PHYSICAL REVIEW E **101**, 022128 (2020)

Dynamic phase transition in the contact process with spatial disorder: Griffiths phase and complex persistence exponents

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(Received 29 July 2019; revised manuscript received 31 December 2019; accepted 30 January 2020; published 24 February 2020)

We present a model which displays the Griffiths phase, i.e., algebraic decay of density with continuously varying exponents in the absorbing phase. In the active phase, the memory of initial conditions is lost with continuously varying complex exponents in this model. This is a one-dimensional model where a fraction r of sites obey rules leading to the directed percolation class and the rest evolve according to rules leading to the compact directed percolation class. For infection probability $p \leqslant p_c$, the fraction of active sites $\rho(t) = 0$ asymptotically. For $p > p_c$, $\rho(\infty) > 0$. At $p = p_c$, $\rho(t)$, the survival probability from a single seed and the average number of active sites starting from single seed decay logarithmically. The local persistence $P_l(\infty) > 0$ for $p \leqslant p_c$ and $P_l(\infty) = 0$ for $p > p_c$. For $p \geqslant p_s$, local persistence $P_l(t)$ decays as a power law with continuously varying exponents. The persistence exponent is clearly complex as $p \to 1$. The complex exponent implies logarithmic periodic oscillations in persistence. The wavelength and the amplitude of the logarithmic periodic oscillations increase with p. We note that the underlying lattice or disorder does not have a self-similar structure.

DOI: 10.1103/PhysRevE.101.022128

I. INTRODUCTION

The understanding of phase transition in equilibrium statistical physics was a major success in theoretical physics in the 1970s. The concepts of scaling and renormalization group were introduced to explain the divergence of characteristic length scales near continuous phase transitions. An extension of these ideas to the nonequilibrium systems is an active area of research with applications ranging from granular matter to epidemics [1–3]. The power laws associated with the continuous transitions are observed in very close vicinity of the critical point. They are found precisely in theoretical models and are a signature of the universality class of associated phase transition. Very few experimental verifications of such power laws are obtained. It requires extreme fine-tuning of experimental parameters. Nonetheless, the power laws in space and time are ubiquitous. This has led to the paradigm of self-organized criticality which often requires adiabatic drive [4–6]. Another reason for power laws can be disorder and inhomogeneities [7–9].

Usually, the power laws have real exponents. A complex exponent will lead to log-periodic oscillatory corrections to the power law. Such oscillations have been obtained or predicted in a few situations [10–12]. For example, observation of log-periodic oscillations in the stock market has been associated with the possibility of a crash. Some understanding of such oscillations at the critical point when underlying lattice or disorder has self-similar characteristics is obtained [10,13,14]. Such oscillations have also been seen in quantum systems [15,16]. In this work, we observe log-periodic oscillations in the memory of initial conditions even when there is no self-similar structure in the lattice or disorder and the system is far away from the critical point. This can be interpreted as an outcome of quenched disorder alone.

Most theoretical studies in the equilibrium and the nonequilibrium phase transitions involve idealized homogeneous systems. It is a useful approximation (like equilibrium). But the real-life systems involve inhomogeneities invariably. They play an important role in several experiments. For example, in the catalytic reactions, the catalytic surface is not homogeneous. The inhomogeneities can change or destroy critical behavior [9]. The theory of spin glasses in equilibrium systems has given useful insights into disordered systems. It has found applications in fields ranging from evolution to computer science [17].

In this work, we focus on the absorbing phase transitions in nonequilibrium processes in the presence of spatial disorder. We take the prototypical and widely studied class of directed percolation (DP). It is characterized by one component order parameter, short-range dynamical rule, no additional symmetries, and no quenched disorder [18,19]. The experimental verification of DP is rare, although it is very well studied in theory and simulations. The experimental verifications are related to the spatiotemporal intermittency [20]. There are some more universality classes for the transition to an absorbing phase. One of them is compact directed percolation (CDP). In this class, if all the neighbors of a given site are active, it becomes active with probability 1 [21].

Certain justifications have been proposed for difficulties in observing DP behavior in experiments [20]. Intrinsic fluctuations may smear out the transition to the truly absorbing phase.

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DP does not take into account possible intermediate phases in the reaction sequence. The nature of the update (parallel or random sequential), can change the nature of the transition. Finally, realistic systems cannot avoid random inhomogeneity of some kind. Not only the experimental values of the exponents may vary from their theoretical prediction, but the critical behavior may be destroyed due to the inhomogeneity [22].

We study the impact of spatially quenched disorder on the DP universality class. We consider a very strong disorder in which the universality class of the underlying system changes completely. Several studies have been carried out to study the effect of spatially quenched disorder in the past. However, a clear picture is still eluding. The studies on the effect of quenched disorder on the DP model show that even very weak randomness can drastically modify the phase diagram and critical behavior [9]. According to the Harris criteria, the quenched disorders are relevant perturbations if $d\nu_\perp < 2$ where d is the dimensionality and $\nu_\perp$ is the correlation length exponent in the spatial direction of the pure system [23]. In particular, we consider disorder in which half the sites obey DP and the other half obey CDP rules. This is a very strong perturbation. For $d = 1$, $\nu_\perp = 1.09$ for DP and $\nu_\perp = 1$ for CDP. Since $\nu_\perp < 2$ in either case, we can expect quenched disorder to be a relevant perturbation.

In the CDP universality class, we observe compact percolation clusters. It is characterized by an additional $\mathbb{Z}_2$ symmetry. Here, an active site with all active neighbors cannot become inactive. The transition is governed by random walks at the end of the string [21].

We find that the presence of random spatial inhomogeneity leads to a change in the critical behavior and ultraslow dynamics in the one-dimensional (1D) model. The system undergoes algebraic decay with changing exponents in a part of the absorbing phase, followed by stretched exponential and exponential decay. The region with generic power laws is known as Griffiths phase. It is known to emerge from rare region effects due to the presence of quenched defects [24]. The rare regions locally favor one phase over the other, i.e., although the bulk is globally in the absorbing phase, the rare regions are locally in the fluctuating phase with over average percolation probability. The dynamics of rare regions are extremely slow which leads to stretched exponential decay or algebraic decay [25].

Of late, there has also been interest in the possibility of determining the nature of phase transition from short-time dynamics. The short-time dynamics of the spin-glass model and the Baxter-Wu model were studied. Novel critical exponents unrelated to known exponents were obtained [26,27]. Other quantifiers such as persistence exponents have also been studied. We studied local persistence for this study which quantifies the loss of memory of the initial conditions. In general, power-law decay of the persistence and well-defined persistence exponents (if any) are obtained only at the critical point. However, for our model, we observe well-defined persistence exponents over the entire fluctuating phase. These exponents are complex and we observe logarithmic periodic oscillations over and above the usual power laws. The oscillations do not average out (by the cancellation of phase) by averaging over disorder and initial conditions.

II. MODEL

We consider the cellular automata model of contact process (CP) originally proposed by Domany and Kinzel [28,29]. This model shows the transition in DP or CDP class. We consider a 1D lattice of length N. Each site i is associated with variable $v_i(t)$ which is 0 or 1 depending whether they are "inactive" or "active." Each site at any time $t+1$ becomes active with certain probability depending on its neighbors' state at the previous time t. The conditional probabilities $P(v_i(t+1)|v_{i-1}(t), v_{i+1}(t))$ are defined as follows: $P(1|0,0) = 0$, $P(1|1,1) = q$, $P(1|1,0) = P(1|0,1) = p$. DP transition can be obtained for $q \neq 1$. Let us consider $p = q < 1$ for simplicity. The order parameter, the fraction of active sites, is given by $\rho(t) = \frac{1}{N}\sum_{i=1}^{N} v_i(t)$. Below the critical probability p_c, the cluster goes to an absorbing phase from which it cannot escape. Above p_c, $\rho(t) > 0$. In this case $\rho(t) \sim \exp(-\lambda t)$ for $p < p_c$ after a brief transient and $\rho(\infty) > 0$ asymptotically for $p > p_c$. At $p = p_c$, $\rho(t)$ decays as a power law $\rho(t) \sim t^{-\delta}$ with $\delta = 0.158$ in 1D. For the above model, $p_c = 0.705$.

For CDP $q = 1$ and $p < 1$. Thus, no inactive sites can be created in a continguous region of active sites. By symmetry, $p_c = 0.5$ and $\rho(\infty)$ jumps from 0 to 1 at p_c. The exponent $\delta = 0$ for CDP.

Now, we consider a model in which a fraction of sites r marked as type A evolve according to CDP rules and the remaining $1-r$ fraction of sites marked as type B evolve according to DP rules. For the contact process modeling diseases, the type A particles can be interpreted as children, elderly, or sick people who are extremely vulnerable and will catch a disease if everyone around them is sick. The rules are symmetric and our update is synchronous. The conditional probabilities of the update are mentioned above. For the particles of type A, $q = 1$, and for type B, $q = p$, $p < 1$. We consider the model for $r = 0.5$.

We compute two quantities: (a) the fraction of active sites $\rho(t)$ and (b) the local persistence $P_l(t)$. $P_l(t)$ is the fraction of sites such that $v_i(t) = v_i(t')$ for $0 \leqslant t' \leqslant t$. These sites have not changed their state *even once* from their initial conditions until time t. This is a non-Markovian quantity. Interestingly, it has been found that this quantity displays a power-law decay at the critical point of dynamic phase transition in some cases. In these cases, $P_l(t) \sim 1/t^{\theta_l}$ at the critical point where θ_l is known as the local persistence exponent. This is a new exponent independent from other critical exponents related to the transition. It is not universal. However, it has been found useful in finding other exponents such as z and $\nu_\parallel$ [30,31].

The quantity $\rho(t)$ is an order parameter for the absorbing state transitions while $P_l(t)$ is an order parameter for the spreading transition. $\rho(\infty) = 0$ implies that there are no active sites in the lattice asymptotically and evolution has effectively stopped. $P_l(\infty) \neq 0$ suggests that some sites do not deviate from their initial conditions even once. The persistent sites partition lattice in various clusters such that there is no spread of information from one cluster to another. Both $\rho(\infty)$ as well as $P_l(\infty)$ can give us information about the phase transition. We average over initial conditions as well as disorder realizations.

The order parameter $\rho(\infty) > 0$ for $p > p_c$. We find that for $r = 0.5$, $p_c = 0.651$ and a finite density of active sites is obtained for $p > p_c$. Normally, all inactive sites become active at some point in time in the active phase and $P_l(\infty)$ asymptotically approaches zero for $p > p_c$. On the other hand, $P_l(\infty) > 0$ for $p \leqslant p_c$. However, it will be shown in the next section that at $p = p_c$, the active sites decay logarithmically. The dynamics is extremely slow and we do not obtain power-law decay of persistence. The persistence decays as power law for $p \geqslant p_s$. For $p_c < p \leqslant p_s$, the decay is slower than power law. The persistence exponent is the largest at $p = p_s$ and reduces as $p \to 1$.

III. SIMULATION AND RESULTS

We simulate the system for 2.5×10^6 sites and average more than 10^3 configurations.

(a) The absorbing state transition: For $p \leqslant p_c = 0.651$, all sites eventually become inactive and the evolution stops. Normally, we observe an exponential decay of $\rho(t)$ for $p < p_c$ and a power-law decay is observed at $p = p_c$. However, in our case, a power-law decay of $\rho(t)$ is observed over a range of parameters below the critical point but not at the critical point, i.e., $\rho(t) \sim 1/t^{\delta}$ with decreasing δ, as $p \to p_c$. The regime in which the power-law decay of $\rho(t)$ with changing exponents is observed is known as the Griffiths phase. The relaxation behavior changes to stretched exponential and eventually to exponential for very small values of $p \ll p_c$. Figure 1(a) shows the density of active sites $\rho(t)$ vs time t on logarithmic scale for various values of p. Relaxation is slower as $p \to p_c$ and $\delta \to 0$. The power-law decay $\rho(t) \sim t^{-\delta}$ implies that $\rho(t)t^{\delta}$ approaches a constant value asymptotically. This expectation is indeed fulfilled. Figure 1(b) shows $\rho(t)t^{\delta}$ with t on logarithmic scale for $p < p_c$, close to p_c. As $p \to p_c$, δ decreases.

This transition has been confirmed by the large lattice simulation mentioned above as well as by simulations starting with a single seed. At $p = p_c$ the relaxation is ultraslow. For a large lattice of size 2.5×10^6, we observe logarithmic decay of $\rho(t)$ at $p = p_c$. The transition is expected to be in the universality class of activated scaling [32–35]. For this class, the proposed behavior is $\rho(t) \sim [\ln(t/t_0)]^{-\bar{\delta}}$. Thus, if we plot $\rho(t)^{-1/\bar{\delta}}$ as a function of $\ln(t/t_0)$ we observe linear behavior only at the critical point. In Fig. 2, we have plotted $\rho(t)^{-1/\bar{\delta}}$ as a function of $\ln(t/t_0)$ with $\bar{\delta} = 0.381$ at $p < p_c$, $p = p_c$, and $p > p_c$. The value $\bar{\delta} \sim 0.381$ is close to the previously obtained value $\bar{\delta} = 0.381\,97$.

Due to logarithmic decay, it is extremely difficult to locate the critical point precisely and further tests are required to locate it. We confirm the above value with single seed simulations. We compute (a) the survival probability (fraction of clusters surviving until time t) $P_s(t)$ and (b) the average number of particles in a cluster starting from single seed $N(t)$. At $p = p_c$, (a) the quantity $P_s(t)$ is expected to decay asymptotically as $P_s(t) \sim [\ln(t/t_0)]^{-\bar{\delta}}$ and (b) $N(t)$ decays as $N(t) \sim [\ln(t/t_0)]^{\Theta}$. Again we plot $P_s(t)^{-1/\bar{\delta}}$ as a function of $\ln(t/t_0)$ and obtain linear behavior only for $p = p_c$ [see Fig. 3(a)]. Similarly, if we plot $N(t)^{1/\Theta}$ as a function of $\ln(t/t_0)$, a linear behavior is expected only at $p = p_c$. We observe that for $t_0 = 0.2$, $\bar{\delta} = 0.381$, and $\Theta = 1.236$, linear behavior is obtained only at $p = 0.651$ [see Fig. 3(b)]. The values of $\bar{\delta}$ and Θ match with those expected in the class of activated scaling [34,35]. Thus, we confirm $p_c = 0.651 \pm 0.0005$ in three different ways using both large lattice simulations as well as single seed simulations.

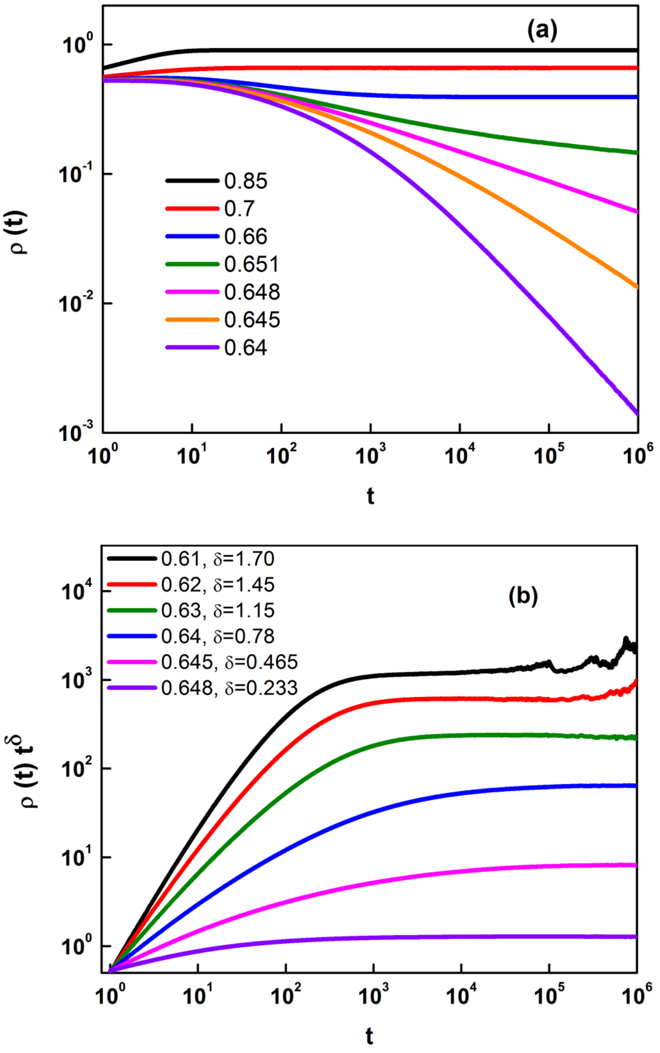

FIG. 1. (a) Overview of time evolution of average density $\rho(t)$ of a 1D system of size $N = 2.5 \times 10^6$ for $r = 0.5$ and $p \geqslant p_c$ in the range 0.64 to 0.85 (from bottom to top), where $p_c = 0.651$. We find that $\rho(t) \sim t^{-\delta}$ for p is close to p_c and $p < p_c$. (b) Log-log plot of $\rho(t) \times t^{\delta}$ vs time in the Griffiths region for a system with $p < p_c$ in the range 0.61 to 0.648 (from top to bottom). Exponent δ changes continuously and reaches 0 as $p \to p_c$.

(b) Persistence: We compute $P_l(t)$ as a function of time for a large system of size $N = 2.5 \times 10^6$. Usually, the absorbing state transition is accompanied by the spreading transition. (In the active phase, we expect the inactive sites can be expected to become active at some time and the active sites will become inactive due to fluctuation.) Since the active sites decay logarithmically at $p = p_c$, the persistence decays very slowly at the critical point and cannot be fitted by a power law (see Fig. 4). However, we observe a clear power-law decay of persistence for $p \geqslant 0.655$. We can even observe the finite-size scaling of persistence at $p = p_s$. For $p_s = 0.655$, we show the asymptotic value of persistence at various system sizes N. We observe a power-law decay of $P_l(\infty)$ as a function of N. If we expect finite-size scaling, we can postulate that $P_l(\infty) \sim$

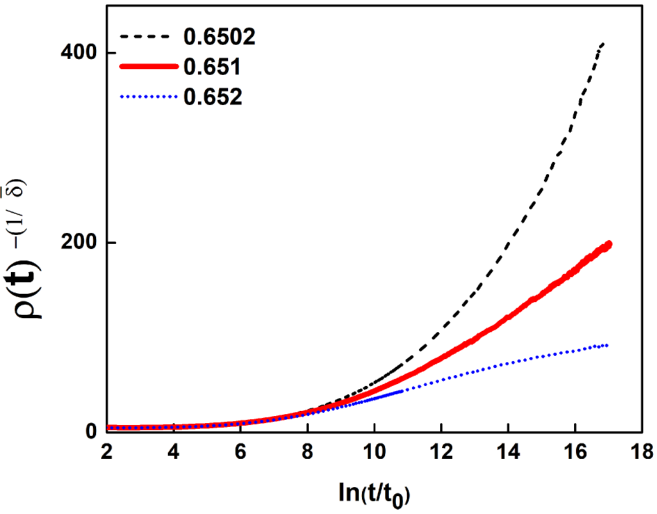

FIG. 2. The time evolution of density of active sites $\rho(t)$ for a system of size 2.5×10^6 with $p < p_c$, $p = p_c$, and $p > p_c$. $\rho(t)$ decays as $[\ln(t/t_0)]^{-\bar{\delta}}$ where $\bar{\delta} = 0.381$ only at $p = p_c$.

$N^{-z\theta}$. We find that $z\theta = 3.18$ and $\theta = 1.92$ implying that $z = 1.656$. ($z < 2$ indicating superdiffusive behavior. For 1D DP, $z = 1.58$.) Figure 5 shows the scaling plot of $P_l(t)\, N^{z\theta}$ with t/N^z. Such scaling is not obtained for other values of p.

In general, persistence shows exponential decay in the active phase. The power-law decay is observed only at the critical point. But our case is different. At p_c, the persistence decays slower than power law. This is because active sites spread logarithmically in time and it takes very long for inactive sites to become active. Thus persistence is dominated by inactive sites that did not become active until that time. We denote the persistence of type A (CDP) sites with initial state 1 and 0 by $P_l(A, 1, t)$ and $P_l(A, 0, t)$, respectively. We define $P_l(B, 1, t)$ and $P_l(B, 0, t)$ in an analogous manner for DP sites of type B. $P_l(t)$ is the sum of these four quantities. The total persistence $P_l(t)$ is found to be dictated by $P_l(A, 1, t)$. The quantity $P_l(A, 1, t)$ deviates from the initial condition very slowly for $p > p_s$. For $p \leqslant p_c$, the evolution effectively stops as soon the system reaches an absorbing state and a finite value of persistence is expected. For $p > p_c$ the persistence is expected to go to zero asymptotically. For $p \geqslant p_s$ it shows power-law decay.

For $p = p_s$ we observe a clear power-law decay of persistence in time with exponent 1.92. For $p \geqslant p_s$, the quantity $P_l(t)$ continues to decay as power law as shown in Fig. 6(a). The decay exponent is smaller for $p > p_s$. Thus the rate at which the system loses the memory of initial conditions is fastest at $p = p_s$. Unlike other cases where we observe exponential decay of persistence in the active phase, the memory of initial conditions decay very slowly even in the active phase. The behavior can be described by $P_l(t) \sim t^{-\theta_l}$ for $p \geqslant p_s$ where θ_l is known as the persistence exponent. Thus $P_l(t)t^{\theta_l}$ is a constant. For larger values of p close to 1, there are systematic oscillations over and above the power-law decay and they can be best described by a complex persistence exponent. The amplitude of oscillations increases as $p \rightarrow 1$ [see Fig. 6(b)].

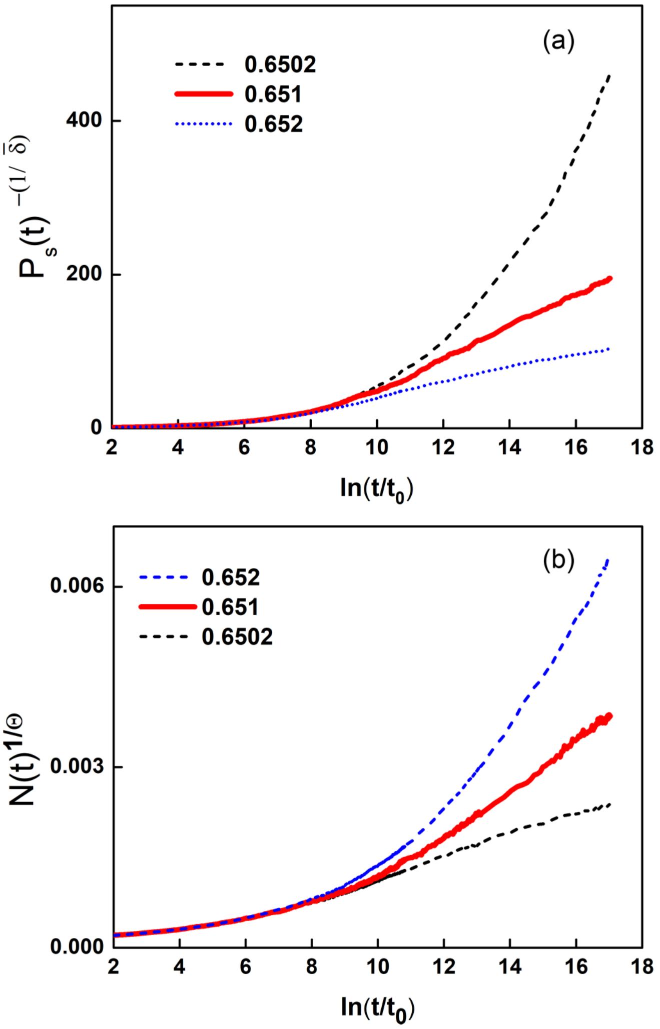

FIG. 3. (a) The time evolution of the survival probability $P_s(t)$ for disorder concentration $r = 0.5$ starting with a single seed, for $p < p_c$, $p = p_c$, and $p > p_c$. $P_s(t)$ decays as $[\ln(t/t_0)]^{-\bar{\delta}}$ with $\bar{\delta} = 0.381$. (b) The time evolution of average number of active sites in a cluster starting with a single seed $N(t)$ for $p > p_c$, $p = p_c$, and $p < p_c$. $N(t)$ decays as $[\ln(t/t_0)]^{\Theta}$ with $\Theta = 1.236$ and $t_0 = 0.2$.

This oscillatory nature of persistence is not reflected in any other quantity. The number of active sites saturates quickly in few time steps. Similarly, the rate at which active sites become inactive and vice versa reaches a constant value quickly. The number of domain walls where active and inactive sites are next to each other does not show any oscillations. If we discriminate between different initial conditions and compute four different quantities, depending on the initial condition and whether the site is of DP character or CDP character, we observe that the persistence is essentially dictated by sites of CDP type that are active initially. All other persistence goes to zero exponentially fast and logarithmic oscillations are due to CDP sites which are active in the beginning. The probability of finding k consecutive CDP sites decays exponentially with k. However, the larger the size, it is more difficult for 0's to invade the center of the CDP cluster.

We simulate the systems of $k + 2$ sites such that the first and $k + 2$th sites are fixed at 0 and we have a cluster of k

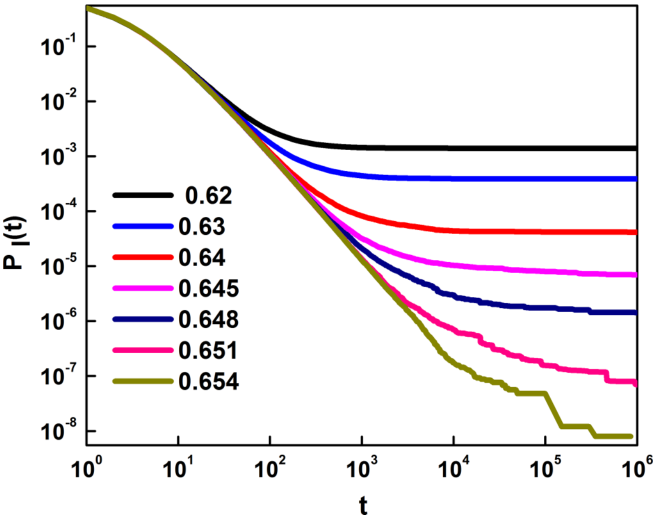

FIG. 4. Time evolution of local persistence $P_l(t)$ for a system of size 2.5×10^6 with $r = 0.5$ and $p < p_s$ in the range 0.62 to 0.654 (from top to bottom).

sites evolving according to CDP rules. These sites are active at the beginning. If all active sites have become inactive at least once, we consider it as a configuration which has not survived. In Fig. 7(a), we have plotted the number of surviving configurations as a function of time for different values of k for $p = 0.96$. We also compute the average time by which all these k sites have become inactive at least once. This time T_k increases exponentially. But there are oscillations over and above the exponential. In Fig. 7(b) we plot T_k the average time taken by $k + 2$ sites to become inactive as a function of the number of sites k in the cluster. Figure 7(c) shows the plot of the relation $T_k/\exp(\gamma_p k)$ with varying k for different values of p. The probability of a CDP cluster of size k decreases exponentially. However, the lifetime of such a cluster increases exponentially. The combination of exponentially rare regions which survive for exponentially long times leads to a power law and the oscillations over and above this exponential lead to log periodicity. For larger p, there are bigger odd-even oscillations in lifetime and the amplitude of log-periodic oscillations in persistence is also bigger.

At longer times, bigger and bigger clusters of CDP sites are invaded fully and the sites become inactive at least once. There is a certain timescale at which say, a cluster of four sites is invaded and after a certain time a cluster of size five is fully invaded. However, apart from an exponential increase in timescales, there is an odd-even oscillation which could be a reason for logarithmic oscillations in persistence.

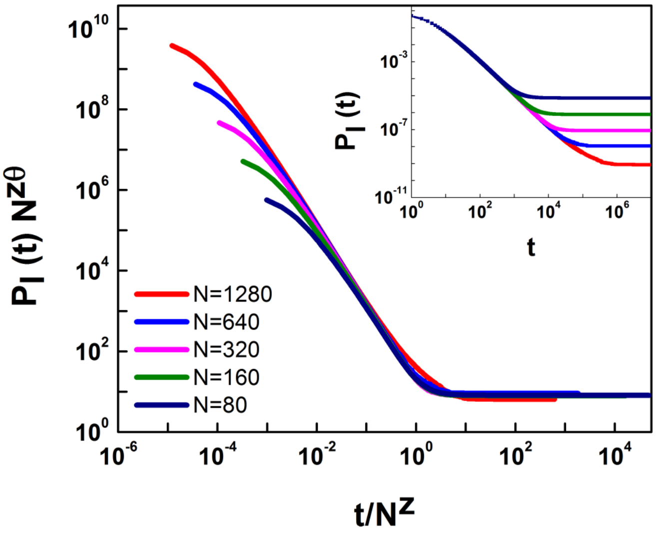

FIG. 5. Scaling plot of the local persistence $P_l(t)$ at $r = 0.5$. with $p = p_s = 0.655$ for different lattice sizes N. The time required for saturation scales as N^z and the saturation value of $P_l(t)$ scales with $N^{-z\theta}$ where N = 80, 160, 320, 640, 1280 (from bottom to top). Inset: Time evolution of $P_l(t)$ with $r = 0.5$, $p = p_s$ for various sizes of lattice N = 80, 160, 320, 640, 1280 (from top to bottom).

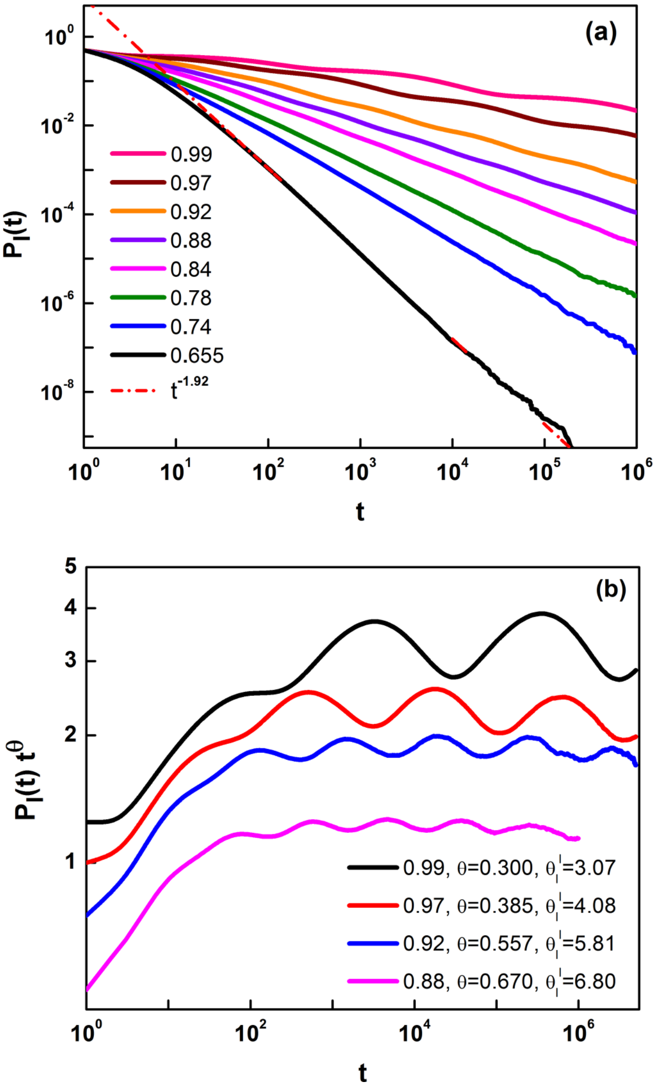

FIG. 6. (a) Time evolution of $P_l(t)$ with $r = 0.5$ and $p \geqslant p_s$. Power law is obtained at $p \geqslant p_s$ in the range 0.655 to 0.99 (from bottom to top). Clearly the persistence exponent θ changes continuously in this range. (b) Log-log plot of $P_l(t) \times t^{\theta}$ with $r = 0.5$ and $p > p_s$, in the range 0.88 to 0.99 (from bottom to top) $p \to 1$. The log-periodic oscillations are clearly evident in this figure. The y axis is multiplied by arbitrary constants for better visualization.

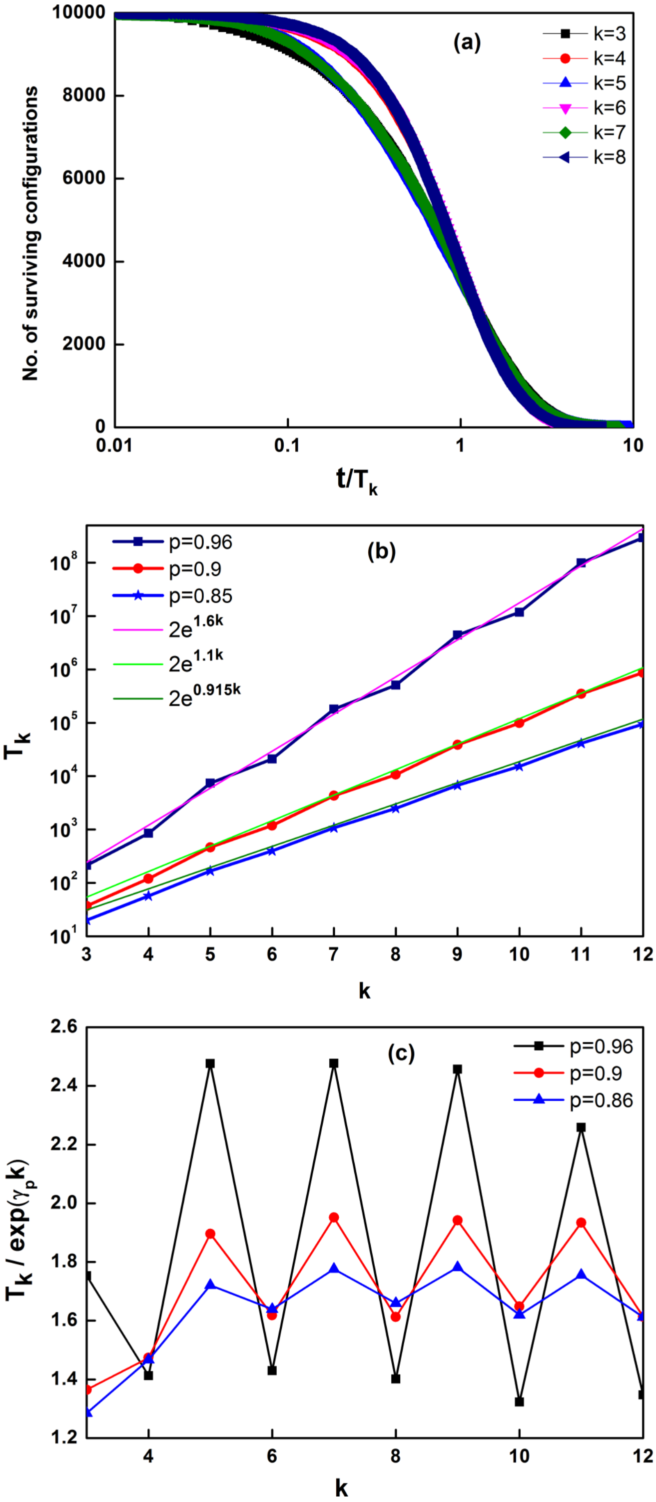

FIG. 7. (a) Starting with 10^4 configurations, we plot the number of surviving configurations vs t/T_k for CDP clusters of k sites surrounded by inactive sites for $p = 0.96$. We consider $k = 3$–8. T_k is the average time taken by a cluster of k CDP sites to become inactive. Clearly, the distribution of lifetimes is different for odd (at the bottom) and even (at the top) values of k. (b) We plot $T_k$vs k. Ignoring odd-even oscillations, it can be fitted as $T_k \propto \exp(\gamma_p k)$ where $\gamma_p = 0.915, 1.1, 1.6$ for $p = 0.85, 0.9, 0.96$. (c) We plot the relation $T_k/\exp(\gamma_p k)$ vs k. The odd-even oscillations are evident.

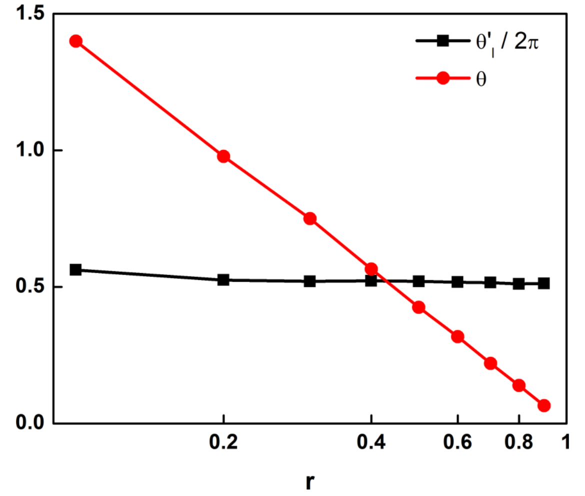

FIG. 8. Semilogarithmic plot of $\mathrm{Re}(\theta_l) = \theta$ and $\mathrm{Im}(\theta_l)/2\pi = \theta'/2\pi$ for $p = 0.96$ and varying disorder fraction r.

A complex exponent would imply that $\theta_l = \theta + i\theta'$. Now the behavior is given by $P_l(t) \sim \mathrm{Re}(At^{-\theta - i\theta'}) \sim At^{-\theta}\cos[\theta' \ln(t)]$ and $P_l(t)t^{\theta} \sim A\cos[\theta' \ln(t)]$. If we plot $P_l(t)t^{\theta}$ as a function of t, we should observe log-periodic oscillations over and above the constant. This is precisely the behavior for large values of p as shown in Fig. 6(b). Since the function is log periodic, it is very difficult to find exact time periodicity. The period of these oscillations decreases and amplitude increases as $p \to 1$. For p close to p_s, the amplitude (if any) is very small, and it is difficult to determine if $\theta' \neq 0$ is close to p_s. Log-periodic oscillations emerge due to the inherent self-similar structure in a variety of studies [11,36]. In our model the self-similarity is absent. The value of θ decreases as we approach $p = 1$. For $p \to 1$, $\theta \to 0$.

For CDP, $P_l(\infty) > 0$ for any p. For DP, $P_l(t)$ will decay exponentially for $p > p_c$. When both types of evolution are possible, naively one may expect that the decay will be slower than exponential due to clusters of CDP sites. One could expect a stretched exponential, power law, or even logarithmic decay. The dynamics will further slow down with an increase of r, leading to a decrease in the real part of the exponent. Figure 8 shows that the real part of the persistence exponent varies as $\log(r)$. Therefore, $\theta \to 0$ as $r \to 1$. However, the imaginary part of the persistence exponent θ' decreases only slightly with an increase in r.

IV. SUMMARY

We studied the contact process when a fraction r of sites on the 1D lattice follows CDP rules, and the rest evolve according to the rules leading to DP universality class. For $r = 0.5$, we observe a transition to the fluctuating phase at critical probability $p_c = 0.651$ for $r = 0.5$. In the absorbing phase, we observe the Griffiths phase over a range of parameters, where the order parameter $\rho(t)$ decays as a power law with continuously varying exponent. For $p \ll p_c$, $\rho(t)$ decays in a stretched exponential manner and eventually shows exponential behavior. For $p > p_c$, $\rho(\infty) > 0$. The slow dynamics in the absorbing phase is due to the rare region effect. The rare region effect decomposes the lattice into several disconnected finite-size clusters. These clusters are active while the bulk is in the inactive phase. Thus the overall activity is the sum of activities of clusters of various sizes. At $p = p_c$, the decay is extremely slow and we observe a logarithmic decay. We confirmed the critical point by large lattice as well as single seed simulation. We have also obtained the survival probability $P_s(t)$ and the average number of active sites in a cluster starting with single seed $N(t)$.

We also study the local persistence $P_l(t)$ in this system. In DP, critical point p_s at which $P_l(\infty) = 0$ coincides with p_c in general. For CDP, the persistence $P_l(t)$ does not go to zero in either phase. In our case, the decay of persistence is slower than power law at $p = p_c$. For $p \geqslant p_s$, $P_l(t)$ decays as power law with a continuously varying complex exponent. The real and imaginary parts of the exponent decrease as $p \to 1$, but amplitude increases. This system does not have a self-similar or fractal disorder and the underlying lattice is not fractal. Interestingly, log-periodic oscillations can be observed due to uncorrelated quenched disorder alone. We have shown that the reason for log-periodic oscillations can be found in lifetimes of active CDP clusters. We have mainly presented results for $r = 0.5$. However, changing r leads to similar results. The above observations may apply to other dynamical rules and topologies.

ACKNOWLEDGMENTS

P.M.G. thanks DST-SERB (Grant No. EMR/2016/006685) for financial assistance and Prof. P. Sen and Prof. M. Burma for discussions.

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