Evolutionary game inspired by Cipolla's basic laws of human stupidity

Joel Kuperman
Facultad de Matemática, Astronomía y Física, Universidad Nacional de Córdoba, Ciudad Universitaria, 5000 Córdoba, Argentina

Donny R. Bárcenas
Instituto Balseiro, R8400AGP Bariloche, Argentina

Marcelo N. Kuperman
Consejo Nacional de Investigaciones Científicas y Técnicas Centro Atómico Bariloche (CNEA) and Instituto Balseiro, R8400AGP Bariloche, Argentina

(Received 6 December 2019; revised manuscript received 14 February 2020; accepted 14 April 2020; published 11 May 2020)

In this work we present an evolutionary game inspired by the work of Carlo Cipolla entitled *The Basic Laws of Human Stupidity*. The game expands the classical scheme of two archetypical strategies, collaborators and defectors, by including two additional strategies. One of these strategies is associated with a stupid player that, according to Cipolla, is the most dangerous one as it undermines the global wealth of the population. By considering a spatial evolutionary game and imitation dynamics that go beyond the paradigm of a rational player we explore the impact of Cipolla's ideas and analyze the extent of the damage that stupid players inflict on the population.

DOI: 10.1103/PhysRevE.101.052307

I. INTRODUCTION

When around 1976 Cipolla formulated the fundamental laws of human stupidity, he was being sarcastic and trying to build a cartoonish image of human society. However, his ideas contained some aspects that constituted an adjusted characterization of the type of behavior displayed in interpersonal relationships. In his work, published in 1988 [1], Cipolla describes personal interactions in terms of benefits and damages derived from any transaction, conceptually going beyond monetary aspects exclusively. He pointed at the concept of stupidity as seen within a social context, and to establish a proper frame for his ideas he classified the behavior that an individual may display within a social context into four groups. These groups are the intelligent (I), the bandit (B), the unsuspecting (U), and the stupid (S). The difference between them arises from the inclination to produce benefits or harms for oneself and for others in any interaction.

It should be pointed out that in Cipolla' s work the concepts of stupidity and intelligence are lax and do not intend to refer to any cognitive abilities of the subjects. Group (I) consists of individuals who, when they interact with others, produce a mutual benefit. Group (B) is composed of selfish individuals who seek individual benefits without hesitating to cause harm to others. Group (U) represents a type of altruistic individual who seeks the wealth of others even at the expense of self-inflicted harm. Finally, the (S) group contains the individuals that not only cause harm to others but also to themselves. To mathematically represent the behavior associated with each group, it is possible to choose two parameters: the gains or losses that an individual causes to him or herself, p, and the gains or losses that an individual inflicts on others, q. These four groups are then defined by the range of values adopted by p and q as follows:

S : $p_s \leqslant 0$ and $q_s < 0$,
U : $p_u \leqslant 0$ and $q_u \geqslant 0$,
I : $p_i > 0$ and $q_e \geqslant 0$,
B : $p_b > 0$ and $q_b < 0$.

Figure 1 shows the location of each strategy on the (p, q) plane. Besides the previous classification of the population into four groups, the central point in Cipolla's work is the enunciation of *The Basic Laws of Human Stupidity*, listed below and quoted from Ref. [1]:

(1) *Always and inevitably everyone underestimates the number of stupid individuals in circulation.*

(2) *The probability that a certain person be stupid is independent of any other characteristic of that person.*

(3) *A stupid person is a person who causes losses to another person or to a group of persons while himself deriving no gain and even possibly incurring losses.*

(4) *Non-stupid people always underestimate the damaging power of stupid individuals. In particular non-stupid people constantly forget that at all times and places and under any circumstances to deal and/or associate with stupid people always turns out to be a costly mistake.*

(5) *A stupid person is the most dangerous type of person.*

Corollary. A stupid person is more dangerous than a pillager.

The mathematical characterization of the four groups together with the fundamental laws inspire us to formulate an evolutionary game that we call Cipolla's game. Each of the four groups described above is associated with a possible strategy and the corresponding payoff matrix is built in terms of the outcome of the interactions between them. The values

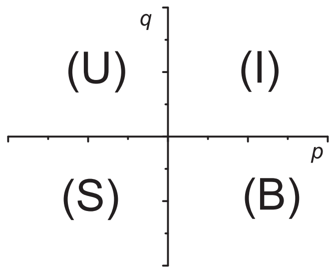

FIG. 1. Location of each strategy in (p, q) space.

of this matrix are loaded in Table I, which indicates which is the payoff of the strategy in the row when competing with the strategy in the column.

Once the strategies and the payoffs are defined, we propose an evolutionary game whose dynamics can be associated with that of the replicator. In the following we will consider that the (B) group is the one that gets the highest self reward p, so being a bandit has certain incentives. As we show later, the resulting game has a unique strict Nash equilibrium, the strategy (B). If we consider a mean-field model described by the usual replicator equations, (B) is the only trivial stable steady state and thus the population converges to a homogeneous group of bandits.

To have a richer dynamics we can consider the subgame in which only the strategies (I) and (B) participate and choose the values of the payoff matrix in order to get a prisoner's dilemma (PD). While this does not add anything to the previous observation regarding the Nash equilibrium when formulating mean-field equations, previous works have shown that, when considering an underlying network defining the topology of the interaction between players, the results can change. It has been observed that a departure from the assumption of a well-mixed population promotes the emergence of cooperation in the classical PD game, at least for certain network topologies and a range of values for the payoffs of the competing strategies [2,3]. Based on these results, one of the objectives of this work is to understand how the topology affects the dynamics of the game. For that we introduce a spatially extended game and consider that the topology of the interactions between players is described by a network. In such a case each player plays with its neighbors and the decision to update its strategy is based only on the local information collected throughout the neighborhood. There is a plethora of network topologies from which we can choose the substrate. In this work we focus on a family of networks that are likely to enhance the effects on the propagation of a cooperative behavior such as (I) due to the local character of the dynamics. These networks, described in Refs. [4] and [5], present a topology that varies according to the value of the disorder parameter. In particular, there are two quantities of interest such as the clustering coefficient and the average path length, though we focus on the former.

TABLE I. Payoff table.

	S	**U**	**I**	**B**
S	$p_s + q_s$	$p_s + q_u$	$p_s + q_i$	$p_s + q_b$
U	$p_u + q_s$	$p_u + q_u$	$p_u + q_i$	$p_u + q_b$
I	$p_i + q_s$	$p_i + q_u$	$p_i + q_i$	$p_i + q_b$
B	$p_b + q_s$	$p_b + q_u$	$p_b + q_i$	$p_b + q_b$

On the other hand, we must define an imitation dynamics associated with the evolution of the distribution of strategies among the population. The simplest assumption is to think of a deterministic imitation. In each round a given player, the focal one, plays with all its neighbors, while each of its neighbors does the same with their own. After that round the focal player analyses its performance or earnings and compares them with that of its neighbors. Then, it adopts the strategy of the player with the highest gain. In the case of a tie the choice is decided at random. This update dynamics is the simplest one, representing a deterministic imitation and closely linked to the replicator dynamics [6]. Adding nondeterministic aspects can lead to more interesting dynamics but will also screen the topological effects.

Either case, deterministic or not, is not considering the nature of the players. The dynamics originally proposed is based on the idea of a rational player, who seeks its own benefit above all. This is directly associated with the characteristics of a (B) player but not with the rest. For example, if we attain to the laws of Cipolla, a player (S) will not be interested in earning a higher profit and could ignore what happens with its neighborhood; that is, it could stay immutable and not change the strategy at all or even imitate the strategy of the neighbor that has caused the higher loss to the rest.

One way to include this in the imitation dynamics is to consider different inclinations not to behave as dictated by rationality according to the nature of the groups. In the following sections we discuss this possibility.

II. MEAN-FIELD RESULTS

In this section we analyze the replicator dynamics, under the assumption of a well-mixed population. First we introduce the payoff matrix

$$A = \begin{pmatrix} p_s + q_s & p_s + q_u & p_s + q_i & p_s + q_b \\ p_u + q_s & p_u + q_u & p_u + q_i & p_u + q_b \\ p_i + q_s & p_i + q_u & p_i + q_i & p_i + q_b \\ p_b + q_s & p_b + q_u & p_b + q_i & p_b + q_b \end{pmatrix}.$$

As stated in the introduction, we choose the values of the payoff matrix so that the subgame (I, B) is a prisoner's dilemma (PD). In this case we need

$$p_b + q_i > p_i + q_i > p_b + q_b > p_i + q_b.$$

Given that $q_b < 0$ and $q_i > 0$ it is enough to choose $p_b > p_i$.

In fact, there is a comparison with a particular case of the PD that is more suitable. In this version called the donation game [7], the game is played by two players. Each one is separately asked whether he or she wishes to give a donation to the other player or not. If the player accepts to be a donor he or she would have to give an amount p and the other player, which will receive a total amount amount $q + p$. Here we associate donation (DO) with cooperation. If both players cooperate they both get q each. However, the most profitable strategy for each of them is to donate nothing (ND). The payoff matrix in Table II illustrates the situation.

TABLE II. Donation game.

	DO	ND
DO	q	$-p$
ND	$p+q$	0

The equations for the evolution of the density of each strategy x_k are

$$\dot{x}_k = x_k([A\vec{x}]_k - A\vec{x}A), \tag{1}$$

where $[A\vec{x}]_k = \sum_j a_{kj}x_j$ and a_{kj} are the elements of A. From now on we associate the subindices 1, 2, 3, 4 with s, u, i, b, respectively.

We can simplify the calculations by making use of one property of the replicator equations that says that the addition of a constant c_k to the kth column of A does not change Eq. (1) (when restricted to the simplex where the relevant dynamics occurs) [6]. We can use then

$$B = \begin{pmatrix} p_s & p_s & p_s & p_s \\ p_u & p_u & p_u & p_u \\ p_i & p_i & p_i & p_i \\ p_b & p_b & p_b & p_b \end{pmatrix},$$

and show that the dynamics is solely defined by the p_k values. Equation (1) can now be written in a much simpler form:

$$\dot{x}_k = x_k\left(p_k - \sum_j x_j p_j\right). \tag{2}$$

This system has four relevant steady solutions corresponding to the survival of a single strategy. The Jacobian of the system is

$$\begin{pmatrix} (1-x_s)p_s - \bar{p} & -x_s p_u & -x_s p_i & -x_s p_b \\ -x_d p_s & (1-x_d)p_u - \bar{p} & -x_u p_i & -x_u p_b \\ -x_i p_s & -x_i \pi_d & (1-x_i)p_i - \bar{p} & -x_i p_b \\ -x_b p_s & -x_b \pi_d & -x_b p_i & (1-x_b)p_b - \bar{p} \end{pmatrix},$$

with $\bar{p} = \sum_j x_j p_j$. Considering that the steady states correspond to only one of the x_k being equal to unity and the rest equal to zero, the eigenvalues for a state when $x_k = 1$ and $x_j = 0$ for $j \neq k$ are

$$(1 - \delta_{k,j})p_j - p_k.$$

It is straightforward to conclude that the only stable steady state, when B has four negative eigenvalues, is the one corresponding to the survival of the strategy with the highest p_k. Thus, when considering a mean-field model, the population converges to a homogeneous group of bandits.

III. DYNAMICS ON NETWORKS

During the last decade many authors began studying evolutionary spatial games to overcome the limitations associated with the assumption that players were always part of a well-mixed population [8–10]. These works showed that the evolutionary behavior and survival of the populations of each strategy might be affected by the underlying topology of links between players [2,3,5,11–13].

The fact that strategies not associated with the Nash equilibrium can survive by forming clusters and gain certain advantage from this has been analyzed in several works where the classical cooperative (C) and noncooperative (D) strategies are considered [9,13–23].

We can gain some intuition about what is happening by the following reasoning: If (C) nodes can exploit the advantages of mutual cooperation, the effect of clustering would be to protect the internal (C) nodes from the presence of the (D) nodes at the border. Since (D) can only get advantage from its interaction with (C), only those defectors located on the border of a group of cooperators can have benefits, while the grouped (C) obtain benefits from the mutual cooperation.

If the (C) nodes at the boundaries of the cluster notice that the cooperators inside do better than the (D) outside they will not be tempted to change their strategy and they might even succeed to expand the cooperative strategy towards the defective population. However, this phenomenon is strongly dependent on the relative values of the payoff of (C) and (D) when playing against (C) and on the structure of the network. The most relevant feature in this regard is the clustering coefficient, which measures the mean connectiveness between the members of a node's neighborhood. Ultimately, it is the existence of local transitive relationships, closely related to clustering [24], that defines the possibility of survival and expansion of small cooperator groups [3].

In this work we consider regular networks with a tunable degree of disorder that translates into different values of clustering and path length. By construction, these networks are regular because all the nodes have the same number of neighbors. To build them we use a modified algorithm based on the one originally proposed in Ref. [4] that maintains the regularity [5].

The usual algorithm of construction of WS networks is as follows: Starting from a regular ordered network with degree k, each link is rewired with a certain fixed probability, preserving one of its adjacent nodes but connected at the other extreme to a random one. Double and self links are not allowed. Although the algorithm conserves the total number of links, at the end of the process the degree of each node is statistically characterized by a binomial distribution. As we are interested in filtering any effect related to changes in the size of the neighborhoods, we modify the original WS algorithm to constrain the resulting networks to a subfamily with a delta-shaped degree distribution. We call this family of networks the k-small world (k-SW) networks, where k indicates the degree of the nodes. The procedure is schematized in Fig. 2.

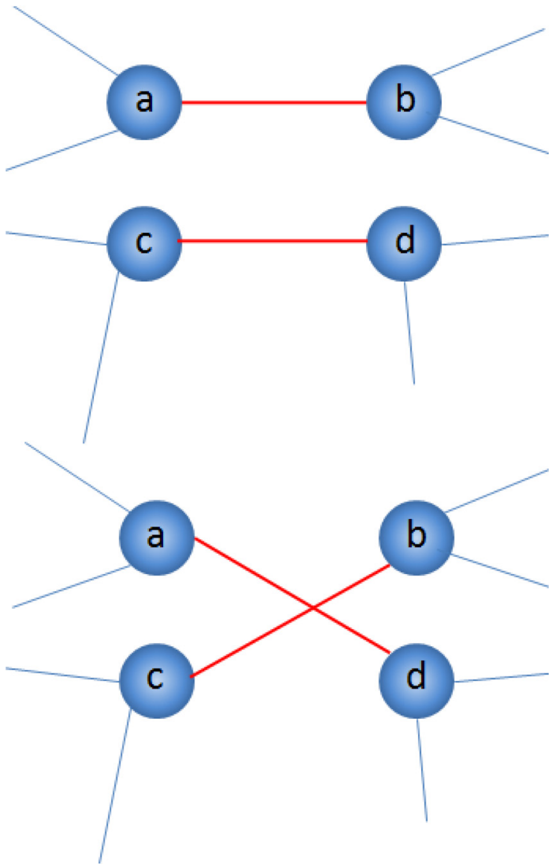

FIG. 2. Algorithm of construction of the k-SW networks. In this example, a single change is depicted. All nodes have degree equal to three. Initially nodes **a** and **b** and nodes **c** and **d** are connected. After the exchange, node **a** is connected to node **d**, and node **c** is connected to node **b**. The degree of the nodes has not changed.

The construction procedure begins again with a regular ordered network which structure is broken by a sequential exchange of the nodes attached to the ends of two randomly chosen links. Starting, for example, from an ordered ring network, each link is subject to the possibility of exchanging one of its adjacent nodes with another randomly chosen link with probability π_d. Thus, to proceed with the reconnection of the network we choose two couples of linked nodes (or partners) rather than one. If we accept to switch the partners, we get two new pairs of coupled nodes. In this way all the nodes preserve their degree while the process of reconnection ensures the introduction of a certain degree of disorder.

The results shown in the present work correspond to networks with $k = 8$. The clustering coefficient starts at $C = 9/14$ for ordered networks to reach $C \approx 10^{-3}$ for the highly disordered networks.

A. Simple deterministic dynamics

We consider first the simplest dynamics. A chosen player plays with it neighbors, who in turn also play with the members of their neighborhoods. After that round, the chosen player imitates the most successful neighbor. But at this point we introduce a slight variation. While the imitation of the most successful will always be the rule for (B), (I), and (U), we will analyze two different behaviors for (S): one in which it imitates the best neighbor as the other strategies and one in which it never changes the strategy. In this case an (S) player remains always as (S). We will call the first case *no-frozen* and the second one *frozen*.

TABLE III. Chosen values for the payoff matrix.

x_i	x_b	x_d	x_e	y_i	y_b	y_d	y_e
1	[1.1, 2]	[−2, −1]	[−2, −1]	1	−1	1	−1

As shown in Refs. [2,3], we need to take into account that, in order for the game to have a nontrivial dynamics and allow the survival of strategies other than the Nash equilibrium such as (I), the quotient $\frac{p_m}{p_i}$ must not exceed a certain threshold value that depends on the topology of the underlying network, especially on the clustering of the nodes and the mean degree. Thus, we fix the values of all the parameters letting p_m vary within a proper range.

The chosen values are given in Table III.

The main goal of this work is to characterize the influence of the (S) strategy on the dynamics of the strategy profile of the population. This is the main rationale to compare the results derived from the *frozen* and *no-frozen* dynamics. Considering the first two laws it would be interesting to analyze the effect of the proportion of (S) players among the population. Therefore we also take different initial fractions of (S) and analyze the effect they may have on the global wealth of the population.

Here we show results corresponding to networks with 10^5 nodes and degree eight, although we have tested different degrees to ensure that this choice does not affect the generality of the results. The only constraint is that the network should be diluted, i.e., a relatively low mean degree.

In all the cases we have verified the convergence to a global steady state, with sometimes negligible local dynamics. Once this steady state is reached, we measure the fraction of individuals in each strategy, x_k. We show that, despite the Nash equilibrium of the game is the pure strategy (B), the spatial effects can make the (I) strategy survive. In most cases, except when the fraction of (S) is maintained fixed, the populations of (S) and (U) disappear.

To analyze the effect of the network topology on the final state we consider several values of π_d and to understand the role of (S); we start with different fractions of its population.

To characterize the steady state we measure the ratio x_i/x_b and the total profit that is being generated in the population due to the interactions, $\langle\epsilon\rangle$. When the strategies (U) and (S) are absent in the steady state both quantities will display exactly the same behavior but when at least one of these two strategies survives we will need both to fully recover the information of what is happening in the system.

Across the numerical calculations we verified that the system quickly reaches the steady state after 10 000 time steps, each one consisting in N rounds of a game between a randomly chosen node and its neighbors. First we point to analyze the effect of the initial population of (S) players, $\rho_s(0)$, and the topology of the network. For this reason we consider several values of $\rho_s(0)$ and π_d. At the beginning

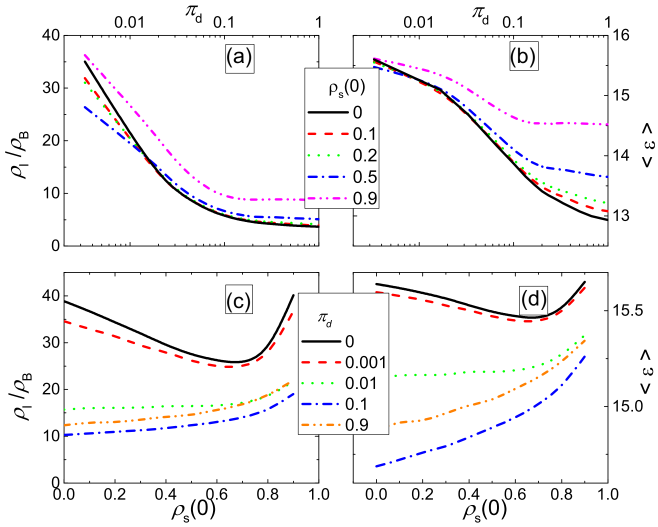

FIG. 3. These plots display the results for the *no-frozen* dynamics. In the top plots we show the results as a function of π_d and different values of $\rho_s(0)$. (a) This plot shows the ratio ρ_i/ρ_b in the steady state. (b) This figure shows the mean gain in the steady state. The bottom plots display the results as a function of $\rho_s(0)$ for different values of π_d. (c) This plot shows the ratio ρ_i/ρ_b in the steady state. (d) This figure shows the mean gain in the steady state. In these plots the full line correspond to $\rho_s(0) = 0$. $p_b = 1.2$.

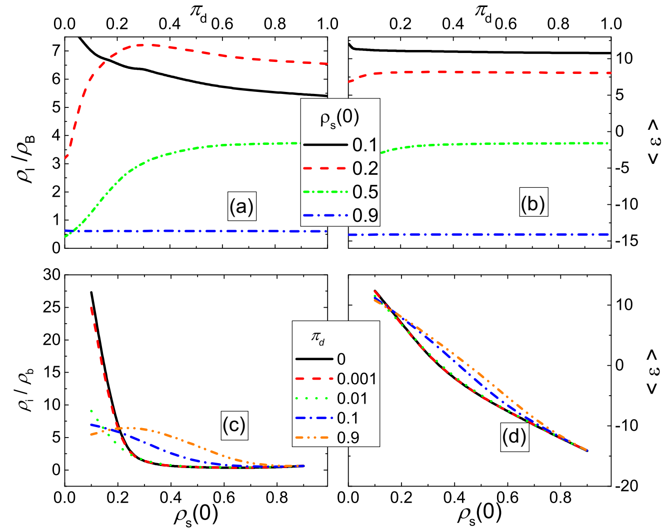

FIG. 4. These plots display the results for the *frozen* dynamics. In the top plots we show the results as a function of π_d and different values of $\rho_s(0)$. (a) This plot shows the ratio ρ_i/ρ_b in the steady state. (b) This figure shows the mean gain in the steady state. The bottom plots display the results as a function of $\rho_s(0)$ for different values of π_d. (c) This plot shows the ratio ρ_i/ρ_b in the steady state. (d) This figure shows the mean gain in the steady state. $p_b = 1.2$.

of the dynamics, the fraction of the rest of the strategies is the same, $[1 - \rho_s(0)]/3$. We have scanned the results for several values of the parameters p_k and q_k, and found two distinct situations. If we take $1.1 < p_b < 2.0$ the (I) strategy can always survive thanks to the advantage it can get from the formation of clusters of (I) individuals that collaborate with each other, giving them advantages over (B). When $p_b > 2$ this advantage disappears and the population of (I) tends to zero. The (S) strategy, when present, does not have this advantage and disappears, just like (U), unless we consider the *frozen* dynamics.

First, we study the *no-frozen* dynamics. Figures 3(a) and 3(b) show the values adopted by the ratio ρ_i/ρ_b and the mean gain of the population $\langle\epsilon\rangle$ in the steady state, respectively, as a function of π_d. We find that, effectively, the (S) and (U) fractions fall to zero and the steady state shows a weak dependence on the initial fraction of (S). The game ends up being a PD and the results qualitatively agree with those obtained in other works for this case. The fraction of (I) decreases as π_d increases [3]. However, the initial fraction of (S) affects the final state in a nontrivial way. Except for the lowest values of π_d, it seems to have an effect contrary to the one predicted by Cipolla, because the increase of $\rho_s(0)$ leads to a steady state with a higher ratio of cooperators and even a higher main global gain. We propose later an explanation for this effect. This can be more clearly observed in Figs. 3(c) and 3(d), where we show the values adopted by ρ_i/ρ_b and $\langle\epsilon\rangle$ as a function of $\rho_s(0)$. The crossover observed in Figs. 3(a) and 3(b) is reflected in the change of slope of the curves according to the values of π_d.

In this analysis we also include the case when $\rho_s(0) = 0$, which helps us to evaluate the effect of $\rho_s(0) \neq 0$. We observe that, for the lowest values of π_d and $\rho_s(0)$, the population is harmed by the presence of (S). This scenario seems to change for higher values of π_d or when $\rho_s(0)$ is high enough. As mentioned before, we provide an explanation after studying the *frozen* case.

In the former example, the populations of (S) and (U) decay to reach extinction.

Next, we may think of an alternative imitation dynamics that might seem to be the closest interpretation of Cipolla's laws. We now consider that the population of (S) does not change its strategy throughout the evolution of the strategies of the rest of the population. Note that the unlikely adoption of the strategy (S) is not forbidden.

The results are shown in Fig. 4, with a correspondence between the panels of Fig. 3 and this one. We see that, in most of the cases when the value of π_d increases, the final fraction of (I), ρ_i, increases too. This is not the case for the lowest values of $\rho_s(0)$, when the results are similar to what has been observed for $\rho_s(0) = 0$.

These results give us a hint of what could be happening that could explain why in the *no-frozen* case, the highest initial fraction of (S) favors the survival of (I). When confronted with an (S) player, the (I) will never change its strategy. The only temptation for a change comes from a possible higher payoff only attainable by a (B) player. Thus, the (S) population is screening or isolating the (I) players, letting them to clusterize and eventually propagate their strategy. In the *no-frozen* case, this transient phenomenon leads an increase in the ratio ρ_i/ρ_b. In the *frozen* case this effect is limited by the permanent presence of (S), which partially inhibits the propagation of both strategies.

But in the presence of an (S) player in the steady state, the ratio ρ_i/ρ_b is not giving us the proper information of the state of the population, because (B) players are potentially being replaced by (S). Thus we analyze the values of $\langle \epsilon \rangle$. We observe that, unlike in the *no-frozen* case, the greater the initial fraction of (S), the worse is the performance of the population. And also, we can see that the mean profit is always lower than in the *no-frozen* case. Thus, the survival of the (S) population results in a clear global damage.

Some of the results shown in Fig. 4 could be explained just by the fact that we are starting with a higher initial number of individuals within the frozen (S) population, leaving us with a trivial effect. Given that the population of (S) is maintained frozen, it is not surprising that the wealth of the population decreases with the initial fraction of (S), but the curves displayed in Fig. 4(d) show a nonlinear dependence, evincing nontrivial effects.

As stated in previous works [2,3], the possibility of survival of (I) depends on the ratio between the payoffs received by strategies (I) and (B) when confronting another (I), that is, $(p_b + q_i)/(p_i + q_i)$. As this ratio grows, the surviving fraction of the (I) population decreases. For both cases we verified that, for $p_b \geqslant 2$, only (B) players survive, except for the frozen population of (S) in the corresponding case.

We note that, in all the cases studied above, the population of (U) disappears.

B. Specific dynamics

In the previous section we considered a differentiated imitation dynamics only for the (S) strategy. Here we explore an expansion of this idea by considering a specific imitation dynamics for each strategy, always inspired by the principles that characterize each of them.

Among the four groups defined by Cipolla only (B) behaves like a rational player, always looking for the individual wealth above all and therefore always imitating the neighbor with the highest profit. On the opposite side, the U group presents an altruistic nature, seeking the benefit of the other. In that sense, we may assume that such players will try to imitate the neighbor who generates the greatest profit for the rest, irrespective of his own profit associated with that change.

In the previous section we consider two possibilities for the imitation behavior of (S): it could or could not change its behavior. In the present case we also consider these two options but, in case it changes its strategy, it will not act as a rational player. We assume that the need to generate damage, regardless of the costs, is rooted in its nature. Following this premise it will imitate the neighbor that produces the greatest loss or minor gain in its neighborhood.

Finally, we consider that the (I) group shows some traces of altruism but not at the cost of self-generating a loss. So it will seek not to suffer a loss but at the same time to be involved with the generation of a global profit. So it will imitate the neighbor who generates the greatest global profit and at the same time does not involve its own loss.

So, as in the previous section, we have a *no-frozen* and a *frozen* case. As will be shown, the results for both cases present a new feature, the survival of the (U) population.

Both cases show results qualitatively very similar to what we obtained for the *frozen* dynamics in the previous example, reflecting that the dynamics chosen for the (S) groups ensures its survival.

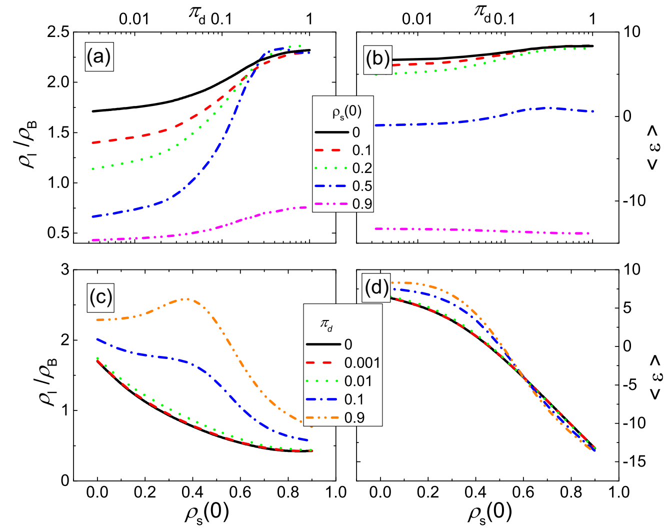

FIG. 5. These plots display the results for the *no-frozen* dynamics and specific imitation behavior. In the top plots we show the results as a function of π_d and different values of $\rho_s(0)$ when the imitation dynamics is differentiated. (a) This plot shows the ratio ρ_i/ρ_b in the steady state. (b) This figure shows the mean gain in the steady state. The bottom plots display the results as a function of $\rho_s(0)$ for different values of π_d. (c) This plot shows the ratio ρ_i/ρ_b in the steady state. (d) This figure shows the mean gain in the steady state. $p_b = 1.2$.

Figure 5 shows the results for the *no-frozen* dynamics. The new imitation behavior adopted by (S) prevents it from changing the strategy, indicating that, even at a local scale, the (S) player is the one causing the greater loss. Despite the similarities, the mean profit of the population is always higher for the *no-frozen* case, mainly due to the fact that the presence of (I) players is higher, as can be observed in Fig. 6. Also, in the *no-frozen* case there is a decrease of the (S) population, reaching steady fractions verifying $\rho_s \approx \rho_s(0)^2$.

The main difference between the former results and the new ones resides in the fact that now, a small population of (U) can persist. This is shown in Fig. 7 where the steady fraction of (U), ρ_u, is depicted. The figures show the *frozen* and *no-frozen* cases, for several values of π_d and as a function of $\rho_s(0)$. We just recall that the clustering coefficient goes down as π_d increases so, for a proper comparison with previous figures, the x axis should be read from right to left.

C. The clustering effect

Throughout this section we have been pointing at the network structure, specifically, the clustering coefficient, as the main item responsible for the observed dynamics. To support this claim we include a figure where the curves are depicted as a function of the mean clustering of networks built by using the same value of π_d. Figure 8 shows the ratio between the population of (I) and (B) for the cases studied above.

It is clear that the effect of clustering is not the same in the case shown in Fig. 8(a) as in the rest. This case corresponds to the *no-frozen* dynamics, where the (S) players can be eventually replaced by (I) or (B) and thereafter the already-known results about higher clustering promoting the

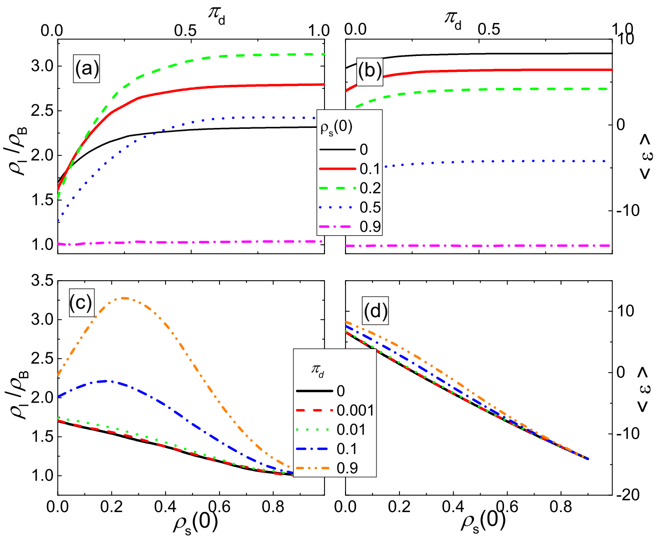

FIG. 6. These plots display the results for the *frozen* dynamics and specific imitation behavior. In the top plots we show the results as a function of π_d and different values of $\rho_s(0)$ when the imitation dynamics is differentiated. (a) This plot shows the ratio ρ_i/ρ_b in the steady state. (b) This figure shows the mean gain in the steady state. The bottom plots display the results as a function of $\rho_s(0)$ for different values of π_d. (c) This plot shows the ratio ρ_i/ρ_b in the steady state. (d) This figure shows the mean gain in the steady state. $p_b = 1.2$.

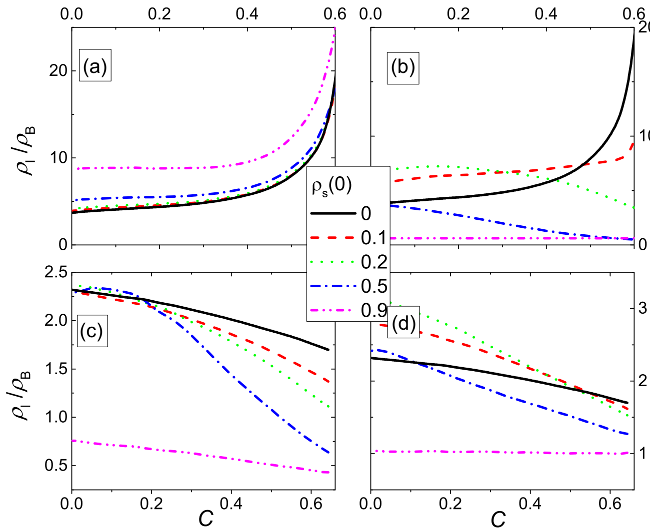

FIG. 8. This plot shows the ratio ρ_i/ρ_b in the steady state for different cases as a function of the clustering coefficient C: (a) frozen and simple dynamics, (b) *no-frozen* and simple dynamics, (c) *frozen* and specific dynamics, (d) *no-frozen* and specific dynamics.

cooperation are recovered [3]. In this case, the higher the clustering, the higher the density of cooperators, since cooperation can only survive the invasion of the noncooperators by forming clusters and taking advantage of mutual cooperation. The figure shows how, in the other cases, a higher clustering seems to inhibit cooperation or the density of (I) players. While the effect is the opposite that described earlier, the underlying phenomenon is the same. If a small group of clustered (I) is invaded by a (B) player, the temptation to defect can propagate. However, in a low clustered network and in the presence of an (S) player, the (I) player survives not by being clustered but by being isolated from the temptation to become a (B) player by the presence of (S) players. In this case any (I) player has no temptation to change its strategy because its payoff is always higher than that of an (S) player. A case that deserves further explanation is the one corresponding to Fig. 8(c). In this case, we are dealing with a non-frozen situation. However, the specific dynamics ensures the survival of a population of (S) players, who play the screening role mentioned above.

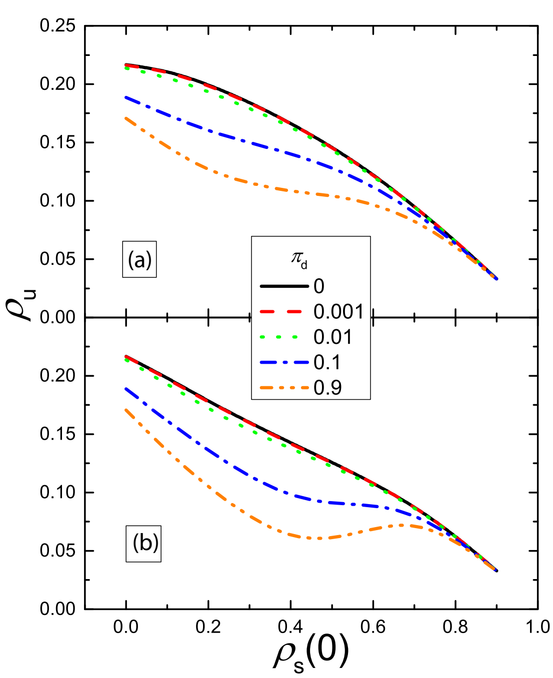

FIG. 7. Steady fraction of (U), ρ_u, as a function of $\rho_s(0)$ for the (a) *frozen* and (b) *no-frozen* cases.

IV. CONCLUSIONS

In this work we present a mathematical interpretation and analysis of the ideas introduced by Cipolla in Ref. [1]. The adopted formalism is based on the formulation of an evolutionary game whose payoff matrix is a direct translation of the definition of the four groups characterizing the nature of human transactions. We have shown that the resulting game has a unique Nash equilibrium and thus the evolution of the strategies under the replicator dynamics leads to a trivial solution corresponding to a homogeneous population of bandits. Based on previous results on spatial cooperative games, we adopted payoff values that let us identify some features of the present game with a prisoner' s dilemma. In addition to this, we explored a spatial version of the game by considering a selected family of underlying regular networks. These networks are characterized by a single disorder parameter and the degree of the nodes.

The analysis of the spatial version of the game presented interesting results that let us reveal the mathematical structure behind the ideas of Cipolla.

According to Cipolla's laws, the number of stupids cannot be estimated. To explore the possibility of a critical fraction of (S) individuals can affect the population, we have explored a range of values in the interval [0,1]. We have found that even the smallest fraction of stupids produces a notable effect. Letting aside some subtleties to be explained later, the overall conclusion is that, as the fifth law establishes, a stupid person is the most dangerous, even more dangerous than a bandit. This is reflected in the fact that, in most cases, a higher fraction of (S) lead to a lower global gain, independently of whether the (S) group can or cannot change its strategy. We found some exceptions where the (S) group seems to exert contradictory effects favoring the propagation of (I) players and leading to a higher mean profit. Before explaining this effect we want to address other results that deserve a closer look and are related to the behavior of the ratio between the (I) and (B) group, and the survival of (U) individuals. We have found that, when the (S) players survive, their steady fraction depends only on $\rho_s(0)$, so the topology of the networks seems to play no role. While this may sound obvious for the frozen dynamics, it is not for the *no-frozen* one. However, the topology of the network is extremely relevant in defining how the initial (S) population will affect evolution and organization of the final state. The (S) initial population together with the topology of the network is what governs the final ratio between the (I) and (B) population, and thus the overall gain of the population. In all the cases, the permanent presence of the (S) group undermines the wealth of the population and only a transient survival can lead to an overall gain. This phenomenon is the result of a screening effect played by the (S) population, as they isolate the (I) players from the (B) ones avoiding the tempting change from (I) to (B). At the same time, during the transient presence of (S), the (I) group strengthens and may start to propagate towards the (B) population. At this point, the (S) populations starts to play the opposite role, as it prevents the (I) group from advancing over the (B) population. This effect is responsible for the nonmonotonic shape of the curves observed in Figs. 4(c), 5(c), and 6(c).

The present four-strategy game can be also analyzed in the context of potential games [25]. These sorts of games present deterministic Nash equilibria. At the same time, a close connection between the dynamics of the strategies of the players and the thermodynamics of a system of spins on a lattice can be done [26,27]. The departure from the Nash equilibrium observed in the present work is solely attributed to the imitation dynamics and to the disordered structure of the underlying network.

One of the distinctive features of the present game is the irruption of a completely disruptive strategy, in the sense that it violates the spirit of any rational player and of the evolutionary dynamics itself. The stupid player plays even against itself. It is worth mentioning some other examples in the literature where such irrational behavior have been proposed and tested. In Ref. [28] the authors study a usual public game with the addition of a new strategy called joker. This strategy is somehow analogous to (S) in the sense that it causes a loss to others even when incurring a self-loss because it cannot share the profits. The remarkable finding of this work is that the joker strategy affects the evolution of the game, even promoting the consolidation of cooperation as a response to the irrational behavior of the joker.

The influence of the topological structure of the underlying network has also been shown and discussed in Refs. [29] and [30]. In Ref. [29] the authors study the evolution of the strategies of a population playing the PD subject to a noisy payoff matrix. The relevance of this work in the context of the present one is that the authors showed that the presence or absence of loops in the links affected the survival of the cooperative behavior. On the other hand, in Ref. [30] the authors consider different network structures to analyze the probability that a given mutant individual can propagate the mutation over the entire population. While the analyzed phenomena could be seen as different from that studied here, the underlying mechanism of propagation is very similar.

In this work we have excluded the possibility that $p_i > p_b$. If such were the case, the structure of the game would be different, leading to a trivial homogeneous population of (I) individuals, even in an extended game. We wanted to explore a situation in which there is a social dilemma and there is a temptation not to adopt a cooperative strategy, such as the PD.

In summary, our work explores the ideas of Cipolla, showing that their implementation as a game may lead to interesting and nontrivial conclusions, in agreement with the proposed laws.

In this work we have only considered deterministic dynamics. The introduction of some stochasticity, not only in the imitation dynamics but also in the possibility of a spontaneous change of strategy of some players, will be analyzed in a future presentation.

[1] C. Cipolla, *Allegro Ma Non Troppo* (Il Mulino, Bologna, 1998).
[2] O. Durán and R. Mulet, Phys. D **208**, 257 (2005).
[3] M. N. Kuperman and S. Risau-Gusman, Phys. Rev. E **86**, 016104 (2012).
[4] D. J. Watts and S. H. Strogatz, Nature (London) **393**, 440 (1998).
[5] M. N. Kuperman and S. Risau Gusman, Eur. Phys. J. B **62**, 233 (2008).
[6] J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics* (Cambridge University Press, Cambridge, 1998).
[7] K. Sigmund, *The Calculus of Selfishness* (Princeton University Press, Princeton, 2010).
[8] M. A. Nowak and R. M. May, Nature (London) **359**, 826 (1992).
[9] M. A. Nowak and R. M. May, Int. J. Bifurcation Chaos Appl. Sci. Eng. **3**, 35 (1993).
[10] G. Szabó and G. Fáth, Phys. Rep. **446**, 97 (2007).
[11] M. A. Nowak, Science **314**, 1560 (2006).
[12] G. Abramson and M. N. Kuperman, Phys. Rev. E **63**, 030901(R) (2001).

[13] C. P. Roca, J. A. Cuesta, and A. Sánchez, Phys. Rev. E **80**, 046106 (2009).
[14] H. Ohtsuki, C. Hauert, E. Lieberman, and M. A. Nowak, Nature (London) **441**, 502 (2006).
[15] M. Doebeli and C. Hauert, Ecol. Lett. **8**, 748 (2005).
[16] C. Hauert and G. Szabó, Am. J. Phys. **73**, 405 (2005).
[17] R. Cong, Y.-Y. Qiu, X.-J. Chen, and L. Wang, Chin. Phys. Lett. **27**, 030203 (2010).
[18] S. Assenza, J. Gómez-Gardeñes, and V. Latora, Phys. Rev. E **78**, 017101 (2008).
[19] C. Lei, J. Jia, X. Chen, R. Cong, and L. Wang, Chin. Phys. Lett. **26**, 080202 (2009).
[20] M. Ifti, T. Killingback, and M. Doebeli, J. Theor. Biol. **231**, 97 (2004).
[21] F. C. Santos, J. F. Rodrigues, and J. M. Pacheco, Phys. Rev. E **72**, 056128 (2005).
[22] J. Vukov, G. Szabó, and A. Szolnoki, Phys. Rev. E **77**, 026109 (2008).
[23] G. Wu, K. Gao, H. Yang, and B. Wang, Chin. Phys. Lett. **25**, 2307 (2008).
[24] S. Wasserman and K. Faust, *Social Network Analysis: Methods and Applications* (Cambridge University Press, Cambridge, 1994).
[25] D. Monderer and L. S. Shapley, Games Econ. Behav. **14**, 124 (1996).
[26] G. Szabó and I. Borsos, Phys. Rep. **624**, 1 (2016).
[27] G. Szabó, K. S. Bodó, B. Allen, and M. A. Nowak, Phys. Rev. E **92**, 022820 (2015).
[28] A. Arenas, J. Camacho, J. A. Cuesta, and R. J. Requejo, J. Theor. Biol. **279**, 113 (2011).
[29] J. Vukov, G. Szabó, and A. Szolnoki, Phys. Rev. E **73**, 067103 (2006).
[30] A. Pavlogiannis, J. Tkadlec, K. Chatterjee, and M. A. Nowak, Sci. Rep. **7**, 82 (2017).