Reply to "Comment on 'Quantum fidelity approach to the ground-state properties of the one-dimensional axial next-nearest-neighbor Ising model in a transverse field' "

O. F. de Alcantara Bonfim*
Department of Physics, University of Portland, Portland, Oregon 97203, USA

B. Boechat† and J. Florencio‡
Departamento de Física, Universidade Federal Fluminense, Niterói, 24210-340, RJ, Brazil

(Received 7 April 2020; revised 16 May 2020; accepted 24 June 2020; published 13 July 2020)

We disagree with the objections raised by Nemati *et al.* regarding the phase transitions reported in our paper, where we used the fidelity method. Contrary to their claims, our fidelity calculations do not depend on energy level crossing between excited states. We obtain the same results just by analyzing the second derivative of the ground-state energy with respect to the interaction energy coupling J_2.

DOI: 10.1103/PhysRevE.102.016102

In their Comment, Nemati *et al.* [1] claim that energy level crossings between low-lying excited states produce false markers of quantum transitions. Their Fig. 1 shows the behavior of the fidelity and those of the first three energy levels as the coupling energy J_2 is varied. The figure shows that whenever the excited levels cross with each other, fidelity goes through a dip. That feature is certainly interesting but hardly can be construed as cause and effect, since fidelity by definition depends only on the system ground-state properties [2]. In the calculations of our paper, we use only ground-state eigenvectors. Level crossings between energies not involving the ground-state have no bearing on the fidelity.

The basic idea behind the fidelity method as stated in Ref. [2] is the following. Consider a Hamiltonian, $H(\lambda)$, where λ is an arbitrary parameter. The fidelity is defined as the overlap between two ground states corresponding to the Hamiltonian for the parameters λ and $\lambda + \delta$.

There are two points to be considered here. First is the behavior of the overlap as a function of the size (N) of the system for a fixed δ, in the limit of $N \to \infty$. It was pointed out by Anderson [3] that at that limit the overlap (fidelity) of the two ground states vanishes for points on each side of the transition—the so-called *Anderson catastrophe*.

Second, for finite N, the ground states from each side of the transition are not truly orthogonal, and hence the overlap of their wave vectors cannot be zero. That causes the finiteness of the peak at the transition. What happens to the excited state energies at the transition, whether they cross between themselves or not, is certainly an interesting point that deserves further investigation, but such crossings cannot serve as criterion for dismissal of quantum phase transitions. Most likely they would reinforce the results from the fidelity susceptibility.

*bonfim@up.edu
†bmbp@if.uff.br
‡jfj@if.uff.br

There are instances where the fidelity can be calculated exactly, namely the Ising model in a transverse field [4] and the anisotropic XY model in a transverse field [5]. We refer to these paper for the comparisons between the two ways of calculating the fidelity and its ability to identify quantum phase transitions.

We can obtain the same transition locations by using the ground-state energy E_0 alone. Without loss of generality, let us consider the simplest example, the case $L = 12$, with $B_x = 0.2$. In Fig. 1 the ground-state energy and its second derivative, E_0'', are shown as functions of J_2. Figure 2 shows the second derivative of the ground-state energy level and the fidelity susceptibility, χ, as functions of J_2. There appears a very close correspondence in the location of the peaks obtained by these two different methods.

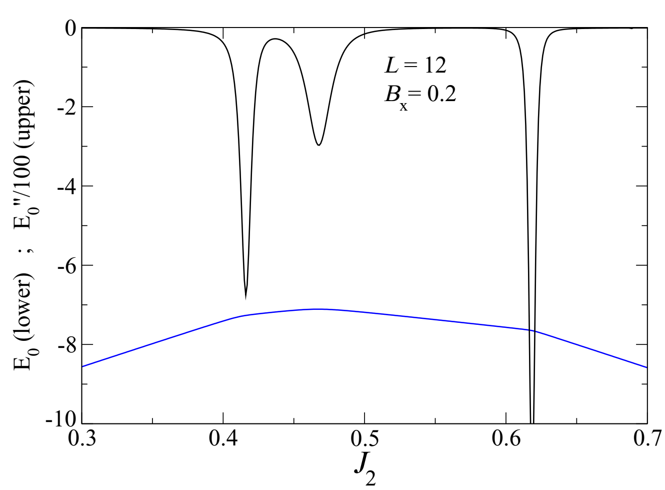

FIG. 1. Ground-state energy (lower curve) and its second derivative (upper curve) as a function of the energy coupling J_2. The curvature of the energy shows dramatic changes at the location of the transitions. Here and in the next figure, we have $L = 12$ and $B_x = 0.2$.

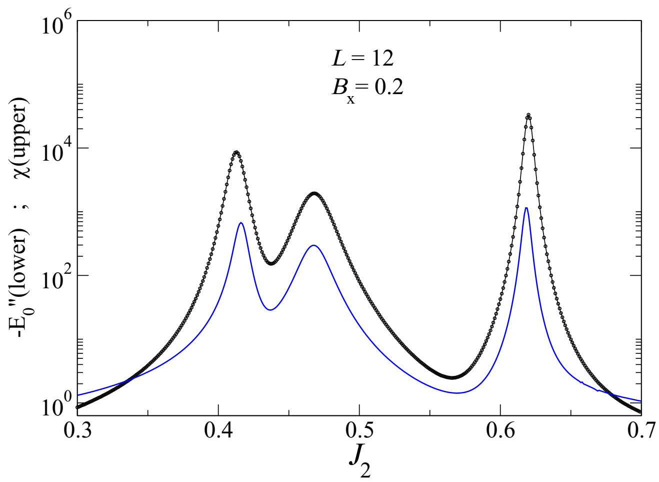

FIG. 2. Fidelity susceptibility (upper curve) and second derivative of the ground-state energy (lower curve) as a function of J_2. Note the correspondence of the peaks of each curve.

It is no accident that the ground-state energy can provide the location of a phase transition, just as the fidelity can. Fidelity, which is basically an overlap between ground-state eigenvectors is not influenced by the excited states at least when $\delta = 0$. In this particular model, Nemati *et al.* obtain level crossings between excited states at the locations of the intermediate peaks of the fidelity susceptibility. That have nothing to do with the ground-state properties of the system. Thus those crossings cannot be used to dismiss the results in our paper.

In other words, fidelity is not a result of what happens amongst excited states. The location of the phase transition can be obtained by using the ground-state eigenvectors, as in the fidelity approach we used in our article, or by the ground-state energy itself, as shown here. Of course, one always needs to identify the nature of the phases involved, as was done in our paper.

This work was supported by FAPERJ and PROPPI-UFF (Brazilian agencies).

[1] S. Nemati, F. K. Fumani, and S. Mahdavifar, Comment on "Quantum fidelity approach to the ground-state properties of the one-dimensional axial next-nearest-neighbor Ising model in a transverse field", Phys. Rev. E **102**, 016101 (2020).

[2] O. F. de Alcantara Bonfim, B. Boechat, and J. Florencio, Quantum fidelity approach to the ground-state properties of the one-dimensional axial next-nearest-neighbor Ising model in a transverse field, Phys. Rev. E **96**, 042140 (2017).

[3] P. W. Anderson, Infrared Catastrophe in Fermi Gases with Local Scattering Potentials, Phys. Rev. Lett. **18**, 1049 (1967).

[4] B. Damski, Fidelity susceptibility of the quantum ising model in a transverse field: The exact solution, Phys. Rev. E **87**, 052131 (2013).

[5] Q. Luo, J. Zhao, and X. Wang, Fidelity susceptibility of the anisotropic XY model: The exact solution, Phys. Rev. E **98**, 022106 (2018).