Parallel propagating electromagnetic waves in magnetized quantum electron plasmas with finite temperature

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We studied parallel propagating electromagnetic waves in a magnetized quantum electron plasma of finite temperature, as an extension of our previous study on a zero temperature plasma. We obtained simple analytic dispersion relations in the long wavelength limit that included the thermal effect as correction terms to the zero temperature results. As in the zero temperature case, the lower branch of the R wave showed significant damping and became ill-defined at short wavelengths. Quantum effects seemed to give qualitative changes, such as the appearance of anomalous dispersion regions, to the classical dispersion relations when $v_F/v_{th} \leqslant 0.2$ for a set of exemplary parameters of $v_F = 0.1c$ and $\omega_{ce}/\omega_{pe} = 0.05$ was used. We also noted that introduction of the Planck constant in the quantum Vlasov equation changed the shape of the anomalous dispersion region qualitatively, by forming a normal dispersion region in the middle of the original single broad anomalous dispersion region.

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I. INTRODUCTION

A quantum electron plasma is characterized by its high density and relatively low temperature compared to the corresponding Fermi energy. Hence, in a quantum electron plasma, the degenerate pressure exceeds or is comparable to the thermal pressure and the (thermal) de Broglie wavelength is larger than the mean interparticle distance. This requires a quantum mechanical treatment for the study of its collective behavior. The importance of theoretical research on quantum electron plasmas was recently recognized with the progress in laser plasma experiments, and the miniaturization of electronic devices [1–4]. It has also long been believed that the interiors of white dwarfs are also in a degenerate state [5].

Various plasma waves have been studied for magnetized and unmagnetized quantum electron plasma, with the aid of fluid or kinetic equations. As can be expected, the quantum mechanical effects appear both through the fluid or kinetic equations and from the degenerate pressure or the Fermi-Dirac distribution function.

The notable difference between quantum mechanical fluid equations and classical ones is the Bohm potential term that appears in the momentum equation with the correction of the order of $\hbar^2$, where $\hbar$ is the reduced Planck constant. Its effect is electrostatic: Only the electrostatic mode and the electromagnetic X mode which have the electric field in the propagating direction are affected by the Bohm potential, whereas the electromagnetic L, R, and O modes, whose electric fields are perpendicular to the propagating direction, are not affected. With the Bohm potential term, the electrostatic wave can propagate even without thermal pressure [6,7].

Kinetic treatment of quantum plasma waves has mostly been carried out using the classical Vlasov equation, with the velocity distribution replaced by the Fermi-Dirac distribution, because of the mathematical difficulty of the quantum Vlasov equation (Wigner equation): The results are such that the Fermi velocity acts like the thermal velocity in the case of classical plasmas, with correction terms added to the corresponding classical results of the pressureless fluid [8,9]. The effects are also manifested as only minor shifts of cutoff frequencies in the case of electromagnetic waves.

However, when the exact quantum Vlasov equation is adopted, qualitative differences are seen. For electrostatic oscillations, the Landau damping rate takes the form of a finite difference in the two points located at $\pm\hbar k/2m_e$ from the resonance velocity, which reduces to the classical one when $\hbar \to 0$ [10,11]. The study of the Landau damping has been extended to the electromagnetic case: A general expression for the dispersion relation of electron plasma waves in electromagnetic wave fields shows that electron and photon Landau damping processes are very similar [12]. The Wigner-Maxwell system is also applied to the electromagnetic Weibel instability and it is found that the quantum mechanical effects suppress the Weibel instability for anisotropic plasmas [13]. Further, in the case of relativistic quantum plasmas with a ring velocity distribution, it is shown that the quantum effect reduces the Weibel instability region whereas the relativistic effect lowers the growth rate [14]. Recently, it has been argued that, in the case of zero temperature Fermi-Dirac distribution in a small range of wave number k just below the R wave exhibits anomalous dispersion in a small range of wave number k just below the singular point that originates from the nonphysical assumption of zero temperature. The wave also shows damping for k above a critical value before it finally becomes ill-defined [15].

Thermal effects on the plasma waves in a degenerate electron plasma have been discussed. In the quantum fluid equations, when the pressure term in the momentum equation

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was modified to incorporate the Fermi-Dirac distribution, the dispersion relation for the electrostatic wave was given in a simple form with a slight modification of the classical thermal term, which becomes proportional to v_{th}^2 in the high temperature limit and v_F^2 in the cold temperature limit, where v_{th} is the thermal velocity and v_F is the Fermi velocity [16]. A kinetic approach has also been taken with the classical Vlasov equation for an electron plasma wave: With a dispersion relation quite similar to that of the fluid treatment in the limit of long wavelength, the damping rate was given in an analytic form as a function of temperature, Fermi energy, and the chemical potential [17]. More exact quantum mechanical treatment with the quantum mechanical Vlasov equation was carried out for electrostatic waves: The dispersion relation for electron plasmas was determined to be identical to the previous fluid results in the limit of long wavelength [18]. Recently, the Landau damping rate for an electrostatic electron wave was calculated by solving the quantum Vlasov equation numerically for a wide range of temperatures and chemical potentials [15].

We would like to extend our previous study of electromagnetic waves propagating in the direction parallel to the ambient magnetic field with zero temperature [15] by including the finite temperature effect in the present paper. In Sec. II, we start with the general form of the susceptibility tensor obtained in our previous paper to apply it to the case of the Fermi-Dirac distribution function with finite temperature. The dispersion relations are obtained in simple analytic forms in the long wavelength limit for the two cases of low degeneracy and high degeneracy in Sec. III. In Sec. IV, discussions compare the zero temperature case for representative values of density, temperature, and the magnetic field. Finally, a summary of the present work is given in Sec. V.

II. DIELECTRIC FUNCTION

In our previous paper, we derived the susceptibility tensor for the parallel propagating waves in a magnetized quantum electron plasma with arbitrary velocity distribution $f_{e0}(\vec{v})$ [15]. The following expressions were obtained from the quantum mechanical Vlasov equation, with the quantum recoil effect included:

$$\overleftrightarrow{\chi} = \chi_R \overleftrightarrow{U} + \chi_L \overleftrightarrow{U}^* + \chi_\parallel \overleftrightarrow{T}, \tag{1}$$

where

$$\begin{aligned}\chi_R = &-\frac{\omega_{pe}^2}{2\omega^2} - \frac{\pi\omega_{pe}^2\omega_{ce}}{2\omega^2}\left(\int \frac{G_{e1}(v_\parallel)dv_\parallel}{\omega_+ - \omega_{ce} - kv_\parallel}\right.\\ &\left.+\int \frac{G_{e1}(v_\parallel)dv_\parallel}{\omega_- - \omega_{ce} - kv_\parallel}\right)\\ &+\frac{\pi\omega_{pe}^2}{2\omega^2}\frac{m_e}{\hbar}\left(\int \frac{G_{e3}(v_\parallel)dv_\parallel}{\omega_+ - \omega_{ce} - kv_\parallel}\right.\\ &\left.-\int \frac{G_{e3}(v_\parallel)dv_\parallel}{\omega_- - \omega_{ce} - kv_\parallel}\right),\end{aligned} \tag{2}$$

$$\begin{aligned}\chi_L = &-\frac{\omega_{pe}^2}{2\omega^2} + \frac{\pi\omega_{pe}^2\omega_{ce}}{2\omega^2}\left(\int \frac{G_{e1}(v_\parallel)dv_\parallel}{\omega_+ + \omega_{ce} - kv_\parallel}\right.\\ &\left.+\int \frac{G_{e1}(v_\parallel)dv_\parallel}{\omega_- + \omega_{ce} - kv_\parallel}\right)\\ &+\frac{\pi\omega_{pe}^2}{2\omega^2}\frac{m_e}{\hbar}\left(\int \frac{G_{e3}(v_\parallel)dv_\parallel}{\omega_+ + \omega_{ce} - kv_\parallel}\right.\\ &\left.-\int \frac{G_{e3}(v_\parallel)dv_\parallel}{\omega_- + \omega_{ce} - kv_\parallel}\right),\end{aligned} \tag{3}$$

and

$$\chi_\parallel = \frac{2\pi\omega_{pe}^2 m_e}{\hbar k^2}\left(\int \frac{G_{e1}(v_\parallel)dv_\parallel}{\omega_+ - kv_\parallel} - \int \frac{G_{e1}(v_\parallel)dv_\parallel}{\omega_- - kv_\parallel}\right), \tag{4}$$

with the definition of the matrices $\overleftrightarrow{U}$ and $\overleftrightarrow{T}$:

$$\overleftrightarrow{U} = \begin{bmatrix} 1 & i & 0\\ -i & 1 & 0\\ 0 & 0 & 0\end{bmatrix}, \overleftrightarrow{T} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}, \tag{5}$$

and

$$\begin{aligned}G_{e1}(v_\parallel) &= \frac{1}{n_0}\int f_{e0}(\vec{v})v_\perp dv_\perp, G_{e3}(v_\parallel)\\ &= \frac{1}{n_0}\int f_{e0}(\vec{v})v_\perp^3 dv_\perp,\end{aligned} \tag{6}$$

where n_0 is the electron density. ω_{pe} and ω_{ce} are the plasma frequency and the gyro frequency, respectively:

$$\omega_{pe} = \sqrt{\frac{4\pi n_0 e^2}{m_e}}, \omega_{ce} = \frac{|e|B_0}{m_e c}. \tag{7}$$

Here, e and m_e are the charge and mass of an electron, respectively, and B_0 is the strength of the ambient magnetic field. Finally, ω_+ and ω_- are the frequencies shifted by $+\hbar k^2/2m_e$ and $-\hbar k^2/2m_e$, respectively:

$$\omega_\pm = \omega \pm \frac{\hbar k^2}{2m_e}. \tag{8}$$

With the aid of the Maxwell's equations, we define the following dielectric functions for the electromagnetic and electrostatic waves propagating in the direction parallel to the ambient magnetic field:

$$\begin{aligned}\epsilon_R = 1 &- \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} + \frac{\pi\omega_{pe}^2\omega_{ce}}{\omega^2}\\ &\times\left(\int \frac{G_{e1}(v_\parallel)dv_\parallel}{v_\parallel - \frac{\omega_+ - \omega_{ce}}{k}} + \int \frac{G_{e1}(v_\parallel)dv_\parallel}{v_\parallel - \frac{\omega_- - \omega_{ce}}{k}}\right)\\ &-\frac{\pi\omega_{pe}^2}{\omega^2}\frac{m_e}{\hbar}\left(\int \frac{G_{e3}(v_\parallel)dv_\parallel}{v_\parallel - \frac{\omega_+ - \omega_{ce}}{k}} - \int \frac{G_{e3}(v_\parallel)dv_\parallel}{v_\parallel - \frac{\omega_- - \omega_{ce}}{k}}\right),\end{aligned} \tag{9}$$

$$\epsilon_L = 1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} - \frac{\pi\omega_{pe}^2\omega_{ce}}{\omega^2} \times \left(\int \frac{G_{e1}(v_\parallel)dv_\parallel}{v_\parallel - \frac{\omega_+ + \omega_{ce}}{k}} + \int \frac{G_{e1}(v_\parallel)dv_\parallel}{v_\parallel - \frac{\omega_- + \omega_{ce}}{k}}\right) - \frac{\pi\omega_{pe}^2}{\omega^2}\frac{m_e}{\hbar}\left(\int \frac{G_{e3}(v_\parallel)dv_\parallel}{v_\parallel - \frac{\omega_+ + \omega_{ce}}{k}} - \int \frac{G_{e3}(v_\parallel)dv_\parallel}{v_\parallel - \frac{\omega_- + \omega_{ce}}{k}}\right), \tag{10}$$

and

$$\epsilon_\parallel = 1 - \frac{2\pi\omega_{pe}^2 m_e}{\hbar k^3}\left(\int \frac{G_{e1}(v_\parallel)dv_\parallel}{v_\parallel - \frac{\omega_+}{k}} - \int \frac{G_{e1}(v_\parallel)dv_\parallel}{v_\parallel - \frac{\omega_-}{k}}\right). \tag{11}$$

ϵ_R and ϵ_L are the dielectric functions for the electromagnetic right (R) and left (L) polarization waves, respectively, and $\epsilon_\parallel$ is the dielectric function for the longitudinal electrostatic wave. The dispersion relations can be obtained by setting $\epsilon_R = 0$ for the R wave, $\epsilon_L = 0$ for the L wave, and $\epsilon_\parallel = 0$ for the electrostatic wave.

Let us take the Fermi-Dirac distribution for $f_{e0}(\vec{v})$ [19]:

$$f_{e0}(\vec{v}) = 2\left(\frac{m_e}{2\pi\hbar}\right)^3 \frac{1}{1 + \exp\left(\beta\left(\frac{1}{2}m_e v^2 - \mu\right)\right)}, \tag{12}$$

where $\beta = 1/k_BT$ (k_B is the Boltzmann constant) and the chemical potential μ is a function of the electron density n_0 and temperature T. By integrating the above Fermi-Dirac distribution function over the entire velocity (energy) space, we obtain the electron density n_0 in terms of the thermal parameter β and the chemical potential μ:

$$n_0 = -\frac{1}{4}\left(\frac{2m_e}{\pi\beta\hbar^2}\right)^{3/2} Li_{3/2}(-e^{\beta\mu}), \tag{13}$$

or equivalently,

$$Li_{3/2}(-e^{\beta\mu}) = -\frac{4}{3\sqrt{3\sqrt{\pi}}}(\beta E_F)^{3/2} = -\frac{4}{3\sqrt{\pi}}\left(\frac{v_F}{v_{th}}\right)^3, \tag{14}$$

where $Li_s(x)$ is the polylogarithm function, which can be defined in two forms, a series expansion and an integral,

$$Li_s(x) = \sum_{n=1}^{\infty}\frac{x^n}{n^s} = \frac{1}{\Gamma(s)}\int_0^\infty \frac{t^{s-1}}{e^t/z - 1}dt. \tag{15}$$

The Fermi energy E_F and the Fermi velocity v_F are defined as

$$E_F = \frac{\hbar^2}{2m_e}(3\pi^3 n_0)^{2/3} = \frac{1}{2}m_e v_F^2, \tag{16}$$

and the thermal velocity v_{th} is defined as

$$v_{th} = \sqrt{\frac{2k_BT}{m_e}}. \tag{17}$$

Hence, the velocity moments $G_{e1}(v_\parallel)$ and $G_{e3}(v_\parallel)$ can also be written in terms of the polylogarithm functions:

$$G_{e1}(v_\parallel) = \frac{1}{2\pi^{3/2}v_{th}}\frac{1}{Li_{3/2}(-e^{\beta\mu})}Li_1(-e^{\beta(\mu - \frac{1}{2}m_e v_\parallel^2)}), \tag{18}$$

and

$$G_{e3}(v_\parallel) = \frac{v_{th}}{2\pi^{3/2}}\frac{1}{Li_{3/2}(-e^{\beta\mu})}Li_2(-e^{\beta(\mu - \frac{1}{2}m_e v_\parallel^2)}). \tag{19}$$

Using the definition of the polylogarithm functions in $G_{e1}(v_\parallel)$ and $G_{e3}(v_\parallel)$, we can express the following integrals that appear in the dielectric functions in terms of the plasma dispersion functions as follows:

$$\int \frac{G_{e1}(v_\parallel)dv_\parallel}{\omega - kv_\parallel} = \frac{1}{2\pi v_{th}}\frac{1}{Li_{3/2}(-e^{\beta\mu})}\sum_{n=1}^{\infty}(-1)^n\frac{e^{n\beta\mu}}{n} \times Z\left(\frac{\omega}{kv_{th}}\sqrt{n}\right), \tag{20}$$

and

$$\int \frac{G_{e3}(v_\parallel)dv_\parallel}{\omega - kv_\parallel} = \frac{v_{\rm th}}{2\pi}\frac{1}{Li_{3/2}(-e^{\beta\mu})}\sum_{n=1}^{\infty}(-1)^n\frac{e^{n\beta\mu}}{n^2} \times Z\left(\frac{\omega}{kv_{\rm th}}\sqrt{n}\right), \tag{21}$$

where $Z(x)$ is the plasma dispersion function defined as [20]

$$Z(x) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{e^{-t^2}}{t - x}dt. \tag{22}$$

Hence, the dielectric functions become

$$\epsilon_R = 1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} + \frac{\omega_{pe}^2}{2\omega^2}\frac{\omega_{ce}}{kv_{\rm th}}\frac{1}{Li_{3/2}(-e^{\beta\mu})}\sum_{n=1}^{\infty}(-1)^n\frac{e^{n\beta\mu}}{n}\left(Z\left(\frac{(\omega_+ - \omega_{ce})\sqrt{n}}{kv_{\rm th}}\right) + Z\left(\frac{(\omega_- - \omega_{ce})\sqrt{n}}{kv_{\rm th}}\right)\right) - \frac{\omega_{pe}^2}{2\omega^2}\frac{m_e v_{\rm th}}{\hbar k}\frac{1}{Li_{3/2}(-e^{\beta\mu})}\sum_{n=1}^{\infty}(-1)^n\frac{e^{n\beta\mu}}{n^2}\left(Z\left(\frac{(\omega_+ - \omega_{ce})\sqrt{n}}{kv_{\rm th}}\right) - Z\left(\frac{(\omega_- - \omega_{ce})\sqrt{n}}{kv_{\rm th}}\right)\right), \tag{23}$$

$$\epsilon_L = 1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pe}^2}{2\omega^2}\frac{\omega_{ce}}{kv_{\rm th}}\frac{1}{Li_{3/2}(-e^{\beta\mu})}\sum_{n=1}^{\infty}(-1)^n\frac{e^{n\beta\mu}}{n}\left(Z\left(\frac{(\omega_+ + \omega_{ce})\sqrt{n}}{kv_{\rm th}}\right) + Z\left(\frac{(\omega_- + \omega_{ce})\sqrt{n}}{kv_{\rm th}}\right)\right) - \frac{\omega_{pe}^2}{2\omega^2}\frac{m_e v_{\rm th}}{\hbar k}\frac{1}{Li_{3/2}(-e^{\beta\mu})}\sum_{n=1}^{\infty}(-1)^n\frac{e^{n\beta\mu}}{n^2}\left(Z\left(\frac{(\omega_+ + \omega_{ce})\sqrt{n}}{kv_{\rm th}}\right) - Z\left(\frac{(\omega_- + \omega_{ce})\sqrt{n}}{kv_{\rm th}}\right)\right), \tag{24}$$

and

$$\epsilon_{\parallel} = 1 - \frac{m_e\omega_{pe}^2}{\hbar k^3 v_{\rm th}}\frac{1}{Li_{3/2}(-e^{\beta\mu})} \times \sum_{n=1}^{\infty}(-1)^n\frac{e^{n\beta\mu}}{n}\left(Z\left(\frac{\omega_+}{kv_{\rm th}}\sqrt{n}\right) - Z\left(\frac{\omega_-}{kv_{\rm th}}\sqrt{n}\right)\right). \tag{25}$$

III. DISPERSION RELATIONS IN THE LONG WAVELENGTH LIMIT

For the long wavelength limit of the dispersion relations of electrostatic and electromagnetic waves, let us assume $|\frac{\omega_\pm-\omega_{ce}}{kv_{\rm th}}| \gg 1$ and $|\frac{\omega_\pm-\omega_{ce}}{kv_F}| \gg 1$ ($|\frac{\omega_\pm}{kv_{\rm th}}| \gg 1$ and $|\frac{\omega_\pm}{kv_F}| \gg 1$ in the electrostatic case). The real part of the plasma dispersion function can be expanded as

$$\mathrm{Re}(Z(x)) \approx -\frac{1}{x} - \frac{1}{2x^3} - \frac{3}{4x^5}\cdots. \tag{26}$$

Hence, the real parts of the dielectric functions are

$$\begin{aligned}\epsilon_{R,r} \approx 1 &- \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pe}^2}{2\omega^2}\frac{\omega_{ce}}{kv_{th}}\left[\left(\frac{1}{\omega_+-\omega_{ce}} + \frac{1}{\omega_--\omega_{ce}}\right)(kv_{th})\right.\\ &\left.+\frac{Li_{5/2}(-e^{\beta\mu})}{2Li_{3/2}(-e^{\beta\mu})}\left(\frac{1}{(\omega_+-\omega_{ce})^3} + \frac{1}{(\omega_--\omega_{ce})^3}\right)(kv_{th})^3 + \cdots\right]\\ &+\frac{\omega_{pe}^2}{2\omega^2}\frac{m_ev_{th}}{\hbar k}\left[\frac{Li_{5/2}(-e^{\beta\mu})}{Li_{3/2}(-e^{\beta\mu})}\left(\frac{1}{\omega_+-\omega_{ce}} - \frac{1}{\omega_--\omega_{ce}}\right)(kv_{th}) + \cdots\right],\end{aligned} \tag{27}$$

$$\begin{aligned}\epsilon_{L,r} \approx 1 &- \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} + \frac{\omega_{pe}^2}{2\omega^2}\frac{\omega_{ce}}{kv_{th}}\left[\left(\frac{1}{\omega_++\omega_{ce}} + \frac{1}{\omega_-+\omega_{ce}}\right)(kv_{th})\right.\\ &\left.+\frac{Li_{5/2}(-e^{\beta\mu})}{2Li_{3/2}(-e^{\beta\mu})}\left(\frac{1}{(\omega_++\omega_{ce})^3} + \frac{1}{(\omega_-+\omega_{ce})^3}\right)(kv_{th})^3 + \cdots\right]\\ &+\frac{\omega_{pe}^2}{2\omega^2}\frac{m_ev_{th}}{\hbar k}\left[\frac{Li_{5/2}(-e^{\beta\mu})}{Li_{3/2}(-e^{\beta\mu})}\left(\frac{1}{\omega_++\omega_{ce}} - \frac{1}{\omega_-+\omega_{ce}}\right)(kv_{th}) + \cdots\right],\end{aligned} \tag{28}$$

and

$$\epsilon_{\parallel,r} = 1 - \frac{m_e\omega_{pe}^2}{\hbar k^3 v_{th}}\left[\left(\frac{1}{\omega_+} - \frac{1}{\omega_-}\right)(kv_{th}) + \frac{Li_{5/2}(-e^{\beta\mu})}{2Li_{3/2}(-e^{\beta\mu})}\left(\frac{1}{\omega_+^3} - \frac{1}{\omega_-^3}\right)(kv_{th})^3 + \cdots\right]. \tag{29}$$

Further, in the long wavelength expansion, we keep only the lowest order corrections of the thermal effect ($|\frac{kv_{th}}{\omega\pm\omega_{ce}}| \ll 1$ for the electromagnetic case and $|\frac{kv_{th}}{\omega}| \ll 1$ for the electrostatic case) as well as the quantum recoil effect ($|\frac{\hbar k^2}{2m_e(\omega\pm\omega_{ce})}| \ll 1$ for the electromagnetic case and $|\frac{\hbar k^2}{2m_e\omega}| \ll 1$ for the electrostatic case) to obtain

$$\epsilon_{R,r} \approx 1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega(\omega-\omega_{ce})}\left[1 + \frac{Li_{5/2}(-e^{\beta\mu})}{Li_{3/2}(-e^{\beta\mu})}\left(\frac{kv_{th}}{\omega-\omega_{ce}}\right)^2 + \frac{\omega_{ce}}{\omega(\omega-\omega_{ce})^2}\left(\frac{\hbar k^2}{2m_e}\right)^2\right], \tag{30}$$

$$\epsilon_{L,r} \approx 1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega(\omega+\omega_{ce})}\left[1 + \frac{Li_{5/2}(-e^{\beta\mu})}{Li_{3/2}(-e^{\beta\mu})}\left(\frac{kv_{th}}{\omega+\omega_{ce}}\right)^2 - \frac{\omega_{ce}}{\omega(\omega-\omega_{ce})^2}\left(\frac{\hbar k^2}{2m_e}\right)^2\right], \tag{31}$$

and

$$\epsilon_{\parallel,r} \approx 1 - \frac{\omega_{pe}^2}{\omega^2}\left[1 + \frac{3Li_{5/2}(-e^{\beta\mu})}{2Li_{3/2}(-e^{\beta\mu})}\left(\frac{kv_{th}}{\omega}\right)^2 + \frac{1}{\omega^2}\left(\frac{\hbar k^2}{2m_e}\right)^2\right]. \tag{32}$$

We note that the above electrostatic dielectric function corresponds to the electrostatic dispersion of Eq. (32) in Ref. [21]. The quantum recoil effects, as represented by the dependence on $\hbar$, appear as a correction proportional to $(\hbar k^2/2m_e)^2$ in all of the above electromagnetic and electrostatic dielectric functions.

A. Low degeneracy plasma

For a large finite thermal velocity in the low degeneracy case ($v_{\rm th} \gg v_F$), we claim that for any natural number s

$$\frac{Li_{s/2}(-e^{\beta\mu})}{Li_{3/2}(-e^{\beta\mu})} = 1 - \frac{4}{3\sqrt{\pi}}\left(\frac{1}{2^{s/2}} - \frac{1}{2^{3/2}}\right)\left(\frac{v_F}{v_{\rm th}}\right)^3 + O\left(\left(\frac{v_F}{v_{\rm th}}\right)^6\right). \tag{33}$$

Then, the dispersion relations for the low degeneracy case are

$$\epsilon_{R,r} \approx 1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega(\omega-\omega_{ce})}\left[1 + \frac{1}{2}\left(1 + \frac{1}{3\sqrt{2\pi}}\left(\frac{v_F}{v_{\rm th}}\right)^3\right)\left(\frac{kv_{\rm th}}{\omega-\omega_{ce}}\right)^2 + \frac{\omega_{ce}}{\omega(\omega-\omega_{ce})^2}\left(\frac{\hbar k^2}{2m_e}\right)^2\right], \tag{34}$$

$$\epsilon_{L,r} \approx 1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega(\omega+\omega_{ce})}\left[1 + \frac{1}{2}\left(1 + \frac{1}{3\sqrt{2\pi}}\left(\frac{v_F}{v_{\rm th}}\right)^3\right)\left(\frac{kv_{\rm th}}{\omega+\omega_{ce}}\right)^2 - \frac{\omega_{ce}}{\omega(\omega-\omega_{ce})^2}\left(\frac{\hbar k^2}{2m_e}\right)^2\right], \tag{35}$$

and

$$\epsilon_{\parallel,r} \approx 1 - \frac{\omega_{pe}^2}{\omega^2}\left[1 + \frac{3}{2}\left(1 + \frac{1}{3\sqrt{2\pi}}\left(\frac{v_F}{v_{\rm th}}\right)^3\right)\left(\frac{kv_{\rm th}}{\omega}\right)^2 + \frac{1}{\omega^2}\left(\frac{\hbar k^2}{2m_e}\right)^2\right]. \tag{36}$$

If $v_F = 0$, all of the dielectric functions have thermal correction terms proportional to $v_{\rm th}^2$: With finite v_F, its coefficient is modified to include a small term proportional to $(v_F/v_{\rm th})^3$.

B. High degeneracy plasma

For the high degeneracy case, $e^{\beta\mu} \gg 1$, and we may use the following asymptotic expansion [22]:

$$Li_s \approx -\frac{1}{\Gamma(s+1)}(\beta\mu)^s\left(1 + \frac{\pi^2 s(s-1)}{6}\left(\frac{1}{\beta\mu}\right)^2 + \cdots\right), \tag{37}$$

$$\frac{Li_{s/2}(-e^{\beta\mu})}{Li_{3/2}(-e^{\beta\mu})} \approx \frac{\Gamma(5/2)}{\Gamma(s/2+1)}(\beta\mu)^{\frac{s-3}{2}}\left(1 + \frac{(s-3)(s+1)\pi^2}{24}\times\left(\frac{1}{\beta\mu}\right)^2 + \cdots\right), \tag{38}$$

$$\mu \approx E_F\left(1 - \frac{\pi^2}{12}\left(\frac{v_{\rm th}}{v_F}\right)^4 + \cdots\right). \tag{39}$$

Applying these expansions to the dielectric functions of Eqs. (31)–(33), we obtain

$$\epsilon_{R,r} \approx 1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega(\omega-\omega_{ce})}\left[1 + \frac{1}{5}\left(1 + \frac{5\pi^2}{12}\left(\frac{v_{\rm th}}{v_F}\right)^4\right)\left(\frac{kv_F}{\omega-\omega_{ce}}\right)^2 + \frac{\omega_{ce}}{\omega(\omega-\omega_{ce})^2}\left(\frac{\hbar k^2}{2m_e}\right)^2\right], \tag{40}$$

$$\epsilon_{L,r} \approx 1 - \frac{k^2c^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega(\omega+\omega_{ce})}\left[1 + \frac{1}{5}\left(1 + \frac{5\pi^2}{12}\left(\frac{v_{\rm th}}{v_F}\right)^4\right)\left(\frac{kv_F}{\omega+\omega_{ce}}\right)^2 - \frac{\omega_{ce}}{\omega(\omega-\omega_{ce})^2}\left(\frac{\hbar k^2}{2m_e}\right)^2\right], \tag{41}$$

and

$$\epsilon_{\parallel,r} \approx 1 - \frac{\omega_{pe}^2}{\omega^2}\left[1 + \frac{3}{5}\left(1 + \frac{5\pi^2}{12}\left(\frac{v_{th}}{v_F}\right)^4\right)\left(\frac{kv_F}{\omega}\right)^2 + \frac{1}{\omega^2}\left(\frac{\hbar k^2}{2m_e}\right)^2\right]. \tag{42}$$

Whereas thermal correction is proportional to v_{th}^2 in the nondegenerate classical limit, the quantum mechanical correction is proportional to v_F^2, and its thermal correction is such that its coefficient is modified to include a small term proportional to $(v_{th}/v_F)^4$. We can reproduce the results of the zero temperature limit in Ref. [15] by taking $v_{th} \to 0$.

IV. DISCUSSION

We obtained the dispersion relations numerically by setting the right-hand sides of Eqs. (31)–(33) for representative values of $v_F = 0.1c$ and $\omega_{ce}/\omega_{pe} = 0.05$, corresponding to $n_e = 6 \times 10^{26}$ cm^{-3} and $B_0 = 4 \times 10^9 G$, which are typical values for a magnetic white dwarf. To obtain thermal effects, we assumed $v_{th} = 0.07c$ for an exemplary case. The results are shown in Fig. 1 , compared with those of the zero temperature case. Figure 1(a) shows the upper branch of the R wave and the L wave, and Fig. 1(b) the lower branch of the R wave. It is seen that including the thermal effect increases the phase and group velocities slightly for both the L wave and the upper branch of the R wave, and it affects the R wave a little more than the L wave. For the lower branch of the R wave, on the other hand, damping is significant and becomes severe at longer wavelength compared with the zero temperature case, and the anomalous dispersion seen in the zero temperature cases does not appear in the case of the present parameters. Hence, we may expect that the anomalous dispersion occurs only in the extreme high degeneracy case. In Fig. 2, we have plotted the dispersion relations of the lower R wave with $v_F = 0.1c$ for $v_{th} = 0, 0.01c(v_{th}/v_F = 0.1)$, and $0.02c(v_{th}/v_F = 0.2)$. We see two anomalous dispersion regions appear when $v_{th}/v_F = 0.1$, whereas only the normal dispersion appears when $v_{th}/v_F = 0.2$. For $v_{th}/v_F \geqslant 0.2$, it seems the quantum recoil effect only gives a minor correction to the classical characteristics of the R wave without a qualitative change such as anomalous dispersion. Quantum effects enter the dispersion relations through the velocity distribution (Fermi-Dirac statistics) as well as through the quantum Vlasov equation. As the Fermi-Dirac distribution introduces two anomalous dispersion regions, separated by a normal dispersion between

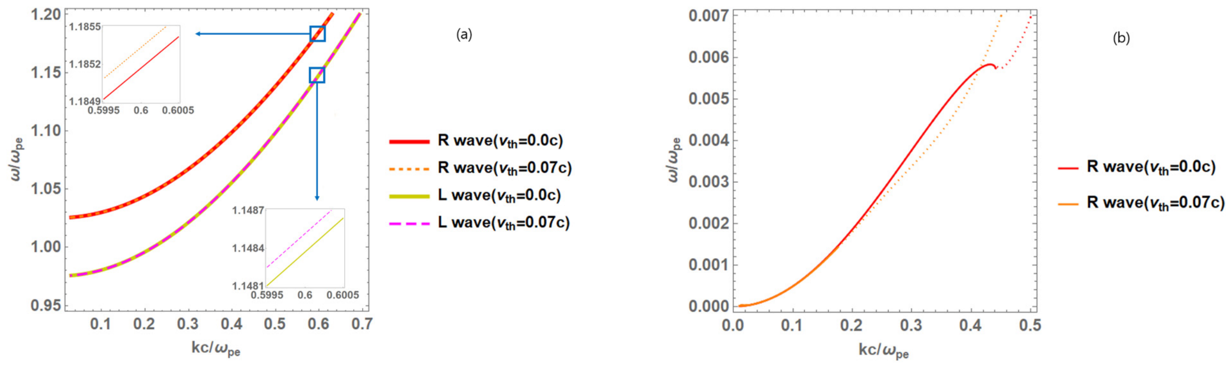

FIG. 1. The dispersion relations for the case of $v_F = 0.1c$, $v_{th} = 0.07c$, and $\omega_{ce}/\omega_{pe} = 0.05$. (a) The upper branch of the R wave and the L wave. (b) The lower branch of the R wave. The dotted line in (b) corresponds to the large values of $\epsilon_i(>5)$, implying severe damping.

them, it should be interesting to examine how the inclusion of the $\hbar$ terms in the Vlasov equation affects the dispersion relation since many previous studies on quantum plasma were based on the classical Vlasov equation, only with the Maxwellian velocity distribution replaced by the Fermi-Dirac distribution. Figure 3 shows the dispersion relations of the lower R wave with zero temperature for the three cases of arbitrary $\hbar$ normalized by the true Planck constant. We see a single region of anomalous dispersion when $\hbar = 0$, whereas a normal dispersion appears at the center and grows as $\hbar$ increases. In other words, the Fermi-Dirac distribution develops one broad kinetic effect and divides into two by forming a normal dispersion region between the two resonance points of $\omega = \omega_{ce} - kv_F \pm \hbar k^2/2m_e$.

V. SUMMARY

In the present paper, we extended our previous study on the parallel propagating electromagnetic waves of a zero temperature quantum electron plasma to the cas of finite temperature. The following are the main results.

(1) We obtained simple analytic dispersion relations for the R and L waves as well as an electrostatic wave in the long wavelength limit. The quantum kinetic effects appeared as a correction proportional to $(\hbar k^2/2m_e)^2$ in all of the electromagnetic and electrostatic dielectric functions.

(2) Further, in the extreme degenerate case, the quantum mechanical correction due to the Fermi-Dirac distribution was seen to be proportional to v_F^2, and its thermal correction was such that its coefficient is modified to include a small term proportional to $(v_{th}/v_F)^4$.

(3) We obtained the dispersion relations numerically for a set of exemplary parameters of $v_F = 0.1$, $v_{th} = 0.07c$, and $\omega_{ce}/\omega_{pe} = 0.05$. Whereas the results of the upper branch of the R wave and the L wave were similar to those of the classical dispersion relations, with only minor corrections, the lower branch of the R wave showed significant damping at short wavelengths and became ill-defined, as in the zero temperature case. Anomalous dispersion was not seen in this rather high temperature case.

(4) Quantum effects seemed to be significant only for $v_{th}/v_F \leqslant 0.2$ for the set of exemplary parameters of $v_F = 0.1c$ and $\omega_{ce}/\omega_{pe} = 0.05$, with qualitative changes in the

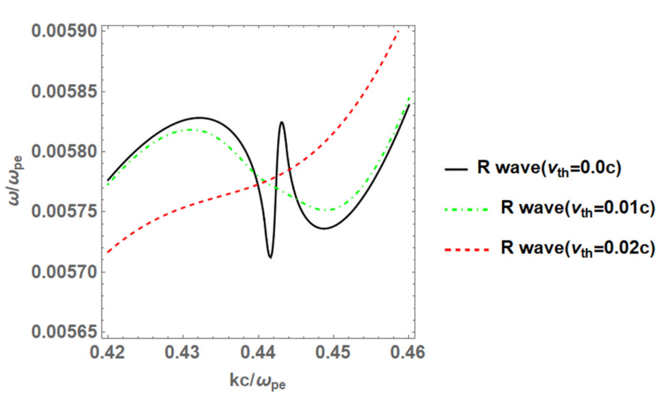

FIG. 2. Temperature effect on the dispersion relation for the lower R wave with $v_F = 0.1c$ and $\omega_{ce}/\omega_{pe} = 0.05$. No anomalous dispersion is seen when $v_{th} = 0.02c$ (red, dashed) whereas the cases of $v_{th} = 0$ (black, solid) and $v_{th} = 0.01c$ (green, dot-dashed) exhibit anomalous dispersion.

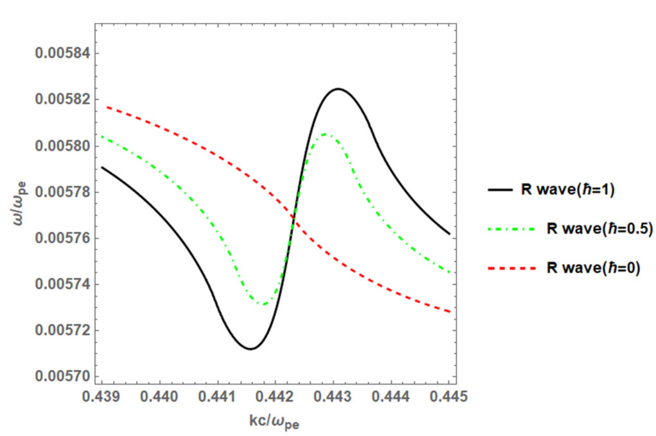

FIG. 3. Change in the anomalous dispersion region according to the values of $\hbar$ for $v_F = 0.1c$, $v_{th} = 0$, and $\omega_{ce}/\omega_{pe} = 0.05$. The $\hbar$ values in the figure are normalized by the true Planck constant.

characteristics of the dispersion relations. With the classical Vlasov equation, a broad single anomalous dispersion region developed at low temperature due to the Fermi-Dirac distribution, whereas it was bisected into two anomalous dispersion regions, separated by the region of normal dispersion when the quantum Vlasov equation was used instead.

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