Nonlinear self-focusing in strongly magnetized pair plasma

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An intense radiation field can modify plasma properties and the corresponding refractive index and lead to nonlinear propagation effects such as self-focusing. We estimate the corresponding effects in pair plasmas for circularly polarized waves, in both unmagnetized and strongly magnetically dominated cases. First, in the unmagnetized pair plasma the ponderomotive force does not lead to charge separation but to density depletion. Second, for astrophysically relevant plasmas of pulsar magnetospheres [and possible loci of fast radio bursts (FRBs)], where the cyclotron frequency ω_B dominates over the plasma frequency ω_p and the frequency of the electromagnetic wave $\omega_B \gg \omega_p, \omega$, we show that (i) there is virtually no nonlinearity due to changing effective mass in the field of the wave; (ii) the ponderomotive force is $F_p^{(B)} = -m_e c^2/4B_0^2 \nabla E^2$, which is reduced by a factor $(\omega/\omega_B)^2$ if compared to the unmagnetized case (B_0 is the external magnetic field and E is the electric field of the wave); and (iii) for a radiation beam propagating along a constant magnetic field in the pair plasma with density $n_\pm$, the ponderomotive force leads to the appearance of circular currents that lead to a decrease of the field within the beam by a factor $\Delta B/B_0 = 2\pi n_\pm m_e c^2 E^2/B_0^4$. Applications to the physics of FRBs are discussed; we conclude that for the parameters of FRBs, the dominant magnetic field completely suppresses nonlinear self-focusing or filamentation.

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I. INTRODUCTION

Neutron stars possess magnetic fields that can approach quantum critical magnetic field [1–3]. In addition, pulsars produce high-intensity coherent emission (giant pulses are especially intense [4]) that may modify the properties of the background plasma. The effects of the backreaction of the radiation field on the background plasma are becoming even more important with the recent discoveries related to fast radio bursts (FRBs) [5–7], particularly identifications of the repeater FRB 121102 [8], FRB 180814 [9], and recent numerous FRBs detected by the Canadian Hydrogen Intensity Mapping Experiment telescope [10,11]. Magnetospheres of neutron stars are one of the main possible loci of the FRBs [12–16].

As we discuss in the present paper, the radiation-plasma interaction in the case of FRBs takes place in an unusual (compared to the more well studied laboratory laser plasma) regime. First, the plasma is likely composed of electron-positron pairs. That eliminates or modifies many effects that arise due to different masses of charge carries even in the unmagnetized case. For example, in the electron-ion plasma the ponderomotive force leads to electrostatic charge separation. In the unmagnetized pair plasma it leads to density depression, while in the highly magnetized plasma it leads to the modification of the background magnetic field.

Second, in Ref. [17] new limitations on the plasma parameters that FRBs impose if compared with pulsars are discussed. High infrared radiation energy densities at the source renewed interest in nonlinear radiative phenomena in plasmas [18–20]. The above-cited works consider nonmagnetized or weakly magnetized plasma. As discussed in Refs. [17,21], the properties of first repeater FRB 121102 require a large magnetic field at the source. For a given observed flux and known distance, the equipartition magnetic field energy density at the source evaluates

$$B_{eq} = \sqrt{8\pi}\frac{\sqrt{\nu F_\nu}D}{c^{3/2}\tau} = 3\times 10^8\ \text{G}, \tag{1}$$

where $\nu \sim 1$ GHz is the observed frequency, $F_\nu \sim 1$ Jy ($1\ \text{Jy} = 10^{-26}\ \text{W m}^{-2}\ \text{Hz}^{-1}$) is the observed flux, $D \sim 1$ Gpc is the distance to the FRB, and $\tau \sim 10^{-3}$ s is the duration of the burst. The resulting cyclotron frequency ω_B is much larger than the observational frequency and mostly likely larger than the local plasma frequency ω_p. (The inherent assumption is that the duration of the bursts $\tau \approx 1$ ms is an indication of the emission size.) Specifically, as argued in Ref. [17], such high magnetic fields *are needed* to avoid high "normal" (noncoherent) radiative losses. Thus, we expect that the magnetization parameter [22] is large

$$\sigma = \frac{\omega_B^2}{\omega_p^2} \gg 1, \tag{2}$$

where $\omega_B = eB/m_e c$ is cyclotron frequency and $\omega_p^2 = 4\pi n e^2/m_e$ is plasma frequency.

Nonlinear plasma effects in this astrophysically specific regime of highly magnetized plasmas remain unexplored. Highlighting these differences is the main goal of the paper. Current reviews on relativistically strong lasers, e.g., Refs. [23–26], do not address this specific regime.

In this work we consider nonlinear self-focusing in a pair plasma, both nonmagnetic and magnetically dominated plasma with $\omega_B \gg \omega_p, \omega$. One expects that in applications to astrophysical FRBs it is the self-focusing (due to transverse modulation of plasma properties) that may playan important role, as opposed to longitudinal modulations such as self-phase modulation and wake-field production [27,28].

Nonlinear effects in a pair plasma of pulsar magnetospheres have attracted interest in the plasma physics community [26,29–31], yet we are not aware of work on self-focusing in the magnetically dominated regime $\omega_B \gg \omega, \omega_p$. Similarly, effects such as thermal self-focusing [32,33] are likely to be unimportant in highly magnetically dominated plasma, with $\sigma \gg 1$ (in this case the magnetic energy dominates even over the rest-mass energy).

II. NONLINEAR SELF-FOCUSING IN PAIR PLASMAS IN THE ABSENCE OF A MAGNETIC FIELD

In the absence of an external magnetic field a particle in a strong radiation field experiences oscillations (quiver) with dimensionless transverse momentum [27,34–38]

$$a_0 \equiv \frac{p_\perp}{m_e c} = \frac{eE}{m_e c\omega}, \tag{3}$$

where E is the electric field in the wave, ω is the frequency of the wave, and the other notation is standard. When $a_0 \geqslant 1$ the transverse oscillating momentum of a particle in a wave becomes relativistic. This corresponds to an energy flux

$$F = \frac{cE^2}{4\pi} = a_0^2 \frac{m_e^2 c^3 \omega^2}{4\pi e^2} = 3 \times 10^{14} a_0^2 \nu_9^2 \text{ erg s}^{-1}\text{ cm}^{-2}, \tag{4}$$

where ν_9 is the frequency in gigahertz.

In unmagnetized plasma the nonlinear effects of the strong laser light can be first accommodated into the changing effective mass of particles [35] so that the refractive index n for a circularly polarized electromagnetic wave with $\omega \gg \omega_p$ becomes

$$n^2 = 1 - \frac{\omega_p^2}{\sqrt{1 + a_0^2}\omega^2}, \quad \omega_p^2 = \frac{4\pi n_\pm e^2}{m_e}, \tag{5}$$

where $n_\pm$ is the total pair plasma density (two times the density of each species). Circularly polarized waves propagating along magnetic fields have been historically used as analytically treatable benchmark problems [35]. Linearly polarized waves are much more complicated and are typically unstable to longitudinal modulation.[1]

Consider a beam of radiation propagating in plasma. The jump of the refractive index between the core of the beam and the background due to the changing mass is then

$$\Delta n \approx \frac{1}{2}\frac{{\omega_p}^2}{\omega^2}\left(1 - \frac{1}{\sqrt{1 + a_0^2}}\right) \approx \begin{cases} \frac{1}{4}\frac{{\omega_p}^2}{\omega^2} a_0^2, & a_0 \ll 1 \\ \frac{1}{2}\frac{{\omega_p}^2}{\omega^2}, & a_0 \gg 1. \end{cases} \tag{6}$$

The refractive index is larger in the core of the beam. If the radiation pattern forms a beam with decreasing power away from the central axis (this can occur also due to fluctuations on the beam intensity), the parameter a_0 decreases away from the center so that a converging lens is formed.

If the beam diameter is d, the beam might be expected to expand by diffraction with an angular divergence of $\theta \sim \lambda/d$, where λ is the wavelength of radiation. However, a higher refractive index inside the beam may lead to internal reflection if the beam power satisfies [39]

$$P_b > P_c = \frac{1.22^2 c}{256 n_2} = 7 \times 10^{-4} \frac{m_e^3 c^5 \omega^2}{e^4 n_\pm}. \tag{7}$$

This is the energy flux for self-focusing in unmagnetized plasma, considering only modification of mass; a weakly nonlinear regime is assumed $a_0 \ll 1$. The corresponding focal length and lensing angle are, respectively,

$$R_f \approx \frac{d}{2}\sqrt{\frac{n_0}{n_2 E^2}} \approx \frac{d}{a_0}\frac{\omega}{\omega_p},$$

$$\theta_f = \frac{d}{R_f} = 2\sqrt{\frac{n_2 E^2}{n_0}}. \tag{8}$$

In the highly nonlinear regime $a_0 \gg 1$, the refractive index inside the beam becomes approximately 1, while outside it is still approximately $1 - \frac{{\omega_p}^2}{2\omega^2}$. Equating the diffraction angle $\sim 1.22\lambda/2d$ to the critical angle of internal total reflection gives a condition on the width of the self-collimating beam

$$d \leqslant 7.6 \frac{c\omega}{\omega_p^2} = 0.6 \frac{c m_e \omega}{e^4 n_\pm}. \tag{9}$$

III. SELF-FOCUSING IN A MAGNETICALLY DOMINANT PAIR PLASMA

A. No nonlinear effects due to quiver momentum

If there is an external magnetic field B_0 such that $\omega_B \gg \omega$, the plasma dynamics changes dramatically. Most importantly, the leading nonlinear effects in the unmagnetized plasmas, induced by the variation of effective mass, disappears.

For $\omega_B \gg \omega$ a particle in a wave experiences linear acceleration not for a fraction of the wave period but for a fraction of the cyclotron gyration. The magnetic nonlinearity parameter is then

$$a_0^{(B)} \equiv \frac{p_\perp}{m_e c} = \frac{eE}{m_e c \omega_B} = a_0 \frac{\omega}{\omega_B} = \frac{E}{B_0}, \tag{10}$$

the ratio of the electric field in the wave to the external magnetic field.

For a wave with energy flux P, the ratio of the electric field in the wave to the external field is

$$\frac{E}{B_0} = 2\sqrt{\pi}\frac{\sqrt{P}}{\sqrt{c}B_0}. \tag{11}$$

It becomes unity for

$$P = \frac{B_0^2 c}{4\pi} = 4 \times 10^{36} b_q^2 \text{ erg cm}^{-2}\text{ s}^{-1}, \tag{12}$$

where we normalized the magnetic field to the quantum critical magnetic field $B_0 = b_q B_Q$, $B_Q = m_e^2 c^3/e\hbar$. This is an unrealistically high energy flux, not likely to be reached: The electric field in the wave is much smaller than the external

[1] Note that in an electron-ion plasma with a density $n_\pm$, a radiation-driven ponderomotive displacement of the electrons with respect to the ions generates the electric field $E_{\rm disp} \approx (\omega_p/\omega)^2 E$. This does not happen in the pair plasma as both species experience the same ponderomotive force.

magnetic field $a_0^{(B)} \ll 1$ [this corrects a typo in Eq. (5) in [21]]. Thus, instead of large-amplitude oscillations a particle experiences an $E \times B$ drift with nonrelativistic velocity $v_\perp/c = a_0^{(B)} \ll 1$. The magnetic nonlinearity is always small $a_0^{(B)} \ll 1$, the quiver velocity is nonrelativistic, and the mass modification in the regime $\omega_B \gg \omega$ is negligible.

B. Ponderomotive force across a magnetic field in a magnetically dominant plasma

In pulsars, and presumably FRBs, emission is likely to be produced by relativistic particles propagating approximately along the local magnetic field [40–43]. Let us assume that the circularly polarized radiation propagates exactly along the external magnetic field. Typically, in pulsar magnetospheres the cyclotron frequency is much higher than the plasma frequency and the radiation frequency (in the plasma frame in the case of relativistic bulk motion).

As demonstrated above, in the case $\omega_B \gg \omega$, instead of large-amplitude oscillations with $p_\perp \sim a_0 m_e c$ particles experience $E \times B$ drift with velocity $(E/B_0)c$, where B_0 is the external magnetic field. In a beam with intensity dependent on distance from the axis we can separate the particle motion into fast oscillations with coordinate-dependent amplitude. Averaging over fast oscillation, the ponderomotive force then becomes

$$\mathbf{F}_p^{(B)} = -\frac{m_e c^2}{4B_0^2} \nabla E^2, \tag{13}$$

where the superscript (B) indicates that estimate is for the case of a strong magnetic field. The expression for $F_p^{(B)}$ is the ponderomotive force in the magnetically dominant plasma.

The ratio of the ponderomotive forces in magnetically dominated plasma and plasma without a magnetic field is

$$\frac{F_p^{(B)}}{F_p} = \left(\frac{\omega}{\omega_B}\right)^2 \ll 1. \tag{14}$$

Thus, the ponderomotive force is reduced by a factor $(\omega/\omega_B)^2$ if compared to the unmagnetized case.

Most importantly, the effect of the ponderomotive force on the background particles is qualitatively different in the magnetically dominated case, as we demonstrate next. If the radiation beam is propagating along the magnetic field and its intensity varies in the perpendicular direction, Eq. (13) gives a force on a particle in the direction perpendicular to the magnetic field. As a result, the particle will experience a drift with velocity

$$v_d = \frac{c}{e} \frac{\mathbf{F}_p^{(B)} \times \mathbf{B}_0}{B_0^2}. \tag{15}$$

The drift is in the azimuthal direction (with respect to the background magnetic field) (see Fig. 1).

Charges of opposite sign rotate in the opposite direction. In a charge-neutral pair plasma with densities $n_\pm$ that will induce a current

$$j_\phi = 2e v_d n_\pm. \tag{16}$$

For a beam of diameter b we can estimate the modification of the magnetic field within the beam from the induction

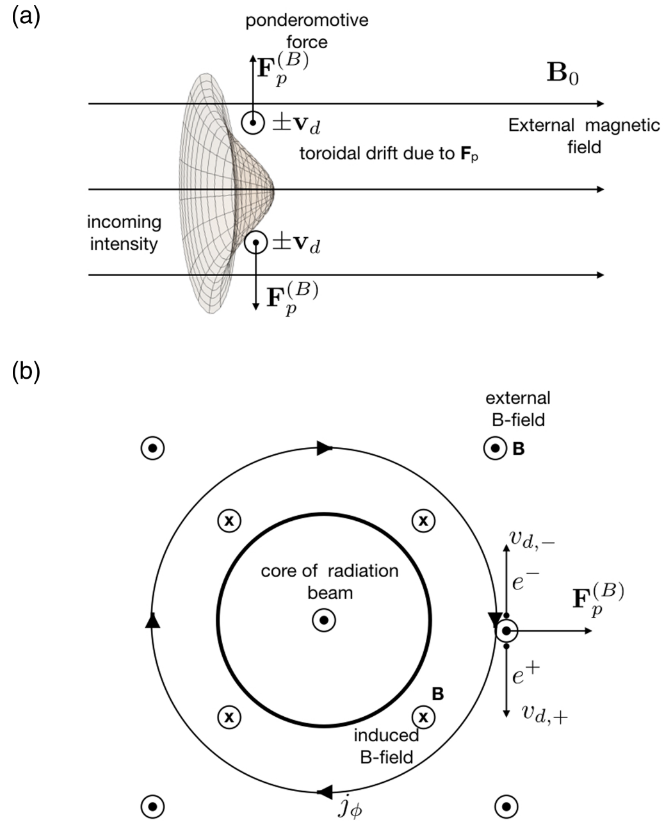

FIG. 1. (a) View from the side. An intense radiation beam is propagating along the magnetic field. The gradients of the field intensity induce the ponderomotive force $\mathbf{F}_p^{(B)}$. In the high external magnetic field B_0 the ponderomotive force leads to azimuthal drift of charged particles $\pm\mathbf{v}_d$ that creates toroidal current and decreases the background magnetic field. (b) View along the direction of the beam. In the core the radiation energy density is high and it induces a ponderomotive force directed away from the center. In the external magnetic field (chosen to be out of the plane) the ponderomotive force leads to charge-dependent drift of particles and generation of the toroidal current (in the clockwise direction). The induced current produces a field counteraligned with the external field.

equation

$$\frac{\Delta B}{b} \approx \frac{4\pi}{c} j_\phi, \quad j_\phi = 2e\left(\frac{m_e c^3}{4eb}\frac{E^2}{B_0^3}\right) n_\pm, \tag{17}$$

where Eqs. (13), (15), and (16) were used and we estimated $\nabla E^2 \approx E^2/d$ in (13). Thus, the magnetic field within the beam will be modified by

$$\Delta B = -2\pi n_\pm m_e c^2 \frac{E^2}{B_0^3} \tag{18}$$

(the magnetic field is smaller in the core).

We can then introduce a magnetic nonlinear intensity parameter $\eta_0^{(B)}$,

$$\eta_0^{(B)} = \frac{\Delta B}{B_0} = 2\pi n_\pm m_e c^2 \frac{E^2}{B_0^4} = \frac{\omega_p^2}{2\omega_B^2} a_0^{(B),2}. \tag{19}$$

Modification of the field becomes of the order of unity at radiative flux

$$P^{(B)} = \frac{B_0^4}{8\pi^2 m_e c n_\pm}. \tag{20}$$

The dimension of $P^{(B)}$ is erg cm^{-2} s^{-1}. When $\eta_0^{(B)} \sim 1$ the radiative power leads to changes of the background magnetic field of the order of unity.

Modification of the magnetic field (18) will lead to the changes of the refractive index within a beam (as we argued above, there is no contribution from changing oscillatory motion of bulk charges). In the linear approximation, in the limit $\omega_B \gg \omega_p, \omega$, the wave dispersion reads [44–47]

$$n^{(B)} = 1 + \frac{1}{2}\left(\frac{\omega_p}{\omega_B}\right)^2 \tag{21}$$

(for parallel propagation; for simplicity we assume cold plasma in its rest frame). In the presence of the strong wave the magnetic field is modified [see Eq. (18)]. Expanding in the wave intensity, we find

$$\begin{aligned} n^{(B)} &\approx 1 + \frac{1}{2}\left(\frac{\omega_p}{\omega_B}\right)^2 + \frac{\omega^2\omega_p^4}{2\omega_B^6}a_0^2 \\ &= 1 + \frac{1}{2}\left(\frac{\omega_p}{\omega_B}\right)^2 + \frac{\omega_p^4}{2\omega_B^4}a_0^{(B),2} \\ &= 1 + \frac{1}{2}\left(\frac{\omega_p}{\omega_B}\right)^2 + \frac{e^2}{2m_e^2c^2}\frac{\omega_p^4}{\omega_B^6}E^2, \end{aligned} \tag{22}$$

where ω_B is defined with the initial background field. The plasma lens has larger refractive index in the core and thus is convergent. (The decease in the magnetic field is due to newly generated internal currents, not expansion; hence the density remains constant.) The critical energy flux for self-collimation in a highly magnetized plasma, when the third term in (22) equals the second, is then

$$P_c^{(B)} = 3 \times 10^{-3}\frac{B_0^6}{m_e^2 c n_\pm \omega^2} \tag{23}$$

and the focal distance and lensing angle

$$\begin{aligned} R_f^{(B)} &= \frac{\omega_B^3}{\omega\omega_p^2}\frac{d}{\sqrt{2}a_0} = \frac{\omega_B^2}{\omega_p^2}\frac{d}{\sqrt{2}a_0^{(B)}}, \\ \theta_f^{(B)} &= \sqrt{2}a_0\frac{\omega\omega_p^2}{\omega_B^3} = \sqrt{2}a_0^{(B)}\frac{\omega_p^2}{\omega_B^2}. \end{aligned} \tag{24}$$

C. Implications for FRBs

Let us use the properties of the first repeater for the estimates of the relevant parameters [21]: flux $F_\nu \approx 1$ Jy, frequency $\nu = 1$ GHz, distance to the FRB $d_{\rm FRB} \approx 1$ Gpc, and duration $\tau = 1$ ms. The electric field of the wave at the source of size $c\tau$ (in cgs units) and the beam power are then

$$\begin{aligned} E &= 2\sqrt{\pi}\frac{d_{\rm FRB}\sqrt{\nu F_n u}}{c^{3/2}\tau} = 2 \times 10^8, \\ P &= \frac{\nu F_\nu d_{\rm FRB}^2}{c^3\tau^2} = 10^{26} \text{ erg s}^{-1}\text{ cm}^{-2} \end{aligned} \tag{25}$$

(the estimate of the electric field is also the value of the equipartition magnetic field [21]) [see Eq. (1)]. The nonlinearity parameters then evaluate to

$$\begin{aligned} a_0^{(B)} &= 2\sqrt{\pi}\frac{d_{\rm FRB}\sqrt{\nu F_\nu}}{c^{3/2}\tau B_0} = 4 \times 10^{-6} b_q^{-1}, \\ a_0 &= 5 \times 10^5. \end{aligned} \tag{26}$$

Thus, the nonlinear effects are suppressed by the magnetic field by some ten orders of magnitude (for quantum field $b_q = 1$).

To proceed further we need to estimate the plasma density. As the sources of FRBs remain mysterious, below we scale density according to two somewhat oppositely extreme limits: (i) the Goldreich-Julian [48] density (with some multiplicity $\kappa_{\rm GJ}$) and (ii) the quantum density of inverse Compton length cubed $n_\pm = \kappa_C\lambda_C^{-3}$, $\lambda_C = \hbar/m_e c$. These two limits exemplify the clean or light magnetospheres of pulsars and heavy pair-loaded magnetospheres one expects in magnetar flares.

1. Pulsarlike scaling

Using Goldreich-Julian [48] scaling for the plasma density

$$n_\pm = \kappa_{\rm GJ}\frac{\Omega B}{2\pi e c}, \tag{27}$$

where $\kappa_{\rm GJ}$ is plasma multiplicity and Ω is the spin frequency of a pulsar, we find

$$\begin{aligned} P_c &= 3 \times 10^3 \kappa_{\rm GJ,6}^{-1} b_q^{-1} P_{-3}^1 \text{ erg s}^{-1}\text{ cm}^{-2}, \\ P_c^{(p)} &= 3 \times 10^5 \kappa_{\rm GJ,6}^{-1}\theta_T b_q^{-1} P_{-3}^1 \text{ erg s}^{-1}\text{ cm}^{-2}, \\ P_c^{(B)} &= 2 \times 10^{60} b_q^4 P_{-3}^2 \kappa_{\rm GJ,6}^{-2} \text{ erg s}^{-1}\text{ cm}^{-2}, \\ \theta_f^{(B)} &= 10^{-16} b_q^{-2} P_{-3}^{-1}\kappa_{\rm GJ,6}. \end{aligned} \tag{28}$$

2. Magnetarlike scaling

Scaling $n = \kappa_C\lambda_C^{-3}$, we find, using (7) and (20),

$$\begin{aligned} P_c &= 5.9 \times 10^{-7}\kappa_C^{-1} \text{ erg s}^{-1}\text{ cm}^{-2}, \\ P_c^{(p)} &= 6 \times 10^{-5}\kappa_C^{-1}\theta_T \text{ erg s}^{-1}\text{ cm}^{-2}, \\ P_c^{(B)} &= 7 \times 10^{40} b_q^6\kappa_C^{-2} \text{ erg s}^{-1}\text{ cm}^{-2}, \\ \theta_f^{(B)} &= 5 \times 10^{-7} b_q^{-3}\kappa_C, \end{aligned} \tag{29}$$

where the magnetic field was scaled to the critical quantum field. The above estimates cover a wide range of densities and magnetic fields. However, there is a clear conclusion: The nonlinear effects are highly suppressed in the magnetically dominant plasma, by some 50 orders of magnitude for both magnetarlike and pulsarlike scaling.

IV. CONCLUSION

In this paper we have given estimates of the nonlinear optical effects in strongly magnetized pair plasma. We found that in magnetically dominated regime $\omega_B \gg \omega, \omega_p$, (i) the relativistic effective mass-changing effects on the wave nonlinearity are completely negligible, (ii) the ponderomotive force is suppressed by a factor $(\omega/\omega_B)^2 \ll 1$ if compared with unmagnetized regime, and (iii) the ponderomotive force induces toroidal currents that modify (decrease) the background magnetic field; the resulting lens is also converging as in the unmagnetized case. Overall, the plasma nonlinearity is highly suppressed in the magnetized case. As a result, effects such as self-collimation and plasma filamentation are not likely to play an important role in pulsar magnetospheresand FRBs.

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