relation

$$\frac{\omega - 2k_z u_\parallel}{\Omega_i} + 2\frac{k_z^2}{k_y^2}\left\{\frac{3}{4}\frac{k_\perp^2 u_\perp^2}{\Omega_i^2} + \frac{k_z^2 u_\parallel^2(2\omega - k_z u_\parallel)}{(\omega - k_z u_\parallel)^3} - \frac{u_\parallel^2}{u_\perp^2}\frac{2k_\perp u_\perp}{\Omega_i} - \frac{k_z u_\parallel(3\omega - 2k_z u_\parallel)}{(\omega - k_z u_\parallel)^2} + 1\right\} > 0. \quad (27)$$

To analyze this switching relation, the conditions under which Eq. (25) has been obtained, in particular, $k_z v_{Ti} \ll \omega$ and $k_\perp u_\perp/\Omega_i \ll 1$, should be considered. Due to inhomogeneity being weak (large L_N), $k_\perp/k_z$ must be too large to satisfy the condition used, $k_z v_{Ti} \ll \omega = k_\perp v_{Te}^2/\Omega_e L_N$. Under such a condition, it should be understood that the terms with the factor $k_z^2/k_\perp^2$ in Eq. (27) are so low that the first term $(\omega - 2\,k_z u_\parallel)/\Omega_i$ determines the conditions for which the beam has the stabilizing or destabilizing effect on the TGI. So it can be implied that, in the case $\omega > 2\,k_z u_\parallel$, the parameter A is positive ($A > 0$) and therefore the ion beam has a stabilizing effect on the TGI. In the inverse case $\omega < 2\,k_z u_\parallel$, the parameter A is negative ($A < 0$) and therefore the ion beam has a destabilizing effect on the TGI.

By implementing the proposed model and equations, the TGI in the presence of a rotating ion beam is investigated in the following section.

IV. RESULTS AND DISCUSSION

For the purpose of investigation, it is interested to see how the TGI would be affected by the presence of energetic oxygen ions (O^+) in the form of a beam including parallel and perpendicular velocity components through the wave-particle interactions. The plasma parameters used for computing the wave frequency and the growth rate of TGI in this study are listed in Table I.

To analyze TGI in the presence of a nonrelativistic rotating ion beam such as O^+, Eqs. (23) and (25) are implemented regarding the oscillation frequency spectrum and the growth rate of the instability, respectively. The inhomogeneous plasma consists of electrons and ions and includes temperature and density gradients. The density and temperature gradients driving TGI are assumed to be perpendicular to the magnetic field, where the magnetic field and inhomogeneity are along the z and x directions, respectively (the gradients in density and temperature are assumed to be in the $\mathbf{x}$ and $-\mathbf{x}$ directions, respectively).

It can be implied from Eq. (23) that the oscillation frequency is dependent on the density inhomogeneity scale length L_N and wavelength. Figure 2 shows the normalized wave frequency of the temperature gradient wave ω_r/Ω_i as a function of normalized wave number ($k_\perp \rho_i$) and normalized density inhomogeneity scale length (L_N/ρ_i) for $T_e/T_i = 1$, where $\rho_i = v_{Ti}/\Omega_i$ is the Larmor radius of the ion.

It can be seen that the magnitude of the wave frequency decreases by increasing the characteristic length of inhomogeneity L_N, and increases by increasing the perpendicular component of the wave vector $k_\perp$. As the diamagnetic drift of electrons and the propagation of the temperature gradient wave are in the $-\mathbf{y}$ direction, the positive wave frequency ($\omega > 0$) means that the phase velocity of the wave is in the

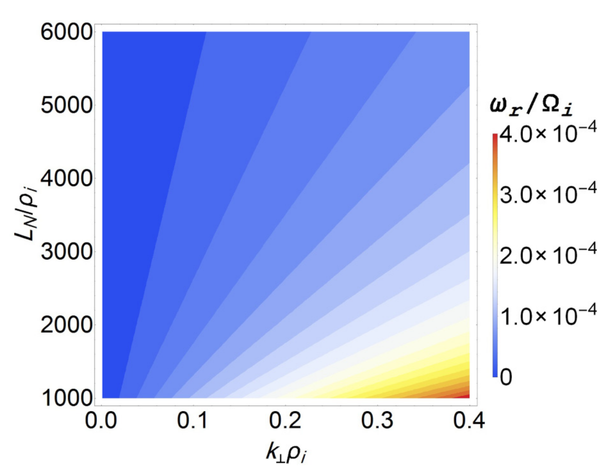

FIG. 2. Wave frequency ω_r/Ω_i as a function of wave number $k_\perp\rho_i$ and density inhomogeneity scale length L_N/ρ_i for $T_e/T_i = 1$.

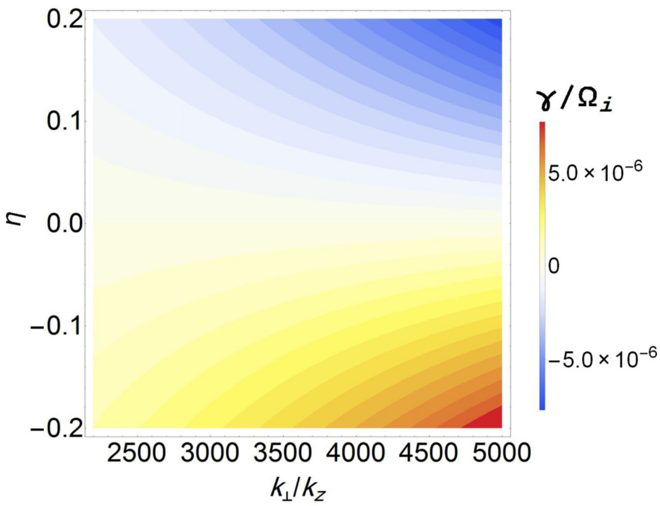

FIG. 3. Growth rate of the TGI in the absence of an ion beam as a function of $k_\perp/k_z$ and the relative parameters $\eta_e = L_N/L_{Te}$ for $k_\perp\rho_i = 0.4$, $T_e/T_i = 1$, $m_e/M_i = 1/(8 \times 1836)$, $L_N/\rho_i = 5 \times 10^3$, $v_{Ti} = 800$ m/s, and $\nu_{\rm eff}/\Omega_i = 1.1$.

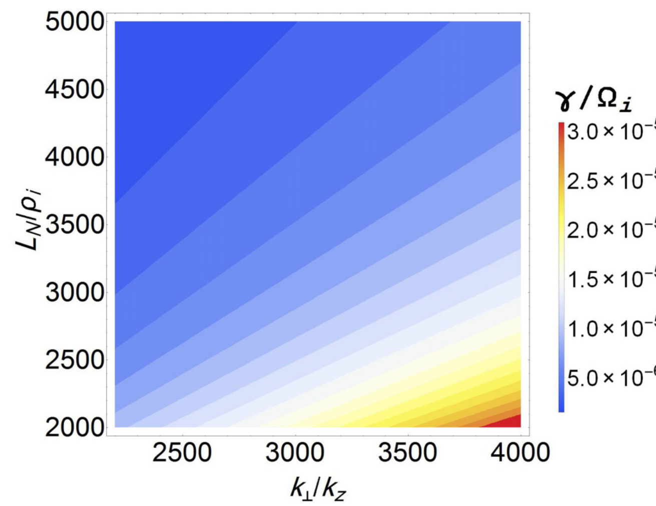

FIG. 4. Growth rate of the TGI in the absence of an ion beam as a function of $k_\perp/k_z$ and the density inhomogeneity scale length L_N/ρ_i for $k_\perp\rho_i = 0.4$, $T_e/T_i = 1$, $m_e/M_i = 1/(8 \times 1836)$, $\eta_e = -0.2$, $v_{Ti} = 800$ m/s, and $\nu_{\rm eff}/\Omega_i = 1.1$.

direction of the electron drift velocity ($\omega = \omega_{Ne} = \mathbf{k} \cdot \mathbf{v}_{de} > 0$) [47]. This refers to the frequency range $|\omega| \gg k_z v_{Ti}$, for which, in high frequency, the electrons are responsible.

According to Eq. (25), the growth rate of the instability depends on the wave vector components and the relative parameters $\eta_e = L_N/L_{Te}$. Figure 3 shows the normalized growth rate of the TGI, γ/Ω_i, in the absence of an ion beam as a function of $k_\perp/k_z$ and the normalized relative parameters ($\eta_e = L_N/L_{Te}$) for $k_\perp\rho_i = 0.4$, $T_e/T_i = 1$, $m_e/M_i = 1/(8 \times 1836)$, $L_N/\rho_i = 5 \times 10^3$, $v_{Ti} = 800$ m/s, and $\nu_{\rm eff}/\Omega_i = 1.1$. It could be noticed from Fig. 3 that the instability occurs in the region that the electron density and temperature gradients are in the opposite directions ($\eta_e < 0$), where the point corresponding to the interchange between stability and instability is $\eta_e = 0$, i.e., without the temperature gradient. Consequently, the temperature gradient is found to be necessary for the instability.

In addition, it can be readily seen from Fig. 3 that the growth rate of the instability increases by increasing the degree of propagation of the TGI with respect to the magnetic field. This treatment is based on the fact that the TGI is created across the gradients and the magnetic field, i.e., along the y direction.

Since Fig. 3 corresponds to the constant characteristic density length of inhomogeneity, it indicates that the TGI is destabilized by the temperature gradients, which, as shown in Fig. 4, are destabilized by the density gradients. Figure 4 shows the normalized growth rate of the TGI (γ/Ω_i) in the absence of an ion beam as a function of $k_\perp/k_z$ and normalized density inhomogeneity scale length L_N/ρ_i for $k_\perp\rho_i = 0.4$, $T_e/T_i = 1$, $m_e/M_i = 1/(8 \times 1836)$, $\eta_e = -0.2$, $v_{Ti} = 800$ m/s, and $\nu_{\rm eff}/\Omega_i = 1.1$.

As it was discussed in Sec. II, the plasma is assumed to be completely ionized, and the Coulomb collisions associated with the electrons are considered in this study. Figure 5 shows the normalized growth rate of the TGI in the absence of an ion beam as a function of $k_\perp/k_z$ and normalized electron collision frequency $\nu_{\rm eff}/\Omega_i$ for $k_\perp\rho_i = 0.4$, $T_e/T_i = 1$, $m_e/M_i = 1/(8 \times 1836)$, $\eta_e = -0.2$, $L_N/\rho_i = 2 \times 10^3$, and $v_{Ti} = 800$ m/s. It can be readily seen that the electron collisions destabilize the TGI. So it can be implied that the particle collisions do not stabilize the drift instabilities of the inhomogeneous plasma and they may even cause their development.

As it can be seen from Figs. 3–5, in the absence of the ion beam, an unstable condition known as TGI is experienced in regions with opposite electron density and temperature gradients ($\eta < 0$), where it is destabilized by the temperature and plasma density gradients and the frequent electron collisions. In addition, the growth of the instability increases by increasing the degree of its propagation with respect to the magnetic field. Now, in order to know how the TGI is

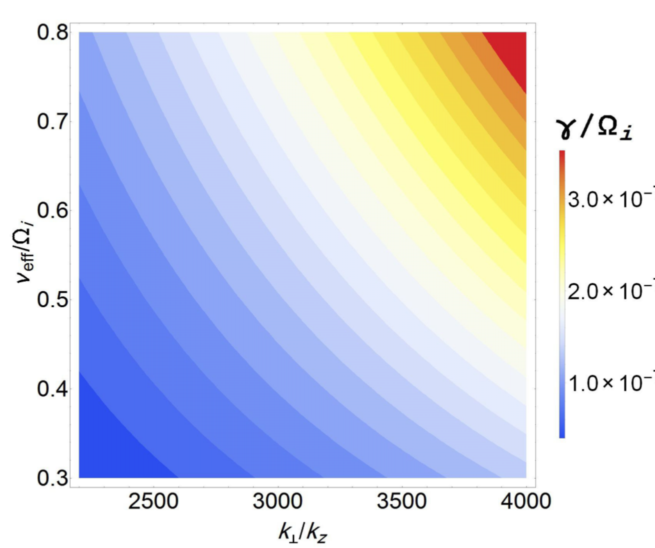

FIG. 5. Growth rate of the TGI in the absence of an ion beam as a function of $k_\perp/k_z$ and electron collision frequency $\nu_{\rm eff}/\Omega_i$ for $k_\perp\rho_i = 0.4$, $T_e/T_i = 1$, $m_e/M_i = 1/(8 \times 1836)$, $\eta_e = -0.2$, $L_N/\rho_i = 2 \times 10^3$, and $v_{Ti} = 800$ m/s.

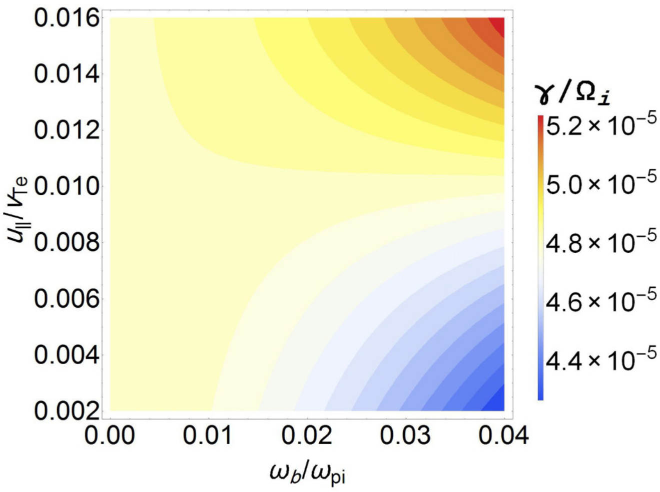

FIG. 6. Growth rate of the instability in the presence of an ion beam as a function of the Langmuir beam frequency ω_b/ω_{pi} and parallel beam velocity $u_{\|}/v_{Te}$ for $k_\perp\rho_i = 0.1$, $k_\perp/k_z = 2\times 10^3$, $T_e/T_i = 1$, $m_e/M_i = 1/(8\times 1836)$, $\eta_e = -0.2$, $L_N/\rho_i = 2\times 10^3$, $v_{Ti} = 800$ m/s, $\nu_{\rm eff}/\Omega_i = 1.1$, and $u_\perp/c = 2\times 10^{-5}$.

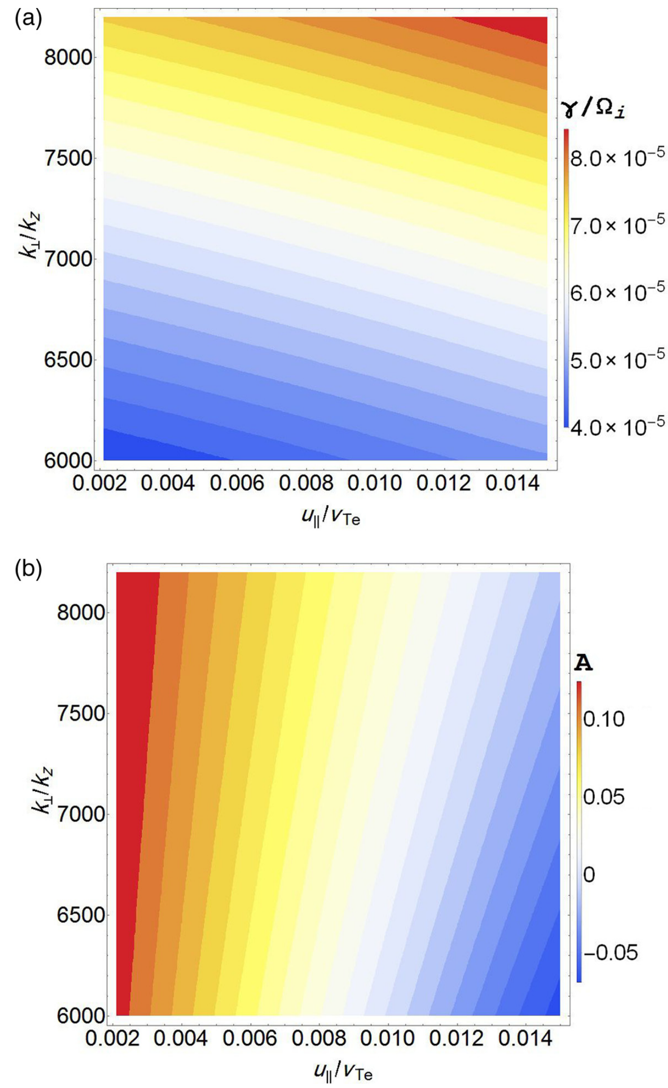

FIG. 7. (a) Growth rate of the instability in the presence of an ion beam and (b) effect of the ion beam on the TGI as a function of the parallel beam velocity $u_{\|}/v_{Te}$ and $k_\perp/k_z$ for $k_\perp\rho_i = 0.1$, $T_e/T_i = 1$, $m_e/M_i = 1/(8\times 1836)$, $\eta_e = -0.2$, $L_N/\rho_i = 2\times 10^3$, $v_{Ti} = 800$ m/s, $\nu_{\rm eff}/\Omega_i = 1.1$, $\omega_b/\omega_{pi} = 0.02$, and $u_\perp/c = 4\times 10^{-5}$.

affected by a rotating ion beam, we continue our analysis by considering the TGI in the presence of energetic oxygen ions (O^+) in the form of a charged particle beam including parallel and perpendicular velocity components. In order to provide a deeper understanding of the effects of a rotating ion beam on the TGI, it is instructive to find the instability growth rate as a function of the ion beam parameters such as velocity, density, and ion species.

Figure 6 shows the normalized growth rate of the instability in the presence of an ion beam as a function of the normalized Langmuir beam frequency ω_b/ω_{pi} and normalized parallel beam velocity $u_{\|}/v_{Te}$ for $k_\perp\rho_i = 0.1$, $k_\perp/k_z = 2\times 10^3$, $T_e/T_i = 1$, $m_e/M_i = 1/(8\times 1836)$, $\eta_e = -0.2$, $L_N/\rho_i = 2\times 10^3$, $v_{Ti} = 800$ m/s, $\nu_{\rm eff}/\Omega_i = 1.1$, and $u_\perp/c = 2\times 10^{-5}$. As expected from the discussion in Sec. III, depending on the parallel beam velocity component, the instability growth rate can be both stabilized and destabilized by the ion beam. In the sufficiently low parallel beam velocity component ($\omega > 2k_z u_{\|}$), the instability growth rate is stabilized by the Langmuir beam frequency ω_b/ω_{pi}, which is due to the interaction between the ion beam and the temperature gradient wave; by increasing the density of the ion beam, the energy transferring from the TGI to the beam increases. Indeed, under this condition, the stabilizing effect of the rotating ion beam exceeds the destabilizing effect due to the free energy of the beam.

The parallel beam velocity component dependence shows that by increasing this ion beam parameter to be sufficiently large ($\omega < 2k_z u_{\|}$), the instability is destabilized by the rotating ion beam. Indeed, by increasing the parallel beam velocity component, a condition is reached in which increasing the free energy of the beam exceeds the reduction of the instability growth rate due to the energy transferring from the TGI to the beam. Therefore, by increasing the density of the ion beam, the TGI is destabilized.

From the analysis of Fig. 6 the effect of the rotating ion beam on the TGI is evident. Under the conditions considered, in the presence of the rotating ion beam, by increasing the density of the ion beam, the growth rate of the instability can decrease more than two times, in the sufficiently low parallel beam velocity component, and it can increase more than one to two times, in the sufficiently high parallel beam velocity component.

Figure 7 shows the growth rate of the instability in the presence of an ion beam [Fig. 7(a)] and the effect of the ion beam on the TGI [Fig. 7(b)] as a function of normalized parallel beam velocity $u_{\|}/v_{Te}$ and $k_\perp/k_z$ for $k_\perp\rho_i = 0.1$, $T_e/T_i = 1$, $m_e/M_i = 1/(8\times 1836)$, $\eta_e = -0.2$, $L_N/\rho_i = 2\times 10^3$, $v_{Ti} = 800$ m/s, $\nu_{\rm eff}/\Omega_i = 1.1$, $\omega_b/\omega_{pi} = 0.02$, and $u_\perp/c = 4\times 10^{-5}$. Figure 7(a) illustrates an increase of the growth rate of the instability with an increase of $k_\perp/k_z$, while in this range of the parallel beam velocity Fig. 7(b) shows the enhancement in the stabilizing effect ($A > 0$) and reduction

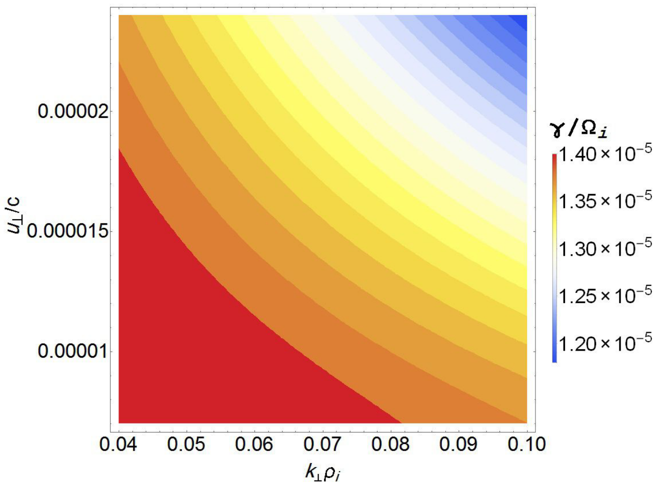

FIG. 8. Growth rate of the instability in the presence of an ion beam as a function of wave number $k_\perp\rho_i$ and perpendicular beam velocity $u_\perp/c$ for $k_\perp/k_z = 2.7\times10^3$, $T_e/T_i = 1$, $m_e/M_i = 1/(8\times 1836)$, $\eta_e = -0.2$, $L_N/\rho_i = 2\times10^3$, $v_{Ti} = 800$ m/s, $\nu_{\text{eff}}/\Omega_i = 1.1$, $\omega_b/\omega_{pi} = 0.1$, and $u_\parallel/v_{Te} = 0.02$.

in destabilizing effect ($A < 0$) of the ion beam with an increase of $k_\perp/k_z$. This result means that the rate of the energy transferring from the temperature gradient wave to the beam is increasing with an increase of $k_\perp/k_z$. It can be noticed that as it was shown in Fig. 3, in the absence of the ion beam, the growth rate of the TGI increases with an increase of $k_\perp/k_z$. Taken together, these imply that the enhancement in producing TGI exceeds the reduction of the growth rate due to the energy transfer to the ion beam and consequently the growth rate of the instability increases with an increase of $k_\perp/k_z$.

According to these findings, we conclude that increasing the degree of propagation of the TGI with respect to the magnetic field improves the interaction of it with the rotating ion beam. Therefore, although the growth rate of the TGI is enhanced by increasing $k_\perp/k_z$ [21], the damping of it can be improved by increasing $k_\perp/k_z$.

As it was explained in Sec. III, due to $k_\perp/k_z$ being large, the terms including the perpendicular velocity component become suppressed and lose their effect. However, in some special conditions such as $\omega = 2k_z u_\parallel$, which leads to elimination of the first term in Eq. (26), the perpendicular velocity component becomes important. We analyze how the instability would be affected by the perpendicular velocity component in the range mentioned. Figure 8 shows the normalized growth rate of the instability in the presence of an ion beam as a function of normalized wave number ($k_\perp\rho_i$) and perpendicular beam velocity ($u_\perp/c$) for $k_\perp/k_z = 2.7\times10^3$, $T_e/T_i = 1$, $m_e/M_i = 1/(8\times1836)$, $\eta_e = -0.2$, $L_N/\rho_i = 2\times10^3$, $v_{Ti} = 800\,\text{m/s}$, $\nu_{\text{eff}}/\Omega_i = 1.1$, $\omega_b/\omega_{pi} = 0.1$, and $u_\parallel/v_{Te} = 0.02$. It can be seen that for a fixed degree of propagation, the growth rate of the instability decreases with an increase in the wave number and perpendicular beam velocity.

Investigating the effects of a rotating ion beam on the instability growth rate reveals that the rotating ion beam could provide a tendency for the TGI towards stabilizing or destabilizing. The stabilizing effect can be due to the interaction between the ion beam and the temperature gradient wave; the energy of the temperature gradient wave transfers to the ion beam. Depending on conditions, the destabilizing effect due to the free energy of the beam may exceed the stabilizing effect of the rotating ion beam.

V. CONCLUSION

The aim of this paper was to investigate the effect of a rotating ion beam on the temperature gradient instability. So in this work the attention was put on the long-wavelength temperature gradient waves ($\omega \leqslant \omega_{d\alpha} \sim k_\perp v_{T\alpha}^2/\Omega_\alpha L_0$) driven by the density and temperature gradients perpendicular to the magnetic field in the presence of a rotating ion beam such as O^+. A nonrelativistic rotating ion beam was taken into consideration that propagates parallel to the magnetic field. The kinetic theory and the zeroth-order approximation of geometrical optics were implemented to derive the dielectric permittivity tensor of a collisional inhomogeneous plasma medium and consequently observe the growth rate of the TGI in the collisional regime.

It was found that in regions where the electron density and temperature gradients are in opposite directions, an unstable situation known as TGI is observed. In addition, it was found that the TGI is destabilized by the frequent electron collisions and the plasma temperature and density gradient, and depending on conditions can be stabilized or destabilized by the characteristics of the ion beam, namely, velocity and density.

The interplay of the stabilizing tendency of the rotating ion beam and the destabilizing tendency of the free energy results in both stabilizing and destabilizing states for the TGI. In the sufficiently low parallel beam velocity component ($\omega > 2k_z u_\parallel$), the rotating ion beam provides a significant stabilizing effect on the instability in the frequency range under consideration in the present study. The stabilizing effect can be due to the interaction between the ion beam and the temperature gradient wave; the energy of the temperature gradient wave transfers to the ion beam and consequently leads to stabilization of the instability growth rate. By increasing the parallel beam velocity component to be sufficiently large ($\omega < 2k_z u_\parallel$), the instability is destabilized by the rotating ion beam. Furthermore, in some special conditions such as $\omega = 2k_z u_\parallel$, the perpendicular velocity component becomes important and provides a stabilizing effect on the TGI.

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