Dynamics of a ratchet gear powered by an active granular bath

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Recent experiments show universal features of ratchet gear dynamics that are powered by different types of active baths. We investigate further for the case of a ratchet gear in a bath of self-propelling granular rods (SPRs). The resulting angular velocity was found to follow a nonmonotonic dependence to the SPR concentration similar to the observation from other active bath systems. This behavior is caused by the interplay of the momentum transfer of the SPRs in the trapping regions of the gear and the mean velocity of the SPRs inside the bath. For all SPR concentrations, we found that the angular velocity is proportional to the product of the number of SPRs pushing the gear and the SPRs mean velocity.

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I. INTRODUCTION

Although the second law of thermodynamics prohibits the existence of a perpetual machine of the first kind, ratchet devices that enable us to extract energy from a nonequilibrium bath are permissible, called Brownian motors [1,2]. Typically, such devices couple a ratchet gear, with broken spatial symmetry, with a nonequilibrium bath of self-propelling particles (SPPs). Thus, an active bath of *Escherichia coli* (*E.coli*) was used to drive microscopic gears to a net work [3–5]. Other experiments using *Bacillus subtilis* [6] and the swarming bacteria *V. alginolyticus* [7] revealed the same effect of net gear rotation. In many recent experiments, ratchet gear rotation was achieved in a bath composed of self-propelling catalytic Janus particles in a hydrogen peroxide solution [8]. If such Brownian motors are used for technological applications such as those in microfluidic devices, understanding how to control the gear rotation is critical [3–8].

Net gear rotation of the above mentioned experiments is shown to be caused by the momentum transfer of the SPP movement to the gear. The gear asymmetry leads to preferred rotation to one side than the other [3,4,8]. Another general feature is that for different types of SPPs, the resulting gear angular velocity was found to increase and then decrease at higher SPP concentrations. For the case of *Bacillus subtilis* powered gears, this is attributed to a decrease in bacterial motility as an effect of quorum sensing and biofilm formation [9,10]. Jamming like states of the *V. alginolyticus* are also thought to cause gear angular velocity to decrease [7]. While it is believed that competition on the availability of fuel (hydrogen peroxide) led to slower movement of Janus particles and thus slower rotation of the gear [8]. A universal physical picture of the gear rotation ω, independent of the type of SPPs comprising the active bath, can thus be constructed from the SPP ensemble average velocity $\langle v \rangle$, as such,

$$\omega \propto \langle v^{\alpha} \rangle. \tag{1}$$

$\alpha = 2$ was proposed in the *B. Subtillis* [10] experiments while $\alpha = 1$ for the case of catalytic Janus particle experiments [8]. Investigations to check the validity of Eq. (1) via single particle tracking for different SPP concentrations are difficult due to the characteristic length scale of the SPPs used in previous studies. Furthermore, control of bacterial concentrations during experiments are oftentimes difficult [6,10].

We thus perform experiments in the macroscopic scale that mimic the microscopic experiments discussed above to elucidate the physical mechanism leading to gear rotation. This is done by replacing the "live" *bacteria* with self-propelled rods (SPRs) under vibration [11]. In this approach for instance, intercell information sharing (e.g., quorum sensing), which is thought important in Ref. [6], can be turned off. Furthermore, the effect of collective motion can be easily measured in the macroscopic case. We note that similar ratchet experiments were done in Ref. [12] where the chaotic motion of granular beads can propel the asymmetric probe along the direction of the asymmetry. Vanes of a symmetric ratchet were made to have a different coefficient of restitution and thereby the ratchet undergoes rotation due to the unequal momentum transfer [13] while a thermal ratchet was demonstrated whose mechanism is caused by Coulomb friction [14]. However, the granular studies mentioned still do not mimic the experiments in Refs. [4,6,7] where an active particle constitutes the bath.

Our results shows spontaneous gear rotation at some appreciable concentration of the SPR, less than the SPR concentration required for collective motion. Nonmonotonic behavior of ω with the SPR concentration is also observed similar to previous studies with $\alpha = 1$ verified for all concentrations.

II. EXPERIMENT METHODOLOGY

The experimental setup consists of a ratchet gear in a bath of SPRs, see Fig. 1(a). The SPR follows the design of Ref. [11], where a spherocylindrical hollow bead (diameter 3 mm and 5 mm long) is attached to a chain of six spherical hollow beads (diameter 1.5 mm). The spherical beads are connected to each other by a flexible link with one of the beads clipped inside the spherocylindrical bead giving a total

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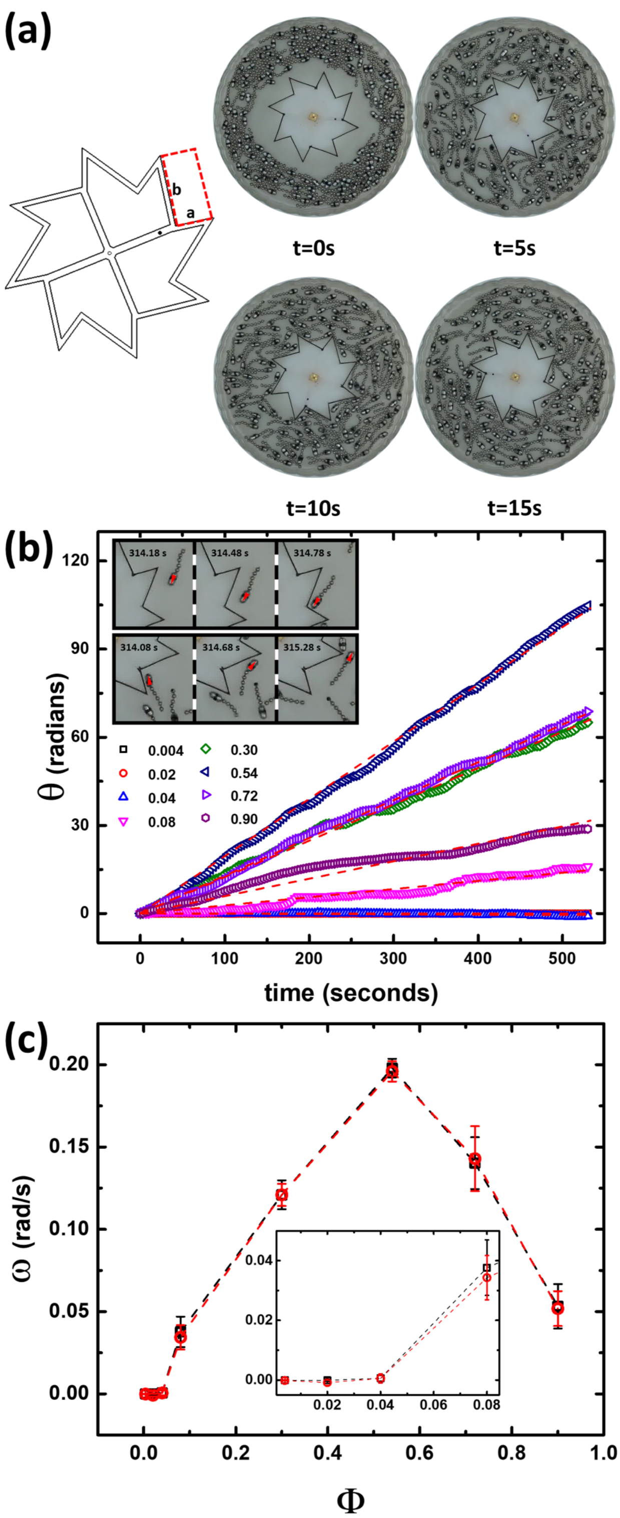

FIG. 1. (a) Sequence of images showing gear rotation for $\Phi = 0.54$. Leftmost image is schematic of the trapping region of the gear. (b) Time series of the angular position of the gear for different Φ. The broken lines are linear fitting. Inset shows two modes of approach of the SPR towards the gear's tooth. (c) Angular speed of the gear for different concentrations of the SPR. □ and ◯ corresponds to the value of ω acquired from linear fitting of $\theta(t)$ and quadratic fitting of $\langle[\theta(t+\Delta t)-\theta(t)]^2\rangle$, respectively. Open symbols and the error bars are the average and standard deviation of three trials, respectively. Inset shows the onset of net rotation occurring at $\Phi > 0.04$.

length of $l = 17 \pm 2$ mm. We varied the concentration of the SPR from $\Phi \sim 0.004$ to $\Phi \sim 0.90$ ($1 \leqslant N_{\mathrm{SPR}} \leqslant 225$). For $0.004 \leqslant \Phi \leqslant 0.54$ the SPRs were initially distributed evenly in the space outside the gear's radius R_{gear} [3]. However, for $\Phi > 0.54$ we arranged the SPRs in a manner that they are well distributed in the chamber.

Ratchet gears were fabricated via laser cutting a PMMA (*Polymethyl methacrylate*) into a gear with eight teeth, an outer diameter of 63 mm, and thickness of 5 mm. The short axis of the gear's teeth is $a = 11$ mm while the long axis was set at $b = 19$ mm—enough to accommodate the entire length of a single SPR. The gear is fixed at the center of the chamber with a spacer that lifts the gear at a distance not more than the diameter of the smaller beads.

The gear and the SPRs are enclosed in a circular chamber made of PMMA ($d = 120$ mm) with a wavelike structure along the boundary to prevent particle aggregation [15]. An electromagnetic shaker (Modal, 2075E) is used to vibrate the entire system (gear and the SPRs) with a driving of $\Gamma \simeq 3.6$ g and frequency of 30 Hz. No net rotation of the gear was observed when $N_{\mathrm{SPR}} = 0$. The experiment was recorded using a digital camera at 60 frames/s. The angle of rotation was computed from the position difference of a point marker and the center of the gear using a home-made particle tracking software with a resolution better than 0.017 rad.

III. RESULTS AND DISCUSSIONS

The persistent motion of the SPRs leads them to occupy the trapping region depicted in Fig. 1(a) [16]. There are two possible docking modes of the SPRs in the trapping region: when the SPR is parallel with the surface of b and when the SPR is parallel with the surface of a as shown in the inset of Fig. 1(b) (sample movie in the Supplemental Material [17]). Both cases cause momentum transfer into the gear. However, the net rotation was observed only in the former case. Most SPRs collide with the gear since the wavelike structure minimizes aggregation of SPRs in an area of the boundary. Only for $\Phi \geqslant 0.8$ that the accumulation of SPR at any point in the boundary persists for a very long time due to close packing. Time series of the angular position of the gear shows a net drift and a stochastic component, caused by the random bombardment of the SPR in both sides of the trapping regions as shown in Fig. 1(b).

We characterized the angular position fluctuations via the mean squared angular displacement $\mathrm{MSAD}(\Delta t) = \frac{1}{N}\sum_{i=1}^{N}[\theta(t+\Delta t)-\theta(t)]^2$, where N is the total number of data set windows for a given time interval Δt. Figure 2(a) shows the calculated MSAD for different SPR concentrations which shows a quadratic growth with Δt that are more pronounced at higher values of SPR concentrations until $\Phi = 0.54$ where it starts to drop down.

Following earlier studies in Refs. [18,19], we model the asymmetric gear rotation with a Langevin type equation given by

$$\frac{d\theta}{dt} = \omega + \zeta(t), \tag{2}$$

where $\zeta(t)$ is a Guassian white noise with $\langle\zeta(t)\rangle = 0$ and is Dirac delta correlated with $\langle\zeta(t)\zeta(t')\rangle = 2D\delta(t-t')$, D

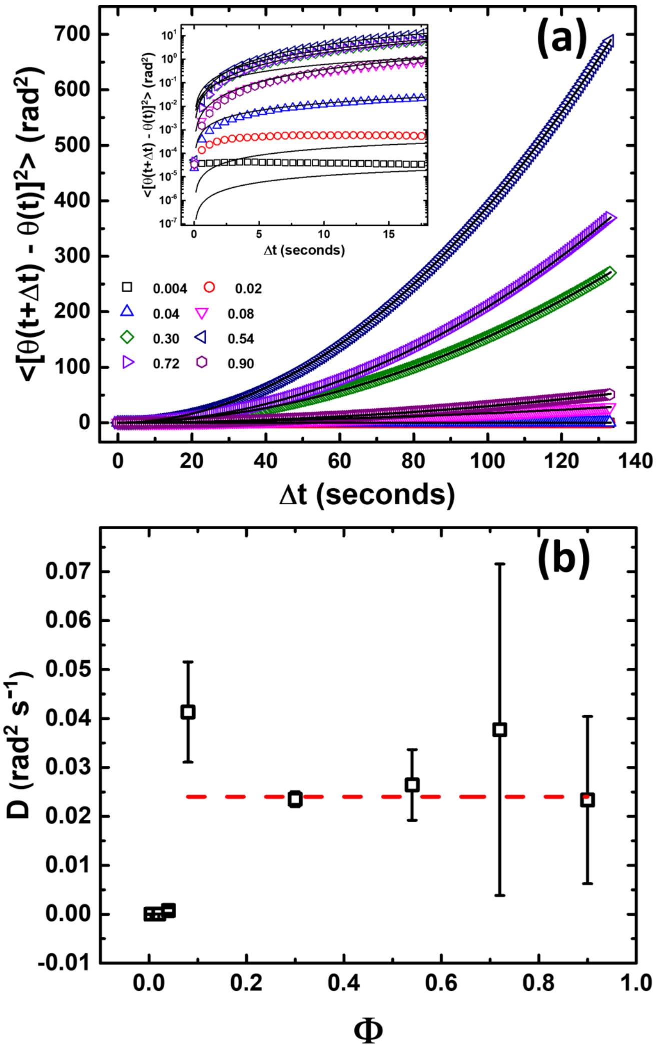

FIG. 2. (a) MSAD of the asymmetric gear with solid line as fitting from Eq. (3). (b) Rotational diffusion coefficient D of the asymmetric gear for different concentrations of the SPRs. Open box and error bar are the average and standard deviation of three trials, respectively. Red dashed line corresponds to the mean value of $\langle D\rangle \sim 0.024\ \frac{\mathrm{rad}^2}{\mathrm{s}}$ for $0.08 \leqslant \Phi \leqslant 0.90$.

being the rotational diffusion coefficient of the asymmetric gear. Equation (2) can be integrated to yield the MSAD as follows:

$$\langle[\theta(t+\Delta t)-\theta(t)]^2\rangle = 2D\Delta t + \omega^2\Delta t^2. \tag{3}$$

The first term corresponds to the short timescale dynamics of the asymmetric gear where fluctuations are assumed to be dominant. The second term corresponds to the long time dynamics where the net rotation of the asymmetric gear is observed with an angular velocity ω. Equation (3) is fitted to the experimental data of MSAD as depicted by the solid lines in Fig. 2(a) where excellent agreement is obtained except for $\Phi \leqslant 0.04$ where the linear approximation at short timescale fails. This is due to the very low chance of an SPR-gear collision and as a consequence $D \sim 0\ \mathrm{rad}^2\,\mathrm{s}^{-1}$ for these concentration ranges as shown in Fig. 2(b). Meanwhile, for $\Phi \geqslant 0.08$ the values of D, on average, approaches a constant value as depicted by the red dashed line at $D \sim 0.024\ \mathrm{rad}^2\,\mathrm{s}^{-1}$. The angular position fluctuations can be approximated as $\Delta\theta \simeq \tan^{-1}[\frac{\langle v\rangle\Delta t}{R_{\mathrm{gear}}}]$ where $\langle v\rangle\Delta t$ is the displacement of the SPR inside the trapping region for a particular time interval and $\langle v\rangle$ is the mean velocity of the SPR in the bath. For an SPR present in the trapping region of the asymmetric gear the rotational diffusion coefficient can be roughly estimated as $D \sim \frac{\Delta\theta^2}{\Delta t} = \frac{[\tan^{-1}(\frac{\langle v\rangle\Delta t}{R_{\mathrm{gear}}})]^2}{\Delta t}$ which on average gives us $D \simeq 0.023\ \mathrm{rad}^2\,\mathrm{s}^{-1}$ which is in strong agreement with the experimental value.

The net angular velocity of the gear (ω), for both MSAD and linear fitting of $\theta(t)$, follows a nonmonotonic dependence with Φ as seen in Fig. 1(c). The strong agreement of the values of ω generated from MSAD and linear fitting of the angular trajectory suggests that the white noise approximation for the angular fluctuations is valid. Three phases in the dynamics of asymmetric gear were observed, initially at low Φ, few SPR collisions with the gear did not result in a noticeable rotation until $\Phi > 0.04$ as shown in the inset of Fig. 1(c). For $0.04 < \Phi \leqslant 0.54$, we observed a linear rise of ω until $\Phi = 0.54$ after which ω decreases for higher SPR concentrations.

The same nonmonotonic behavior of ω with Φ is also observed for gears in active baths composed of catalytic Janus particles and bacteria [6,8], we thus deduced that this dynamics of ratchet gears in an active bath is universal. For catalytic Janus particles powered gears, the interplay of drag and concentration of H_2O_2 resulted in the observed peak in ω [8]. While in bacterial powered gears, gear dynamics is a consequence of the collective motion of the bacteria and the effective viscosity of the medium due to the presence of the bacteria [6]. In contrast, collective motion for our system only occurs at higher concentration, $\Phi = \frac{3\pi w}{2l} \simeq 0.8$ ($w = 3$ mm; $l = 17 \pm 2$ mm) [11]. On the other hand, our observation reveals that the torque required us to rotate the gear is dependent on the number of SPRs in the trapping regions, we conjecture that in its most basic form, the observed universal behavior can be captured by just looking at the average number of SPRs trapped in the trapping region of the gear and the mean velocity of the SPRs as presented in Eq. (1).

First, we make measurements on the average number of SPRs inside the trapping regions of the gear that move towards the direction of rotation at some time interval. Increasing the SPR concentration increases the number of SPRs trapped in the trapping regions up to $\langle N_{\mathrm{trap}}\rangle \simeq 18$. This implies that the trapping regions accommodates approximately two SPRs at maximum, see Fig. 3(a). SPR velocities were measured from individual SPR trajectories, $v = \frac{\Delta r}{\Delta t}$, where Δr is the net displacements of the SPRs after some time duration Δt. In particular, we choose $\Delta t = [1\ \mathrm{s}, 4\ \mathrm{s}]$ since these timescales were comparable to the dwell time of the SPR inside the trapping regions of the gear especially for concentrations of the SPRs where the onset of rotation is seen. The SPR velocity average $\langle v\rangle$ were then extracted as the mean of the SPR velocity distribution. We plot $\langle v\rangle$ for different SPR concentrations in Fig. 3(b) and found $\langle v\rangle$ to decrease for increasing Φ. This can be understood by looking at the diffusion of a single SPR in the bath, increasing SPR concentrations hinders an SPR to move freely as its neighboring SPRs act as barriers which results in $\langle v\rangle \propto -\Phi$ [11].

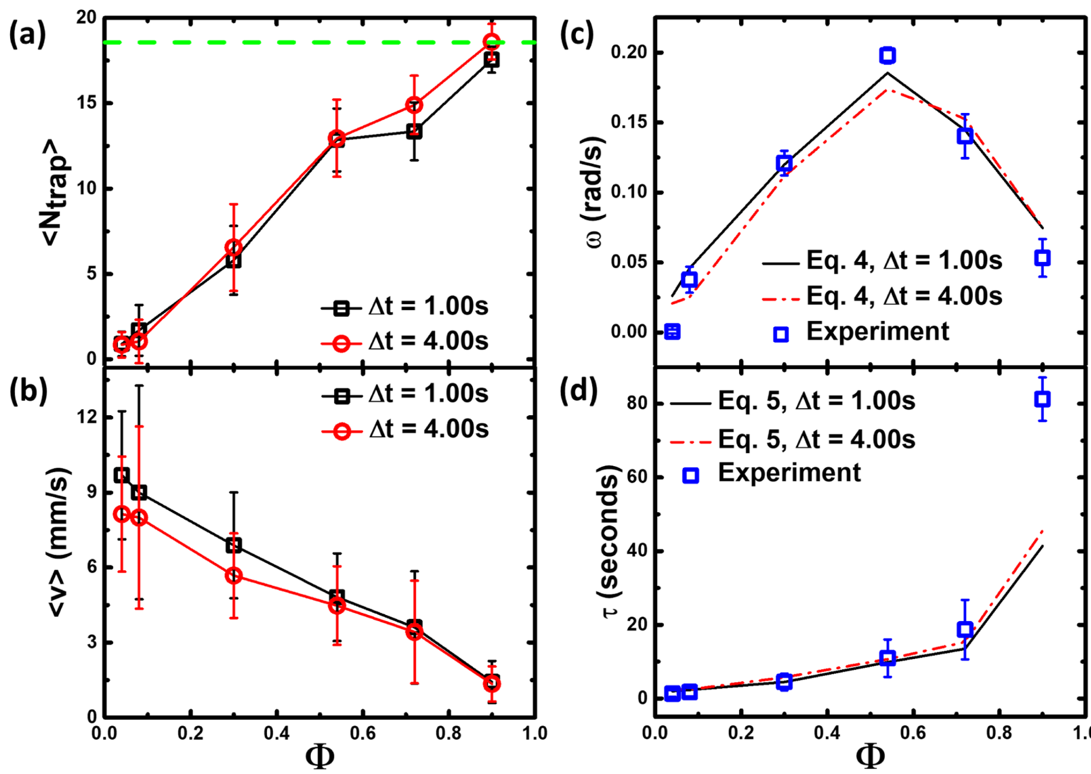

FIG. 3. (a) The mean number of the SPRs in the trapping regions of the gear, error bar is the standard deviation for three trials. (b) Mean velocity of the SPRs in the entire system for varied SPR concentrations, error bar is the standard deviation for 150 SPRs. (c) Experimental data of ω compared to theoretical estimate, Eq. (4). (d) Dwell time statistics of the SPRs in the trapping region of the gear as compared to estimate, Eq. (5).

We now proceed in estimating ω from the net torque exerted by the SPRs to the gear. The net torque is the sum of the total torques delivered by each SPR in the trapping regions $T \leqslant \sum_{i=1}^{\langle N_{\rm trap}\rangle} R_{\rm gear} f_i$, where $R_{\rm gear}$ is the radius of the gear and f_i is the net force exerted by the SPR. Since $R_{\rm gear} \gg$ SPR head diameter and assuming constant force, then the net force exerted by the SPRs can be estimated as $F \simeq \langle N_{\rm trap}\rangle f$. We can write the net torque as $T \simeq \langle N_{\rm trap}\rangle R_{\rm gear} f$. The SPR-gear collision results in a change of the SPR momentum proportional to its prior velocity which gives us $f \sim \frac{m_{\rm SPR}\langle v\rangle}{\Delta t}$. The SPR induces torque that leads to an increase in the kinetic energy of the gear after a time interval Δt by K.E. $= T\omega\Delta t$. We can also estimate the kinetic energy of the gear as K.E. $= \frac{1}{2} I\omega^2$, where I is the gear moment of inertia which is estimated to be that of a thin disk $I = \frac{1}{2} m_{\rm gear} R_{\rm gear}^2$ [3,4,6]. Therefore, the angular velocity of the gear is given by

$$\omega = \beta\left(\frac{4m_{\rm SPR}}{m_{\rm gear}R_{\rm gear}}\right)\langle N_{\rm trap}\rangle\langle v\rangle, \tag{4}$$

where β is a correction factor due to the disk approximation of I and also that of friction of the gear with spacer. We plotted in Fig. 3(c) the experimentally obtained values of ω with that of Eq. (4), which shows excellent agreement with the experimental measurements where $m_{\rm SPR} = 0.23$ grams, $m_{\rm gear} = 1.6$ grams, and $\beta = \frac{1}{3}$ for optimal fitting. Two differing dependence of ω with $\langle v\rangle$ have been proposed with the energetic nature of $\omega \propto \langle v^2\rangle$ for the bacterial experiments [6] and the mechanistic picture of $\omega \propto \langle v\rangle$ for the catalytic Janus powered gears [8]. For small SPR concentrations, both dependencies should yield similar results for ω. However, energetic assumptions for higher Φ is expected to yield lower estimates due to jamming and collective motion [6,11,20].

We also perform an estimate on the SPR dwell time data in Fig. 3(d). The total time an SPR spends inside a trapping region is the sum of the time it needs to disentangle from another SPR and the time it needs to travel the entire length of the trapping region. Using the model proposed by Kudrolli [11], we thus have

$$\tau = \frac{l}{\langle v\rangle} + \frac{\frac{\langle N_{\rm trap}\rangle}{8} b}{\langle v\rangle}. \tag{5}$$

We plotted the experimentally measured τ and compared it with the predictions of Eq. (5) for different Φ, see Fig. 3(d). Our estimate shows very good agreement with experimental data for $\Phi < 0.8$. Close packing becomes so prevalent for $\Phi \geqslant 0.8$ that it takes a long time for a single SPR to move past its nearest neighboring SPR and thus the time needed for an SPR to disentangle with another SPR is $\gg \frac{l}{\langle v\rangle}$ [11,20].

IV. CONCLUSION

Our experiments have obtained the same empirical result on the SPP concentration dependence of the ratchet gear rotation with that of other active matter systems [6,8]. The universality of the dynamics is attributed to the total pressure exerted by the SPR on the trapping region of the gear. Such swim pressure, as introduced by Brady *et al.* [21], on the other hand is governed by the number of SPRs on the trapping region and its mean bulk speed. For a myriad of systems, it is expected that $\langle v \rangle \propto -\Phi$ due to steric neighbor interaction and not just because of the availability of fuel as discussed in the catalytic Janus experiments [8]. For instance, light activated Janus particles will not have the fuel problems [22]. The next step is to understand the thermodynamics of active ratchet gears, independent of the active matter system, of which the swim pressure again is believed to have a central role [23].

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