Mitigation of multispecies Weibel instability in a finite non-neutral magnetized beam-plasma system

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Compressing and charge neutralization of the heavy-ion beam in an inertial fusion reactor are considered as pivotal processes for increasing the energy gain. For this purpose, the ion beam is usually transported through the plasma channel so that multispecies Weibel instability can grow in this system. Recently, a small solenoidal magnetic field has been added to this method to have additional control of these processes. On the other hand, charge and current may not be completely neutralized in this transport; as a result, the fractional charge and current neutralization can affect the growth rate of this instability. In this work, the dispersion equation has been obtained in cylindrical, cold, magnetized, non-neutral plasma in the macroscopic fluid frame. Numerical results show that if the electron cyclotron frequency of the background plasma normalized by plasma frequency is larger than the ion beam velocity normalized by the speed of light, as well as the current fraction being smaller than 1/2, the eigenmodes of multispecies Weibel instability can be completely stabilized. Moreover, these results are valid when the percent deviation from the charge-neutral state is positive. In the negative regime of percent deviation, the instability increases drastically so that the system can be completely unstable only for more than 2%. Therefore, selecting the radius of the ion beam smaller than the electron skin depth of the plasma and the beam pulse duration much longer than the plasma oscillation time are proposed for quenching of this instability.

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I. INTRODUCTION

Neutralization of the ion beam charge and current states by the background plasma is an important issue in broad research areas which involves the transport of fast particles in plasma, especially in astrophysics [1–4], accelerators [4,5], and inertial confinement fusion [6–9]. The successful transport and focusing of the relativistic ion beam in the inertial fusion reactor core on the submillimeter-sized target is a challenging task. It is usually difficult to reduce the focal spot of the incident ion such as lithium, potassium, or cesium from several centimeters to a few millimeters [10,11]. The latest proposed technique to tackle this problem has been applied in the Neutralized Drift Compression eXperiment (NDCX) project at the collaborative program of LBNL, LLNL, and PPPL. In this project, an ion beam with an initial particle density of $10^{10} - 10^{14}\,\mathrm{cm}^{-3}$ was injected into a cylindrical waveguide that previously was filled by a cold, collisionless plasma having the temperature 1–10 eV [12,13]. In this process, the density and the temperature of the beam reach 100 times the initial value, while the radius of the beam has decreased from a few centimeters to several millimeters [14]. Recently, the application of a solenoidal magnetic field in a more advanced project, NDCX-II, provides additional control on the focusing of the ion beam. Here the magnetized plasma responses to the injection of the intense ion beam excite the micro-instabilities such as multispecies two-stream and Weibel instabilities [15–18]. Moreover, the Whistler instability and helicon waves may grow in this medium, provided that $\omega_{\mathrm{ce}}/\omega_{\mathrm{e}} \geqslant 2\,\beta_{\mathrm{b}}$, where ω_{ce}, ω_{e}, and β_{b} are the cyclotron frequency, plasma electron frequency, and ion beam velocity to the speed of light, respectively [19].

This research has been focused on the weak magnetic field ($\omega_{\mathrm{ce}}/\omega_{\mathrm{e}} \leqslant 2\,\beta_{\mathrm{b}}$); consequently, there are no Whistler and helicon waves in this medium. However, there still exists concern about the linear growth rate of the multispecies two-stream and Weibel instabilities in the non-neutral magnetized plasma. In most cases, the two-stream instability exhibits a much faster growth rate than the Weibel instability; however, it finally reduces the beam quality by pinching [20]. An overview of multispecies Weibel instability in the absence of the external magnetic field may be found in Ref. [15]. There, by using the fluid model, the growth rate of this unstable mode in the finite neutral state was discussed, and for the nonconstructive situation of the system, the optimum lengths of the plasma channel were also derived. Moreover, other studies on the magnetized infinite plasma in the neutral state, especially Ref. [18] on the NDCX project, have confirmed that the axial external magnetic field can moderate the growth rate of the Weibel instability [21,22].

There is some evidence that charge and current states in the beam-plasma system may deviate from the neutral state. The electron response time to the injection of the ion beam is estimated with the electron plasma frequency ($T_{\mathrm{response}} = 2\pi/\omega_{\mathrm{pe}}$) [23]. So, if the pulse duration of the incident ion beam is smaller than the response time, the ion beam can experience the charge and current non-neutrality during the compression stage. The application of a small magnetic field of ~100 G may destabilize the beam neutralization. Moreover, it has been pointed out that the axial magnetic field satisfying the condition of $\omega_{\mathrm{ce}}/\omega_{\mathrm{e}} \geqslant \beta_{\mathrm{b}}$ strongly affects the degrees of charge and current neutralization [24,25]. In a non-magnetized plasma, for a thin beam ($Z_{\mathrm{b}} n_{\mathrm{b}} \leqslant n_{\mathrm{p}}$), the plasma

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is assumed to be charged quasineutralized, where n_b, Z_b, and n_p are density and charge of the beam and density of the plasma, respectively. Consequently, the beam-plasma system experiences a slight deviation from the fully charge-neutral state. Moreover, if the beam radius is larger than the electron skin depth of the plasma ($r_b \geqslant c/\omega_e$), the neutralized current state may be established. However, it has been proved that the current state must be non-neutral in the opposite region ($r_b \leqslant c/\omega_e$) [26,27]. In previous studies, it was shown that small deviations in charge and current neutralization may affect the growth rate. However, the arbitrary level of the non-neutral beam-plasma system immersed in the external magnetic field has not been addressed analytically so far.

To get a deeper insight into the stable transport in the real beam-plasma system, it is necessary to generalize our previous formulation to include the condition of the charge and current non-neutrality. So any modification relative to the neutral state may shift the key physical parameters, and thus it proves the sensitivity of the Weibel instability mode to non-neutrality conditions. Because of the higher inertia, the rotation of ion species around the magnetic field may be ignored. Moreover, the induced axial magnetic field by the electron return current is small compared to the external magnetic field [28].

Here we have considered a mono-energetic cesium beam without initial rotation, which carries 1–50 GeV energy in an argon plasma. Then the growth rate of the multispecies Weibel instability for the non-neutral finite ion beam-plasma system at the different magnetic fields strength has been examined. Finally, using the optimum parameters of the system, forbidden parameters to mitigate this instability are determined. For this purpose, in Sec. II the beam-plasma dispersion equation is extracted in the fluid model using the laws of mass and momentum conservations. In Sec. III we first solve numerically this equation and then evaluate the effects of non-neutrality and solenoid magnetic field strength on the growth rate of multispecies Weibel instability. Finally, we will discuss and compare them with the relevant studies.

II. MACROSCOPIC FLUID MODEL AND EIGENVALUE EQUATION

Non-neutrality of the plasma and application of the external magnetic field can lead to the rotation of the beam-plasma components. Moreover, plasma particles can rotate in a non-neutral plasma even in the absence of the external magnetic field [29]. Hence, taking particle rotation into account is inevitable in non-neutral plasma. Fortunately, one can neglect the rotational effects of the ions due to their high inertia, while rotation of the electrons can have significant effects on the growth of the multispecies Weibel instability. For adding the rotation of the beam-plasma components to the dispersion equation, first, the equilibrium force balance equation of the particles is written, and then the eigenvalue equation of the system is derived.

A. Equilibrium force balance equation

In this section, before proceeding to the eigenvalue equation, the equilibrium force balance equation is investigated for a heavy ion beam without initial rotation that is injected into a cold, multicomponent, non-neutral plasma column aligned parallel to a uniform applied magnetic field $B_0\,\mathbf{e}_z$ in Fig. 1. This investigation has been done in steady state ($\partial/\partial t = 0$) and cylindrical symmetry ($\partial/\partial\theta = 0$). The results of this section are useful for the macroscopic fluid description of the magnetized beam-plasma system.

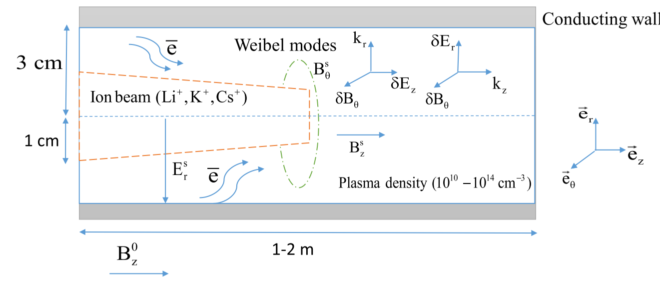

FIG. 1. Schematic of a cylindrical non-neutral plasma column confined radially by an applied magnetic field $B_0\,\mathbf{e}_z$. The lack of equilibrium charge neutrality produces a radial self-electric field $E_r(r)\mathbf{e}_r$, and the plasma current in the axial direction produces an azimuthal self-magnetic field $B^s{}_\theta\,\mathbf{e}_\theta$. The azimuthal current associated with the equilibrium rotation of the plasma components generally produces a diamagnetic contribution $B^s{}_z(r)\mathbf{e}_z$ to the total axial magnetic field $B(r) = B_0 + B^s{}_z(r)$ [15,24].

As shown in Fig. 1, as soon as the ion beam enters the plasma, a radial electric field and an azimuthal magnetic field are induced, which attempt to decay and compress the ion beam, respectively. However, the radial force is always focusing, because the electron flow velocity in the return current is always smaller than the beam velocity, $\beta_e \leqslant \beta_b$ [30]. Moreover, the solenoidal magnetic field can change the radial force from focusing to defocusing in $\omega_{ce}/\omega_e \geqslant \beta_b$ because the radial electric field grows faster than the azimuthal magnetic field [23]. Nevertheless, it has been assumed that the radial force balance would happen for a component of the plasma in this work. This balance on the jth component fluid element in the weak region of the solenoid magnetic field is given by

$$\frac{\gamma_j m_j\, v_{\theta j}^2}{r}\,\mathbf{e_r} = q_j\big\{E_r^s(r)\,\mathbf{e_r} + \beta_{\theta j}\,\big[B_0 + B_z^s(r)\big]\mathbf{e_r} - \beta_{zj}\,B_\theta^s(r)\,\mathbf{e_r}\big\}. \tag{1}$$

In the above relation, q_j is the electric charge of the particle, m_j is mass of the particle, $B^s{}_z(r)$ is the axial induced magnetic field, and $v_{\theta j}$ is the azimuthal velocity of the jth particle. Moreover, $\beta_{\theta j}$ and β_{zj} are the azimuthal and axial velocities for jth particle normalized by the speed of light, respectively, and $\mathbf{e}_r$, $\mathbf{e}_\theta$, and $\mathbf{e}_z$ are the radial, azimuthal, and axial unit vectors, respectively. This induced electromagnetic field can be obtained from Maxwell's equations:

$$\nabla \times \mathbf{B}^s(r) = \sum_{j=i,b,e} 4\pi q_j n_j(r)(\beta_{zj}\,\mathbf{e_z} + \beta_{\theta j}\,\mathbf{e_\theta}), \tag{2}$$

$$\nabla \cdot \mathbf{E}^s(r) = \sum_{j=i,b,e} 4\pi q_j n_j(r). \tag{3}$$

In the above equations, $n_j(r)$ is the density of the jth particle in the plasma. For simplicity, the density is assumed constant for every component. The axial velocity of the particles is allowed to be relativistic, while the azimuthal velocity of particles has been considered nonrelativistic $[(v_{\theta j}/c)^2 \ll 1]$. Therefore, the axial induced magnetic field can be neglected $|B^s{}_z| \ll |B_0|$. The above assumptions and induced electromagnetic fields from Eqs. (2) and (3) simplify Eq. (1) as follows:

$$\omega_{rj}^2 + \varepsilon_j\, \omega_{cj}\, \omega_{rj} + \sum_{k=i,b,e} \frac{2\pi\, q_j q_k n_k}{\gamma_j\, m_j}(1 - \beta_j \beta_k) = 0,$$

$$r\, \omega_{rj} = v_{\theta j}(r), \quad \omega_{cj} = \frac{q_j\, B_0}{\gamma_j\, m_j\, c},$$

$$\gamma_j = \left(1 - \beta_{zj}^2 - \beta_{\theta j}^2\right)^{-1/2} \beta_{zj}^2 \gg \beta_{\theta j}^2. \tag{4}$$

Here ω_{rj} is the angular velocity of particles at r, ω_{cj} is the cyclotron frequency, γ_j is the Lorentz factor, and ε_j denotes the sign of q_j. Equation (4) plays an important role in simplifying the eigenvalue equation.

B. Fluid model

In the following analysis of the macroscopic fluid model, using the linear continuity equation and the momentum conservation equations in three dimensions in the cylindrical coordinates, the eigenvalue equation of the system is begun:

$$\frac{\partial}{\partial t} n_j(\mathbf{x},t) + \nabla \cdot [n_j(\mathbf{x},t)\mathbf{v}_j(\mathbf{x},t)] = 0, \tag{5}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v_j}(\mathbf{x},t) \cdot \nabla\right) \mathbf{p_j}(\mathbf{x},t) = q_j\left[\mathbf{E}(\mathbf{x},t) + \frac{\mathbf{v_j}(\mathbf{x},t) \times \mathbf{B}(\mathbf{x},t)}{c}\right]. \tag{6}$$

In these equations, system quantities depend only on the radial coordinate. The equations can be written for the ion beams and background plasmas commonly used in NDCX. However, throughout the paper, the cesium ion beam $^{133}Cs^+$ and argon plasma $^{40}Ar^+$ have been applied. Since the ion beam energy is in the range of 1–50 GeV, the equations are written in the relativistic form.

Although the density and temperature of the ion beam increase after compression in the channel, equations have been obtained for the equilibrium beam radius. Moreover, density and temperature have been assumed radially uniform and cold; as a result, the electron pressure tensor force on the beam body can be neglected: $\nabla P(r) = [-\nabla n(r) kT_\perp = 0]$.

Polarization of electric and magnetic perturbation waves forms a multispecies Weibel instability; therefore, if the electric and magnetic field perturbation is given as $\delta E(r,t) = \delta E_r(r)e^{-i\omega t}\mathbf{e}_r + \delta E_z(r)e^{-i\omega t}\mathbf{e}_z$ and $\delta B(r,t) = \delta B_\theta(r)e^{-i\omega t}\mathbf{e}_\theta$, axial and radial Weibel modes can be propagated in the medium. As shown in Fig. 1, the polarization of azimuthal magnetic field perturbation with the radial electric field perturbation forms the axial propagation Weibel modes. Moreover, the polarization of azimuthal magnetic field perturbation with the axial electric field perturbation generates the radial propagation of this instability. Note that polarization of the radial electric field perturbation waves with axial magnetic perturbation waves or polarization of the radial magnetic field perturbation waves with axial electric field perturbation waves forms the azimuthal component of this instability. Fortunately, the radial and axial magnetic field perturbations are much lower than the azimuthal magnetic field due to the nonrelativistic azimuthal velocity of the particles. Therefore, this component of the instability is smaller than others, and the system can be assumed azimuthally symmetric as stated at the beginning of the Sec. II A. Also, the density and momentum perturbations of a fluid element of species j are considered as $\delta n_j(r,t) = \delta n_j(r)e^{-i\omega t}$ and $\delta \mathbf{p}_j(r,t) = \delta \mathbf{p}_j(r)e^{-i\omega t}$, respectively. Due to these conditions and the use of Eqs. (5) and (6), the linearized fluid model equations are written as follows:

$$i\omega\, \delta n_j(r) = \frac{1}{r}\frac{\partial}{\partial r} r[n_j\, \delta v_j(r)], \tag{7}$$

$$\begin{aligned} &- i\omega\, \delta p_{rj} - \omega_{rj}(\delta p_{\theta j} + m_j\, \gamma_j\, \delta v_{\theta j}) - m_j\, \gamma_j\, \delta v_{\theta j}\, \omega_{cj} \\ &= -q_j\left[\frac{v_{0zj}\, \delta B_\theta^s(r)}{c}\right], \end{aligned} \tag{8}$$

$$-i\omega\, \delta p_{\theta j} - \omega_{rj}(\delta p_{rj} + m_j\, \gamma_j\, \delta v_{rj}) - m_j\, \gamma_j\, \delta v_{rj}\, \omega_{cj} = 0, \tag{9}$$

$$-i\omega\, \delta p_{zj} = q_j\, \delta E_z(r). \tag{10}$$

The second step in calculating an eigenvalue equation is the use of the Maxwell equations:

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{11}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}(r,t) + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}. \tag{12}$$

By combining Eqs. (4) and (7)–(12), the system's eigenvalue equation is obtained as follows:

$$\frac{1}{r}\frac{\partial}{\partial r} r\left\{1 + \sum_{j=b,e,i} \frac{\omega_{pj}^2\, \beta_j^2}{\omega^2 - (\omega_{cj} + 2\,\omega_{rj})^2} + \frac{\left[\sum_{j=b,e,i} \frac{\omega_{pj}^2\, \beta_j}{\omega^2 - (\omega_{cj} + 2\,\omega_{rj})^2}\right]^2}{1 - \sum_{j=b,e,i} \frac{\omega_{pj}^2\, \beta_j^2}{\omega^2 - (\omega_{cj} + 2\,\omega_{rj})^2}}\right\}\frac{\partial}{\partial r}\delta E_z + \left\{\frac{\omega^2}{c^2} - \sum_{j=b,e,i} \frac{\omega_{pj}^2}{\gamma_j^2\, c^2}\right\}\delta E_z = 0. \tag{13}$$

In this equation, $\omega_{pj} = (4\pi q^2{}_j n_j/\gamma_j m_j)^{1/2}$ is independent of the radius, leading to a Bessel differential equation. For simplicity, the following variables are defined for solving this equation for inside and outside of the beam. It is important to note that the beam ions and background electrons move axially in the beam-plasma system, but the background electrons and ions are axially motionless in space between the beam and waveguide:

$$T_{\rm i}^2 = \left(\frac{\omega^2}{c^2} - \sum_{j=b,e,i} \frac{\omega_{\rm pj}^2}{\gamma_{\rm j}^2 c^2}\right)\left\{1 + \sum_{j=b,e,i} \frac{\omega_{\rm pj}^2\,\beta_{\rm j}^2}{\omega^2 - (\omega_{\rm cj} + 2\,\omega_{\rm rj})^2} + \frac{\left[\sum_{j=b,e,i} \frac{\omega_{\rm pj}^2\,\beta_{\rm j}}{\omega^2-(\omega_{\rm cj}+2\,\omega_{\rm rj})^2}\right]^2}{1 - \sum_{j=b,e,i} \frac{\omega_{\rm pj}^2\,\beta_{\rm j}^2}{\omega^2-(\omega_{\rm cj}+2\,\omega_{\rm rj})^2}}\right\}^{-1},\quad 0 \leqslant r \leqslant r_{\rm b},$$

$$T_{\rm O}^2 = -\left(\frac{\omega^2}{c^2} - \sum_{j=e,i} \frac{\omega_{\rm pj}^2}{\gamma_{\rm j}^2 c^2}\right),\quad r_{\rm b} \leqslant r \leqslant r_{\rm w}. \tag{14}$$

${T_{\rm i}}^2$ and ${T_{\rm O}}^2$ in Eq. (14) indicate inside and outside of the ion beam, respectively, and $r_{\rm b}$ and $r_{\rm w}$ are the radius of the ion beam and waveguide, respectively. By inserting Eq. (14) in Eq. (13), it can be easily recognized that Eqs. (15) and (16) are Bessel's equations of order zero:

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\delta E_{\rm z}^{\rm I} + T_{\rm i}^2\,\delta E_{\rm z}^{\rm I} = 0,\quad 0 \leqslant r \leqslant r_{\rm b}, \tag{15}$$

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\delta E_{\rm z}^{\rm II} - T_{\rm O}^2\,\delta E_{\rm z}^{\rm II} = 0,\quad r_{\rm b} \leqslant r \leqslant r_{\rm w}. \tag{16}$$

The solutions of the Eqs. (15) and (16) must be regular at $r = 0$, continuous at $r = r_{\rm b}$, and vanish at the conducting wall. These solutions can be obtained as follows:

$$\delta E_{\rm z}^{\rm I}(r) = A\,J_0(T_{\rm i}\,r),\qquad 0 \leqslant r \leqslant r_{\rm b}, \tag{17}$$

$$\delta E_{\rm z}^{\rm II}(r) = A\,J_0(T_{\rm i}\,r_{\rm b})\,\frac{K_0(T_{\rm O}r_{\rm w})I_0(T_{\rm O}r) - K_0(T_{\rm O}r)I_0(T_{\rm O}r_{\rm w})}{K_0(T_{\rm O}r_{\rm w})I_0(T_{\rm O}r_{\rm b}) - K_0(T_{\rm O}r_{\rm b})I_0(T_{\rm O}r_{\rm w})},\qquad r_{\rm b} \leqslant r \leqslant r_{\rm w}, \tag{18}$$

where A is a constant, $J_0(T_{\rm i}r)$ is the ordinary Bessel function of the first kind of order zero, $I_0(T_{\rm O}r)$ is the modified Bessel function of the first kind of order zero, and $K_0(T_{\rm O}r)$ is the modified Bessel function of the second kind of order zero. The only remaining boundary condition for the continuity of the tangential electric field perturbation, $\delta E_{\rm z}(r)$, from Eqs. (17) and (18) in $r = r_{\rm b}$, gives the dispersion relation:

$$\left\{1+\sum_{j=b,e,i}\frac{\omega_{\rm pj}^2\,\beta_{\rm j}^2}{\omega^2-(\omega_{\rm cj}+2\,\omega_{\rm rj})^2} + \frac{\left[\sum_{j=b,e,i}\frac{\omega_{\rm pj}^2\,\beta_{\rm j}}{\omega^2-(\omega_{\rm cj}+2\,\omega_{\rm rj})^2}\right]^2}{1-\sum_{j=b,e,i}\frac{\omega_{\rm pj}^2\,\beta_{\rm j}^2}{\omega^2-(\omega_{\rm cj}+2\,\omega_{\rm rj})^2}}\right\}(-J_1(T_{\rm i}\,r_{\rm b}))T_{\rm i} = J_0(T_{\rm i}\,r_{\rm b})T_{\rm O}\left[\frac{K_0(T_{\rm O}r_{\rm w})I_1(T_{\rm O}r) + K_1(T_{\rm O}r)I_0(T_{\rm O}r_{\rm w})}{K_0(T_{\rm O}r_{\rm w})I_0(T_{\rm O}r_{\rm b}) - K_0(T_{\rm O}r_{\rm b})I_0(T_{\rm O}r_{\rm w})}\right], \tag{19}$$

where $J_1(T_{\rm i}r)$ is the Bessel function of the first kind of the first order, and $I_1(T_{\rm O}r)$ and $K_1(T_{\rm O}r)$ are modified Bessel functions of the first and second kind of the first order, respectively. Equation (19) constitutes a closed transcendental dispersion relation that determines the complex oscillation frequency for the electromagnetic perturbations in the magnetized plasma.

For the case where the beam-plasma system extends to the conducting wall ($r_{\rm b} = r_{\rm w}$), the solution of Eq. (19) is simplified to $J(T_{\rm i}r_{\rm b}) = 0$. In this relation, $p_{\rm n0}$ is the nth zero of $J(p_{\rm n0}) = 0$ and $n = 1, 2, 3, \ldots$. Therefore, this transcendental Eq. (19) is simplified to this polynomial ($T_{\rm i}^2\,{\rm r}_{\rm b}^2 = p_{\rm n0}^2$) :

$$\left(\frac{\omega^2}{c^2} - \sum_{j=b,e,i} \frac{\omega_{\rm pj}^2}{\gamma_{\rm j}^2 c^2}\right)\left\{1 + \sum_{j=b,e,i} \frac{\omega_{\rm pj}^2\,\beta_{\rm j}^2}{\omega^2 - (\omega_{\rm cj} + 2\,\omega_{\rm rj})^2} + \frac{\left[\sum_{j=b,e,i} \frac{\omega_{\rm pj}^2\,\beta_{\rm j}}{\omega^2-(\omega_{\rm cj}+2\,\omega_{\rm rj})^2}\right]^2}{1 - \sum_{j=b,e,i} \frac{\omega_{\rm pj}^2\,\beta_{\rm j}^2}{\omega^2-(\omega_{\rm cj}+2\,\omega_{\rm rj})^2}}\right\}^{-1} = \frac{p_{\rm n0}^2}{r_{\rm b}^2}. \tag{20}$$

The rotation term, $(\omega_{\rm cj} + 2\omega_{\rm rj})^2$, has been added to Eq. (20) due to the external magnetic field and non-neutrality charge and current in this system. If beam-plasma is assumed neutralized and not magnetized, Eq. (20) is simplified to Eq. (29) in Ref. [15]. $\omega_{\rm rj}$ can be eliminated from Eq. (20) using Eq. (4):

$$\omega_{\rm rj}^{\pm} = -\frac{\varepsilon_{\rm j}\omega_{\rm ci}}{2}\left\{1 \pm \left[1 - \sum_{k=e,b,i} \frac{8\pi\,q_{\rm j}q_{\rm k}n_{\rm k}}{\gamma_{\rm j}m_{\rm j}\,\omega_{\rm cj}^2}(1 - \beta_{\rm j}\beta_{\rm k})\right]^{\frac{1}{2}}\right\}. \tag{21}$$

Variables of this equation have been already defined in Eq. (4). Equation (21) can be written as follows:

$$(2\omega_{\rm rj} + \omega_{\rm cj})^2 = (\omega_{\rm rj}^{+} - \omega_{\rm rj}^{-})^2 = \left[\omega_{\rm cj}^2 - \sum_{k=i,e,b} \frac{8\pi\,q_{\rm j}q_{\rm k}n_{\rm k}}{\gamma_{\rm j}\,m_{\rm j}}\big(1 - \beta_{\rm j}\beta_{\rm k}\big)\right]. \tag{22}$$

For the non-neutral beam-plasma system, fractional charge and current neutralization can be defined as in Eq. (23), where ions are axially motionless in fractional current neutralization:

$$f=\frac{n_e-n_i}{n_b},\quad f_m=\frac{n_e\beta_e}{n_b\,\beta_b},\quad \alpha=\frac{n_b}{n_i}. \tag{23}$$

In Eq. (23), f, f_m, and α are the fractional charge neutralization, fractional current neutralization, and beam-to-plasma density ratio, respectively. Note that $f=1$ corresponds to $\sum n_k q_k=0$ and $E^s{}_r(r)=0$, whereas $f_m=1$ corresponds to $\sum n_k\beta_k q_k=0$ and $B^s{}_\theta(r)=0$. An interesting case for researchers is the neutral state. For example, in this research, the neutral state leads to two angular frequency modes for ion beam, which are $\omega_{rb}^-=-\omega_{cb}$, $\omega_{rb}^+=0$. For uniform density, inserting Eq. (23) into Eq. (22) readily gives Eqs. (24)–(26) for the beam ions and the background plasma electrons, and the ions as follows:

$$(2\omega_{rb}+\omega_{cb})^2=\left\{\omega_{cb}^2-2\omega_{pb}^2\left[(1-f)-\beta_b^2(1-f_m)\right]\right\}, \tag{24}$$

$$(2\omega_{re}+\omega_{ce})^2=\left(\omega_{ce}^2-2\omega_{pe}^2\left\{\frac{\alpha(f-1)}{(f\,\alpha+1)}-\left[\frac{\alpha\beta_b\,f_m}{(f\,\alpha+1)}\right]\left(\frac{f_m-1}{f_m}\right)\right\}\right), \tag{25}$$

$$(2\omega_{ri}+\omega_{ci})^2=\left[\omega_{ci}^2-2\omega_{pi}^2\alpha(1-f)\right]. \tag{26}$$

The term proportional to $(1-f)$ in Eq. (24) is associated with electric self-field effects, and the ion beam is defocusing whenever $f<1$. On the other hand, the term proportional to $-\beta_b{}^2(1-f_m)$ is associated with magnetic self-field effects, and the ion beam is focusing whenever $f_m<1$. The net self-field contribution to the beam rotation is focusing, provided that $\beta_b{}^2(1-f_m)>(1-f)$. Whenever this condition is satisfied, the rotation of the ion beam exists even in the absence of an axial magnetic field [29]. This condition is satisfied in this paper.

The dimensionless variables for solution Eq. (20) are defined as follows. In this equation, A_b and A_i denote the atomic mass number of beam and background ions, respectively:

$$x=\frac{\omega}{\omega_{pe}},\ Z=\frac{r_b}{c/\omega_{pe}},\ \Omega=\frac{\omega_{ce}}{\omega_{pe}},\ \alpha=\frac{n_b}{n_i},\ R=\frac{m_e}{m_H}=\frac{1}{1836},\ A_b=\frac{m_b}{m_H},\ A_i=\frac{m_i}{m_H},\ \gamma_j=\frac{1}{\sqrt{1-\beta_j^2}},\ \beta_j=\frac{v_j}{c}\quad j=e,b, \tag{27}$$

$$A=\frac{\alpha^2\beta_b f_m^2}{(1+\alpha f)^2\left[x^2-\left(\Omega^2-2\left\{\frac{\alpha(f-1)}{(f\alpha+1)}-\left[\frac{\alpha\beta_b f_m}{(f\alpha+1)}\right]^2\left(\frac{f_m-1}{f_m}\right)\right\}\right)\right]},$$

$$B=\frac{\left(\frac{\gamma_e}{\gamma_b}\right)R\,\alpha\,\beta_b}{A_b(1+\alpha f)\left\{x^2-\left(\frac{R^2}{A_b^2}\left(\frac{\gamma_e}{\gamma_b}\right)^2\Omega^2-\frac{2R\alpha}{A_b(1+\alpha f)}\left(\frac{\gamma_e}{\gamma_b}\right)\left[(1-f)-\beta_b^2(1-f_m)\right]\right)\right\}},$$

$$C=\frac{\gamma_e R\,\alpha}{A_i(1+\alpha f)\left[x^2-\left(\frac{R^2}{A_i^2}\gamma_e{}^2\,\Omega^2-\frac{2R\alpha(1-f)}{A_i(1+\alpha f)}\gamma_e\right)\right]},$$

$$1+\beta_b(A+B)+\frac{(A+B)^2}{(1-A-B-C)}+\frac{Z^2}{\gamma_e{}^2\,p_{n0}^2}+\frac{R\,Z^2}{A_i\,(1+\alpha f)\,p_{n0}^2}+\frac{R\,\alpha\,Z^2}{\gamma_b{}^2\,(1+\alpha f)A_b\;p_{n0}^2}-\frac{x^2Z^2}{p_{n0}^2}=0. \tag{28}$$

In the neutral state in the absence of the external magnetic field, the growth rate of small mode numbers in a small radius of the beam ($r_b<r_w$) is somewhat larger than the filled waveguide ($r_b=r_w$) [15]. The results of this study in the magnetized plasma could confirm it in the next section; therefore, Eq. (28) could be trusted as a simple equation for this system.

For very large mode number ($n\to+\infty$, $p_{n,0}\to+\infty$) or a very small wavelength ($\lambda\to 0$) of instability, in the nonmagnetized ($\Omega=0$) neutral plasma ($f=f_m=1$), and nonrelativistic regime ($\gamma_b\to 1$), Eq. (28) can be simplified as follows:

$$1+\frac{\alpha^2\beta^2}{(1+\alpha)^2x^2}+\frac{R\,\alpha\,\beta^2}{A_b(1+\alpha)x^2}+\frac{\left[\frac{R\,\alpha\,\beta}{A_b(1+\alpha)(x^2)}+\frac{\alpha\,\beta}{(1+\alpha)\,x^2}\right]^2}{\left[1-\frac{R\,\alpha}{A_b(1+\alpha)x^2}-\frac{1}{x^2}-\frac{R\,\alpha}{A_i(1+\alpha)x^2}\right]}=0. \tag{29}$$

Since x is considered very small with attention to the numerical results in the next section ($|x|\sim 10^{-4}$), the growth rate of multispecies Weibel instability can be obtained from Eq. (29):

$$x=\frac{\mathrm{Im}(\omega)}{\omega_e}\simeq\sqrt{\frac{\alpha^2\beta^2}{(1+\alpha)^2}+\frac{R\,\alpha\,\beta^2}{A_b(1+\alpha)}-\frac{\left[\frac{R\,\alpha\,\beta}{A_b(1+\alpha)}+\frac{\alpha\,\beta}{(1+\alpha)}\right]^2}{\left[\frac{R\,\alpha}{A_b(1+\alpha)}+1+\frac{R\,\alpha}{A_i(1+\alpha)}\right]}}. \tag{30}$$

In Eq. (30), the variables are the same as defined in Eq. (27). For instance, for the cesium and potassium beams, with $\alpha = \beta = 0.2$ in the argon background, the growth rate of instability may be obtained: ${x_{cs}}^+ = 1.46 \times 10^{-4}$ and ${x_k}^+ = 2.59 \times 10^{-4}$. Although Eq. (30) cannot show non-neutrality and axial magnetic field effects, it is a useful approximation for calculating the growth rate of multispecies Weibel instability because it can be used as the initial guess for the numerical solution of Eq. (19).

III. NUMERICAL RESULTS AND DISCUSSION

As mentioned earlier, for optimum compression and ignition of the fusion targets, the ion beam not only must be neutralized but also must be compressed to the submillimeter radius. For these purposes, the heavy ion is transported through the plasma column. Recently the weak solenoid magnetic field $(\omega_{ce}/\omega_e \leqslant \beta_b)$ has been used for the improvement of it [23]. Unfortunately, multispecies Weibel instability may happen when the ion beam is transported through the magnetized plasma. Charge and current in this system are usually assumed to be neutral, but this neutralization is almost impossible due to the mentioned reasons in Sec. I.

For considering the non-neutrality effect on multispecies-Weibel instability in magnetized plasma, the cesium beam with 1–50 GeV energy without initial rotation has been injected into the plasma. By combining the linear continuity equation and the momentum conservation equations in three dimensions in the cylindrical coordinates, the dispersion equation of this system was obtained in Eq. (19). This full dispersion equation was simplified to a polynomial in Eq. (28) in the beam-plasma-filled waveguide state. Numerical solution of Eqs. (19) and (28) can clarify the effects of the non-neutral charge and current and axial magnetic field on the growth rate of the instability. Note that only slow-wave solutions $[\mathrm{Im}(\omega) > 0]$ that reflect the propagation of the Weibel-like modes are considered.

Although it is almost impossible to have charge neutrality, it is commonly found in more investigations. If the injected ion beam pulse duration (τ_b) is much smaller than the background electron oscillation time $(\tau_b \gg 2\pi/\omega_e)$, charge neutrality of the system is usually assumed more than 99%. On the other hand, charge neutrality is very sensitive to the external magnetic field strength. However, it has been proved that the effect of the magnetic field on charge neutrality can be neglected in the weak magnetic field region $(\omega_{ce}/\omega_e \leqslant \beta_b)$ [27]. Moreover, a small degree of charge non-neutrality may drastically increase the growth rate of this instability.

In Fig. 2 using the above assumptions would give charge non-neutrality of the system, but it is noted that the current is non-neutral because the beam radius is smaller than the electron skin depth of the plasma $(Z = 1/2)$. However, the growth rate has been obtained in the fully neutral system $(f = f_m = 1)$ for the comparison.

Figures 2 and 3 illustrate the growth rate of the multispecies Weibel instability for the cesium ion beam with energy of 2.6 GeV $(\beta = 0.205)$ in a relative density $\alpha = 0.2$ in the beam-plasma-filled waveguide state $(r_b = r_w)$. This assessment has been done for the charge-neutral state in Fig. 2 and the current-neutral state in Fig. 3. The effects of the external magnetic field and the fractional current neutralization have been shown in Fig. 2 in four states. In case (1), the system is neutral in current and nonmagnetized $(f_m = 1,\ \Omega = 0)$. In this state, the growth rate is more than in the others. In case (2) $(f_m = 1,\ \Omega = 0.2)$, the external magnetic field has been added to the system. Case (3) $(f_m = 0.5,\ \Omega = 0)$ shows that the fractional current neutralization reduces instability in nonmagnetized plasma more than does case (2). In case (4) the hybrid effect of the external magnetic field and the current fraction $(f_m = 0.5,\ \Omega = 0.2)$ reduce instability severely. Hence the two first eigenmodes have been completely stabilized $[\mathrm{Im}(\omega) = 0]$, and instability has decreased almost 13 times relative to case (1) for other modes $(1.78 \times 10^{-4}/1.33 \times 10^{-5} \sim 13.38)$. It should be emphasized that all

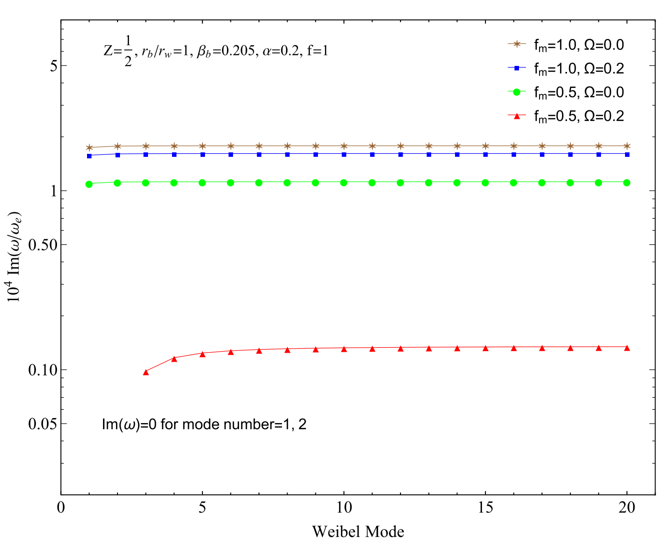

FIG. 2. Variation of the growth rate of multispecies Weibel instability for a $^{133}Cs^+$ beam in $^{40}Ar^+$ plasma for different axial magnetic field strength (Ω) and the fractional current neutralization (f_m).

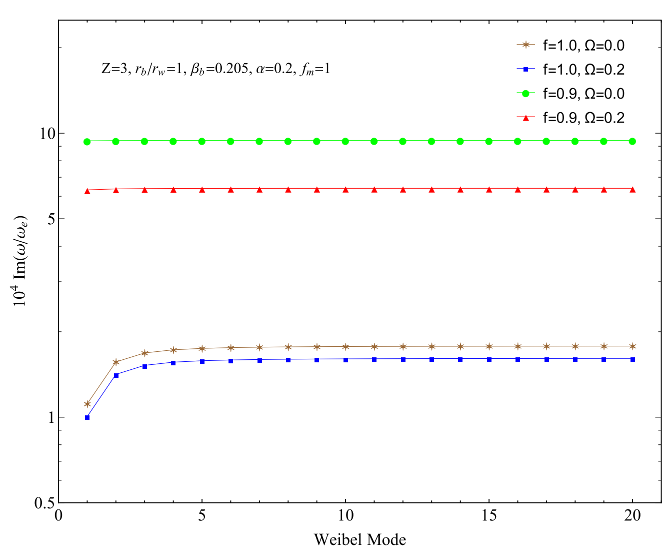

FIG. 3. Variation of the growth rate of multispecies Weibel instability for a $^{133}Cs^+$ beam in $^{40}Ar^+$ plasma for different axial magnetic field strength (Ω) and the fractional charge neutralization (f).

eigenmodes are completely stabilized due to a slight reduction in the current fraction relative to case (4). This point has been checked for $f_{\mathrm{m}} = 0.49$.

In Fig. 3 four different states have been analyzed. The effects of the fractional charge neutralization on the instability in the magnetized plasma also have been tested. Since the current in the beam-plasma system is neutral ($f_{\mathrm{m}} = 1$), it has been assumed that the beam radius is larger than the electron skin depth of the plasma ($Z = 3$), unlike Fig. 2. In case (1), there is no external magnetic field ($\Omega = 0$), and plasma is fully neutral ($f_{\mathrm{m}} = f = 1$). This case is similar to case (1) in Fig. 2. In case (2), the existence of the axial magnetic field reduces the instability from 1.77×10^{-4} to 1.6×10^{-4}. In case (3), the system has deviated 10% from the full neutral state ($f = 0.9$) in nonmagnetized plasma ($\Omega = 0$); as a result, the growth rate of instability increases almost 10 times relative to previous cases. If the axial magnetic field ($\Omega = 0.2$) is added to the background plasma, case (3) is converted to case (4). Due to this change, the growth rate of the instability decreases from 9.43×10^{-4} to 6.39×10^{-4}. Furthermore, the ion beam can be defocused in this state because of $(1 - f) \geqslant \beta^2(1 - f_{\mathrm{m}})$. Finally, note that Figs. 2 and 3 show localized filaments that pinch the ion beam if they are thinner than the skin depth of plasma. The physics of large Weibel modes is interesting and can be elucidated with attention to filament size. As the instability grows, the filaments with a typical length scale $\sigma \sim r_{\mathrm{b}}/p_{\mathrm{n},0}$ carry a current $I \sim q n_{\mathrm{b}} \beta_{\mathrm{b}} c \sigma^2$. This current produces a magnetic field which pinches the ion beam. In the cold theory or this work, the pinching force has a maximum value that is approximately constant in large Weibel modes; consequently, the growth rate of instability in high mode numbers or small filaments can be saturated [31]. In the absence of the temperature, pinching continues for any width of filaments. However, if the thermal effect is added to this system, filaments are expanded; as a result, the pinching force must exceed the pressure force for the instability to grow.

Figure 4 shows the eigenfunction of $\delta E_z(r)$ that has been normalized by $\delta E_z(0)$ for some eigenmodes. Note that the growth rates, obtained by solving Eq. (19), have been substituted into Eqs. (17) and (18) when the ion beam has a radius smaller than the waveguide radius. For beam-plasma-filled waveguide, the instability wave is cut off at the waveguide wall. This case can be seen in $n = 2$, $r_{\mathrm{b}}/r_{\mathrm{w}} = 1$. For $n = 4$ and $Z = 3$, if the beam radius is larger than the skin depth of the plasma, the eigenfunction descends according to the geometric factor between the beam and waveguide space. This factor can be defined from Eq. (18) as follows:

$$g_1 = \frac{K_0(T_{\mathrm{O}} r_{\mathrm{w}}) I_0(T_{\mathrm{O}} r) - K_0(T_{\mathrm{O}} r) I_0(T_{\mathrm{O}} r_{\mathrm{w}})}{K_0(T_{\mathrm{O}} r_{\mathrm{w}}) I_0(T_{\mathrm{O}} r_{\mathrm{b}}) - K_0(T_{\mathrm{O}} r_{\mathrm{b}}) I_0(T_{\mathrm{O}} r_{\mathrm{w}})}. \tag{31}$$

However, for $n = 4$ and $Z = 1/3$, eigenfunction is cut off in $r = r_{\mathrm{b}}$, and instability may not propagate outside the beam. It should be emphasized that the current is not neutral for $f_{\mathrm{m}} = 1/3$ because the beam radius is smaller than the skin depth of the plasma.

Figure 2 showed the decreasing effect of current neutrality on the instability growth rate. Hence in Figs. 5 and 6, for all variations of the current fraction at different intensities of the external magnetic field ($\Omega = 0$, 0.07, 0.2) for eigenmodes in

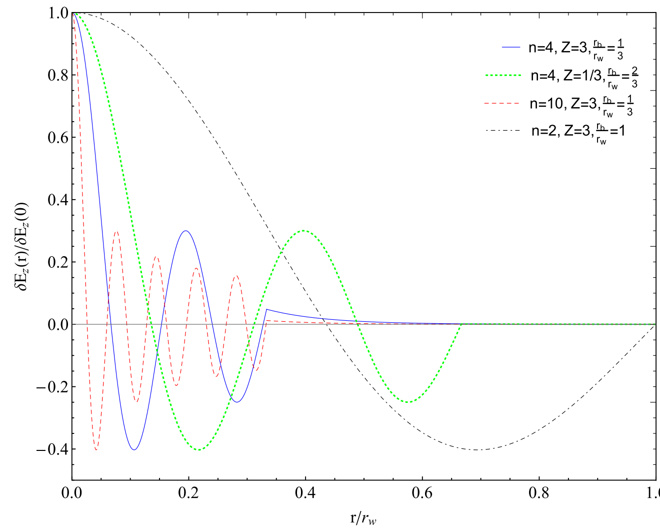

FIG. 4. Eigenfunction of $\delta E_z(r)$ relative to $\delta E_z(0)$ versus the ratio of the beam radius to the waveguide radius in $\alpha_{\mathrm{Cs}^+} = \beta_{\mathrm{Cs}^+} = 0.2$, $f = f_{\mathrm{m}} = 1$ for all states, except for $Z = 1/3$, where f_{m} has been replaced by 1/3.

$n = 1$ and $n \to +\infty$, this point has been checked. Figure 5 presents the results for $n = 1$. In the absence of the external magnetic field ($\Omega = 0$), there is instability for all values of the current fraction, but external magnetic field ($\Omega = 0.07$, 0.2) would create forbidden values ($0 \leqslant f_{\mathrm{m}} \leqslant 0.8$) that the instability cannot propagate in this system [$\mathrm{Im}(\omega) = 0$]. It is important to note that the maximum region has been obtained for $\Omega \geqslant 0.2$. For this intensity of the magnetic field, $\Omega \geqslant 0.2$, this region is constant. If the system deviates only 2% from the charge neutrality, this region decreases to ($0 \leqslant f_{\mathrm{m}} \leqslant 0.2$). For convenience, the percent deviation from the charge-neutral state has been defined as follows: $\Delta f = (f - 1) \times 100\%$. For example, $\Delta f = -2\%$ is equivalent to $f = 0.98$ in Fig. 5. This means that the density difference of background

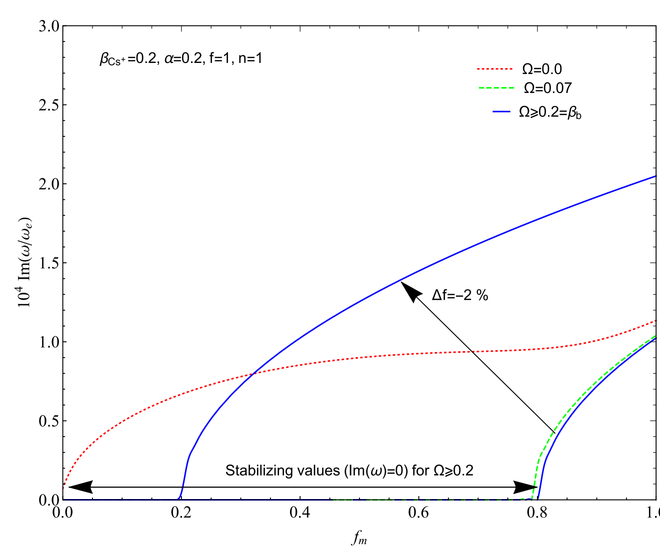

FIG. 5. Weibel instability growth rate for various fractional current neutralization for eigenmode $n = 1$, $f = 1$, and $\alpha = \beta = 0.2$.

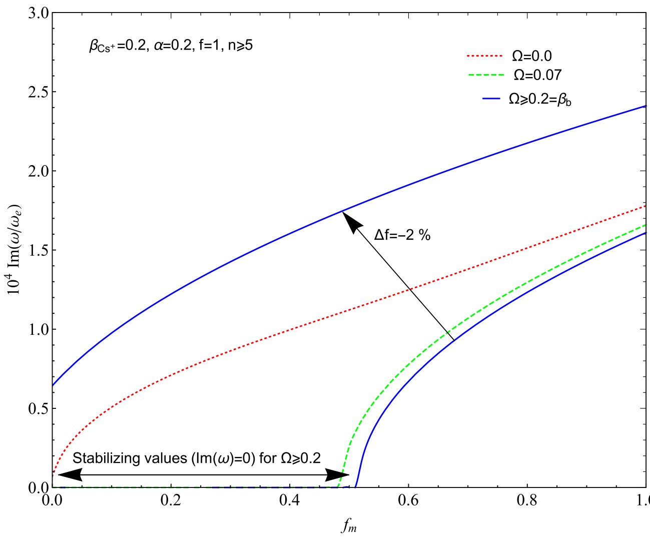

FIG. 6. Weibel instability growth rate for various fractional current neutralization for eigenmodes $n \geqslant 5$, $f = 1$, and $\alpha = \beta = 0.2$.

electrons and ions is smaller than the ion beam density; hence background electrons cannot neutralize the ion beam charge. Consequently, the radial electric field can be amplified, and the growth rate of instability increases. In the opposite region ($\Delta f \geqslant 0$), the electron population can completely neutralize the ion beam charge; as a result, multispecies Weibel instability must be completely stabilized. In Figs. 5 and 6, $\Delta f > 0$ has not been considered because instability is not virulent in this region. Also, the percent deviation from the current neutrality has been applied in Fig. 7. This parameter can be defined as follows: $\Delta f_{\mathrm{m}} = (f_{\mathrm{m}} - 1) \times 100\%$. For this parameter, there is no positive state because the return current of electrons is always smaller than the beam current [30]. Note that f and f_{m} have been defined in Eq. (23).

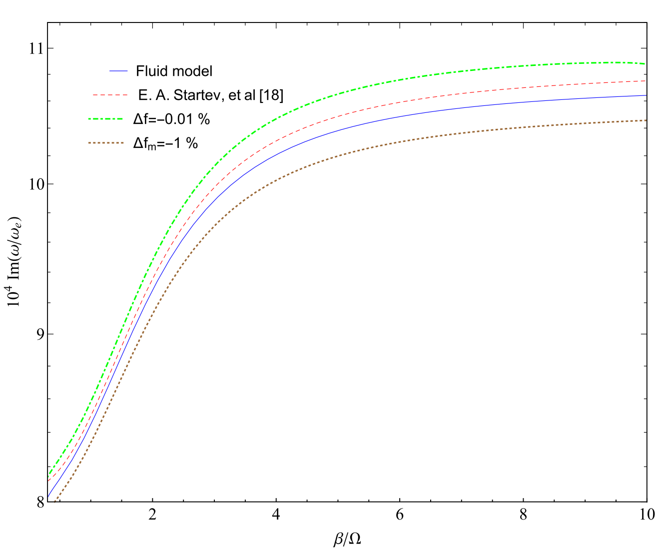

FIG. 7. Comparison of the growth rate of Weibel instability in a neutral and non-neutral magnetized plasma in the fluid model with Ref. [18] for $\alpha = 0.2$, $\beta = 0.125$, $r_{\mathrm{b}}\omega_{\mathrm{e}}/c = 3$, and $n = 7$.

Figure 6 illustrates other radial modes' behavior. If the radial mode numbers are increased to large values ($n \to +\infty$) in the magnetized plasma, the same result is obtained for other modes that are larger than five ($n \geqslant 5$). Strength of the external magnetic field from the condition $\Omega \geqslant \beta_{\mathrm{b}}$ can be calculated as $B_0 \geqslant 320.7 \times 10^{-5}\, \gamma_{\mathrm{b}}\beta_{\mathrm{b}}\sqrt{n_{\mathrm{e}}}$ G. In this relation, n_{e}, β_{b}, and γ_{b} have been defined in Sec. II A and Eq. (27). For instance, in a system with the properties of Fig. 6 and background plasma density $n_{\mathrm{e}} = 10^{10}\,\mathrm{cm}^{-3}$, the external magnetic field intensity must be larger than 64.14 G. It should be emphasized that increasing density and velocity of the ion beam increase the growth rate of the instability, but the forbidden values remain unchanged. As it is seen, a small deviation from charge neutrality ($\Delta f = -2\%$) can eliminate the forbidden values for these modes. With these interpretations, the multispecies Weibel instability can be completely stabilized if the electron cyclotron frequency normalized by the plasma electrons frequency is larger than beam velocity normalized by the speed of light ($\Omega \geqslant \beta_{\mathrm{b}}$). Also, fractional current neutralization must be smaller than $1/2(f_{\mathrm{m}} \leqslant 0.5)$. Of course, these conditions are valid when the system is charge-neutral ($f = 1$).

For charge neutrality of the system, the beam pulse duration (τ_{b}) must be selected longer than the oscillation period of the plasma electrons ($\tau_{\mathrm{b}} \gg 2\pi/\omega_{\mathrm{e}}$). Moreover, for minimizing instability, the current must be non-neutral. Therefore, the beam radius must be selected smaller than the electron skin depth of the background plasma ($r_{\mathrm{b}} \leqslant c/\omega_{\mathrm{e}}$).

Figure 7 compares the growth rate of this instability in the macroscopic fluid model with Fig. 4 in Ref. [18]. In this reference, the solution of the Eq. (9) gives the growth rate versus β/Ω in $ck_{\perp}/\omega_{\mathrm{e}} = 20$. $k_{\perp}$, c, and ω_{e} are vertical wave number of the instability, the speed of light, and plasma electron frequency, respectively. In this study, for further adaption, the parameters have been chosen the same as Ref. [18] except for the wave number. We have purposely selected $k_{\perp} = 20\,\omega_{\mathrm{e}}/c$ in Ref. [18] and $p_{\mathrm{n},0}/r_{\mathrm{b}} = \omega_{\mathrm{e}}/3c$ for $n = 7$ in the fluid model because of separating these diagrams. Therefore, $p_{\mathrm{n},0}/r_{\mathrm{b}}$ can be defined as an effective wave number of the instability ($k_{\perp} = p_{\mathrm{n},0}/r_{\mathrm{b}}$). If this equality is established, two diagrams can be adapted. It should be noted that the beam ions and the background ions are proton and helium, respectively. Moreover, the dispersion equation is valid only in $k^2c^2/{\omega_{\mathrm{e}}}^2 \gg 1$ for very small wavelengths in Ref. [18], while the fluid model considers the entire spectrum of the instability in magnetized, finite, non-neutral plasma. The effects of increasing and decreasing the charge and current fraction on the growth rate in the fluid model for a slight deviation ($\Delta f = -0.01\%$, $\Delta f_{\mathrm{m}} = -1\%$) from the neutral state is completely obvious.

Although instability was investigated in a cold, collisionless plasma, it should be emphasized that compression of the beam might increase the temperature of the real system. However, it can be neglected because the growth rate of this instability usually decreases at a higher temperature. Moreover, the beam radius diminishes during transport, but this investigation has been carried out in a force balance equilibrium in an arbitrary radius and has not considered the temporal evolution of the system. Furthermore, the external magnetic field causes the diamagnetic effect in plasma. Fortunately, it has been proved that its effect is not significant; for example, when the external field strength is set at 900 G, the self-magnetic field reaches 37 G [28]. Consequently, it is negligible compared to the external magnetic field.

IV. CONCLUSIONS

To summarize, in the present paper, we have studied multispecies Weibel instability of the heavy-ion beam in the cold, non-neutral plasma in the cylindrical waveguide. The application of a weak axial magnetic field can control the growth of this instability. The fractional charge and current neutralization can induce a radial electric field and azimuthal magnetic field that have an increasing and decreasing effect on the growth rate of this instability. Moreover, the conducting wall of the waveguide led to the quantization of the instability wave. Hence the growth rate was obtained for these eigenmodes by the solution of the dispersion Eq. (19). This investigation presents a more general state relative to Ref. [15] that has considered this instability in the neutral state in the absence of external magnetic field and Ref. [18] that was limited to small wavelengths in infinite plasma. Furthermore, it has been proved that the results of this paper have good agreement with cited papers in the same regions. Moreover, findings of Ref. [15] have proposed a constraint on the length of the waveguide for escaping from instability as follows: $\mathrm{Im}(\omega)/\omega_e \leqslant v_b/L$. Here v_b and L are the beam velocity and waveguide length, respectively. This means that the interaction time of the beam with plasma must be smaller than the time that instability can be dominant in the system, while our results can release it from this constraint by forbidden values.

In the initial analysis, simplifying the dispersion equation led to a good approximation of the growth rate in Eq. (30). Moreover, numerical results anticipate that if electron cyclotron frequency normalized by the plasma frequency is larger than the velocity of the ion beam normalized by the speed of light ($\Omega \geqslant \beta$) and the response current of electrons is half the ion beam current ($f_m = 1/2$), multispecies Weibel instability may not propagate in this medium. Furthermore, to validate this result, the system must be fully charge-neutral ($f = 1$). It is noted that decreasing this fraction relative to the neutral state ($f = 1$) increases the growth rate severely. Consequently, these achievements can determine the characteristics of the beam-plasma system for avoiding multispecies Weibel instability. Selecting the beam radius smaller than the skin depth of the plasma ($r_b \leqslant c/\omega_e$) causes that charge current not to be neutral ($f_m < 1$). Furthermore, if the pulse duration of the ion beam (τ_b) is selected very longer than the plasma oscillation time ($\tau_b \gg 2\pi/\omega_e$), the charge of the system can be neutralized ($f \sim 1$). In this work, the temperature and density of the components of the plasma have been assumed cold and uniform. Hence one can neglect pressure tensor. The diamagnetic effect in plasma was not so noticeable because the induced axial magnetic field was so much smaller than the solenoid magnetic field. Consequently, these achievements can be reliable.

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