

Erratum: Modeling the quasistatic energy transport between nanoparticles [Phys. Rev. E **92**, 062138 (2015)]

George Y. Panasyuk and Kirk L. Yerkes

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In Eq. (49), a factor ϵ_{vk} in the left-hand side was omitted. Corrected, Eq. (49) is

$$\frac{d}{dt}E_v = \sum_{k=1}^N \epsilon_{vk} \dot{E}_{vk} = -[J_v^{(1)} + J_v^{(2)}], \quad (49)$$

where

$$\epsilon_{vk} = \frac{C}{\hat{h}(z_k)} [Z_v(z_k) - A_{vk}^{(2)}(0)(2z_k^2)^{-1}], \quad (1)$$

with $\hat{h}(z_k)$, $Z_v(z_k)$, and $A_{vk}^{(p)}(t)$ determined from (23), (28), and (46), respectively; $C = 2\hat{\gamma}D^2/\pi$. As one can check, $\epsilon_{vk} = O(1)$. Corrected relations (54)–(56) are

$$\sum_{k=1}^N \epsilon_{1k} \dot{E}_{1k} = - \sum_{k=1}^N [(j_{1k} + j_{11k})E_{1k} + j_{12k}E_{2k}], \quad (54)$$

$$\sum_{k=1}^N \epsilon_{2k} \dot{E}_{2k} = - \sum_{k=1}^N [(j_{2k} + j_{22k})E_{2k} + j_{21k}E_{1k}], \quad (55)$$

and

$$\sum_{k=1}^N (\epsilon_{1k} \dot{E}_{1k} + \epsilon_{2k} \dot{E}_{2k}) = 0. \quad (56)$$

Correspondingly, relations (60) and (61) become

$$\epsilon_{1k} \dot{E}_{1k} = R_{1k}E_{1k} - R_{2k}E_{2k} \equiv -J_k \quad (60)$$

and

$$\epsilon_{2k} \dot{E}_{2k} = R_{2k}E_{2k} - R_{1k}E_{1k} \equiv J_k, \quad (61)$$

where the coefficients R_{1k} and R_{2k} are correct and determined from (62) and (63). Finally, due to the corrected Eq. (56) and the mode independence, the relation between $\delta E_{1k}(t)$ and $\delta E_{2k}(t)$ becomes $\epsilon_{2k} \delta E_{2k}(t) = -\epsilon_{1k} \delta E_{1k}(t)$.

The origin of the factors ϵ_{vk} can be explained as follows (see, also, [1]). Using formulas (A1)–(A5) from the Appendix and relation (38) for E_{vk} , one can reduce the expression for

$$E_v = \sum_{i=1}^{N_v} \left\langle \frac{p_{vi}^2}{2m_{vi}} + \frac{m_{vi}\omega_{vi}^2 x_{vi}^2}{2} \right\rangle \quad (2)$$

to

$$2E_v = \sum_k E_{1k} \sum_{i=1}^{N_v} (e_{vi}^k)^2 + \sum_k E_{2k} z_k^{-2} \sum_{i=1}^{N_v} (w_{vi} e_{vi}^k)^2, \quad (3)$$

where

$$\sum_{i=1}^{N_v} (e_{vi}^k)^2 = \frac{C}{\hat{h}(z_k)} A_{vk}^{(2)}(0) = \frac{C}{\hat{h}(z_k)} Z_v(z_k), \quad (4)$$

and

$$\begin{aligned} \sum_{i=1}^{N_v} (w_{vi} e_{vi}^k)^2 &= -\frac{C}{\hat{h}(z_k)} A_{vk}^{(4)}(0) \\ &= \frac{C}{\hat{h}(z_k)} [z_k^2 Z_v(z_k) - A_{vk}^{(2)}(0)]. \end{aligned} \quad (5)$$

Thus, after substitution of (4) and (5) into (3), the resulting expression for E_v reads

$$E_v = \sum_k (\epsilon_{1k} E_{1k} + \epsilon_{2k} E_{2k}), \quad (6)$$

where ϵ_{vk} are determined from (1). Reducing the left-hand side of (5) using the approach used to derive the approximate expression (29) for $Z_v(z_k)$, one can simplify the expression for

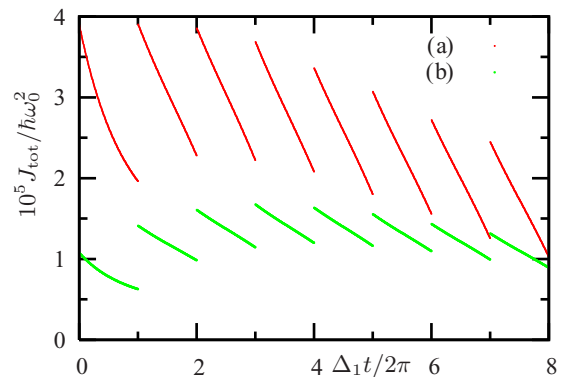


FIG. 7. Time dependences of J_{tot} between identical nanoparticles when $\hat{\gamma}/\omega_0 = 0.1$, $D/\omega_0 = 1$, and $\Delta_1/\omega_0 = \Delta_2/\omega_0 = 0.001$ and $\alpha = 0.01$ (a) $k_B \bar{T}/\hbar\omega_0 = 0.2$ and (b) $k_B \bar{T}/\hbar\omega_0 = 0.15$.

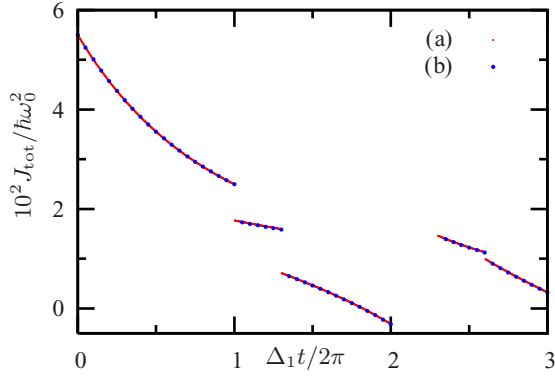


FIG. 8. Time dependences of J_{tot} between unequal nanoparticles when $\hat{\gamma}/\omega_0 = 0.1$, $D/\omega_0 = 1$, and $\Delta_1/\omega_0 = 0.001$, $\Delta_2/\omega_0 = 0.0013$ and $\alpha = 0.01$. (a) $k_B\bar{T}/\hbar\omega_0 = 0.2$ and (b) $k_B\bar{T}/\hbar\omega_0 = 0.1$.

$\epsilon_{\nu k}$ to

$$\epsilon_{\nu k} = \frac{\Delta_{\nu_1} \sin^2(\pi z_k / \Delta_{\nu_1})}{\Delta_1 \sin^2(\pi z_k / \Delta_1) + \Delta_2 \sin^2(\pi z_k / \Delta_2)}, \quad (7)$$

where $\nu_1 = 1$ if $\nu = 2$, and $\nu_1 = 2$ if $\nu = 1$. If $\Delta_1 = \Delta_2$, $\epsilon_{1k} = \epsilon_{2k} = 1/2$.

The main idea underlying the derivations of the coefficients j_{1k} , j_{11k} , j_{12k} , j_{2k} , j_{22k} , and j_{21k} is the same as in [1]. It is based on an observation that each time when t passes t_{nm} from (65), the time integrals, such as

$$\int_0^t ds g(s) \sin(w_{\nu i} s) \quad \text{and} \quad \int_0^t ds g(s) \cos(w_{\nu i} s), \quad (8)$$

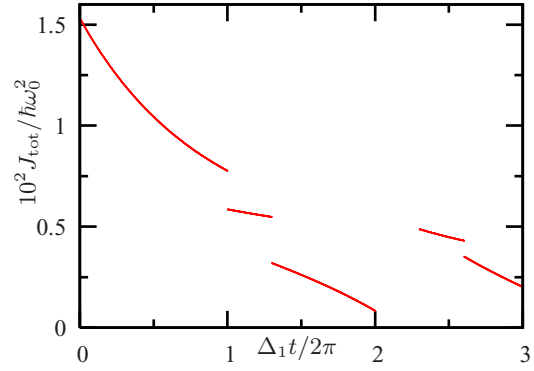


FIG. 9. Time dependences of J_{tot} between Pt nanoparticles when $\hat{\gamma}/\omega_0 = 0.13$, $D/\omega_0 = 0.35$, and $\Delta_1/\omega_0 = 0.001$, $\Delta_2/\omega_0 = 0.0013$, $k_B\bar{T}/\hbar\omega_0 = 0.1$, and $\alpha = 0.01$.

change their values and stay approximately unchanged until next time moment $t_{n'm'}$. This happens because the solution kernel $g(t)$ from (22) is noticeably nonzero only at times within time intervals $[t_{nm} - \tau, t_{nm} + \tau]$ near t_{nm} , where τ is the characteristic microscopic time from (24). Details of this derivation can be found in Appendix B from [1]. In this work, we also developed another way to derive the same coefficients. It is based on the accurate time averaging of expressions (36) and (37). As we found, the produced coefficients essentially coincide with the ones obtained in the way described above. The latter way, however, is preferable because it is much faster than the accurate time averaging.

Although these corrections do not qualitatively change the main results of the article (no revisions of the Abstract and Conclusions are needed), they lead to some modifications of the energy current between the nanoparticles, which are reflected in Figs. 7–9.

[1] G. Y. Panasyuk, K. L. Yerkes, and T. J. Haugan, *Phys. Rev. E* **99**, 032141 (2019).