Excited state of spiral waves in oscillatory reaction-diffusion systems caused by a pulse

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(Received 29 October 2019; revised manuscript received 1 March 2020; accepted 9 April 2020; published 29 April 2020)

Previous studies claim that the dynamic behaviors of spiral waves are uniquely determined by the nature of the medium, which can be determined by control parameters. In this article, the authors break from the previous view and present an alternate stable state of spiral waves, named the excited state. The authors find that two states of the spiral wave switch to each other after a one-off pulse is applied to the medium. The dynamic behaviors of the two states are quite different, specifically, the spiral tip trajectory of the original spiral, which is named the ground-state spiral as observed in the previous studies, is a point, while the spiral tip trajectory of the excited-state spiral is a circle. Moreover, the authors study the trajectories of the spiral tip of spiral waves in both states after the pulse is applied and find two types of trajectories, a spiral trajectory and a spiral-inward-petal trajectory. The frequency of the spiral wave in the excited state is less than that in the ground state. The findings enrich the dynamics of pattern formation.

DOI: 10.1103/PhysRevE.101.042219

I. INTRODUCTION

Spiral waves are typical spatiotemporal patterns in reaction-diffusion systems driven far from thermodynamic equilibrium. They exist extensively in excitable and oscillatory systems having spacial extensions, such as reacting chemical systems [1–4], aggregating starving slime mold cells [5,6], liquid crystals subjected to electric or magnetic fields [7], catalytic reactions on a platinum surface [8,9], and cardiac tissue [10,11]. The study of the dynamics of spiral waves has attracted great interest not only because of its nonlinear and far-off characteristics but also because of its extensive applications and destructions. For instance, spiral waves and turbulence may be the main mechanisms of tachycardia and ventricular fibrillation, respectively [12,13]. Many types of spiral waves, such as meandering [14,15], ripple-armed [16,17], segmented [18,19], super-armed [20,21], zigzag-armed [22], multiarmed [23], stepped super-armed [24], multi-stable stepped [25], super multi-armed and segmented (SMAS) [26], and bistable Yin-Yang (BYY) [27] spirals, have been reported in the past three decades. Generally, the dynamic behavior of a spiral wave transits from one type to another with the change of the values of control parameters [28]. Further, Barkley *et al.* [14] and Cai *et al.* [29] have discovered the dynamic behaviors of spiral waves in parameter space based on the Barkley model and FitzHugh-Nagumo model, respectively. Mahanta *et al.* have obtained different tip trajectories of spiral waves with different parameters [30]. Previous works hold that the dynamic behaviors of spiral waves can be determined by the control parameters; i.e., the characteristic of a spiral wave depends only on the properties of the medium [28–30].

In addition, the application of a continuous external periodic force can also change the dynamic behavior of the spiral wave [31–34]. For example, periodic illumination can cause the spiral tip in the Belousov-Zhabotinsky reaction to meander, and as the frequency of illumination changes, the pattern of the tip trajectory changes [31,32]; even if a very weak periodic disturbance is applied in a small area around the spiral tip, the spiral wave becomes unstable [34–36]. However, when the periodic force is removed, the spiral wave will return to its original state [33]. In summary, previous studies show that the dynamic behaviors of spiral waves are uniquely determined by the nature of the medium without external forces.

In mathematics, a single spiral wave is only a special solution of a reaction-diffusion equation, which raises a question: Is the special solution, i.e., a single spiral wave with a certain dynamic behavior, unique when the control parameters of the system are fixed? To answer this question, we apply a one-off pulse to a spiral wave to examine whether the final state (dynamic behavior) of the spiral wave can be changed. If the final state of the spiral wave can be changed, it means that we have overturned the previous research and found an alternative state of the spiral wave. Amazingly, we found that the state of the spiral wave stably remains in another state after a one-off pulse is applied, and the two states of the spiral wave switch to each other by applying a one-off pulse to the medium. The rest of the article is organized as follows. Section II shows the method. Section III shows the numerical results, including four subsections: apply a pulse to the spiral wave in the ground state; apply a pulse to the spiral wave in the excited state; the change process near the spiral tip; and the oscillation of local points in the excited-state spiral wave. Section IV presents the analytical results, and last, a discussion and conclusions are presented in Sec. V.

II. METHOD

Spiral waves are usually studied theoretically by means of the complex Ginzburg-Landau equation (CGLE), a universal

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model describing the evolution of nearly coherent waves [37]. According to the problem involved, it can appear in various forms, including complex coefficients and nonlinearities [38,39]. The CGLE has been widely applied to different physical, biological, and chemical systems, such as electrohydrodynamic convection in liquid crystal, transversely extended laser, Bose-Einstein condensate, planar gas discharge, fluid and chemical turbulence, bluff body motion, plasma surface-wave oscillation, etc. [40–42], and is closely related to the Gross-Pitaevsky or nonlinear Schrödinger equation [43,44].

The CGLE system describes spatially extended media, in which the homogeneous system is oscillatory and is near a supercritical Hopf bifurcation [45,46]. It has the form

$$\frac{\partial A}{\partial t} = A + (1 + i\alpha)\nabla^2 A - (1 + i\beta)|A|^2 A + P, \tag{1}$$

where α and β are real control parameters, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $A(\vec{r}, t)$ is the complex variable, and P is an external pulse, which reads

$$P = \begin{cases} p, & \text{if} \quad t' \leqslant t \leqslant t' + \delta t, \\ 0, & \text{else}, \end{cases} \tag{2}$$

where p is a constant and δt represents a very short period of time, during which time the system has no time to respond to the pulse. Since the duration of the pulse is extremely short, the effect of the pulse on the variable A can be separated from Eq. (1) and calculated separately. The contribution of the pulse P to the variable A is

$$\Delta A = \int P dt = \int_{t'}^{t'+\delta t} p dt = p\delta t \equiv kA_0, \tag{3}$$

where k is a real number for controlling the pulse strength, and $A_0 = 0.95$ is the amplitude of the real part $\mathrm{Re}(A)$ of the variable A (when there is a spiral wave in the system). Consequently, the impulsive differential system [47] of Eq. (1) has the form

$$\begin{aligned} &\frac{\partial A}{\partial t} = A + (1 + i\alpha)\nabla^2 A - (1 + i\beta)|A|^2 A, \ t \neq t', \\ &\Delta A = kA_0, \ t = t'. \end{aligned} \tag{4}$$

Various values of the parameter k were considered; the other parameters were fixed: $\alpha = 3.00$ and $\beta = 0.05$. Those chosen parameters were also discussed in Refs. [48,49], which shows that, this way can form a rigidly rotating spiral wave with the radius of the trajectory of the spiral tip being zero.

Our model is integrated on 400×400 grid points with *no-flux* boundary conditions. A nine-point finite difference scheme is applied to compute the Laplacian term $\nabla^2 A$, and then the discrete equation is calculated via the *four-order Runge-Kutta* method. The space steps Δx and Δy are both 1, and the time step Δt is 0.01. The spiral waves in this article are shown in the Re(A) field. FORTRAN and MATLAB codes to generate the figures are available from the authors on request.

III. NUMERICAL RESULTS

This section is divided into four subsections. In Sec. III A, we generate a common spiral wave, named the ground-state spiral wave, in the medium. Then one-off pulses with different strengths are applied to the ground-state spiral wave. The influence of the pulse and the influence of the strength of the pulse on the dynamic behavior of the ground-state spiral wave are both studied in detail. At last, we discovered an alternative state of the spiral wave, named the excited state. In Sec. III B, one-off pulses with different strengths are applied to the excited-state spiral wave to study the influence of the pulse and the influence of the strength of the pulse on the dynamic behavior of the excited-state spiral wave. At last, we realized the conversion between the ground state and the excited state. The change process near the spiral tip and the oscillation of local points in the excited-state spiral wave are studied in Secs. III C and III D, respectively.

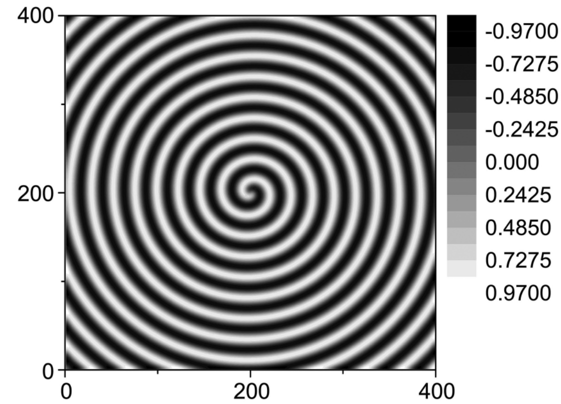

FIG. 1. The ground state of the spiral wave in the medium. The wavelength of the spiral wave is 23.7 grid points, and the frequency is 0.4132×10^{-1}. The image shown is 400×400 grid points.

A. Apply a pulse to the spiral wave in the ground state

In the two-dimensional system, when the initial condition is $\mathrm{Re}(A)(1:200,\ 1:400) = 0.5$, $\mathrm{Re}(A)(201:400,\ 1:400) = -0.5$, $\mathrm{Im}(A)(1:400,\ 1:200) = 0.5$, and $\mathrm{Re}(A)(1:400,\ 201:400) = -0.5$, the system can generate a rigidly rotating spiral wave with the radius of the spiral-tip trajectory being zero (Fig. 1), which is the same as that in Ref. [48]. The frequency of the spiral wave is 0.4132×10^{-1}. We named the state of this spiral wave *the ground state*.

Then we apply a pulse with $k = 0.100$ to the spiral wave in Fig. 1 to examine the effect of the pulse on the spiral wave [Fig. 2(a)]. The results show that, when the pulse is applied,

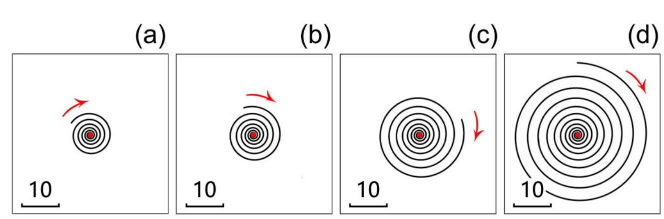

FIG. 2. The trajectory of the spiral tip with different values of k: (a) $k = 0.100$, (b) $k = 0.200$, (c) $k = 0.300$, and (d) $k = 0.375$. The red arrow indicates the direction of motion for the spiral tip, and the red point indicates the final location of the spiral tip. The trajectories have been reasonably digitally manipulated for beautification. Each image shown is 40×40 grid points.

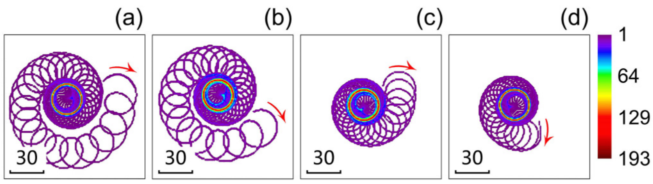

FIG. 3. The trajectory of the spiral tip with different values of k: (a) $k = 0.379$, (b) $k = 0.400$, (c) $k = 0.450$, and (d) $k = 0.800$. The color bar indicates the number of overlapping layers. The red arrow indicates the direction of motion for the spiral tip, and the red circle indicates the final trajectory of the spiral tip. Each image shown is 120×120 grid points.

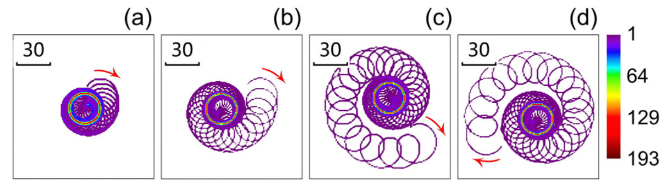

FIG. 5. The trajectory of the spiral tip with different values of k: (a) $k = 0.050$, (b) $k = 0.100$, (c) $k = 0.200$, and (d) $k = 0.250$. The color bar indicates the number of overlapping layers. The red arrow indicates the direction of motion for the spiral tip, and the red circle indicates the final trajectory of the spiral tip. Each image shown is 120×120 grid points.

the original spiral wave temporarily loses stability, the spiral tip moves along a spiral trajectory and eventually reaches the center of the spiral trajectory, and then the spiral wave returns to its original state [Fig. 2(a)]. Next, we increase the pulse strength by increasing the value of k to study the effect of pulse strength on the dynamic behavior of the spiral wave. When k is increased (less than $\bar{k}_1 = 0.379$), as described in Fig. 2(a), the spiral tip moves along a spiral trajectory and eventually returns to its original state. In addition, the maximum radius of the corresponding spiral trajectory increases as the parameter k increases [Figs. 2(a)–2(d)].

When k is greater than or equal to $\bar{k}_1 = 0.379$, the trajectory of the spiral tip is no longer like the spiral trajectory shown in Fig. 2, but rather has a spiral-inward-petal trajectory [Fig. 3(a)]. The change process of the spiral wave after being affected by the pulse is shown in Movie 1 of the Supplemental Material [50]. The spiral tip moves along the spiral-inward-petal trajectory and eventually enters a circular trajectory with a radius of 25 grid points for clockwise motion. As k continues to be increased, the area occupied by the corresponding trajectory decreases [Figs. 3(b)–3(d)].

Surprisingly, when the spiral wave reaches its steady state, the spiral wave does not return to its original state, but enters a new state (Fig. 4). We named this state *the excited state* of the spiral wave. Note that the trajectory of the spiral tip is

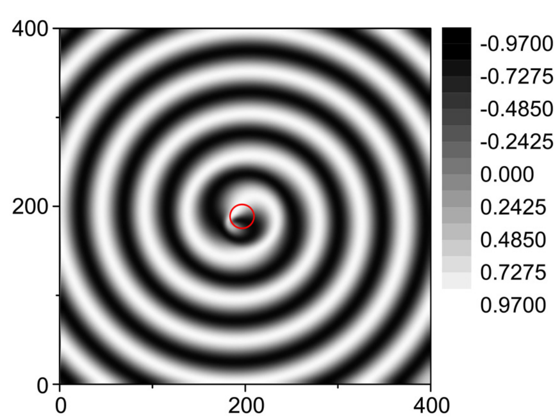

FIG. 4. The exited state of the spiral wave in the medium. The red circle is the final trajectory of the spiral tip, which is the same as the red circles in Fig. 3. The wavelength of the spiral wave is 47.5 grid points, and the frequency is 0.1623×10^{-1}. The image shown is 400×400 grid points.

no longer a point, but rather a circle with a radius of 25 grid points. The dynamics of the spiral wave in the excited state is shown in Movie 2 of the Supplemental Material [50]. The frequency of this spiral wave is 0.1623×10^{-1}.

B. Apply a pulse to the spiral wave in the excited state

Next, we apply a pulse to the excited-state spiral wave in Fig. 4 to examine the effect of the pulse on the excited-state spiral wave. When k is equal to 0.050, the excited-state spiral wave temporarily loses stability, and the spiral tip moves along the spiral-inward-petal trajectory and finally enters the circular trajectory with a radius of 25 grid points for clockwise motion [Fig. 5(a)]. As k is increased, the area occupied by the corresponding trajectory increases [Figs. 5(b)–5(d)]. The spiral wave that returns to the steady state is still the excited-state spiral wave in Fig. 4.

As k continues to be increased, when k is greater than or equal to $\bar{k}_2 = 0.251$, the trajectory of the spiral tip is no longer like the spiral-inward-petal shown in Fig. 5, but is changed back to the spiral trajectory [Fig. 6(a)]. The tip of the spiral wave moves along the spiral trajectory and eventually converges toward the center of the spiral trajectory. When k is increased, the maximum radius of the corresponding spiral trajectory increases [Figs. 6(b)–6(d)]. The spiral wave returning to the steady state becomes the ground state of the spiral wave in Fig. 1.

Below, we conduct a detailed study of the spiral wave in the excited state.

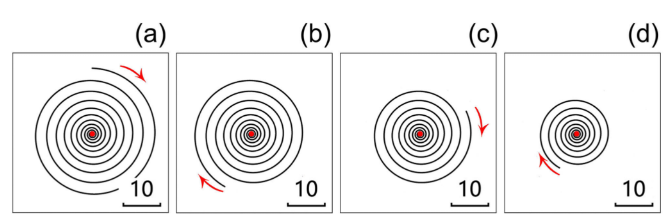

FIG. 6. The trajectory of the spiral tip with different values of k: (a) $k = 0.251$, (b) $k = 0.300$, (c) $k = 0.450$, and (d) $k = 0.600$. The red arrow indicates the direction of motion for the spiral tip, and the red point indicates the final location of the spiral tip. The trajectories have been reasonably digitally manipulated for beautification. Each image shown is 40×40 grid points.

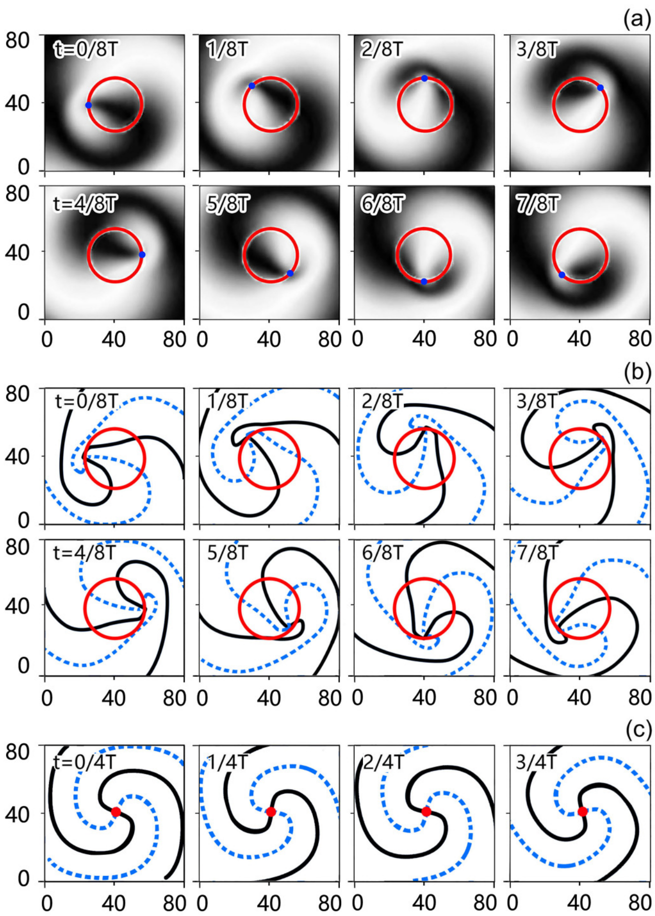

FIG. 7. The areas near the spiral core. (a) The changing process of the central region of the excited-state spiral wave in one period. (b) The distribution of local points where Re(A) or Im(A) is 0 of the excited-state spiral wave in one period. The black lines are composed of local points where Re(A) is equal to 0, and the dotted blue lines are composed of local points where Im(A) is equal to 0. (c) The distribution of local points where Re(A) or Im(A) is 0 of the ground-state spiral wave in one period. The red circles indicate the trajectories of the spiral tips, and the blue points in panel (a) indicate the spiral tips.

C. The change process near the spiral tip

We observed the change process near the spiral tip and recorded snapshots near the spiral tip in one period [Fig. 7(a)]. During the clockwise rotation of the spiral tip along the trajectory (the red circle), the shape of the spiral tip changes periodically with a period of change of half the period of the spiral wave. For example, the shape of the spiral tip with $t = (0/8)T$ is the same as that with $t = (4/8)T$, and the shape of the spiral tip with $t = (1/8)T$ is the same as that with $t = (5/8)T$. In order to obtain more accurate state characteristics of the region near the spiral tip, we marked the local points where the real or imaginary part is zero [Fig. 7(b)]. The solid black and the dotted blue lines, named zero-lines, are composed of local points where Re(A) and Im(A) are equal to 0, respectively. Figure 7(b) shows the corresponding snapshots of the ones in Fig. 7(a). The dynamic process in Fig. 7(b) is shown in Movie 3 of the Supplemental

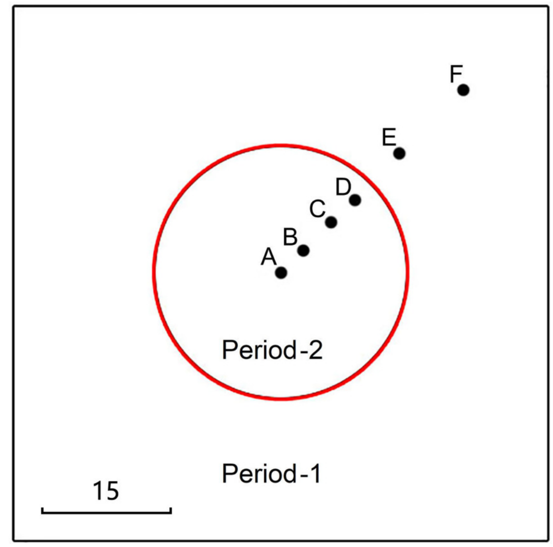

FIG. 8. Oscillation characteristics of local points in a medium. The red circle indicates the trajectory of the spiral tip, and the black points indicate the selected local points. Point A is the center of the circular trajectory. The local points within the red circle are in the period-2 oscillatory state except one local point in the center of the circular trajectory, and the local points outside the red circle are in the single-periodic oscillation state.

Material [50]. As shown in Fig. 7(b) or Movie 3, one can find that, during the movement of the wave tip along the orbit, there is always one zero-line (the solid black or dotted blue line) tangent to the circular trajectory (the red circle). Moreover, a surprising phenomenon can be found in Figs. 7(a) and 7(b) that, in one period of the spiral wave, local points outside the circular trajectory oscillate once; however, local points within the circular trajectory oscillate twice. This shows that the oscillations inside and outside the circular trajectory are different. In order to compare with the spiral wave in the ground state, we also recorded the zero-lines near the spiral center [Fig. 7(c)]. The shapes of both the zero-lines do not change during the rotation of the spiral wave. The dynamic process in Fig. 7(c) is shown in Movie 4 of the Supplemental Material [50].

D. The oscillation of local points in the excited-state spiral wave

We studied the oscillation of local points inside and outside the trajectory and found that the local points within the circular trajectory are in the period-2 oscillation state (except the local point at the center of the circle), while the local points outside the circular trajectory are in the single-period oscillation state (Fig. 8). Although the oscillation forms are different, the oscillation periods of the local points inside and outside the trajectory are the same (except the local point at the center of the circle).

In order to detail the oscillation characteristics of the local points, we selected six representative local points and recorded the time series (Fig. 9). Within the circular trajectory, from the center of the circle to the outside, the period-2 characteristic of the oscillation becomes more obvious [Figs. 9(a)–9(d)], while the point A at the center of the circle completely

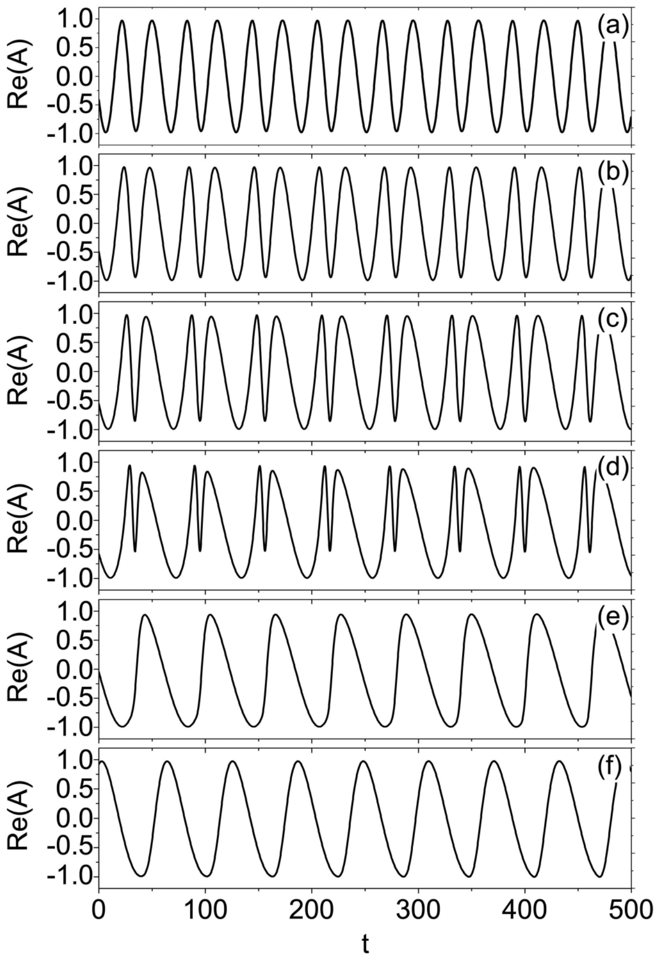

FIG. 9. Time series of the selected points in Fig. 8. (a)–(f) are the time series of the points A-F in Fig. 8, respectively.

loses the characteristics of the period-2 oscillation state, making its period half of the period of other local points. The local points outside the circular trajectory are in a single-period oscillation state [Figs. 9(e) and 9(f)].

IV. ANALYTICAL RESULTS

For the sake of discussion, Eq. (1) without the external pulse P is rewritten as

$$\frac{\partial A(\rho,\theta,t)}{\partial t} = \Phi[A(\rho,\theta,t)] + \Psi[A(\rho,\theta,t)], \tag{5}$$

with

$$\Phi[A(\rho,\theta,t)] = A(\rho,\theta,t) - (1+i\beta)|A(\rho,\theta,t)|^2 A(\rho,\theta,t),$$
$$\Psi[A(\rho,\theta,t)] = (1+i\alpha)\nabla^2 A(\rho,\theta,t), \tag{6}$$

where $\Phi[A(\rho,\theta,t)]$ and $\Psi[A(\rho,\theta,t)]$ are the reaction term and the diffusion term, respectively; (ρ,θ) represents the polar coordinate with the spiral core as the origin coordinate; and t is the time.

Under periodic boundary conditions, Eq. (5) has the following traveling wave solutions:

$$A(\rho,\theta,t) = A_0 \exp[i(\vec{k}'\vec{r} - \omega t)], \tag{7}$$

where

$$k' = \frac{2\pi m}{L} \quad (m = 0, \pm 1, \pm 2, \ldots),$$
$$A_0 = \sqrt{1-k'^2},$$
$$\omega = \beta + (\alpha-\beta)k'^2. \tag{8}$$

Here, L represents the system size, k' $(0 \leqslant k' \leqslant 1)$ represents the wave number, and $\omega \equiv 2\pi\nu$ represents the angular frequency.

The single spiral wave solutions shown in Fig. 1 can be written as

$$A(\rho,\theta,t) = F(\rho)\exp\{i[n\theta - \omega t + \psi(\rho)]\}, \tag{9}$$

where the integer n is the topological charge that $n = +1$ (or -1) represents the phase changes by 2π when rotating a circle along the spiral anticlockwise (or clockwise). $F(\rho)$ represents the amplitude of the spiral wave, and the real number $\psi(\rho)$ represents the phase of the spiral wave. The asymptotic behaviors of $F(\rho) \equiv |A|$ and $\psi(\rho)$ are

$$\rho \to 0, \quad F(\rho) \sim \psi(\rho) \sim \rho,$$
$$\rho \to \infty, \quad F(\rho) \sim \sqrt{1-k'^2},\ \psi(\rho) \sim k'. \tag{10}$$

In the region far from the center of the spiral wave, the spiral wave is a plane wave with uniform amplitude.

With the same control parameters, another special solution, i.e., the single spiral wave solutions shown in Fig. 4, can be written as

$$A(\rho,\theta,t) = \begin{cases} F(\rho)\exp\{i[n\theta - \omega t + \psi(\rho)]\}, & \text{if} \quad \rho > r_t, \\ F(\rho)H(\rho,\theta,t), & \text{if} \quad \rho \leqslant r_t, \end{cases} \tag{11}$$

where r_t is the radius of the tip trajectory, and (ρ,θ) represents the polar coordinate with the center of the tip trajectory as the origin coordinate. The asymptotic behaviors of $H(\rho,\theta,t)$ are

$$\begin{aligned} &\rho \to 0, \quad H(\rho,\theta,t) \sim \exp[-i2\omega t + \varphi(\theta)], \\ &\rho \to r_t, \quad H(\rho,\theta,t) \sim \exp[-i\omega t + \varphi(\theta)], \end{aligned} \tag{12}$$

where ω is the rotating frequency of the spiral wave in Fig. 4, and $\varphi(\theta)$ represents the phase of the pattern within the tip trajectory.

The fixed point of the partial differential equation, Eq. (5), needs to satisfy the condition

$$\Phi[A(\rho,\theta,t)] = 0,$$
$$\Psi[A(\rho,\theta,t)] = 0, \tag{13}$$

or

$$\Phi[A(\rho,\theta,t)] + \Psi[A(\rho,\theta,t)] = 0,$$
$$\Phi[A(\rho,\theta,t)] \neq 0, \quad \Psi[A(\rho,\theta,t)] \neq 0. \tag{14}$$

In the ground-state spiral wave shown in Fig. 1, during the rotation of the spiral wave, the position of spiral tip does not change. As shown in Fig. 7(c), the real and imaginary parts of the value in the spiral center are both 0, therefore

$$\Phi[A(\rho_0,\theta_0)] = 0, \tag{15}$$

where $(\rho_0 \equiv 0, \theta_0 \equiv 0)$ is the coordinate of the spiral center. As shown in Eq. (7), in the traveling wave solutions, the peaks and troughs are symmetrical with respect to the equilibrium point, i.e.,

$$\sin(\vec{k}'\vec{r}) = -\sin[\vec{k}'(-\vec{r})]. \quad (16)$$

Moreover, the zero-lines are symmetrical about the spiral center. Therefore in the area around the spiral tip,

$$A(\rho, \theta) + A(\rho, \theta + \pi) = 0. \quad (17)$$

Accordingly,

$$\Psi[A(\rho_0, \theta_0)] = 0. \quad (18)$$

The value of the local point in the spiral center satisfies Eq. (13); the spiral center is stationary.

However, in the excited-state spiral wave shown in Fig. 4, the zero-lines are not symmetrical about the spiral center (the zero-lines have no center of symmetry) [Fig. 7(b)]. Therefore, in the area around the spiral tip,

$$A(\rho, \theta) + A(\rho, \theta + \pi) \neq 0, \quad (19)$$

and therefore,

$$\Psi[A(\rho_0, \theta_0)] \neq 0. \quad (20)$$

The value of the local point in the spiral tip does not satisfy Eqs. (13) and (14); the spiral tip cannot be stationary.

V. DISCUSSION AND CONCLUSION

The classical [1–4] and the meandering [14,15] spiral waves have been discovered before, and previous studies have claimed that these two types of spirals exist in different parameter spaces. In other words, the transformation of these two types of spirals can only be achieved by changing the values of control parameters [14,28]. Indeed, previous studies have dissevered the inherent relation between the two states of a spiral wave. However, the results of this article break with the previous view. We found that these two types of spirals switch to each other after a one-off pulse is applied to the medium. In addition, for ease of understanding, we called the state of the classical spiral wave the ground state and the state of the meandering spiral wave the excited state.

The previous studies [14,28] showed that the spiral tip orbit transits from one type to another by changing the parameter values generally. For instance, Barkley reported that the dynamics of spiral waves are organized around the codimension-2 bifurcation where Hopf eigenmodes interact with eigenmodes and found the spiral tip orbits in the parameter space. However, in our study, we found that the spiral tip orbits with the same parameter values can be different.

Reference [51] reported that the spiral tip meanders rather than follows a periodic circular orbit under certain conditions. Meandering spirals have been extensively studied theoretically [14,52,53] and experimentally [15,54]. References [14,52,53,55,56] have shown that the meandering is not an erratic motion and the spiral tip moves in epicycloidlike orbits or hypocycloidlike orbits that are quasiperiodic in time [55]. In this article, we discovered an alternative type of orbit of the meandering which is an erratic motion.

Reference [57] asserts that the rotation frequency of a spiral wave is determined solely by medium properties. However, our study shows that two different rotation frequencies of spiral waves, i.e., the ground state and the excited state of a spiral wave, exist in the same medium. With a certain operation, two kinds of spiral waves can exist in the same medium at the same time.

Finally, in other studies [14,15,28,30,57], the rotation direction of the spiral wave is the same as that of the tip. Specifically, when the spiral wave rotates clockwise (or counterclockwise), the spiral tip moves along a clockwise (or counterclockwise) orbit. However, in this study, the rotation direction of the spiral wave is opposite to that of the wave tip.

We established an alternative connection between the classic and the meandering spiral waves, found spiral tip orbits with the same parameter values can be different, discovered an alternative type of spiral tip orbit of the meandering spiral wave, reported two types of spiral waves can exist in the one medium at the same time, and discovered an alternative rotation mode of spiral tip for meandering spiral waves. In summary, we discovered an alternative type of state of spiral waves, i.e., the excited state, and realized the conversion between the ground state and the excited state of the spiral wave. The dynamic behaviors of the excited-state spiral wave and the original ground-state spiral wave are completely different, which are under the same control parameters. In other words, we found the dynamic behaviors of spiral waves cannot be determined only by the control parameters. The formation mechanism of the excited-state spiral wave was analyzed in the article. Our findings enrich the dynamics of pattern formation.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China under Grants No. 11875042 (C.G.), No. 11505114 (C.G.), and No.11805128 (T.W.) and the Shanghai project for construction of top disciplines under Grant No. USST-SYS01 (H.Y., C.G., and T.W.).

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