Reply to "Comment on 'Shear modulus of two-dimensional Yukawa or dusty-plasma solids obtained from the viscoelasticity in the liquid state' "

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In the preceding Comment [Khrapak and Klumov, preceding Comment, Phys. Rev. E **101**, 057201 (2020)], a completely different method is used to calculate the shear modulus of two-dimensional Yukawa solids. Here the results of the shear modulus of two-dimensional Yukawa solids obtained in a previous work [Wang *et al.*, Phys. Rev. E **99**, 063206 (2019)] are compared with the results in the preceding Comment. It is found that the two empirical fits from the two methods are in extremely close agreement, supporting the consistency of these two methods.

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In the preceding Comment [1], the authors mainly focus on the expression of the shear modulus for two-dimensional (2D) Yukawa solids, which is

$$G(\kappa)/(Q^2/4\pi\epsilon_0 a^3) = (0.211 - 0.0389\kappa^{1.11})^2 \quad (1)$$

in our paper [2], obtained directly from a phenomenological fitting of the theoretical transverse sound speeds for 2D Yukawa solids in a previous publication [3]. The authors of Ref. [1] mention that there is "no comparison with the available theoretical results on the high-frequency elastic moduli of two-dimensional Yukawa fluids" in our paper, so they want to "provide [such] a comparison . . . and to indicate the correct way of interpreting the numerical results" in their Comment [1].

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The authors in [1] compare our results with prior theory and find good agreement. They present a completely different method to calculate the shear modulus of 2D Yukawa solids. We plot both our results [2] and the results in Ref. [1] in Fig. 1 for a comparison. We find that our candidate for the shear modulus [2] of Eq. (1) almost completely overlaps with the fit of Eq. (4) in [1]. From this comparison of the results of Eq. (4) in [1] with our fitting of Eq. (1) it is clear that the two empirical fits are in extremely close agreement, supporting the consistency of two methods.

We would like to mention that the method in [1] is thorough and the results derived are consistent with our results. Due to the complexity of strongly coupled dusty plasmas, these physical quantities probably cannot be directly derived without a phenomenological fitting. Thus, it is reasonable to use the phenomenological fitting expression (1) in [2] or to use the fit of the Madelung coefficient of Eq. (4) in Ref. [1] from

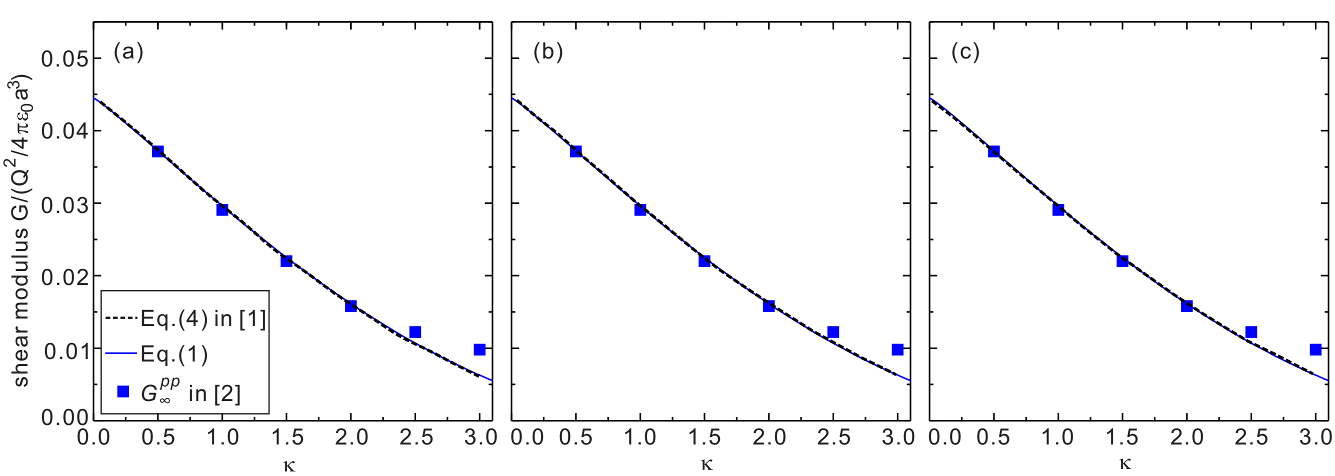

FIG. 1. Infinite-frequency shear modulus G_∞ of 2D Yukawa liquids versus the screening parameter κ for different values of the coupling parameter: (a) $\Gamma = 8$, (b) $\Gamma = 20$, and (c) $\Gamma = 68$. The squares correspond to the potential portion of the infinite-frequency shear modulus G_∞^{pp} from our simulation results in [2]. The solid curve is the candidate for the shear modulus for 2D Yukawa crystals from our phenomenological fitting of the transverse sound speed [2]. The black dashed curve is the fit of Eq. (4) in [1]. Clearly, our phenomenological fitting data almost completely overlap with the fit of Eq. (4) in [1], suggesting that our choice of the candidate for the shear modulus for 2D Yukawa crystals agree well with the final conclusion of [1].

the prior theory. Since both methods contain some approximations, it is reasonable that the shear modulus values obtained from these two methods are slightly different. We think that our phenomenological fitting of Eq. (1) is compact and easy to use. In fact, quantifying the shear modulus of dusty plasma solids is pretty difficult, since it depends on the conditions [4] of the temperature and the κ value. The measurement of the shear modulus of dusty plasma solids in experimental is still lacking.

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