Spontaneous and engineered transformations of topological structures in nonlinear media with gain and loss

B. A. Kochetov,[1] O. G. Chelpanova,[2] V. R. Tuz,[1,3,*] and A. I. Yakimenko[2]

[1]State Key Laboratory of Integrated Optoelectronics, College of Electronic Science and Engineering, International Center of Future Science, Jilin University, 2699 Qianjin Street, Changchun 130012, China

[2]Department of Physics, Taras Shevchenko National University of Kyiv, 64/13, Volodymyrska Street, Kyiv 01601, Ukraine

[3]Institute of Radio Astronomy, National Academy of Sciences of Ukraine, 4, Mystetstv Street, Kharkiv 61002, Ukraine

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In contrast to conservative systems, in nonlinear media with gain and loss the dynamics of localized topological structures exhibit many unique features that can be controlled externally. We propose a robust mechanism to perform topological transformations changing characteristics of dissipative vortices and their complexes in a controllable way. We show that a properly chosen potential carries out the evolution of dissipative structures to regime with spontaneous transformation of the topological excitations or drives generation of vortices with control over the topological charge.

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I. INTRODUCTION

Formation of vortices, excitations that possess a rotational flow around a point of phase singularity, is a general phenomenon observed in various fields of both classical and quantum physics, including acoustics, fluid dynamics, solid-state physics, Bose-Einstein condensation (BEC), etc. [1–4]. In optics the unprecedented attention has been attracted to the vortex formation in nonlinear systems, where linear spreading due to the dispersion or diffraction is balanced by nonlinear focusing [5,6]. Nowadays vortices are a subject of numerous studies in nonequilibrium polariton condensates [7–14].

Besides the studies on vortex existence and stability [9,10] as well as vortex-antivortex dynamics [15,16], tremendous efforts have been aimed to get the full control over the formation of vortices with required topological properties using a broad optical pump [7,8], chiral polaritonic lenses [11], and a ring-shaped incoherent optical pump [12–14]. Formation in a predefined way of stable vortices with an arbitrary topological charge from a small-amplitude noise is possible, when an incoherent pump is locally applied to the exciton-polariton condensate [13]. Moreover, very recently the selective transfer of topological charge between two vortices with unit positive and negative charges [13] has been realized experimentally in the condensate due to the presence of an elliptically shaped incoherent control beam [17]. Further elaboration of this mechanism based on control of the vortex multistability [18] has allowed one to switch a topological state with the charge $m = \pm1, \pm2, \pm3$ to another vortex.

Despite significant progress in the vortex formation in diverse physical systems, getting control over vorticity of multidimensional dissipative solitons in a predefined way is still a challenging problem for many hydrodynamic and optical applications. In this work we propose a mechanism to perform nontrivial topological transformations by applying an external potential which changes topological characteristics of dissipative vortices in a controllable way. We demonstrate this mechanism in the framework of the complex Ginzburg-Landau equation (CGLE) with a potential term, which covers nonlinear phenomena far from equilibrium in many physical systems including mode-locked and fiber lasers, nonlinear optical waveguides, semiconductor devices, Bose-Einstein condensates, reaction-diffusion systems, etc. [19–24]. We show that the properly chosen potential performs preassigned transformations of topological structures.

The rest of the paper is organized as follows: In Sec. II we formulate the problem in the framework of the complex Ginzburg-Landau equation with a potential term and describe the scheme of its numerical solution. Results of the systematic analysis of different transformations of dissipative topological structures are presented in Sec. III. This section is divided into two subsections where we distinguish regimes of spontaneous and completely controlled (engineered) transformations of topological structures. Finally, in Sec. IV we summarize the paper.

II. MATHEMATICAL DESCRIPTION

Apart from the theory of active optical media with spatially modulated refractive indexes [25–30], the cubic-quintic CGLE with a potential term appears in theoretical description of the propagation of light beams through nonlinear magneto-optic planar waveguides [31–33]. In such systems the potential accounts for the influence of external magnetic field upon the dynamics of guided light beams existing in the form of dissipative solitons and vortices. Having adopted the notations used in the theory of nonlinear optical waveguides, we write the cubic-quintic CGLE supplemented by a potential

*tvr@jlu.edu.cn; tvr@rian.kharkov.ua

term with an explicit coordinate dependence as follows:

$$i\frac{\partial \Psi}{\partial z} + i\delta\Psi + \left(\frac{D}{2} - i\beta\right)\nabla^2\Psi + (1 - i\varepsilon)|\Psi|^2\Psi - (\nu - i\mu)|\Psi|^4\Psi + Q(x, y, z)\Psi = 0, \quad (1)$$

where $\Psi(x, y, z)$ is the slowly varying field envelop, which is the complex function of two transverse (x and y) and the longitudinal (z) coordinates. The coefficients of Eq. (1) are assumed to be positive quantities that result in their unambiguous interpretation. Namely, D is the normalized diffraction coefficient, δ is the linear absorption coefficient, β is the linear diffusion coefficient, ε is the coefficient of nonlinear cubic gain, ν accounts for the self-defocusing effect, and μ defines quintic nonlinear losses. The potential $Q(x, y, z)$ is a real function, which accounts for a specific conservative force applied externally that influences over the evolution of the complex envelop $\Psi(x, y, z)$. It is given in the form,

$$Q(x, y, z) = \sum_{n=1}^{N_Q} q_n(x, y, z)[\theta(z - a_n) - \theta(z - b_n)], \quad (2)$$

where N_Q is the number of control manipulations, θ is the Heaviside step function, and $a_n < b_n$ are the points on the z axis where the potential is switched between different states.

For a given set of coefficients taken from relatively wide ranges of values, Eq. (1) with zero potential has numerous stable solutions in the form of dissipative vortices with different topological charges. Each vortex corresponds to certain attractor in the phase space of Eq. (1) with zero potential. To excite the vortex one can use an arbitrary initial condition, which starts a phase trajectory somewhere within the basin of attraction. In all our simulations we use the same fixed set of coefficients of Eq. (1) [31,32]: $D = 1$, $\beta = 0.5$, $\delta = 0.5$, $\mu = 1$, $\nu = 0.1$, and $\varepsilon = 2.5$. For this set of coefficients one can use the following initial condition to excite a given vortex:

$$\Psi_0(x, y) = A_m \exp\left(im\varphi - \frac{r^2}{w^2}\right), \quad (3)$$

where φ is the azimuthal angle, $A_m = 1$ is the amplitude, $w = 3$ is the effective radius, and $r = \sqrt{x^2 + y^2}$ is the radial coordinate. The integer m defines the vortex topological charge, which corresponds to the circulation of the gradient of the phase on a closed curve surrounding the vortex core. First we consider transformations between lowest-order states $m = 0$, $m = 1$, and $m = -1$ which correspond to three different cases when we initially excite the fundamental soliton, vortex, and antivortex with unit topological charges, respectively. Finally we summarize our findings for transformation for higher-order dissipative vortices and discuss general conditions for mutual transformation of the topological excitations in media with gain and loss.

We solve Eq. (1) numerically using the exponential time differencing method and its two Runge-Kutta modifications of the second- and fourth-order accuracy [34] as well as the split-step Fourier method of the second-order accuracy [35].

III. TOPOLOGICAL TRANSFORMATIONS

Being formed a vortex cannot change its waveform as long as parameters in Eq. (1) are fixed. On the other hand, having applied the nonzero potential we can perturb the vortex pushing out its phase trajectory from the original basin of attraction to another one. As soon as the basin of attraction has been changed we switch off the potential and the released soliton waveform evolves into a new appearance. Such mechanism of induced waveform transitions between different one-dimensional dissipative solitons has been thoroughly studied in Refs. [36,37]. Here our goal is to show that this mechanism is quite general and provides tremendous opportunities to control the vortex formation and perform transformations between their different topological charges. In what follows, we distinguish two different mechanisms related to spontaneous and engineered transformations.

A. Spontaneous transformations

It was previously revealed [36] that a one-dimensional nonlinear system with gain and loss can exhibit chaotic behavior being driven by an external potential. Nevertheless, some control over the finite state is possible by choosing the position of switching off the external potential to release the soliton from chaotic to stationary behavior. This mechanism is related to spontaneous transformations of solitons.

In order to illustrate such transformations for vortices we consider a single manipulation potential ($N_Q = 1$). This potential is homogeneous in the y direction and constructed as a superposition of two potential wells with minima at $x = \pm x_0$ separated by potential barrier with $x = 0$ and $x_0 = 10$:

$$q_1(x) = \text{sech}(x - x_0) - \text{sech}(x) + \text{sech}(x + x_0). \quad (4)$$

To monitor the appearance of the topological excitations we use the core detection technique [38]. It allows us to determine a proper value of b_1 when the potential should be switched off. For the same purpose we also calculate energy $\mathcal{E} = \int |\Psi|^2 d^2\mathbf{r}$, angular momentum $\mathbf{L} = -\frac{i}{2}\int [\mathbf{r} \times (\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)]d^2\mathbf{r}$, and normalized momentum of inertia $\mathcal{I} = \int |\Psi|^2 r^2 d^2\mathbf{r}/\mathcal{E}$. In general, energy $\mathcal{E}$, angular momentum $\mathbf{L}$, and momentum of inertia $\mathcal{I}$ are functions of the z coordinate rather than conserved quantities. However, for any localized structure these characteristics are always finite. Moreover, in a stationary case they take constant values suitable for monitoring the dynamics of system (1).

For each z we calculate the topological charge of the system: $\Delta m = m_+ - m_-$, where m_+ and m_- are the numbers of vortices and antivortices, respectively, and $L = L_z/\mathcal{E}$ is the relative angular momentum. Using the information about topological structures available for different z, we predict the range of b_1, for which the finite state acquires a particular topological charge.

As illustrated in Figs. 1–3 in some cases this method allows one to control a type of topological structure which appears after the relaxation process. Figure 1 shows that the number of vortices detected in the wave field changes irregularly, but the analysis of the angular momentum guides one to choose a proper moment when the potential should

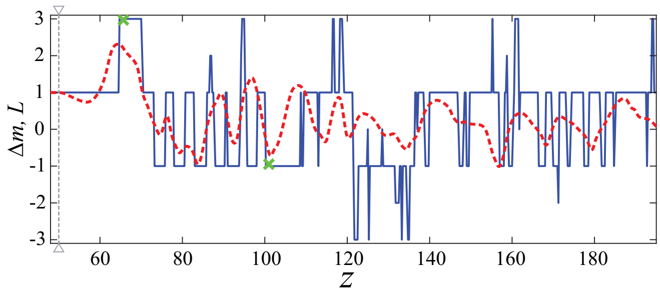

FIG. 1. Transformation of the vortex structure induced by external potential (4). Shown are evolution in the z direction of topological charge Δm (solid blue line) and relative angular momentum L (dashed red line). The green crosses mark the positions $z = b_1$ where the potential should switch off to form a single-charged antivortex (see Fig. 2) and three-charged dissipative vortex soliton (see Fig. 3).

be switched off. For instant, it turns out that the manipulation governed by the potential (4) with $a_1 = 50$ and $b_1 = 101$ transfers the charge of vortex from $m = +1$ to $m = -1$ (Fig. 2). As it is seen from Fig. 1 for $z = 65.5$ the number of detected vortex cores anticipates approaching the basin of the three-charged vortex structure. Indeed, the same potential (4) with $a_1 = 50$ and $b_1 = 65.5$ transforms the vortex with $m = 1$ to the three-vortex rotating cluster, that finally evolves into the three-charged ($m = 3$) vortex (Fig. 3) in good agreement with the prediction of the angular momentum analysis.

In fact, a wide variety of different topological excitations can emerge and decay in highly nonequilibrium state. Therefore, a detailed consideration of spontaneous transformations of dissipative structures in a regime of strong two-dimensional turbulence may be a relevant extension of this work, which might help to illuminate fundamental properties of the turbulence in classical and quantum physical systems.

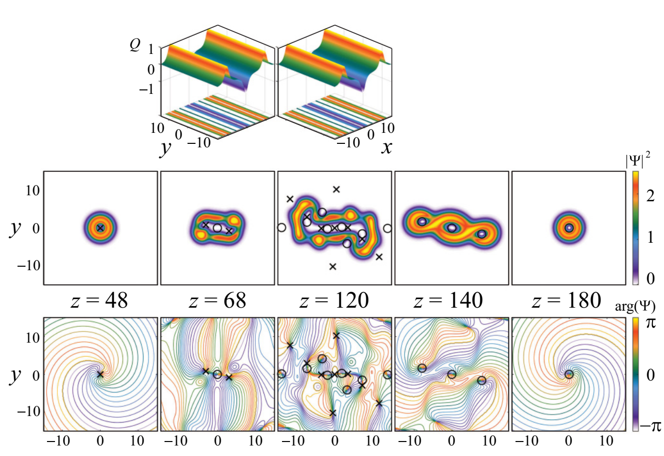

FIG. 2. Vortex to antivortex transformation obtained with control duration of potential action ($a_1 = 50$ and $b_1 = 101$). Snapshots of the spatial distribution of potential (top panel), density (middle panel), and phase (bottom panel) for different z. Crosses (circles) mark the vortex (antivortex) cores.

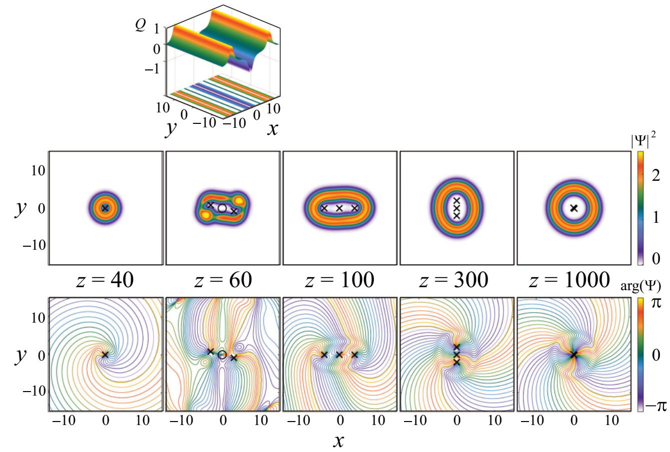

FIG. 3. The same as in Fig. 2 but for the transformation of the vortex to the three-charged vortex ($a_1 = 50$ and $b_1 = 65.5$).

B. Engineered transformations

Although effective, the mechanism of spontaneous transformations has several drawbacks. It is not always possible to control the final state of irregular evolution induced by simple potential with chosen parameter b_1. Also the number of possible transformations is restricted by available topological structures which emerge and decay in the perturbed wave field. In what follows we describe how the potential can be used to obtain complete control over the mutual transformations of different types of dissipative solitons for formation of complicated topological structures. We relate this mechanism to engineered transformations of dissipative solitons induced by external control.

Due to the topological reasons it is possible to get a single vortex only from the periphery of the wave beam, and only vortex-antivortex pairs can be excited inside the wave beam. The system with gain and loss conserves neither energy nor angular momentum, which allows topological transformation forbidden for conservative systems. Here we demonstrate how the well-known methods of the vortex generation in conservative systems can be modified to generate dissipative vortices. Furthermore, we suggest novel approaches for creation of topological excitations in the controllable dissipative systems.

A variable in the z-direction control potential is used to create a rotating repulsive barrier, similar to stirring the wave beam used in Bose-Einstein condensates [39–43]:

$$q_{st}(x, y, z) = -h\theta(\vartheta(x, y, z)) \exp\left[-\vartheta^2(x, y, z)/r_0^2\right], \quad (5)$$

where r_0 is the width of the stirrer, h is the depth of potential, and $\vartheta(x, y, z) = y\cos\Omega z - x\sin\Omega z$. Here Ω is the angular velocity of the stirrer.

The amplitude is chosen to be $h = 0.6$, so that the potential is comparable with other terms in Eq. (1) and the width is $r_0 = 2$. We use an additional repelling core in the potential: $q_1 = q_c + q_{st}$, where $q_c = -2\text{sech}(r)$ produces off-center flows at the wave beam axis and imposes the central toroidal hole. The structure of the potential implies formation of vortex-antivortex pairs near the wave beam axis. Depending on the direction of rotation of the stirrer, a vortex or an antivortex leaves the central region and moves to the periphery.

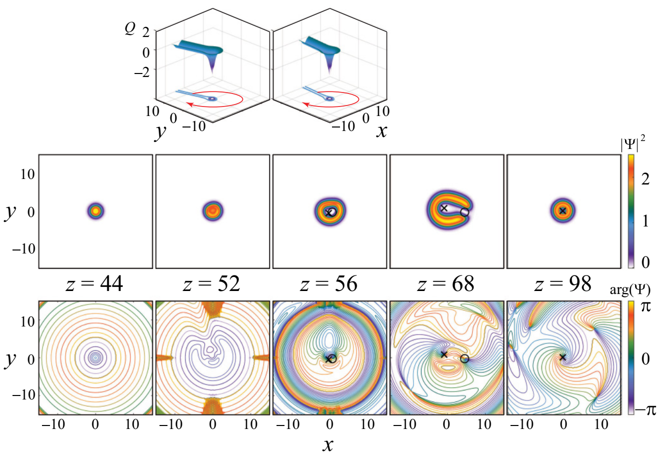

FIG. 4. The same as in Fig. 2 but for the transformation of the fundamental soliton to the vortex induced by the repulsive central core and rotating barrier. Note that *clockwise* rotation of the barrier drives the phase slip and formation of the vortex with topological charge $m = +1$.

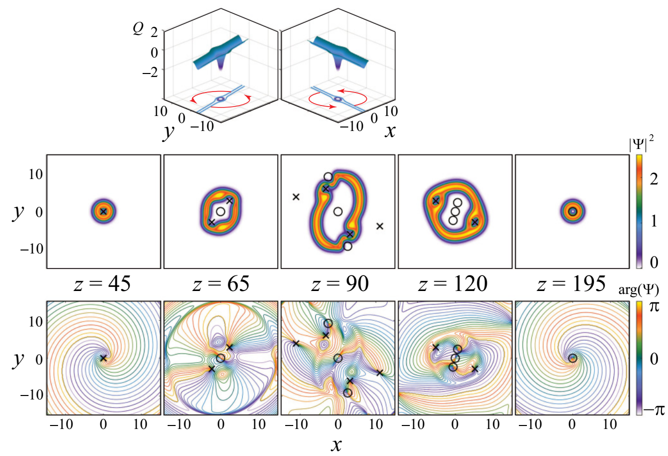

FIG. 6. Controllable transformation of the vortex to the antivortex using rotating potential combined with the repulsive core at the beam axis. Note that the *anticlockwise* rotating potential changes intensity and phase field distributions so that two antivortices come inside the central toroidal hole.

Remarkably, in similar setup during generation of the persistent current in a toroidal atomic BEC the rotating weak link induces a drift of the vortex core in the opposite direction: from external periphery to the axis (see, e.g., Refs. [41–43]). Snapshots of the spatial distribution of the potential, density, and phase for $\Omega = -0.086$, $a_1 = 50$, and $b_1 = 64$ are plotted in Fig. 4. One can estimate the speed required for vortex formation as $\Omega_v = L/\mathcal{I}$, which gives the stationary vortex with charge $m = 1$ when $\Omega = 0.13$. In our simulations the transformation occurs for $|\Omega| \leqslant 0.115$ in accordance with this simple estimation. Figure 5 summarizes our findings for generation vortices with rotating potential and presents the effective angular velocities Ω_v as the function of topological charge. As might be expected, in our simulations we observed that the nonrotating barrier ($\Omega = 0$) induces decay of the stationary vortex state to the fundamental soliton with $m = 0$ (corresponding figure is not presented here).

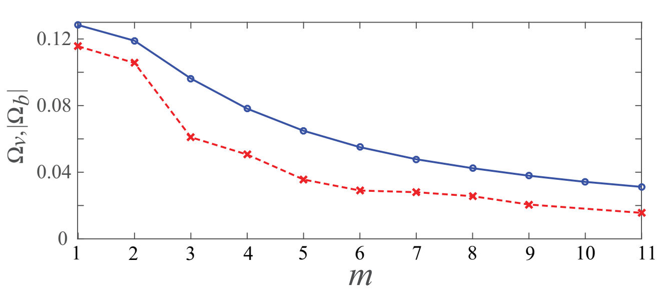

FIG. 5. Effective angular velocity of the stationary vortex $\Omega_v = L/\mathcal{I}$ for different topological charges m (blue circles with solid blue line) and absolute value of the *maximum* angular velocity $\Omega_b = \max(\Omega)$ of the rotating potential, which can transform a nonrotating ground state into an m-charged vortex (red crosses with dashed red line). Note that the effective vortex velocity Ω_v gives a good estimation from above for the maximum barrier angular velocity Ω_b ($|\Omega_b| \leqslant \Omega_v$).

Repulsive rotating potential combined with the repulsive core can be used to change the vortex charge to an arbitrary targeted vortex state. In particular, to transform the topological charge from $+1$ to -1 we use the potential of the form $q_{\rm st} = -h\exp[-\vartheta^2(x, y, z)/r_0^2]$ with $\Omega = 0.05$, $a_1 = 50$, and $b_1 = 100$. The corresponding snapshots of the spatial distribution of the potential, density, and phase for different z are shown in Fig. 6.

Thus, one can use a rotating repulsive potential to excite a vortex state from the fundamental soliton in analogy with excitation of the persistent currents in atomic BEC [39–43]. However, open dissipative systems with controlling potential suggest novel approaches for engineering nonlinear topological structures. The main idea is to create spatially separated vortex-antivortex pairs and then induce a decay of part of the vortex excitations. Using this approach one can create not only a single vortex but also generate much more complicated topological structures. To illustrate this let us discuss a transformation of fundamental soliton to a cluster of two antivortices rotating with the relative angular momentum $L_z/\mathcal{E} = -1.5$. For this transformation we perform four ($N_Q = 4$) control manipulations with potential (2): (i) First, the initial waveform (3) evolves to the fundamental soliton. (ii) Then at $z = 50$ potentials $q_1 = -2[\text{sech}(r) + \text{sech}(10 - r)]$ and $q_2 = -2[\text{sech}(\sqrt{(x-5)^2 + y^2}) + \text{sech}(\sqrt{(x+5)^2 + y^2})]$ turn on. Here term $-2\text{sech}(r)$ creates a repulsive potential at the axis of the wave beam, and induces formation of two vortex-antivortex pairs. Terms $-2\text{sech}(10 - r)$ and q_2 localize the pulse. (iii) After $z = 77$ term q_2 is nullified and potential $q_3 = -\text{sech}(2x) - \text{sech}(2y)$ is applied to separate vortices and antivortices from each other. The term $q_4 = 2\text{sech}(0.75y - 0.75x)$ is added after $z = 125$ to destroy two vortices. (iv) At $z = 145$ all potentials are switched off and the resulting wave field evolves into the rotating antivortices cluster.

Snapshots of spatial distribution of potential Q, intensity $|\Psi|^2$, and phase $\arg(\Psi)$ for different z are plotted in Fig. 7. This vortex cluster is a stable bound state of two

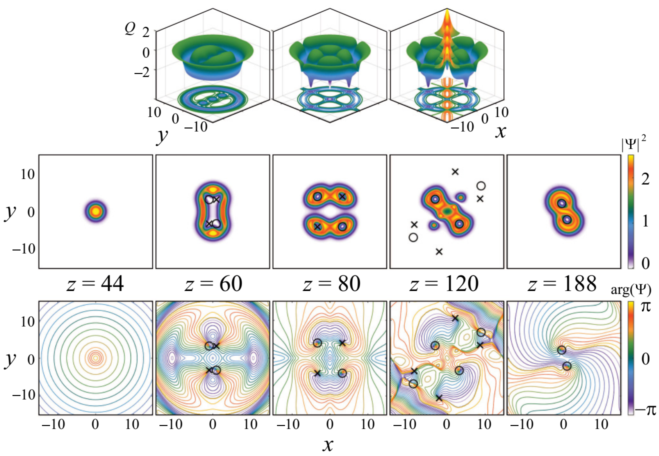

FIG. 7. The same as in Fig. 2 but for the transformation of the fundamental soliton to the rotating cluster composed of two antivortices. The effective topological charge of the cluster is $L_z/\mathcal{E} = -3/2$. In the first stage the soliton is transformed into two coupled dipoles, then the dipoles are spatially separated. Finally the attractive potential destroys the vortices, and the remaining two out-of-phase antivortices form the clockwise rotating antivortex cluster.

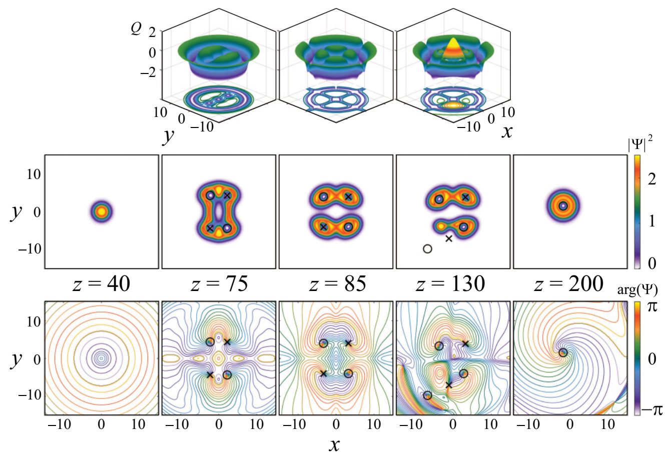

FIG. 8. The same as in Fig. 7 but for the transformation of the fundamental soliton to the antivortex. At the first stage the soliton is transformed into two coupled dipoles, then the dipoles are spatially separated. Finally the attractive potential destroys two vortices and one antivortex.

out-of-phase antivortices. In practice, formation of the bound state of two vortex solitons is a challenging task since it requires fine tuning of their phase difference. Remarkably, in our case the phase tuning occurs automatically for dynamical transformation of the two pairs of two out-of-phase dipoles. We note that introducing some modifications in the described manipulations one can also release the vortex and antivortex from the fundamental soliton.

To obtain the antivortex from the soliton one can change only the last manipulation. For example, q_4 manipulation can be added as follows: $q_4(x, y) = 2\text{sech}(0.5\sqrt{(x+4)^2 + (y+4)^2})$ at $z = 125$. This term destroys one vortex. After turning off the potential at $z = 150$ two antivortices and one vortex unite and evolve into the antivortex. Snapshots of spatial distribution of potential Q, intensity $|\Psi|^2$, and phase arg[Ψ] for different z are plotted in Fig. 8 (we encourage the reader to see animation of the described above transformations in the Supplemental Material [44]).

IV. CONCLUSIONS

In summary, we analyzed dynamical transformations of different topological structures using the model based on (2 + 1D) CGLE. It turns out that in a highly nonequilibrium state, driven by an external potential, various topological excitations emerge and decay. In the nonlinear media with gain and loss the energy of the vortex excitations is not conserved which permits fascinating transformations of topological structures not accessible in conservative systems.

We have analyzed spontaneous transformations of topological structures induced by external potentials. Our systematic analysis of evolution of the phase defects in the wave front opens an avenue on control over transformations of different topological structures by matching the duration of the action of the external potential during propagation of the wave beam. However, complex and irregular character of the phase transformation in the open dissipative system restricts the applicability of control over transformation of the topological structures based only on the potential duration. Moreover, generation of the vortex solitons from the fundamental solitons by simple potential is prohibited by conservation of topological charge in the system. We found a series of external potentials that drive completely controllable transitions of the dissipative soliton or vortex into the structures with required topological charge. Moreover, using the method developed in this work, we demonstrate formation of complex solitonic and vortex structures. The proposed method of controllable transformation of topological structures may open new prospects for future applications in optical signal processing.

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[44] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.100.062202 for animation of waveform transformations presented in Figs. 2–4 and 6–8 of the manuscript.