Evolution of cooperation under punishment

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Punishment has been considered as an effective mechanism for promoting and sustaining cooperation. In most existing models, punishment always comes as a third strategy alongside cooperation and defection, and it is commonly assumed to be executed based on individual decision rules rather than collective decision rules. Differently from previous works, we employ a democratic procedure by which cooperators cast votes independently and simultaneously for whether to impose punishment on defectors, and we establish a relationship between the cooperators' willingness to punish defectors (WTPD) and whether the punishment is inflicted on defectors. The results illustrate that the population can evolve to full cooperation under consensual punishment. It is noteworthy that, compared with autonomous punishment, whether consensual punishment is more in favor of cooperation crucially depends on the minimum number of votes required for punishment execution as well as the cooperators' WTPD. Our findings highlight the importance of collective decision making in the evolution of cooperation and may provide a mathematical framework for explaining the prevalence of democracy in modern societies.

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I. INTRODUCTION

Cooperation is a conundrum from evolutionary perspective as it defies the basic principles of natural selection [1–10]. Exploring the evolution of cooperation has been one of the most challenging topics [11,12]. Evolutionary game theory provides an excellent theoretical framework for exploring the evolutionary dynamics of cooperation [13–17]. So far, various metaphors have been proposed to capture individuals' interactions, e.g., the prisoner's dilemma game [18–20] and public goods game [21,22]. The former is often employed to model pairwise interactions, while the latter one is always adopted to describe the interactions among multiple individuals. By virtue of evolutionary game theory, the evolution of cooperation has been investigated, and accordingly several cooperation-promoting mechanisms have been proposed [23–27]. Punishment, as a prominent cooperation-promoting mechanism, has received increasing attention [28–36]. Punishers impose a fine on the punished at a cost [37–43]. In most existing models, punishment always comes as a third strategy alongside cooperation and defection [44–50].

Distinct from individual decision-making rules, which are commonly adopted in the existing models of punishment execution, collective decision making is also an important form of decision making, such as a voting mechanism. Actually, a voting mechanism is often introduced to make collective decisions in various institutions, e.g., the collegial panel in international judicial systems, the Organization of the Petroleum Exporting Countries (OPEC), and the environmental quality councils [51–55]. So far, several experiments have been carried out to investigate the effect of collective decision rules on the level of cooperation. Decker *et al.* [56] have studied the effect of different voting rules on the level of cooperation driven by punishment. In their experiment, individuals can propose their punishment intensity for each group member and choose which rule (majority or unanimity) is required to decide the execution of punishment. The result illustrates that the unanimity rule is much preferred. Casari and Luini [57] have made a comparative analysis between different punishment institutions based on a public goods experiment. Under the consensual rule, an individual is punished only when the punishment is requested by two or more individuals. The results show that the consensual punishment institution can lead to a higher cooperation level than that under autonomous individual punishment. Moreover, Ambrus and Greiner [58] have explored the effect of consensual punishment on cooperation by virtue of experiments where individuals simultaneously make the decision about who should be punished. An individual would be punished as long as it receives at least three votes in the group of five individuals. The results illustrate that the consensual punishment with majority

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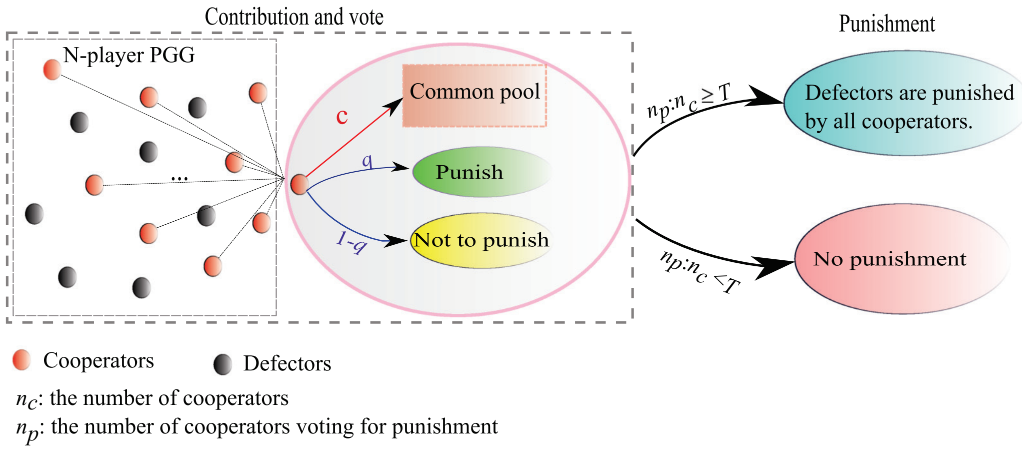

FIG. 1. Schematic diagram of the model. In an infinite and well-mixed population, N individuals are randomly selected to participate in a public goods game with strategies cooperation and defection. Cooperators each contribute c ($c > 0$) to the common pool and make a decision about whether to employ punishment for defectors simultaneously. However, defectors contribute nothing to the common pool. By q we denote the average degree of cooperators' willingness to punish defectors (WTPD). Peer punishment is employed and imposed on defectors by all cooperators as long as a lowest degree of consensus T ($T \in [0, 1]$) is reached, i.e., $\frac{n_p}{n_c} \geqslant T$, and otherwise the punishment is not executed.

rule can lead to a higher cooperation level. Pfattheicher *et al.* [59] have revealed the advantage of consensual punishment based on experiments where individuals participate in the punishment decision-making process equally. The punishment is executed only when a majority of people have voted for its execution. Compared with the autonomous punishment, this kind of consensual punishment is more effective in reducing antisocial punishment and increasing fairness perceptions. Van Miltenburg *et al.* [60] have also investigated the effect of individual and collective decision rules on cooperation under punishment and reward based on a laboratory experiment within groups of four actors. In this model, punishment can take place in one of three experimental conditions: (1) individual (i.e., punishment is executed as long as there exists at least one actor willing to punish), (2) majority (i.e., punishment is implemented as long as there exist at least two actors willing to punish), and (3) unanimity (i.e., an actor is punished only when all three remaining group members propose to punish simultaneously). The result illustrates that, different from the results observed in Refs. [58,59], the level of cooperation is higher in the case of the individual condition than majority, and much higher under majority than unanimity.

Although increasing attention has been paid to exploring the effect of consensual punishment on cooperation, most results are obtained based on experiments, and theoretical studies based on analytical models are relatively rare. Moreover, individuals in most preceding models cannot vote for or against the punishment system, but vote for whether the punishment for another individual should or should not be executed. Thus, to further explore the effect of collective decision making on the evolution of cooperation, we establish an analytical model where individuals play the public goods game with the strategies cooperation and defection. Cooperators invest a cost in the common pool, and they are enrolled in a vote to decide whether to employ punishment for defectors simultaneously. A variable q is introduced to capture the average degree of cooperators' willingness to punish defectors (WTPD). Peer punishment is employed and imposed on defectors by all cooperators when the consensus required for employing punishment is reached. Otherwise punishment is not allowed, and no one has to bear the loss caused by punishment. Based on this model, we investigate the evolutionary dynamics of cooperation under consensual punishment and make a comparative analysis between autonomous and consensual punishments in promoting cooperation. Our results reveal that whether consensual punishment is more effective than autonomous punishment in promoting cooperation critically depends on the degree of cooperators' WTPD and the minimum number of votes required for punishment execution.

II. MATERIALS AND METHODS

Consider an infinite and well-mixed population where N individuals are randomly chosen to play a public goods game with strategies cooperation or defection (see Fig. 1). Defectors (D) contribute nothing to the common pool, while cooperators (C) each contribute c ($c > 0$) to the common pool and make a decision about whether to employ punishment for defectors simultaneously and independently. The total amount of contributions are multiplied by an enforcement factor r ($1 < r < N$) and then equally distributed among all participants. Although cooperators prefer to punish defectors as they dislike free riding [45], some cooperators may prefer to shun punishment to maximize their interests [37]. Thus, we introduce a variable q ($q \in [0, 1]$) to capture the average degree of cooperators' WTPD. The number of cooperators that actually vote for punishment is subject to a binomial distribution.

Whether defectors are punished or not critically depends on the cooperators' consensus on punishing defectors. Let us define the number of cooperators as n_C, and the number of cooperators voting for punishment as n_P. Peer punishment would be employed as long as a lowest degree of consensus T ($T \in [0, 1]$) is reached, i.e., $\frac{n_P}{n_C} \geqslant T$. Under this condition, cooperators impose a fine β on each defector at a cost α. Namely, cooperators each have to invest a cost of punishment proportional to the number of defectors $(N-n_C)\alpha$, and defectors each have to suffer a fine proportional to the number of cooperators $n_C\beta$. In particular, peer punishment for defectors is not employed and no one has to suffer the loss caused by punishment for $\frac{n_P}{n_C} < T$.

Denote the fraction of cooperators and defectors by x and $1-x$, respectively. Accordingly, the replicator dynamics of cooperation is given by

$$\dot{x} = x(\pi_C - \bar{\pi}), \tag{1}$$

where $\dot{x}$ is the gradient of selection, and π_i ($i =$ C,D) denotes the expected payoff of cooperators or defectors. $\bar{\pi}$ is the average payoff of the population, $\bar{\pi} = x\pi_C + (1-x)\pi_D$. Then the evolutionary dynamics of cooperation can be obtained as

$$\dot{x} = x(1-x)(\pi_C - \pi_D). \tag{2}$$

Whether cooperation can dominate defection critically depends on the sign of $\pi_C - \pi_D$. Namely, cooperation is selected as long as the expected payoff of cooperators is greater than that of defectors.

III. RESULTS AND DISCUSSION

A. Evolutionary dynamics of cooperation under consensual punishment

Generally, the average payoff of cooperators (π_C) can be given by

$$\begin{aligned}\pi_C = {} & \frac{(N-1)rcx}{N} + \frac{rc}{N} - c \\ & - q\alpha\sum_{k=0}^{N-2}(N-k-1)\binom{N-1}{k}x^k(1-x)^{N-1-k} \\ & \times \sum_{m=\max\{0,\lceil Tk+T\rceil-1\}}^{k}\binom{k}{m}q^m(1-q)^{k-m} \\ & - (1-q)\alpha\sum_{k=0}^{N-2}(N-k-1)\binom{N-1}{k}x^k(1-x)^{N-1-k} \\ & \times \sum_{m=\lceil Tk+T\rceil}^{k}\binom{k}{m}q^m(1-q)^{k-m}.\end{aligned} \tag{3}$$

We defer the details of calculating π_C to Appendix A. The terms in the right-hand side of the above equation are the expected payoff obtained from the public goods game, the cost of punishment when the cooperator votes for punishment, and the cost of punishment when the cooperator does not opt for punishment.

Similarly, the average payoff of defectors (π_D) can be obtained as (see details in Appendix B)

$$\begin{aligned}\pi_D = {} & \frac{(N-1)rcx}{N} - \beta\sum_{k=1}^{N-1}k\binom{N-1}{k}x^k(1-x)^{N-1-k} \\ & \times \sum_{m=\lceil Tk\rceil}^{k}\binom{k}{m}q^m(1-q)^{k-m}.\end{aligned} \tag{4}$$

The first term in the right-hand side of the above equation is the expected payoff from the public goods game, and the other one is the fine of punishment.

According to the replicator equation, cooperation is more advantageous than defection as long as the average payoff of cooperators is greater than that of defectors, namely, $f(x) = \pi_C - \pi_D > 0$. For $T = 0$, punishment is imposed on defectors as long as there exists at least one cooperator in the group. In this case, the cooperators' WTPD negligibly affects the evolution of cooperation, and the model degenerates into the one where the population is composed of punishers and defectors. It implies that a sufficiently low T ($T = 0$) can induce the same level of cooperation as that in the population where cooperators propose to punishment completely ($q = 1$). Then we have $f(x) = c(-1 + r/N) + (1-N)\alpha + x(N-1)(\alpha+\beta)$. For $x = 0$, $f(0) = c(-1 + r/N) + (1-N)\alpha$. For $x = 1$, $f(1) = c(-1 + r/N) + (N-1)\beta$. Obviously, $f(0) < 0$. Then, knowing that the expression of $f(x)$ is a strictly increasing function of x for $x \in (0, 1)$, we find that whether there exists an interior equilibrium critically depends on the sign of $f(1)$. Specifically, there exists only one interior equilibrium when $\beta > \frac{(N-r)c}{N(N-1)}$ ($f(1) > 0$). Under this condition, there are three equilibria including an interior equilibrium and two boundary equilibria. The interior equilibrium is unstable, and the other two are stable. It is suggested that the public goods game under consensual punishment with $T = 0$ can be transformed into a coordination game.

The lowest degree of consensus for employing punishment (T) plays a pivotal role in the evolution of cooperation (see Fig. 2). Generally, a lower T is more beneficial to the evolution of cooperation as it requires a lower consensus on punishing defectors. As shown in Fig. 2, the population can evolve to full cooperation for a low T ($T = 0.01$) except for a sufficiently low degree of cooperators' WTPD. Increasing the lowest degree of consensus T allows the condition for employing punishment to be much harsher, which weakens the promoting effect of consensual punishment on cooperation. As T increases, a much higher degree of cooperators' WTPD is required for the equilibrium at $x = 1$ to be attracting. For a high T, the $x = 1$ cannot be the steady state unless the cooperators' WTPD is sufficiently high. As shown in Fig. 3, there are some branch points [61] depending on the value of T. The population can evolve to full cooperation under consensual punishment when there is an unstable interior equilibria in the evolutionary dynamics. For instance, the population can evolve to full cooperation for $q \geqslant 0.143$ when the consensual punishment with majority rule ($T = 0.5$) is introduced, and it can evolve to full cooperation for $q > 0.32$ when consensual punishment with super-majority rule ($T = 2/3$) is employed. For $T = 0.7$, the state $x = 1$ can be attracting as long as $q \geqslant 0.321$. In particular, for $T = 1$, we have $f(x) = x(1-x)(c(-1+\frac{r}{N}) + (-1+N)q[1+(-1+q)x]^{(-2+N)}[(-1+x)\alpha + x\beta])$. In this case, a fine is imposed on defectors only when all cooperators agree on punishing defectors. The condition for employing punishment seems much harsher; however, the population can evolve to full cooperation as long as the degree q is high enough [see Fig. 2(h)]. In other words, cooperation can also be facilitated by consensual punishment with unanimity as long as the cooperators' WTPD is sufficiently high (see also Fig. 3).

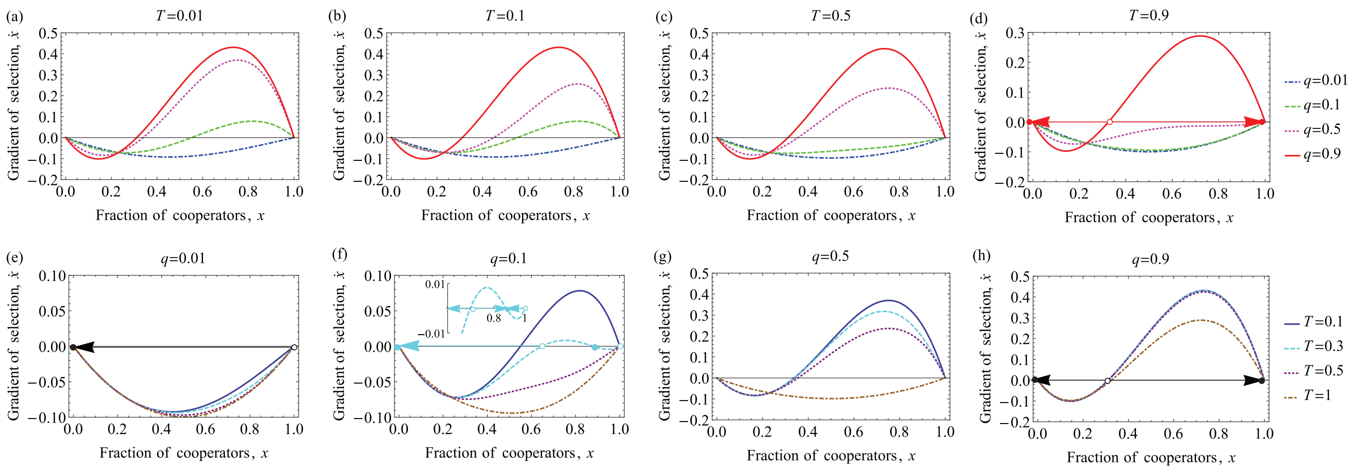

FIG. 2. Gradient of selection $\dot{x}$ for the public goods game under consensual punishment. By q we denote the degree of cooperators' willingness to punish defectors (WTPD), and by T the lowest degree of consensus for employing punishment. Stable (unstable) equilibria are depicted with solid (open) circles, and the expected direction of evolution is depicted with arrows. Parameters: $r = 3,\ c = 1,\ \alpha = 0.3,\ \beta = 1,\ N = 5$.

On the other hand, a higher degree of cooperators' WTPD is more conducive in promoting cooperation. For a low q ($q = 0.01$), there exists only one steady state $x = 0$ regardless of the lowest degree of consensus T ($T > 0$) [see Fig. 2(e)]. As the value of q increases, there exist some interior equilibria in the evolutionary dynamics. As shown in Fig. 2, the population can evolve to full cooperation when the cooperators' WTPD is great enough. Under these conditions, there exists an interior unstable equilibrium. The equilibrium becomes much closer to the steady state $x = 0$ as the value of q increases. It is worth noting that the evolutionary dynamics can have more than one interior equilibrium in the model; see the illustrated cases for $T = 0.3$ in Fig. 2(f). In this case, there is a mixed state of cooperation and defection where an unstable interior equilibrium is at the lower fraction of cooperators and a stable interior equilibrium is at the higher fraction of cooperators. Whether there exists a mixed state of cooperation and defection critically depends on the model parameters as shown in Table I. For instance, when consensual punishment with unanimity ($T = 1$) is introduced, there exists a mixed state of cooperation and defection for $q \in [0.614, 0.622]$ when $r = 2$, and for $q \in [0.535, 0.562]$ when $r = 3$ (other parameters are $N = 5,\ c = 1,\ \alpha = 0.3$, and $\beta = 1$).

As shown in Fig. 4, the basin of attraction of full cooperation increases with increasing enforcement factor r. It suggests that the enforcement factor has a positive impact on the evolution of cooperation. The increase of the group size leads to the expansion of the basin of attraction for the cooperative equilibrium. A higher cost of punishment can induce a smaller basin of full cooperation while a higher fine of punishment can lead to a greater cooperative basin of attraction. Namely, the cost of punishment plays a negative impact on the evolution of cooperation while the fine of punishment has a positive impact. Interestingly, cooperation can also be facilitated by inefficient consensual punishment ($\beta < \alpha$) as shown in Fig. 4(d). It is due to the fact that the

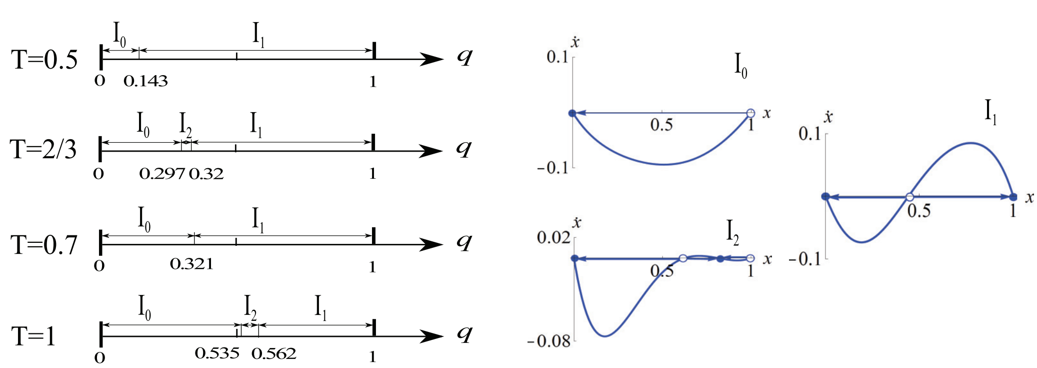

FIG. 3. Bifurcation of equilibrium for varying parameters. There are some branch points depending on the degree of consensus required for employing punishment T. For instance, there exist two bifurcations which subdivide the range of parameter q into three regions, I_0, I_1, and I_2 for $T = 2/3$. The branch-point parameters are 0.297 and 0.320, respectively. When q is varied in I_0 ($q < 0.297$), there are two boundary equilibria with one stable equilibrium at $x = 0$. As q increases, two interior equilibria including one stable interior equilibrium occur for $0.297 \leqslant q \leqslant 0.32$. The state $x = 0$ is still stable. When q is varied in I_1, there exists only one unstable interior equilibrium. The state $x = 1$ becomes stable. Stable (unstable) equilibria are depicted with solid (open) circles, and the expected direction of evolution is depicted with arrows. All parameters are identical to those in Fig. 2.

TABLE I. Effects of model parameters on the value of q_s.[a]

	$r(N=5,\ \alpha=0.3,\ \beta=1)$			$\beta(N=5,\ r=3,\ \alpha=0.3)$		
T_s	1.2	2	3	0.3	1.5	2
0.251–1/3	0.165–0.205	0.133–0.179	0.091–0.142	0.288–0.291	0.059–0.114	0.044–0.097
0.501–2/3	0.398–0.409	0.358–0.373	0.297–0.320	–	0.242–0.275	0.209–0.248
0.751–1.00	0.659–0.660	0.614–0.622	0.535–0.562	–	0.439–0.508	0.368–0.472

[a]A mixed state of cooperation and defection exists across the parameter space (T_s, q_s). For a constant T $(T \in T_s)$, there exists a mixed state of cooperation and defection for $q \in q_s$. Parameter: $c = 1$.

punishment is imposed on defectors by all cooperators once the consensus required is reached.

B. Comparative analysis of consensual and autonomous punishments

Now let us make a comparative analysis between consensual punishment and autonomous punishment. In the population without democratic procedure, whether defectors are punished depends only on the cooperators' WTPD. Namely, the punishment would be imposed on defectors individually as long as there is at least one cooperator intending to punish defectors. Under this condition, we have the average payoff of cooperators as

$$\pi'_C = \frac{(N-1)xrc + rc - Nc}{N} - q\alpha(N-1)(1-x). \tag{5}$$

The first term in the right-hand side of the above equation is the expected payoff obtained from the public goods game, and the other one is the cost of punishment when the cooperator opts for punishment. The average payoff of defectors (π'_D) under autonomous punishment is given by

$$\pi'_D = \frac{(N-1)xrc}{N} - \beta qx(N-1). \tag{6}$$

Accordingly, the difference in payoff between cooperators and defectors can be obtained as $g(x) = \pi'_C - \pi'_D$. Thus, cooperation can dominate defection as long as $g(x) = c(r/N-1) - (N-1)q\alpha + x(N-1)q(\alpha+\beta) > 0$. Specifically, we have $g(0) = c(r/N-1) - (N-1)q\alpha < 0$, and $g(x)$ is a strictly increasing function of x as $(N-1)q(\alpha+\beta) > 0$ for $x \in (0,1)$. Then the evolutionary dynamics of cooperation depend on the sign of expression $g(1)$. When $g(1) > 0$ ($q > \frac{[N-r]c}{N[N-1]\beta}$), there exists an unstable interior equilibrium and two stable boundary equilibria in the population. Otherwise, there is no interior equilibrium, and $x = 0$ is the only one steady state.

Compared with autonomous punishment, consensual punishment can be more effective in facilitating cooperation in some scenarios. For instance, the population under autonomous punishment cannot evolve to full cooperation unless $q > 0.1$ for $N = 5$, $r = 3$, $c = 1$, $\beta = 1$. However, for $q = 0.1$, $x = 1$ can be the steady state under consensual punishment as long as the lowest degree of consensus T is low enough [see Fig. 2(f)]. Figure 5 shows the evolutionary dynamics of cooperation under autonomous and consensual punishments. In general, neither autonomous punishment nor consensual punishment $(T > 0)$ can facilitate cooperation when the value of q is very low (see Fig. 5). As the value of q increases, the cooperative basin of attraction enlarges. On the other hand, when the democratic procedure is employed to decide the utilization of punishment, the cooperative basin of attraction shrinks with the increase of the lowest degree of consensus T [cf. Figs. 5(b) and 5(c)].

Whether consensual punishment is more effective than autonomous punishment in promoting cooperation critically depends on the lowest degree of consensus for employing punishment and the cooperators' WTPD. Specifically, when the lowest degree of consensus T is low enough, the cooperative basin of attraction under consensual punishment is always greater than that under autonomous punishment. It suggests that a lower degree of consensus required for employing punishment can induce a higher level of cooperation than that under autonomous punishment. Interestingly, there exists a crossover of the two curves (for autonomous punishment and consensual punishment) relying upon the degree of cooperators' WTPD as the lowest degree of consensus T increases [see Fig. 5(d)]. At the crossover (q_c, T_c), the effect of consensual punishment on cooperation is the same as that of autonomous punishment on cooperation. For $T = T_c$, cooperation can be more facilitated by consensual punishment as long as $q > q_c$, and otherwise autonomous punishment is more effective in promoting cooperation. In particular, when the lowest degree of consensus T is sufficiently high,

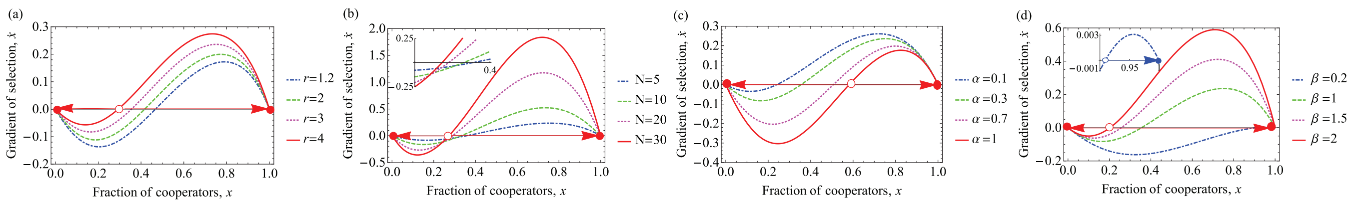

FIG. 4. Effects of parameters on the gradient of selection $\dot{x}$. Stable (unstable) equilibria are depicted with solid (open) circles, and the expected direction of evolution is depicted with arrows. Parameters: $T = 0.5$, $q = 0.5$, and $c = 1$. (a) $N = 5$, $\alpha = 0.3$, and $\beta = 1$. (b) $r = 3$, $\alpha = 0.3$, and $\beta = 1$. (c) $N = 5$, $r = 3$, and $\beta = 1$. (d) $N = 5$, $r = 3$, and $\alpha = 0.3$.

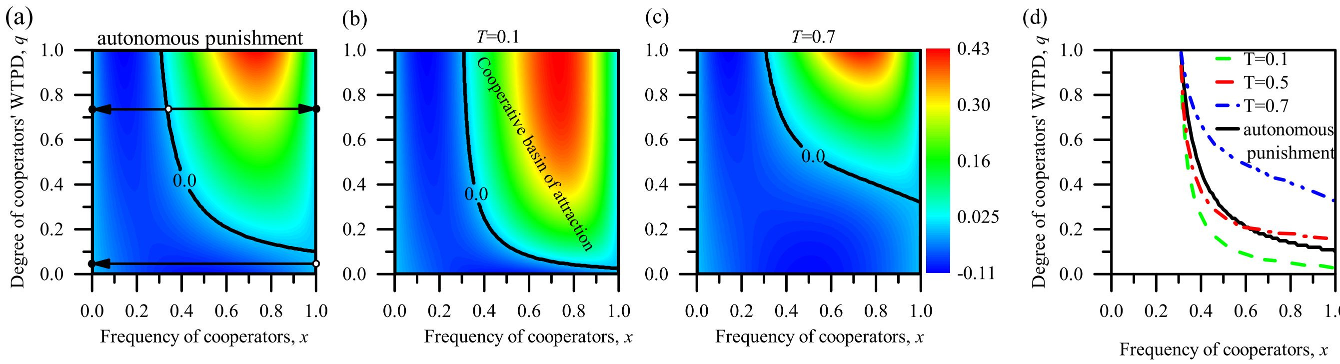

FIG. 5. Comparison of autonomous and consensual punishments for varying parameters. The color in panels (a)–(c) shows the value of the gradient of selection $\dot{x}$. By q we denote the degree of cooperators' willingness to punish defectors (WTPD), and by T the lowest degree of consensus for employing punishment. The solid and open circles depict the stable and unstable equilibria, respectively. The expected direction of evolution is depicted with arrows. All parameters are identical to those in Fig. 2.

autonomous punishment, compared with consensual punishment, can induce a higher level of cooperation. It is due to the fact that the likelihood to reach a broad consensus on employing punishment is minimal. Alternatively, autonomous punishment can be executed as long as there is at least one individual intending to punish defectors. Moreover, we also find that the value of q_c increases with the cost of punishment and the size of group, while it decreases with the fine of punishment as well as the enforcement factor of the public goods game (see Table II). Increasing the cost of punishment or the size of group enlarges the parameter scope where autonomous punishment is more effective than consensual punishment in facilitating cooperation. On the other hand, a greater fine of punishment and a higher enforcement factor are more in favor of consensual punishment when compared with autonomous punishment.

These results can be intuitively understood as follows. Compared with autonomous punishment, consensual punishment can be more effective in facilitating cooperation owing to the fact that consensual punishment is executed by all cooperators once the lowest degree of consensus is reached. Meanwhile, the second-order social dilemma stemming from autonomous punishment can be perfectly avoided when consensual punishment is employed in the population. Cooperation can be promoted by both of the two punishment mechanisms except for a sufficiently low degree of cooperators' WTPD. As the degree of cooperators' WTPD increases, the evolution of cooperation is more facilitated by punishments, and the lowest degree of consensus for employing punishment can be reached with a higher probability. Compared with autonomous punishment, a lower consensus required for employing punishment can be reached with a higher probability, which allows consensual punishment to be more effective in promoting cooperation. On the other hand, autonomous punishment turns to be more effective when a broader consensus is required for employing punishment. For an intermediate degree of consensus required for employing punishment, a higher degree of cooperators' WTPD allows consensual punishment to be more conducive in facilitating cooperation. However, a lower degree of cooperators' WTPD negatively affects the evolution of cooperation under consensual punishment, and then autonomous punishment can induce a higher level of cooperation. Accordingly, there exists a scenario where the level of cooperation under consensual punishment is the same as that under autonomous punishment.

Similarly, we have also investigated the evolutionary dynamics of cooperation in another model where the shared punishment (see Ref. [40]) is introduced once the consensus on punishment is reached. Namely, defectors each have to suffer a fixed fine, and the total cost of punishment is equally shared by all cooperators when the consensus on punishment is reached. Compared with the peer punishment, the average payoffs differ only in the punishment terms (see details in Appendix C). Although we have not presented the actual result, the qualitatively similar results can also be achieved when the shared punishment is introduced in our model. This observation is in consistent with our expectation as the replicator dynamics with peer and shared punishment are equivalent [40].

TABLE II. Effects of parameters on the value of q_c.[a]

	r				α			β			N		
	1.2	2	3	4	0.1	0.7	1	0.5	1.5	2	10	20	50
q_c	0.29	0.27	0.21	0.14	0.19	0.25	0.26	0.29	0.15	0.08	0.37	0.42	0.45

[a]For $q > q_c$, consensual punishment with majority rule ($T = 1/2$), compared with autonomous punishment, is more in favor of cooperation. Otherwise, autonomous punishment is more conducive in promoting cooperation. Parameters: $c = 1$, $N = 5$, $\alpha = 0.3$, $\beta = 1$ for column r, $N = 5$, $r = 3$, $\beta = 1$ for column α, $N = 5$, $r = 3$, $\alpha = 0.3$ for column β, and $r = 2$, $\alpha = 0.3$, $\beta = 1$ for column N.

IV. CONCLUSIONS

Understanding the evolution of cooperation has been one of the most challenging topics. Autonomous punishment, as an effective mechanism in promoting and sustaining cooperation, has attracted increasing attention. However, the well-known second-order social dilemma and antisocial punishment negatively affect the evolution of punishment as well as cooperation. Accordingly, several researchers have explored the evolution of cooperation under consensual punishment recently. In autonomous punishment, the punishment is executed based on individuals' decisions. However, consensual punishment is decisively different from autonomous punishment as the collective decision rules are employed to make decisions on punishment.

Noteworthily, existing studies on consensual punishment have been concentrated mainly on experiments, and the effect of cooperators' attitude to punishment on the evolution of cooperation is grossly neglected. To shed some light on this issue, we propose an analytical model where the punishment for defectors may be employed depending on the result of a vote. It differs decisively with the existing experiments in the following items: (1) the vote is introduced only for cooperators to decide whether to employ punishment for defectors and (2) the link between cooperators' WTPD and whether the punishment is inflicted on defectors is established.

The evolution of cooperation under consensual punishment has been explored based on the replicator dynamics. Our results illustrate that consensual punishment can facilitate cooperation such that the population can evolve to full cooperation. Compared with autonomous punishment, whether consensual punishment is more in favor of cooperation critically depends on the cooperators' WTPD and the lowest degree of consensus for employing punishment. Specifically, consensual punishment is more effective than autonomous punishment in facilitating cooperation when the lowest degree of consensus is low, while autonomous punishment turns to be more conducive when the lowest degree of consensus is very high. When the lowest degree of consensus for employing punishment is neither low nor high, the cooperators' WTPD plays a pivotal role in the comparison between autonomous and consensual punishments. For an intermediate degree of consensus required for employing punishment, there exists a critical degree of cooperators' WTPD at which the effect of consensual punishment on cooperation is the same as that of autonomous punishment. Consensual punishment can be more conducive than autonomous punishment as long as the degree of cooperators' WTPD is greater than the critical value, and otherwise autonomous punishment can induce a higher level of cooperation. Our findings highlight the importance of collective decision rules in the emergence of cooperation and may provide an explanation of the phenomenon that democratic mechanisms are so prevalent in modern societies.

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APPENDIX A: CALCULATION OF π_C

In this section, we depict the details in calculating the payoff of cooperators. Let x denote the fraction of cooperators represented by (C) in the population. Then we have

$$\begin{aligned}
\pi_{\rm C} = {}& q\Bigg(\sum_{k=0}^{N-2}\binom{N-1}{k}x^k(1-x)^{N-1-k}\Bigg\{\sum_{m=0}^{\bar{m}-2}\binom{k}{m}q^m(1-q)^{k-m}\left(\frac{k+1}{N}rc-c\right)\\
&+\sum_{m=\max\{0,\bar{m}-1\}}^{k}\binom{k}{m}q^m(1-q)^{k-m}*\left[\frac{k+1}{N}rc-c-\alpha(N-k-1)\right]\Bigg\}+x^{N-1}(rc-c)\Bigg)\\
&+(1-q)\Bigg(\sum_{k=0}^{N-2}\binom{N-1}{k}x^k(1-x)^{N-1-k}\Bigg\{\sum_{m=0}^{\bar{m}-1}\tbinom{k}{m}q^m(1-q)^{k-m}\left[\frac{k+1}{N}rc-c\right]\\
&+\sum_{m=\bar{m}}^{k}\binom{k}{m}q^m(1-q)^{k-m}\left[\frac{k+1}{N}rc-c-\alpha(N-k-1)\right]\Bigg\}+x^{N-1}(rc-c)\Bigg)\\
={}& q\Bigg[\sum_{k=0}^{N-1}\binom{N-1}{k}x^k(1-x)^{N-1-k}\left(\frac{k+1}{N}rc-c\right)\\
&-\alpha\sum_{k=0}^{N-2}(N-k-1)\binom{N-1}{k}x^k(1-x)^{N-1-k}\sum_{m=\max\{0,\bar{m}-1\}}^{k}\binom{k}{m}q^m(1-q)^{k-m}\Bigg]
\end{aligned}$$

$$+ (1-q)\left[\sum_{k=0}^{N-1}\binom{N-1}{k}x^k(1-x)^{N-1-k}\left(\frac{k+1}{N}rc - c\right)\right.$$
$$\left. - \alpha\sum_{k=0}^{N-2}(N-k-1)\binom{N-1}{k}x^k(1-x)^{N-1-k}\sum_{m=\bar{m}}^{k}\binom{k}{m}q^m(1-q)^{k-m}\right]$$
$$= q\left[\frac{(N-1)rcx}{N} + \frac{rc}{N} - c - \alpha\sum_{k=0}^{N-2}(N-k-1)\binom{N-1}{k}x^k(1-x)^{N-1-k}\sum_{m=\max\{0,\bar{m}-1\}}^{k}\binom{k}{m}q^m(1-q)^{k-m}\right]$$
$$+ (1-q)\left[\frac{(N-1)rcx}{N} + \frac{rc}{N} - c - \alpha\sum_{k=0}^{N-2}(N-k-1)\binom{N-1}{k}x^k(1-x)^{N-1-k}\sum_{m=\bar{m}}^{k}\binom{k}{m}q^m(1-q)^{k-m}\right].$$

Here $\bar{m} = \lceil Tk + T\rceil$. Simplifying,

$$\pi_C = \frac{(N-1)rcx}{N} + \frac{rc}{N} - c - q\alpha\sum_{k=0}^{N-2}(N-k-1)\binom{N-1}{k}x^k(1-x)^{N-1-k}\sum_{m=\max\{0,\bar{m}-1\}}^{k}\binom{k}{m}q^m(1-q)^{k-m}$$
$$- (1-q)\alpha\sum_{k=0}^{N-2}(N-k-1)\binom{N-1}{k}x^k(1-x)^{N-1-k}\sum_{m=\bar{m}}^{k}\binom{k}{m}q^m(1-q)^{k-m}.$$

The first term in the right-hand side of the above equation is the expected payoff from the public goods game, the second term is the cost of punishment when the cooperator opts for punishment, and the last one the cost of punishment when the cooperator does not opt for punishment.

APPENDIX B: CALCULATION OF π_D

Now let us depict the calculation details of the payoff of defectors. Similarly, we have

$$\pi_{\mathrm{D}} = \sum_{k=0}^{N-1}\binom{N-1}{k}x^k(1-x)^{N-1-k}\left[\sum_{m=0}^{\lceil Tk-1\rceil}\binom{k}{m}q^m(1-q)^{k-m}\frac{krc}{N} + \sum_{m=\lceil Tk\rceil}^{k}\binom{k}{m}q^m(1-q)^{k-m}\left(\frac{krc}{N} - k\beta\right)\right]$$
$$= \sum_{k=0}^{N-1}\binom{N-1}{k}x^k(1-x)^{N-1-k}\frac{krc}{N} - \beta\sum_{k=0}^{N-1}k\binom{N-1}{k}x^k(1-x)^{N-1-k}\sum_{m=\lceil Tk\rceil}^{k}\binom{k}{m}q^m(1-q)^{k-m}.$$

Thus,

$$\pi_{\mathrm{D}} = \frac{(N-1)rcx}{N} - \beta\sum_{k=0}^{N-1}k\binom{N-1}{k}x^k(1-x)^{N-1-k}\sum_{m=\lceil Tk\rceil}^{k}\binom{k}{m}q^m(1-q)^{k-m}.$$

The first term in the right-hand side of the above equation is the expected payoff from the public goods game, and the other one is the fine of punishment.

APPENDIX C: AVERAGE PAYOFFS UNDER CONSENSUAL AND AUTONOMOUS SHARED PUNISHMENT

First, let us depict the average payoffs for cooperators (π_{C_s}) and defectors (π_{D_s}) under consensual shared punishment. The average payoffs differ only in the punishment terms. Then we have

$$\pi_{\mathrm{C_s}} = \frac{(N-1)rcx}{N} + \frac{rc}{N} - c - q\alpha\sum_{k=0}^{N-2}\frac{N-k-1}{k+1}\binom{N-1}{k}x^k(1-x)^{N-1-k}\sum_{m=\max\{0,\bar{m}-1\}}^{k}\binom{k}{m}q^m(1-q)^{k-m}$$
$$- (1-q)\alpha\sum_{k=0}^{N-2}\frac{N-k-1}{k+1}\binom{N-1}{k}x^k(1-x)^{N-1-k}\sum_{m=\bar{m}}^{k}\binom{k}{m}q^m(1-q)^{k-m}$$

and

$$\pi_{\mathrm{D_s}} = \frac{(N-1)rcx}{N} - \beta \sum_{k=1}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \sum_{m=\lceil Tk \rceil}^{k} \binom{k}{m} q^m (1-q)^{k-m}.$$

Meanwhile, the average payoffs of cooperators (π'_{C_s}) and defectors (π'_{D_s}) under autonomous shared punishment are given as

$$\begin{aligned}
\pi'_{\mathrm{C_s}} &= (1-q) \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \left[\frac{(k+1)rc}{N} - c \right] \\
&\quad + q \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \left[\frac{(k+1)rc}{N} - c - \alpha \sum_{m=0}^{k} \frac{N-k-1}{m+1} \binom{k}{m} q^m (1-q)^{k-m} \right] \\
&= \frac{(N-1)xrc}{N} + \frac{rc}{N} - c - q\alpha \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \sum_{m=0}^{k} \frac{N-k-1}{m+1} \binom{k}{m} q^m (1-q)^{k-m}
\end{aligned}$$

and

$$\begin{aligned}
\pi'_{\mathrm{D_s}} &= \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \left[\frac{krc}{N} - \beta \sum_{m=1}^{k} \binom{k}{m} q^m (1-q)^{k-m} \right] \\
&= \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k} \left[\frac{krc}{N} - \beta \sum_{m=0}^{k} \binom{k}{m} q^m (1-q)^{k-m} + \beta (1-q)^k \right] \\
&= \frac{(N-1)xrc}{N} - \beta \left[1 - \sum_{k=0}^{N-1} (1-q)^k \binom{N-1}{k} x^k (1-x)^{N-1-k} \right].
\end{aligned}$$

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