Reply to "Comment on 'Partial equivalence of statistical ensembles in a simple spin model with discontinuous phase transitions'"

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(Received 30 July 2020; accepted 21 August 2020; published 15 September 2020)

In the present Reply we show that a Comment casting doubts on the results of our recent paper [Fronczak, Fronczak, and Siudem, Phys. Rev. E **101**, 022111 (2020)] results from a misunderstanding of the assumptions of our model and from overinterpretation based on this misunderstanding.

DOI: 10.1103/PhysRevE.102.036102

The submitted Comment on our paper [1] results from a misunderstanding of the assumptions of our model and from overinterpretation based on this misunderstanding.

The main objection raised in the Comment is that ergodicity is broken in our model and therefore the microcanonical treatment is inappropriate. The author writes the following: "As pointed out by Mukamel *et al.* [2], starting from an initial macroscopic realization of the system is not able to move to the other macroscopic realization via local dynamics."

With reference to this objection, we would like to point out that in order to talk about breaking of ergodicity one needs to specify the model dynamics (in the sense of its evolution over time). Unlike the mentioned paper by Mukamel *et al.*, our paper does not specify any dynamics. We even write the following: "...we cut ourselves off from the question of whether the model is physically realistic or not. We just treat it as a toy model having some nontrivial properties resulting [solely from its Hamiltonian]..." (see Sec. II A). We assume that our model is ergodic by definition. This assumption is expressed by Eq. (5), which defines the microcanonical entropy of the model. In other words, Eq. (5) is true by assumption (by definition). The author of the Comment is wrong to suggest its incorrectness as resulting from ergodicity breaking.

The author's criticism would only be justified if we wrote in our paper that the dynamics of our model is "local." We did not write this anywhere. Furthermore, it is known that nonlocal dynamics allows the system to visit such areas of state space that could be inaccessible to the system realizing local dynamics. In this discussion, issues related to local and nonlocal dynamics and their consequences for ergodicity breaking are even more important, because the Hamiltonian system considered in our paper is motivated by social systems (see [3]). In such systems, nonlocal dynamics is even more common.

To sum up, in our opinion, the criticism given in the Comment is unfounded.

This work has been supported by the National Science Centre of Poland (Narodowe Centrum Nauki) under Grant No. 2015/18/E/ST2/00560 (A.F. and G.S.).

[1] A. Fronczak, P. Fronczak, and G. Siudem, Partial equivalence of statistical ensembles in a simple spin model with discontinuous phase transitions, Phys. Rev. E **101**, 022111 (2020).

[2] D. Mukamel, S. Ruffo, and N. Schreiber, Breaking of Ergodicity and Long Relaxation Times in Systems with Long-Range Interactions, Phys. Rev. Lett. **95**, 240604 (2005).

[3] A. Fronczak, P. Fronczak, and A. Krawiecki, A minimal exactly solved model with the extreme Thouless effect, Phys. Rev. E **93**, 012124 (2016).

