Rapid Communications

Breaking of symmetry of interacting dissipative solitons can lead to partial annihilation

Orazio Descalzi [1,2,*] and Helmut R. Brand [2]

[1] *Complex Systems Group, Facultad de Ingeniería y Ciencias Aplicadas, Universidad de los Andes, Avenida Monseñor Álvaro del Portillo 12.455, Las Condes, Santiago, Chile*

[2] *Department of Physics, University of Bayreuth, 95440 Bayreuth, Germany*

(Received 23 December 2019; accepted 5 April 2020; published 23 April 2020)

We show that for a large range of approach velocities and over a large interval of stabilizing cubic cross-coupling between counterpropagating waves, a collision of stationary pulses leads to a partial annihilation of pulses via a spontaneous breaking of symmetry. This result arises for coupled cubic-quintic complex Ginzburg-Landau equations for traveling waves for sufficiently large values of the stabilizing cubic cross-coupling and for large enough approach velocities of the pulses. Briefly, we show in addition that the collision of counterpropagating pulses in a system of two coupled cubic Ginzburg-Landau equations with nonlinear gradients (Raman effect) might also lead to partial annihilation, indicating that this breaking of symmetry is generic. Systems of experimental interest include surface reactions, convective onset, biosolitons, and nonlinear optics.

DOI: 10.1103/PhysRevE.101.040201

The field of pattern formation analyzing the influence of spatial degrees of freedom on the spatiotemporal behavior of macroscopic nonequilibrium systems [1] has led to a bridge from physics to chemical reactions and biological systems. Solitonic behavior almost free of dissipation is a well-established feature of macroscopic systems in fields such as fluid dynamics, plasma physics, and nonlinear optics [2]. In contrast, the study of dissipative solitons [3], localized structures stabilized by a balance between nonlinearity and dispersion as well as gain and loss of energy and/or matter, is currently under intense experimental and theoretical investigations.

Experimental dissipative systems for which solitonlike behavior has been observed in driven nonequilibrium systems include biological phenomena [4] and chemical reactions [5]. In other experiments collisions between pulses were found to give rise to bound states of pulses, and to partial annihilation for which only one pulse survives the collision. These systems include surface reactions [6], and binary fluid convection near convective onset [7,8]. In the field of nonlinear optics, pulse generation has been described [9], while in an excitable system, namely, the electro-oxidation of CO on Pt, solitonlike behavior and backfiring was experimentally observed. This pulse dynamics was reproduced with a three-component reaction-diffusion model [10].

To model dissipative solitons frequently the approaches of envelope equations valid near the onset of an instability [11,12,13] or order parameter equations containing the most important terms of the appropriate symmetry are used [1,14,15]. Using envelope equations describing the electric field inside a laser with a saturable absorber dissipative solitons in nonlinear optics were predicted [16] and observed [17].

To obtain bound states, interpenetration and complete annihilation of dissipative solitons (DSs), coupled cubic-quintic complex Ginzburg-Landau equations, which have stable dissipative soliton solutions [18], have been used [19,20].

As a complement to experiments and modeling with partial differential equations we mention that particle-based simulations have been carried out to show the solitonlike behavior of traveling bands of interacting deformable self-propelled particles [21].

The phenomenon of partial annihilation of colliding pulses experimentally observed in Refs. [6–8] turns out to be more challenging. In Ref. [22] it has been modeled using reaction-diffusion equations with a defect zone. Using one pulse for collisions, which is not yet fully in its asymptotic shape, it has been shown that partial annihilation is also possible [23] (compare also Ref. [6]). The first more general approach to the question of partial annihilation has been given in Ref. [24], where it was shown that a small amount of additive noise applied near the boundaries between different outcomes of collisions, such as interpenetration and annihilation, leads to partial annihilation. This effect has been further elucidated in Ref. [24] for FitzHugh-Nagumo-type equations as they are used to model nerve pulse propagation and chemical reactions [25,26]. In addition, we showed in Ref. [24] that the interval over which one can get partial annihilation can be increased by adding noise. In the present Rapid Communication, we demonstrate that for noise intrinsic to the method (of fixed amplitude $\sim 10^{-16}$) one can obtain partial annihilation for a large region of parameter space in approach velocity and cross-coupling between the counterpropagating waves. This is due to a spontaneous breaking of symmetry for a sufficiently large approach velocity and cross-coupling strength of the counterpropagating dissipative solitons. Noise only serves to induce the left or right direction. We are thus in a rather different part of the phase diagram and the phenomena described in Fig. 2 of the present Rapid Communication are for fixed intrinsic noise.

*Corresponding author: odescalzi@miuandes.cl

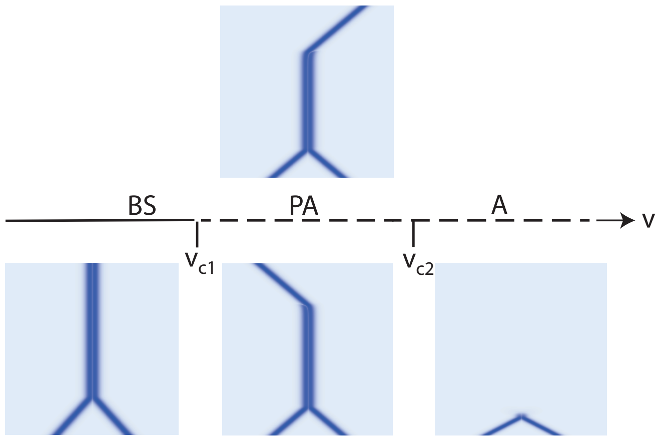

FIG. 1. Sketch of the "phase diagram" as a function of approach velocity v in the region of interest in this Rapid Communication for fixed value c_r. The region of partial annihilation arises between stationary bound states and complete annihilation. BS denotes stationary bound states, A annihilation, and PA partial annihilation. The characteristic outcomes of collisions have been illustrated by x-t plots where time is plotted on the ordinate and the spatial coordinate on the abscissa.

Here, we demonstrate for the collisions of stationary dissipative solitons in the framework of two coupled subcritical cubic-quintic Ginzburg-Landau equations that partial annihilation arises over a large parameter regime between stationary bound states and complete annihilation due to spontaneously broken left-right symmetry for moderate values of the approach velocity and over a large range of stabilizing cubic cross-coupling between counterpropagating waves.

Briefly, at the end of this Rapid Communication, we show that the collision of counterpropagating pulses in a system of two coupled cubic Ginzburg-Landau equations with nonlinear gradients (Raman effect in nonlinear optics) can also lead to partial annihilation, indicating that this behavior is generic.

We study two coupled complex subcritical cubic-quintic Ginzburg-Landau equation for counterpropagating waves,

$$\partial_t A - v\partial_x A = \mu A + (\beta_r + i\beta_i)|A|^2 A + (\gamma_r + i\gamma_i)|A|^4 A + (c_r + ic_i)|B|^2 A + (D_r + iD_i)\partial_{xx} A, \qquad (1)$$

$$\partial_t B + v\partial_x B = \mu B + (\beta_r + i\beta_i)|B|^2 B + (\gamma_r + i\gamma_i)|B|^4 B + (c_r + ic_i)|A|^2 B + (D_r + iD_i)\partial_{xx} B, \qquad (2)$$

where $A(x,t)$ and $B(x,t)$ are complex fields and where we have discarded quintic cross-coupling terms for simplicity. A and B are slowly varying envelopes, β_r is positive, and γ_r is negative in order to guarantee that the bifurcation is subcritical, but saturates to quintic order.

We have carried out our numerical studies for the following values of the parameters, which we kept fixed for the present purposes: $\mu = -0.50$, $\beta_r = 1$, $\beta_i = 0.8$, $\gamma_r = -0.1$, $\gamma_i = -0.6$, $D_r = 0.125$, $D_i = 0.5$, and $c_i = 0$. Positive values of D_i correspond to the regime of anomalous linear dispersion in nonlinear optics. The time step dt used was typically $dt = 0.005$ and as a grid spacing we took $dx = 0.08$ and $N = 1250$ leading to a box size of $L = 100$. Integration of Eqs. (1)

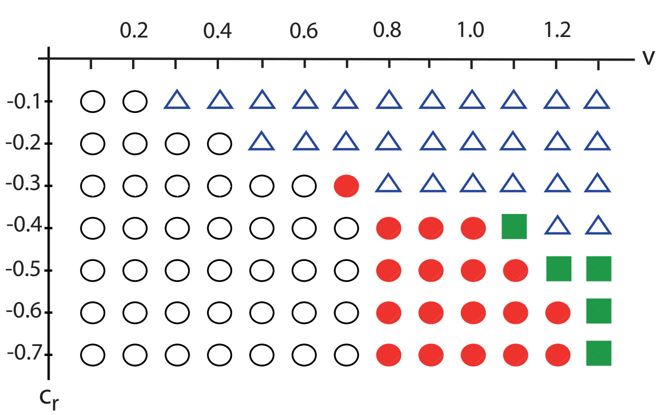

FIG. 2. Phase diagram for interacting stationary DS for $\mu = -0.5$ in the plane approach velocity vs negative (stabilizing) cubic cross-coupling between counterpropagating waves in the presence of a round-off error equivalent to a noise strength of 10^{-16}. Parameters are $\mu = -0.50$, $\beta_r = 1$, $\beta_i = 0.8$, $\gamma_r = -0.1$, $\gamma_i = -0.6$, $D_r = 0.125$, $D_i = 0.5$, and $c_i = 0$. Open black circles (○) denote stationary bound states, open blue triangles (△) interpenetration, solid red circles (●) partial annihilation, and solid green squares (■) annihilation.

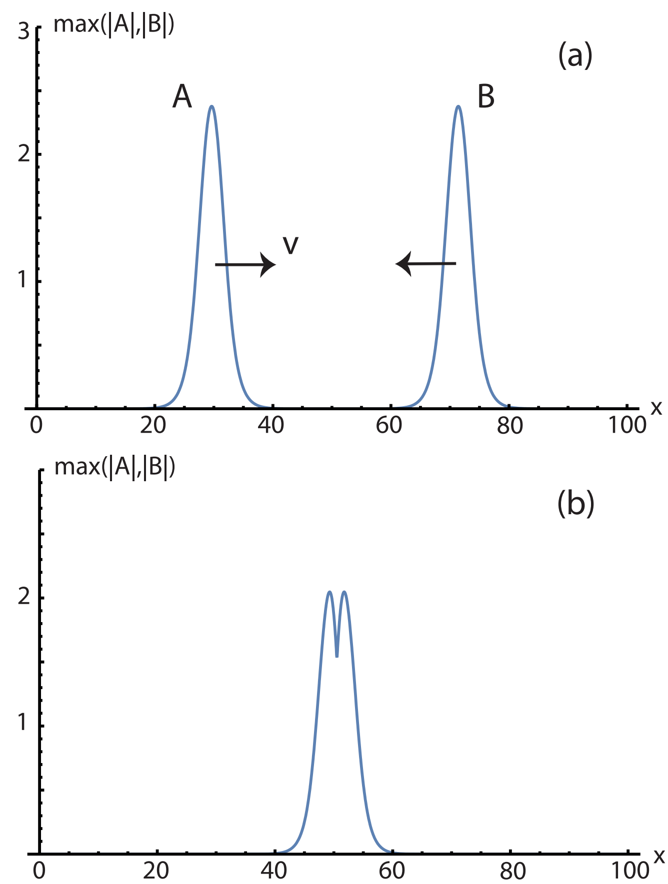

FIG. 3. The process of partial annihilation: The max($|A|$, $|B|$) is plotted in (a) for the initial approach of the pulses, and (b) during the interaction. $c_r = -0.7$ and $v = 0.8$. Parameters are as in Fig. 2.

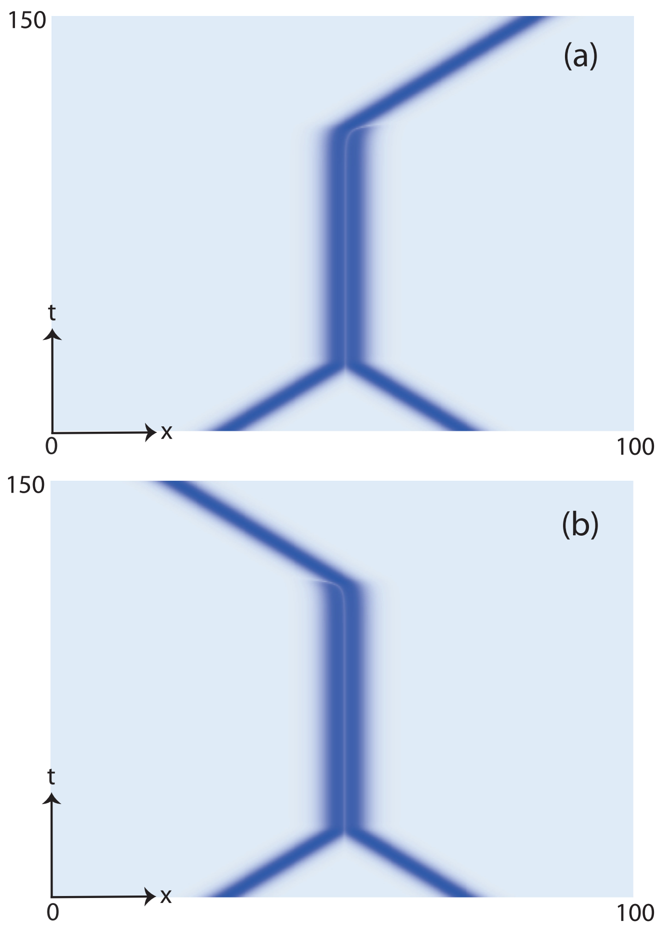

FIG. 4. The process of partial annihilation: (a) and (b) show x-t plots of the partial annihilation process with an interaction time $T_{\rm int} = 80$. $c_r = -0.7$ and $v = 0.8$, when the surviving pulse is moving to the right and to the left, respectively. Parameters are as in Fig. 2.

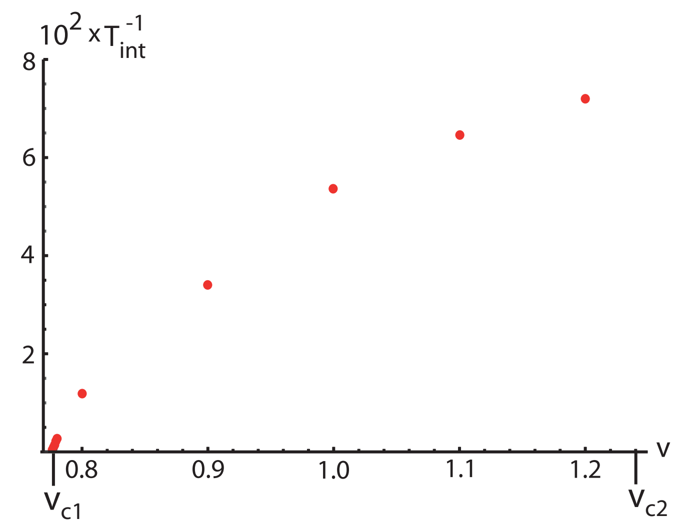

FIG. 5. The inverse of the interaction time in the presence of a round-off error equivalent to a noise strength of 10^{-16} is plotted as a function of approach velocity v for fixed $c_r = -0.7$. The two critical velocities mark the critical velocity for the transition to stationary bound states v_{c1} and the transition to complete annihilation v_{c2}. Parameters are as in Fig. 2.

and (2) was performed using fourth-order Runge-Kutta finite differencing.

In Fig. 1 we sketch the "phase diagram" as a function of approach velocity v in the region of interest in this Rapid Communication for a fixed value c_r. The region of partial annihilation (PA) arises between stationary bound states (BS) and complete annihilation (A). The characteristic outcomes of collisions have been illustrated by x-t plots where time is plotted on the ordinate and the spatial coordinate on the abscissa.

In Fig. 2 we present the phase diagram for interacting stationary DS for $\mu = -0.5$ in the plane approach velocity versus negative (stabilizing) cubic cross-coupling between counterpropagating waves c_r, which demonstrates the four results of collisions discussed here. As we will show below, the amplitude of intrinsic noise due to the numerical method used entering the picture here is about 10^{-16} compared to a pulse amplitude of order 1. As characteristic outcomes of the collisions we find interpenetration for small stabilizing values of the cubic cross-coupling and large enough approach velocity. This result is entirely intuitive: If the two pulses hardly interact with a short interaction time, they easily interpenetrate. For small values of the approach velocity and sufficiently negative values of the stabilizing cubic cross-coupling, a bound state of stationary DSs emerges. If the stabilizing cross-coupling is large enough and the approach velocity sufficiently high, annihilation results. The reduction of the pulse area during the collision is such that the critical area for the reemergence of a DS is not reached from below. The outcome, which is of central interest here, is the partial annihilation arising over a large range of values for both the approach velocity and the stabilizing cubic cross-coupling. We will argue in the following that this region is reflecting a spontaneous left-right symmetry breaking. This observation is corroborated by the fact that the ratio of pulses surviving going to the left and to the right approaches unity in the limit of a large number of collisions.

In Fig. 3 we have plotted two snapshots elucidating the process of partial annihilation. We have plotted the maximum of $|A|$ and $|B|$ for a moderate approach velocity $v = 0.8$ and for $c_r = -0.7$. The initial state has been prepared such that both pulses are in their asymptotic shape in time before the interaction. Figure 3(a) shows the initial approach while Fig. 3(b) is taken during the interaction. Just inspecting the snapshots visually, one does not notice during the interaction whether the pulse traveling to the left or to the right will survive the interaction.

In Fig. 4 we show the x-t plot corresponding to the process of partial annihilation for the same parameters used to to generate Fig. 3. From these x-t plots a "deterministic interaction time" $T_{\rm int}$ (in the presence of a round-off error equivalent to a noise strength of 10^{-16}) with the value $T_{\rm int} = 80$ can be extracted.

This interaction time turns out to be rather sensitive to the approach velocity. In Fig. 5 we have plotted the inverse of the deterministic interaction time $T_{\rm int}$ for the whole range of existence of partial annihilation for $c_r = -0.7$. As one can

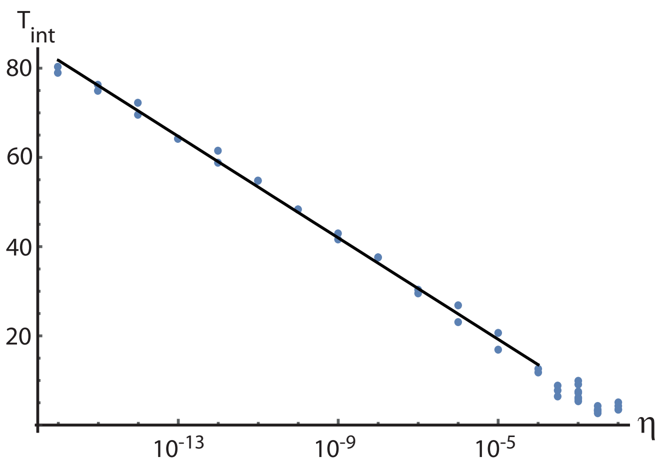

FIG. 6. The interaction time $T_{\rm int}$ is plotted as a function of additive noise η. This plot is obtained for $c_r = -0.7$ and $v = 0.8$. Parameters are as in Fig. 2.

see, the interaction time decreases monotonically by more than two orders of magnitude as the approach velocity is increased. It reaches a value of more than 2000 as the transition to stationary bound states is approached.

To study the influence of noise on the interaction time $T_{\rm int}$, we have incorporated into Eqs. (1) and (2) additive noise of strength η, δ correlated in space and time. We have chosen a location in the phase diagram close to the boundary between stationary bound states and partial annihilation: $c_r = -0.7$ and $v = 0.8$. In Fig. 6 we show the interaction time as a function of the strength of additive noise over 14 orders of magnitude. Inspecting Fig. 6, one can read off a linear decay in a linear-logarithmic plot over at least 12 orders of magnitude. This behavior indicates an activation energy dominated process in the spirit of Kramers escape over a barrier. We note that the data point for $\eta = 10^{-16}$ has also been obtained without additional noise indicating the intrinsic noise level of our numerical method.

One feature characterizing the transition from bound states to annihilation is the magnitude of the interaction time before the partial annihilation occurs, as is plotted in Fig. 5. An additional way to study the regime where partial annihilation arises is by means of the area of the interacting pulses as a function of time; we define $I_A(t) = \int |A(x,t)|dx$, $I_B(t) = \int |B(x,t)|dx$. In Fig. 7 we show the time evolution of $I_A(t)$ and $I_B(t)$ for two different values of the velocity field, one closer to the boundary to bound states ($v = 0.8$) and one close to complete annihilation ($v = 1.22$). As one can see, there are substantial temporal oscillations of both amplitudes for the larger value of v. In Fig. 7(b) we have indicated by Δ the amplitude of the oscillations of $I_A(t)$ and $I_B(t)$ before partial annihilation arises. Δ is plotted as a function of v-v_{c1} in Fig. 8. We conclude that the maximum Δ increases by more than two orders of magnitude monotonically as v_{c1} is approached. In Fig. 9 we have plotted the amount of additive noise needed to induce the transition from stationary bound states to partial annihilation as a function of the velocity for $T = 2000$. As v_{c1} is approached, the amount of additive noise required decreases

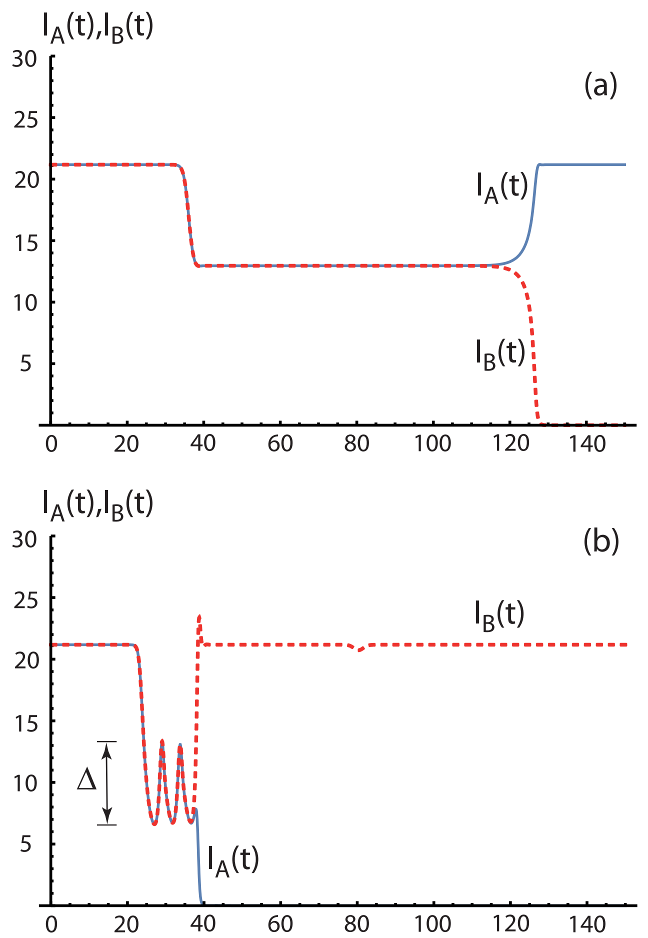

FIG. 7. The time evolution of both pulses is plotted for (a) $c_r = -0.7$, $v = 0.8$ and (b) $c_r = -0.7$, $v = 1.22$. The maximum amplitude of the oscillation of $I_A(t)$ and $I_B(t)$, Δ, is indicated. Parameters are as in Fig. 2.

linearly and only in the immediate vicinity a deviation from this linearity emerges.

A second system showing a breaking of symmetry leading to partial annihilation consists of two coupled complex cubic Ginzburg-Landau equations with nonlinear gradient terms for counterpropagating waves,

$$\partial_t A - v\partial_x A = \mu A + (\beta_r + i\beta_i)|A|^2 A - iR(|A|^2)_x A + (c_r + ic_i)|B|^2 A + (D_r + iD_i)\partial_{xx} A, \quad (3)$$

$$\partial_t B + v\partial_x B = \mu B + (\beta_r + i\beta_i)|B|^2 B + iR(|B|^2)_x B + (c_r + ic_i)|A|^2 B + (D_r + iD_i)\partial_{xx} B, \quad (4)$$

where $A(x,t)$ and $B(x,t)$ are complex fields. A and B are slowly varying envelopes, β_r is positive, and R corresponds to the Raman term in nonlinear optics. Recently, it was shown that a cubic Ginzburg-Landau equation with nonlinear gradient terms has stable stationary dissipative solitons over a fairly large range in parameter space [27].

We have simulated Eqs. (3) and (4) for the following values of the parameters: $\mu = -0.012$, $\beta_r = 0.3$, $\beta_i = 1.0$, $R = 0.1$, $c_i = 0$, $D_r = 0.6$, $D_i = 0.5$. The time step dt used was $dt =$

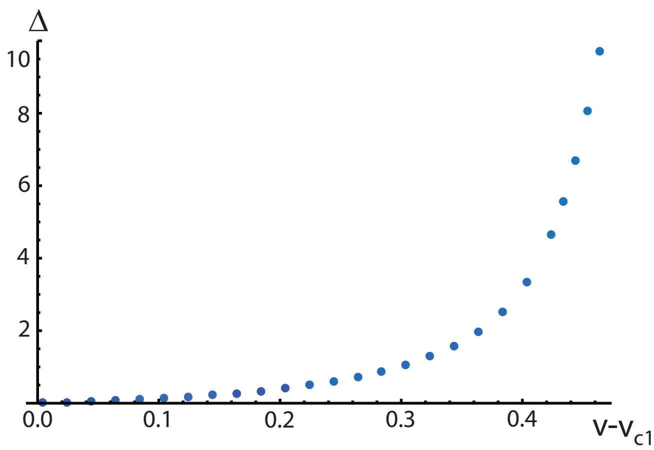

FIG. 8. The maximum amplitude of the oscillation of $I_A(t)$ and $I_B(t)$, Δ, is plotted as a function of the distance from the critical velocity v_{c1}.

0.003, the grid spacing $dx = 0.06$, $N = 1250$, leading to a box size of $L = 75$.

For these values one single pulse moves with a velocity $v_0 = 1.863$. By setting in Eqs. (3) and (4) the group velocity $v = v_0 - v'$, we show in Fig. 10 the x-t plots corresponding to the process of partial annihilation for two coupled complex Ginzburg-Landau equations with nonlinear gradients. During the interaction we observed a meandering oscillatory bound state.

In conclusion, we have shown in this Rapid Communication that partial annihilation of dissipative solitons can arise over a large parameter region for the approach velocity and a stabilizing interaction of the counterpropagating pulses due to spontaneously broken left-right symmetry in driven dissipative systems.

In Ref. [24] we have shown that the noise intrinsic to the numerical method can lead at the boundaries between different outcomes of collisions to partial annihilation (for example, between annihilation and interpenetration). In

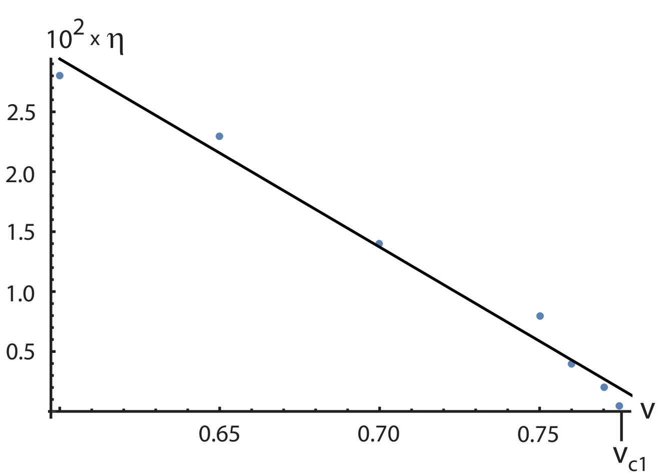

FIG. 9. The amount of additive noise η needed to induce the transition form stationary bound states to partial annihilation is plotted as a function of the velocity for $c_r = -0.7$ and $T = 2.000$.

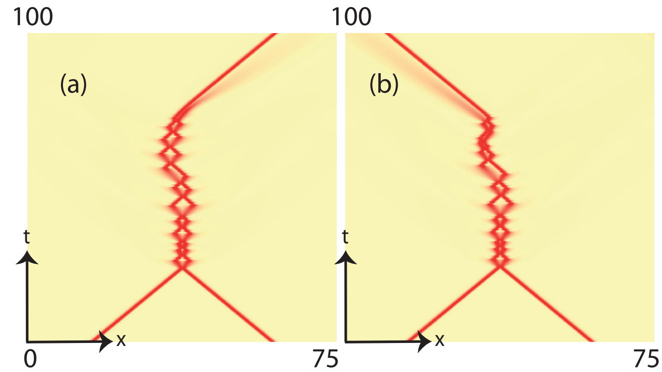

FIG. 10. For $c_r = -0.2$ and $v' = 0.93$, the process of partial annihilation for Eqs. (3) and (4). (a) x-t plot showing the surviving pulse moving to the right. (b) x-t plot showing the surviving pulse moving to the left.

addition, we showed in Ref. [24] that the interval over which one can get partial annihilation can be increased by adding noise.

In the present Rapid Communication we demonstrate that for noise intrinsic to the method (of fixed amplitude $\sim 10^{-16}$), one can obtain partial annihilation for a large region of parameter space in approach velocity and cross-coupling between the counterpropagating waves. This is due to a spontaneous breaking of symmetry for a sufficiently large approach velocity and cross-coupling strength of the counterpropagating dissipative solitons. Noise only serves to induce the left or right direction. We are thus in a rather different part of the phase diagram and the phenomena described in Fig. 2 of the present Rapid Communication are for fixed intrinsic noise.

There is a large number of pattern-forming experimental systems for which such a behavior can be expected. Experimental areas of interest include surface reactions, convective onset, biosolitons, and nonlinear optics.

A natural possibility to generalize the analysis presented will be the application to time-dependent DSs oscillating with one frequency, two frequencies, or chaotically [28]. While interactions between such time-dependent DSs have been studied [29,30], the parameter region in question has not been covered so far. Even more promising appears to be the analysis of colliding exploding DSs. Exploding DSs, which have been found first in Ref. [31] and then also detected experimentally in Ref. [32], are a rather special class of DSs. While they stay spatially localized as a function of time, they show a complex spatiotemporal dynamics. As a function of time they undergo several changes: They start with an unstable pulse shedding radiation or "phonons" [31,33,34], then, in one of the wings of the unstable pulse perturbations grow to generate another pulselike object. These two pulses—both unstable—interact and form a rather broad, high, and highly unstable pulse. In a last step, the latter collapses [31,33,34] to form the unstable pulse reminiscent in shape of a stable fixed shaped pulse. The overall behavior is characterized by a cycle time that fluctuates around an average. While single exploding DSs have been studied theoretically in some detail [35–39], more recently there is also growing interest in their experimental properties in various nonlinear optical systems [40,41]. However, there appears to be so far only one study dealing with the interaction of exploding DSs [42], thus opening the door to another field of investigations.

O.D. wishes to acknowledge the support of FONDECYT (Project No. 1200357) and Universidad de los Andes through FAI initiatives. H.R.B. thanks the Deutsche Forschungsgemeinschaft for partial support of this work.

[1] M. C. Cross and P. C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1993).

[2] A. C. Newell, *Solitons in Mathematics and Physics* (Society for Industrial and Applied Mathematics, Philadelphia, 1985).

[3] *Dissipative Solitons: From Optics to Biology and Medicine*, edited by N. Akhmediev and A. Ankiewicz (Springer, New York, 2008).

[4] H. Kuwayama and S. Ishida, Sci. Rep. **3**, 2272 (2013).

[5] H. H. Rotermund, S. Jakubith, A. von Oertzen, and G. Ertl, Phys. Rev. Lett. **66**, 3083 (1991).

[6] A. von Oertzen, A. S. Mikhailov, H. H. Rotermund, and G. Ertl, J. Phys. Chem. B **102**, 4966 (1998).

[7] P. Kolodner, Phys. Rev. A **44**, 6448 (1991).

[8] P. Kolodner, Phys. Rev. A **44**, 6466 (1991).

[9] J. Peng and H. Zeng, Commun. Phys. **2**, 34 (2019).

[10] P. R. Bauer, A. Bonnefont, and K. Krischer, Sci. Rep. **5**, 16312 (2015).

[11] A. C. Newell and J. A. Whitehead, J. Fluid Mech. **38**, 279 (1969).

[12] L. A. Segel, J. Fluid Mech. **38**, 203 (1969).

[13] H. R. Brand, P. S. Lomdahl, and A. C. Newell, Phys. Lett. A **118**, 67 (1986); Physica D **23**, 345 (1986).

[14] J. Swift and P. C. Hohenberg, Phys. Rev. A **15**, 319 (1977).

[15] H. Sakaguchi and H. R. Brand, Physica D **117**, 95 (1998).

[16] H. R. Brand and R. J. Deissler, Physica A **204**, 87 (1994).

[17] V. B. Taranenko, K. Staliunas, and C. O. Weiss, Phys. Rev. A **56**, 1582 (1997).

[18] O. Thual and S. Fauve, J. Phys. (France) **49**, 1829 (1988).

[19] H. R. Brand and R. J. Deissler, Phys. Rev. Lett. **63**, 2801 (1989).

[20] R. J. Deissler and H. R. Brand, Phys. Rev. A **44**, R3411 (1991).

[21] S. Yamanaka and T. Ohta, Phys. Rev. E **90**, 042927 (2014).

[22] M. Bar, M. Eiswirth, H. H. Rotermund, and G. Ertl, Phys. Rev. Lett. **69**, 945 (1992).

[23] R. J. Deissler and H. R. Brand, Phys. Lett. A **146**, 252 (1990).

[24] O. Descalzi, J. Cisternas, D. Escaff, and H. R. Brand, Phys. Rev. Lett. **102**, 188302 (2009).

[25] T. Ohta, Y. Hayase, and R. Kobayashi, Phys. Rev. E **54**, 6074 (1996).

[26] M. Argentina, P. Coullet, and V. Krinsky, J. Theor. Biol. **205**, 47 (2000).

[27] M. Facão and M. I. Carvalho, Phys. Rev. E **92**, 022922 (2015).

[28] R. J. Deissler and H. R. Brand, Phys. Rev. Lett. **72**, 478 (1994).

[29] R. J. Deissler and H. R. Brand, Phys. Rev. Lett. **74**, 4847 (1995).

[30] O. Descalzi and H. R. Brand, Chaos **28**, 075508 (2018).

[31] J. M. Soto-Crespo, N. Akhmediev, and A. Ankiewicz, Phys. Rev. Lett. **85**, 2937 (2000).

[32] S. T. Cundiff, J. M. Soto-Crespo, and N. Akhmediev, Phys. Rev. Lett. **88**, 073903 (2002).

[33] N. Akhmediev, J. M. Soto-Crespo, and G. Town, Phys. Rev. E **63**, 056602 (2001).

[34] O. Descalzi and H. R. Brand, Phys. Rev. E **82**, 026203 (2010).

[35] N. Akhmediev and J. M. Soto-Crespo, Phys. Rev. E **70**, 036613 (2004).

[36] O. Descalzi, C. Cartes, J. Cisternas, and H. R. Brand, Phys. Rev. E **83**, 056214 (2011).

[37] J. M. Soto-Crespo, N. Akhmediev, N. Devine, and C. Mejia-Cortes, Opt. Express **16**, 15388 (2008).

[38] C. Cartes, O. Descalzi, and H. R. Brand, Phys. Rev. E **85**, 015205(R) (2012).

[39] O. Descalzi, N. Akhmediev, and H. R. Brand, Phys. Rev. E **88**, 042911 (2013).

[40] A. F. J. Runge, N. G. R. Broderick, and M. Erkintalo, Optica **2**, 36 (2015).

[41] M. Liu, A.-P. Luo, Y.-R. Yan, S. Hu, Y.-C. Liu, H. Cui, Z.-C. Luo, and W.-C. Xu, Opt. Lett. **41**, 1181 (2016).

[42] O. Descalzi and H. R. Brand, Eur. Phys. J. B **88**, 219 (2015).