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Evolution of cooperation driven by self-recommendation

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Cooperators increase the fitness of others at a cost to themselves. Thus cooperation should not be favored by natural selection in a well-mixed population. It challenges the evolutionists since cooperation is widespread. Information spreading has been revealed to play a key role in the emergence of cooperation. Individuals, however, are typically assumed to be passive in the information spreading. Here we assume that individuals self-recommend themselves to those that are about to have new neighbors. Individuals with higher tendencies of self-recommendation are likely to have more neighbors. In this way, individuals are active to spread the information. We analytically obtain a critical cost-to-benefit ratio, below which cooperation emerges. It reveals quantitatively how eloquent cooperators have to be compared with defectors to ensure that cooperation takes over the population. It also indicates that individuals need to be open enough to the self-recommendation to enhance cooperation level. In addition, the critical cost-to-benefit ratio represents the viscosity of the population, measuring how close cooperators are to each other. Our results highlight the role self-recommendation plays in cooperation.

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I. INTRODUCTION

Cooperators forgo their interest to benefit others. Thus it cannot be favored based on evolutionary theory in the well-mixed population. Cooperation, however, is ubiquitous, ranging from genes to multicellularities in biology. In addition, human society is based upon cooperation as well. It has taken decades to fill the gap between evolutionary theory and widespread cooperation [1–3]. It provides a convenient paradigm to study the evolution of cooperation [4]. In particular, the Prisoners' Dilemma (PD) has been extensively adopted as a metaphor to study the emergence of cooperation [5,6]. In the simplified PD game, a cooperator offers its opponent a benefit b at a personal cost of c ($b > c > 0$), whereas a defector offers nothing. As a result, it is best to defect irrespective of the coplayer's decision. Beyond the analysis on the static PD, evolutionary game theory takes natural selection into account as the driving force of evolution. The replicator equation [7], which describes the dynamics of cooperation level in the well-mixed population, shows that defection is the only evolutionary stable strategy (ESS) [8,9] of the PD game. Therefore, both static and dynamic analyses show that cooperation cannot be achieved, even though mutual cooperation is optimal for the group interest. Thus, the PD game captures the conflict between group and individual interests, the so-called *social dilemma*.

To resolve this social dilemma, many mechanisms have been proposed, and one of the key factors is information. Information plays an important role in the evolution of cooperation. The decision making processes are based on a variety of information, such as the historical behaviors and payoffs of partners [10]. For example, individuals with imitation rule not only make use of the information of their own payoffs, but also take the opponents' payoffs into account when making decisions [11]. In contrast, individuals with aspiration rule only make use of the information of their own payoffs to make decisions. Due to this difference, a strategy favored by an imitation rule can be disfavored by an aspiration-based one [12]. Thus, it is of importance what information individuals possess in decision making. Once the information is available, the processes of information transmission and acquisition determine who obtains the true information. This process is highly complex in the real world, and it is still unclear how the processes of information transmission and acquisition reshape the evolutionary dynamics.

For information transmission, people commonly self-recommend to actively spread information. In human societies, commercial advertisement is of self-recommendation [13]. Enterprises make advertisements on the mass media including TV, newspapers, and internet to recommend their products to potential consumers. In the animal world, courtship display is also of self-recommendation, that animals attract mates by showing off their beauty or strength [14,15]. For example, peacocks spread their beautiful tails to attract peahens [16]. Individuals self-recommending themselves are essentially active to deliver information and draw attention from others.

For information acquisition, it is of great importance how individuals respond to the self-recommendation. In fact, it can be even more difficult to figure out who the cooperator is, if exaggerated or even false advertising is present. For example, the products or services can be not as good as stated in advertisements, and consumers can be cheated.

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These motivate us to examine how self-recommendation affects the emergence of cooperation. We assume that both cooperators and defectors are able to recommend themselves when an individual decides to have a new neighbor. Their capabilities of self-recommendation are characterized by self-recommendation tendencies. The larger the self-recommendation tendency is, the more eloquent the individual is, and the more likely it is connected. On the other hand, we introduce stubbornness to capture how convincible an individual is. The individual with larger stubbornness value is less likely to accept the self-recommendation.

Our results show that cooperation is more likely to prevail, provided that (i) cooperators are more eloquent than defectors and (ii) individuals who are making decisions to alter social partners are more convincible. The results show that the self-recommendation reshapes the evolutionary fate of cooperation.

II. MODEL

We consider a structured population of N individuals. Initially the population is located on a regular network, in which nodes represent individuals and links represent social ties between individuals. We assume that the population size N is much larger than the average degree $\langle d\rangle$ of the network ($N \gg \langle d\rangle$). In other words, each individual's neighborhood has a limited size. Each individual is either a cooperator (C), denoted by a unit column vector $\mathbf{s} = (1, 0)^T$, or a defector (D), denoted by $\mathbf{s} = (0, 1)^T$, where T indicates the transpose. The payoff matrix $\mathbf{Q}$ is given by

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}, \end{array} \tag{1}$$

where $b > c > 0$, and c/b is the cost-to-benefit ratio.

At each time step, either the strategy of an individual or the structure of the network is updated [17–19]. Let ω be the probability of strategy update, then $1-\omega$ corresponds to the probability of network update. The probability ω captures the relative time scales of the two processes.

A. Strategy update

We adopt the Fermi updating rule [20,21]. At each step of strategy update, a focal individual F is randomly selected from the population, and its accumulated payoff is calculated as $\Phi_F = \sum_{i\in\Delta_F} \mathbf{s}_F^T\mathbf{Q}\mathbf{s}_i$, where Δ_F represents the neighborhood of individual F, and $\mathbf{Q}$ is the payoff matrix. The strategies of individuals F and i are represented by $\mathbf{s}_F$ and $\mathbf{s}_i$, respectively. Then another individual G is randomly selected among the neighborhood of F. The accumulated payoff of G is given by $\Phi_G = \sum_{i\in\Delta_G} \mathbf{s}_G^T\mathbf{Q}\mathbf{s}_i$. The focal individual F compares its accumulated payoff Φ_F with Φ_G and switches to the strategy of G with probability $\{1 + \exp[-\beta(\Phi_G - \Phi_F)]\}^{-1}$. Non-negative β controls the intensity of selection, which corresponds to an inverse temperature in statistical physics [21]. Small β implies weak selection. In this case, individuals imitate others' strategies with probability approximately 1/2, even when the opponent gains much more than the focal individual. In particular, zero selection intensity corresponds to the neutral drift [22]. Large β means strong selection. In this case, individual F is almost sure to adopt the strategy of individual G, provided individual G gains even slightly more than individual F. The infinitely large selection intensity mirrors the perfect rationality in economics [23].

B. Network update

At each step of network update, a link is randomly selected from the network and the link breaks off with probability k. If it is broken, an individual between the two end points of the link is picked randomly as an active individual. The active individual is to reform a new link, i.e., rewire to a new individual who is not in its current neighborhood. On the one hand, we assume that all the individuals in the population are informed that the active individual is searching for a new partner. And all the qualified potential neighbors, who are not in the active individual's neighborhood, recommend themselves to the active individual based on their tendencies of recommendation. For simplicity, we assume positive constants R_C and R_D to capture the tendencies of self-recommendation for cooperators and defectors, respectively. On the other hand, we also assume the stubbornness p of an active individual. With probability p, the active individual does not take account of others' self-recommendation [24]. In this case, a new neighbor is randomly chosen regardless of self-recommendation. Otherwise, the active individual does respond to the self-recommendation with probability $1-p$. The active individual rewires to an individual with a probability proportional to its tendency of self-recommendation. Consequently, the active individual is likely to be attracted by eloquent individuals, those with large tendencies of recommendation.

The network update captures the process of information transmission and its acquisition. For the information transmission and acquisition, self-recommendation is adopted. And the self-recommendation process is similar to the classic Moran process [25]: The Moran process assumes that (i) individuals with higher fitness are more likely to reproduce; (ii) individuals with large fitness are very likely to reproduce only if the selection intensity is strong; and (iii) the population size is constant. The self-recommendation process assumes that (i) individuals with higher recommendation tendency are more likely to be rewired; (ii) individuals with high recommendation tendency are very likely to be rewired only if the probability to react to the self-recommendation is high; and (iii) the number of links keeps constant. In this way, the tendency of the self-recommendation mirrors the fitness of an individual, and links mirror the offsprings. The probability to react to the self-recommendation mirrors the selection intensity. Therefore, individuals are competing for having neighbors in the self-recommendation process as individuals are competing for reproducing offsprings in the Moran process.

III. ANALYSIS

In this section, we make use of mean-field analysis to show how cooperative behavior is reshaped by the self-recommendation, which drives the topology to evolve. Noteworthy, both the strategy and network structure evolve. It gives rise to a coupled dynamics, which is typically challenging to solve [26]. We overcome this by assuming $\omega \ll 1$, i.e., the network update is much more frequent than the strategy update (see Appendix A). Thus, the network structure keeps evolving, and reaches its stationary regime before individuals update their strategies. In this case, the linking dynamics is captured by a Markov chain. The resulting stationary distribution of the Markov chain $\vec{\pi}_0$ quantitatively indicates the fraction of *CC*, *CD*, and *DD* links of the network, respectively. The stationary distribution is given by

$$\vec{\pi}_0 = (\pi_{CC}, \pi_{CD}, \pi_{DD}) = \chi^{-1}(\gamma_3\gamma_4, \gamma_1\gamma_4, \gamma_1\gamma_2), \tag{2}$$

where

$$\begin{aligned}
\gamma_1 &= pkx_D(x_C R_C + x_D R_D) + (1-p)kx_D R_D,\\
\gamma_2 &= \tfrac{1}{2}kx_D p(x_C R_C + x_D R_D) + \tfrac{1}{2}(1-p)kx_D R_D,\\
\gamma_3 &= \tfrac{1}{2}kx_C p(x_C R_C + x_D R_D) + \tfrac{1}{2}(1-p)kx_C R_C,\\
\gamma_4 &= pkx_C(x_C R_C + x_D R_D) + (1-p)kx_C R_C,
\end{aligned} \tag{3}$$

and $\chi = (\gamma_3\gamma_4 + \gamma_1\gamma_4 + \gamma_1\gamma_2)$ is the normalization factor (see Appendix A).

If the active individuals are stubborn, i.e., $p = 1$, the stationary distribution becomes $(x_C^2, 2x_C x_D, x_D^2)$. This is the same as that in the well-mixed population. If the active individuals are not stubborn at all, i.e., $p = 0$, the stationary distribution $\vec{\pi}_0$ is given by $(\alpha_C^2, 2\alpha_C\alpha_D, \alpha_D^2)$ with $\alpha_s = R_s x_s (R_C x_C + R_D x_D)^{-1}$, $s \in \{C, D\}$. It implies that the self-recommendation reshapes the population structure. It acts like the well-mixed population with a rescaled frequency of cooperators, i.e., from x_C to α_C. To be precise, it implies that (i) π_{CC} is a monotonically increasing function of R_C/R_D, that is, the fraction of *CC* links increases with R_C/R_D; (ii) π_{DD} is a monotonically decreasing function of R_C/R_D, and the fraction of *DD* links decreases with R_C/R_D; and (iii) π_{CD}, i.e., the fraction of *CD* links increases at first and then decreases with R_C/R_D. Note that there are few *CD* links, provided R_C/R_D is large enough.

The stationary regime of the network structure allows us to estimate the average accumulated payoff of both cooperators and defectors [Eqs. (B1) and (B2)]. For large populations, the model can be approximately captured by a stochastic differential equation [see Eq. (B3)]. In the limit of population size $N \to +\infty$, the stochastic term of the stochastic differential equation vanishes and the mean-field equation for the fraction of cooperation is given by

$$\dot{x_C} = x_C(1 - x_C)\{(R_C - R_D)[(1-p)b - c]x_C - R_D c\} \tag{4}$$

(see Appendix B for the detailed calculation).

On the other hand, given a two-strategy pairwise game payoff matrix $(a_{ij})_{2\times 2}$, the corresponding replicator equation is given by $\dot{x} = x(1-x)[(a_{11} - a_{12} - a_{21} + a_{22})x + (a_{12} - a_{22})]$. Letting $a_{11} - a_{12} - a_{21} + a_{22} = (R_C - R_D)[(1-p)b - c]$ and $a_{12} - a_{22} = -R_D c$ would transform Eq. (4) into a replicator equation with a 2×2 game (a_{ij}). The two equations give rise to a linear equation of a_{ij}, $i, j \in \{1, 2\}$. Standard algebra analysis shows that there is a solution for the linear equation. We additionally assume that $a_{12} = -R_D c$ and $a_{22} = 0$. The solution is unique. In this way, we transform Eq. (4) to the replicator dynamics $\dot{x}_C = x_C(1 - x_C)(\tilde{f}_C - \tilde{f}_D)$ [7], in which $\tilde{f}_C$ and $\tilde{f}_D$ are determined by the transformed payoff matrix $\tilde{\mathbf{Q}}$:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R_C(1-p)(b-c) & -R_D c \\ R_D(1-p)b + R_C pc & 0 \end{pmatrix}, \end{array} \tag{5}$$

In other words, self-recommendation essentially changes the interaction between cooperators and defectors [4]. The transformed payoff matrix Eq. (5) captures the interaction between cooperators and defectors shifted by self-recommendation.

IV. RESULTS

In this section, we make use of the replicator equation with the transformed game Eq. (5) to investigate how self-recommendation tendencies (R_C and R_D) and stubbornness (p) affect the evolutionary dynamics of cooperation. Additionally, we investigate the robustness of the results with respect to the parameters which are absent in the transformed game Eq. (5) via simulations.

A. More eloquent cooperators and less stubborn individuals promote cooperation

We resort to the transformed matrix Eq. (5) to shed light on the evolutionary fate of cooperation. If the opponent is a defector, then the effective payoff of a defector, i.e., zero, must be greater than that of a cooperator, i.e., $-R_D c < 0$. If the opponent is a cooperator, a cooperator is better off than a defector if and only if $R_C(1-p)(b-c) > R_D(1-p)b + R_C pc$ holds. In other words, cooperation is a strict Nash equilibrium of the transformed matrix, provided that both

$$R_C > R_D \quad \text{and} \quad p < 1 - \frac{c}{b}\frac{1}{1 - \frac{R_D}{R_C}} \tag{6}$$

hold, which is equivalent to the inequality $R_C(1-p)(b-c) > R_D(1-p)b + R_C pc$. In this case, the transformed matrix is a coordination game, in which individuals are better off to do what others do. Both $x_C^* = 1$ and 0 are stable fixed points for Eq. (4), separated by the unstable internal fixed point

$$x_C^* = \frac{cR_D}{(R_C - R_D)[(1-p)b - c]}. \tag{7}$$

Therefore, if Eq. (6) is fulfilled, cooperation dominates the population provided that the initial fraction of cooperators exceeds the critical value x_C^* by Eq. (7). Otherwise, defection takes over the population. The critical value x_C^* via simulation is in good agreement with the theoretical prediction (Fig. 1). It should be pointed out that our model has two homogeneous absorbing states (all C and all D). For each realization of our simulations, the running time is long enough for the process to end up with homogeneous populations. The average final frequency of cooperators (in Fig. 1) is the estimation of fixation probability of cooperation starting from a given initial fraction of cooperators. It does not reflect the fixation probability ratio between cooperators and defectors, although it is commonly used to characterize whether cooperation is favored in finite populations. In addition, it is quite unlikely to observe the coexistence of two strategies in the final state in our simulation, which could be explained by the structure of the transformed payoff matrix in Eq. (5). The matrix corresponds to either

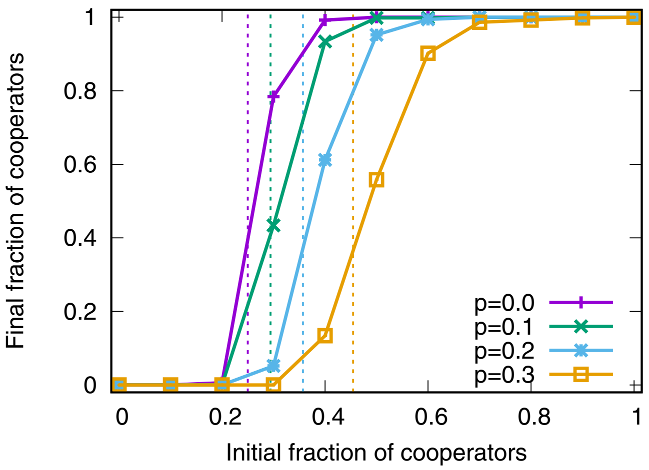

FIG. 1. The final fraction of cooperators as a function of initial fraction of cooperators. The symbols indicate the simulation results and the dashed lines represent the internal unstable fixed point x_C^* given by Eq. (7). The simulation results are in agreement with our theoretical predictions. If the initial value is smaller than the critical fraction of cooperators x_C^*, defection dominates the population. Otherwise, cooperation is taking over the population. This mirrors a coordination game with cooperation a stable Nash equilibrium. Each data point in this figure is averaged over 500 independent runs. In each simulation, we run time up to 10^6 generations. We have checked that all realizations end up with homogeneous populations. The final fraction of cooperators (in the simulation) is the estimation of fixation probability of cooperation starting from a given initial fraction of cooperators. Parameters: Prisoners' Dilemma with $b = 3$ and $c = 1$; the tendencies of self-recommendation of cooperators and defectors, $R_C = 3$ and $R_D = 1$, respectively; population size $N = 1000$; average degree $\langle d \rangle = 20$; probability of a strategy update $\omega = 10^{-3}$; link-breaking probability $k = 1$; and selection intensity $\beta = 10$.

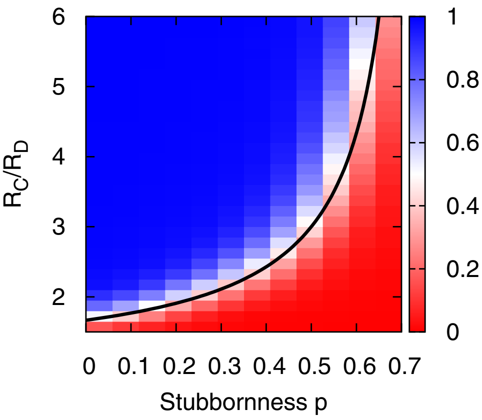

FIG. 2. The final fraction of cooperators as a function of R_C/R_D and stubbornness p. It is shown that cooperation prevails if the cooperators are more eloquent, i.e., R_C/R_D is sufficiently large, and if individuals are open to accept the self-recommendation, i.e., p is small. The simulation, i.e., the heat map, is in agreement with our theoretical prediction, i.e., the black line determined by Eq (7). Parameters: Prisoner's Dilemma with $b = 3$ and $c = 1$, $N = 1000$, $\langle d \rangle = 20$, $\omega = 10^{-3}$, $k = 1$, and $\beta = 10$; the initial fraction of cooperators is 0.5.

a dominant game or a coordination-like game. In both cases the fixation time is of order $N \ln N$ provided large population size N [27]. That is, our running time is long enough for the simulation to get fixation.

Based on Eq. (6), two conditions are required to promote cooperation: (i) the cooperators should be better than defectors at self-recommendation, i.e., $R_C > R_D$; and (ii) individuals should be less stubborn and more open to self-recommendation, i.e., $p < 1 - \frac{c}{b}\frac{1}{1-\frac{R_D}{R_C}}$.

For (i), it indicates that cooperation emerges only if cooperators are more eloquent than defectors. On the one hand, if $R_C > R_D$ cooperators are more active and are more likely to be selected as a new neighbor. This would make cooperators clustered together. The clustered cooperators interact more often with each other and gain higher payoffs. Thus they would outperform defector neighbors in payoff, and eventually take over the population [17]. Noteworthy, eloquent cooperators are taking risks because they would attach to defectors from time to time. However, the resulting clustered cooperators expand so quickly that the risk is under control. On the other hand, let us consider the situation that $R_C > R_D$ does not hold. We focus on a special case $R_C = R_D$, i.e., cooperators and defectors are equally eloquent. In this case, all the potential new neighbors of the active individual have the same likelihood to be selected. In other words, there is no preferential attachment in the linking dynamics. The stationary population regime ends up with $\pi_0 = (x_C^2, 2x_Cx_D, x_D^2)$, which is the same as that in the well-mixed population. The resulting replicator equation is $\dot{x_C} = -cx_C(1 - x_C)$, which is exactly the replicator dynamics of the original PD game. As a result, defection dominates the population regardless of the stubbornness p. Therefore $R_C > R_D$ is necessary for the emergence of cooperation.

For (ii), individuals should be open and less stubborn, i.e., $p < 1 - \frac{c}{b}\frac{1}{1-\frac{R_D}{R_C}}$ (see Fig. 2). To illustrate this, let us consider two extreme cases: $p = 1$ and 0. When $p = 1$, i.e., the active individual is so stubborn that it does not react to the self-recommendation by anybody, the active individual randomly chooses new neighbors. The self-recommendation does not work. The resulting stationary population regime is the same as that of the well-mixed population. Thus the approximated replicator equation is $\dot{x_C} = -cx_C(1 - x_C)$, as in the well-mixed population. Therefore, defection dominates the population. Let us resort to the other extreme case $p = 0$, i.e., individuals are not stubborn at all, and take into account the self-recommendation. The transformed payoff matrix becomes

$$\begin{pmatrix} R_C(b-c) & -R_Dc \\ R_Db & 0 \end{pmatrix}. \tag{8}$$

The transformed payoff matrix Eq. (8) shows that (a) when a defector meets a defector, the effective payoff of each defector is still zero as in the original PD game Eq. (1); (b) when a defector meets a cooperator, both individuals obtain R_D times the payoff of the original PD game; and (c) when a cooperator meets a cooperator, each cooperator gets R_C times the payoff of the original PD game. Noteworthy, the group interest of two cooperators $2R_C(b - c)$ outperforms that of a cooperator and a defector $R_D(b - c)$, provided $2R_C > R_D$. Thus cooperation

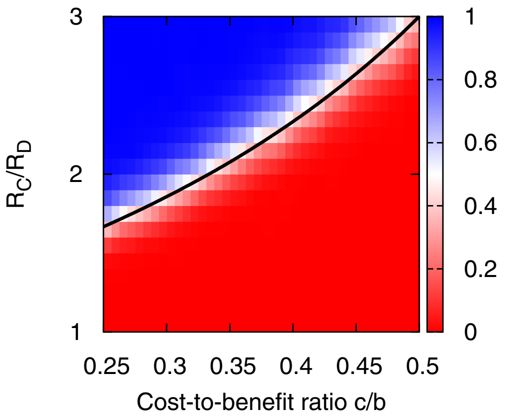

FIG. 3. The final fraction of cooperators as a function of R_C/R_D and cost-to-benefit ratio c/b. The heat map represents the simulation result and the black line indicates the theoretical boundary predicted by $x^* = cR_D/[(b-c)(R_C - R_D)] = 0.5$. We can see that the simulation is in agreement with our theoretical prediction. It is shown that a small cost-to-benefit ratio facilitates the cooperation. Parameters: $N = 1000$, $\langle d \rangle = 20$, $\omega = 10^{-3}$, $k = 1$, and $\beta = 10$; the initial fraction of cooperators is 0.5 and the stubbornness $p = 0$.

could still be a social optimum as in the original PD game Eq. (1). For the emergence of cooperation, it is essential to compare $R_C(b-c)$ and $R_D b$. In the original PD game, $b-c$ is less than b, whereas $R_C(b-c)$ can be larger than $R_D b$ as long as R_C is sufficiently larger than R_D. To be precise, if

$$\frac{c}{b} < 1 - \frac{R_D}{R_C}, \tag{9}$$

cooperators take over the whole population as long as the initial fraction of cooperators exceeds $x_C^* = cR_D/[(b-c)(R_C - R_D)]$. The cost-to-benefit ratio c/b is smaller than 1, thus $R_C > R_D$ is necessary if Eq. (9) holds, or cooperation prevails only if cooperators are much more eloquent than defectors (see Fig. 3). For p between zero and one, the interaction between cooperators and defectors can be captured by Eq. (5). As p increases, the effective game moves from Eq. (1), in which cooperation is not a Nash equilibrium, to Eq. (8), in which cooperation becomes an ESS.

B. Robustness of theoretical predictions

All the above results are based on the replicator equation of the transformed matrix Eq. (5). Equation (5) is determined by the cost-to-benefit ratio, the tendencies of self-recommendation, and the probability to react to the self-recommendation. However, the population size N, average degree $\langle d \rangle$, as well as the frequency of the strategy updates ω are absent in the transformed matrix. Here we investigate the robustness of our theoretical predictions with respect to these parameters via simulations.

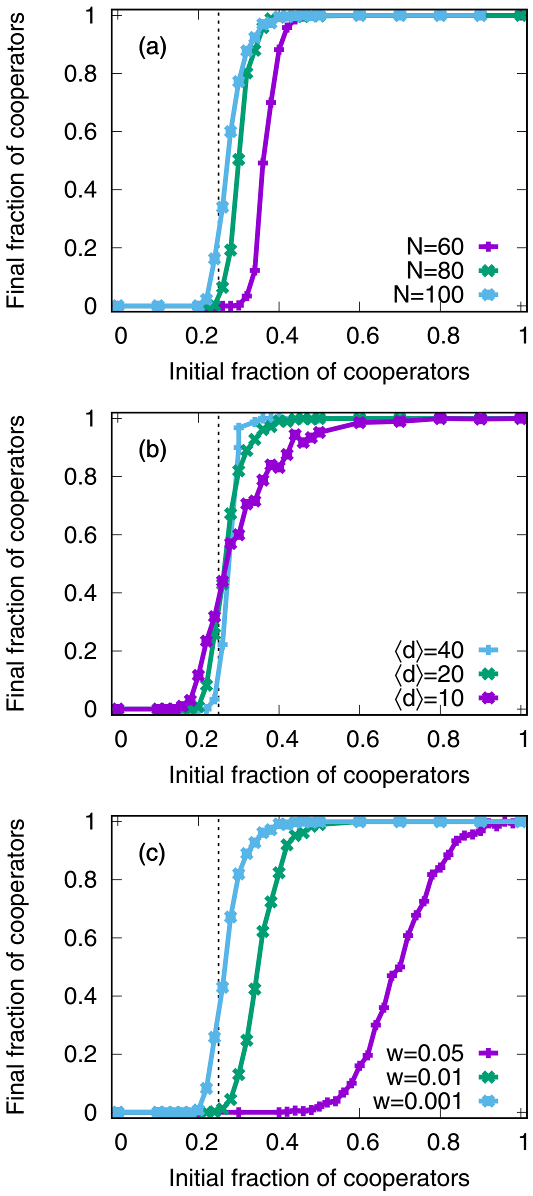

FIG. 4. Illustration of the robustness of our theoretical result. (a) Simulation results for different population sizes; the critical transition from all defection to all cooperation is shifted to the right with the decrease of N. (b) Simulation results for different average degrees; the case with smaller average degree shows more fluctuations than the one with larger average degree. (c) Simulation results are quite sensitive to the frequency of the strategy updates ω; the critical transition is shifted to the right with the increase of ω; the default parameters are the same as in Fig. 1.

First, we find that the larger the population size is, the better the agreement is shown between simulation results and theoretical prediction [Fig. 4(a)]. Noteworthy, our theoretical prediction is based on the mean-field analysis assuming that the population size is large enough. Therefore, it is not surprising to see the disagreement when the population size is small. Note that the critical value is shifted to the right with the decrease of N [Fig. 4(a)]. Actually, given the average degree [$\langle d \rangle = 20$ in Fig. 4(a)], smaller population size corresponds to denser network structure, which could suppress the assortment of cooperators during the network update.

Second, we investigate the average degree $\langle d \rangle$ [Fig. 4(b)]. We can see that the smaller $\langle d \rangle$ is, the more fluctuated the simulation result is. In other words, the transition region from all defection to all cooperation becomes less sharp as $\langle d \rangle$ decreases. This phenomenon seems a little bit counterintuitive because our theoretical approximation is based on the assumption that the whole population size is much larger than the average degree of the network. It means that, given the same population size, the case with smaller average degree should better agree with the theoretical prediction. However, it should be noted that smaller average degree increases the stochastic of the linking dynamics, because smaller $\langle d\rangle$ results in less number of total links ($L = N\langle d\rangle/2$) in the system. In this way, properly increasing the value of $\langle d\rangle$ can reduce the fluctuations around the expected numbers of CC, CD, and DD links in the stationary regime, making the simulation results agree with our theoretical prediction better.

Finally, let us discuss the parameter ω, i.e., the frequency of the strategy updates [Fig. 4(c)]. We find that the simulation results are sensitive to ω. In particular, the critical transition from all defectors to all cooperators increases with the increase of ω. Noteworthy, our method assumes that the network update is much faster than the strategy update ($\omega \ll 1$); it thus makes sense that the simulation with smaller ω shows better agreement with our theoretical prediction. As ω becomes larger, the strategy will update more frequently, and it is not long enough for the network to reach the stationary regime. Therefore, the population structure reshaped by the self-recommendation would be of less benefit for cooperators to form clusters, and then would inhibit the emergence of cooperation.

V. CONCLUSIONS AND DISCUSSIONS

Active information spreading is ubiquitous, ranging from commercial advertisements [13] to election campaigns [28]. Those with high tendencies of self-recommendation are likely to convince others. It is similar to reputation, where cooperators are typically assumed to have a high score of reputation [10,29–32]. Both self-recommendation and reputation play their role in communication. Neither can work unless the observers recognize. However, they remarkably differ from each other. Individuals take the initiative to attract others when they recommend themselves. Those who self-recommend are active. Reputation is at work in a passive way. Individuals with a high score of reputation could be too shy to be recognized.

Based on our model, we find that eloquent cooperators are necessary to promote cooperation. The intuition is not straightforward, because an eloquent cooperator does not imply that cooperation is at an advantage: If the active individual is a cooperator, an eloquent cooperator with high R_C would be more likely to become the active cooperator's potential new neighbor. Once the new link is established, it benefits both the active cooperator and the eloquent cooperator. If the active individual is a defector, however, an eloquent cooperator once again would be more likely to connect with the active defector. This new link would only benefit the active defector rather than the cooperator. In fact, the eloquent cooperators can form clusters, although they occasionally attach to defectors. The clustered cooperators gain much more than their defector neighbors. Thus the occasional attached defector would open an avenue for the cooperator cluster to spread. Eventually cooperation takes over the whole population [33]. Therefore, our results echo the so-called network reciprocity that cooperators can prevail by forming cooperative clusters [4]. Figure 5 illustrates how the degree distributions of cooperators and defectors change with the evolution of strategy. It is shown that both degree distributions of cooperators and defectors behave like Poisson distributions, yet the average degree of cooperators is larger than that of defectors at different levels of cooperation x_C, revealing the cluster of cooperators induced by self-recommendation.

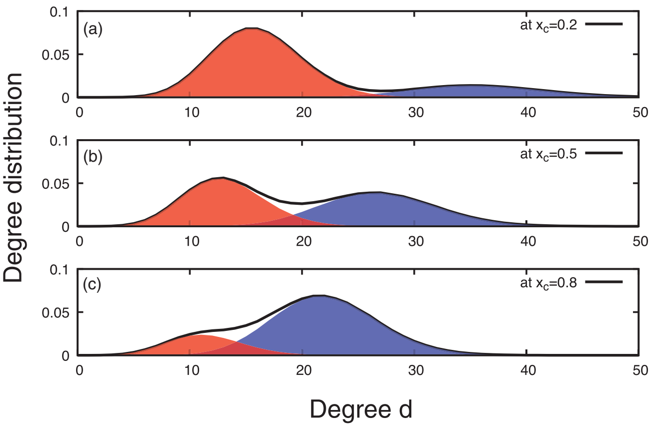

FIG. 5. Degree distributions of the network. From the initial to final states during the coevolutionary dynamics of network and strategy, the degree distributions change and strongly depend on the cooperator frequency. We measure the degree distributions whenever the level of cooperation (x_C) reaches certain levels ($x_C = 0.3$, 0.5, and 0.8). In each panel, the degree distributions of both cooperators (blue) and defectors (red) are shown. The degree distribution of the whole population (black line) shows a mixture distribution of red and blue. The distributions for 500 realizations at each given level of cooperation are averaged. The default parameters are the same as in Fig. 1 with the stubbornness $p = 0.3$ and initial state of $x_C = 0.4$. The average degree is 20 which keeps constant during the simulation.

In addition, we find that the observers should be not too stubborn, and they should be open to the self-recommendation. In particular, if individuals are 100% open to the self-recommendation, cooperation becomes the stable Nash equilibrium, provided $\frac{c}{b} < 1 - \frac{R_D}{R_C}$. The larger R_C is, the larger $1 - \frac{R_D}{R_C}$ is, and the more likely it is to form a cooperative cluster. In other words, $1 - \frac{R_D}{R_C}$ represents the social viscosity, which is similar to [17,18,34].

Our model is a simple start to investigate how the recommendation reshapes the evolutionary dynamics. For simplicity, we assume that tendency of self-recommendation is determined by its trait (cooperation or not) in this paper. The disentanglement between tendency of self-recommendation and its trait should be taken into account for a more sophisticated model. Research along this line is in progress.

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APPENDIX A: MARKOV LINKING DYNAMICS AND ITS STATIONARY DISTRIBUTION

For the linking dynamics, there are three different types of social ties: cooperator-cooperator (*CC*), cooperator-defector (*CD*), and defector-defector (*DD*) links. Based on the presented rewiring rule with self-recommendation, the network updating process can be modeled as a discrete-time Markov chain in the state space of $\{CC, CD, DD\}$ [35,36]. Let us denote x_C and x_D to be the fraction of cooperators and defectors in the population.

We take the transition from *CD* to *DD* as an example. This transition happens only when a *CD* link is broken off (with probability k) and the D individual is selected as the active individual (with probability 1/2), and then rewire to another defector (with probability x_D if D does not respond or with probability α_D if it does respond). As a result, the transition probability is either $k\frac{1}{2}x_D$ or $k\frac{1}{2}\alpha_D$, depending on whether or not the active individual responds to the self-recommendation by others. According to the Law of Total Probability, the transition probability from *CD* to *DD* is given by $pk\frac{1}{2}x_D + (1-p)k\frac{1}{2}\alpha_D$. All the rest of the entries of the transition matrix $\mathbf{M}_0$ are obtained in the same argument.

We thus end up with the transition matrix

$$\mathbf{M}_0 = p\mathbf{M}_1 + (1-p)\mathbf{M}_2, \tag{A1}$$

where

$$\mathbf{M}_1 = \begin{array}{c} \\ CC \\ CD \\ DD \end{array} \begin{array}{c} \begin{array}{ccc} CC & CD & DD \end{array} \\ \begin{pmatrix} 1-kx_D & kx_D & 0 \\ kx_C/2 & 1-(k/2) & kx_D/2 \\ 0 & kx_C & 1-kx_C \end{pmatrix} \end{array} \tag{A2}$$

and

$$\mathbf{M}_2 = \begin{array}{c} \\ CC \\ CD \\ DD \end{array} \begin{array}{c} \begin{array}{ccc} CC & CD & DD \end{array} \\ \begin{pmatrix} 1-k\alpha_D & k\alpha_D & 0 \\ k\alpha_C/2 & 1-(k/2) & k\alpha_D/2 \\ 0 & k\alpha_C & 1-k\alpha_C \end{pmatrix} \end{array} \tag{A3}$$

with $\alpha_C = x_C R_C/(x_C R_C + x_D R_D)$ and $\alpha_D = x_D R_D/(x_C R_C + x_D R_D)$.

Actually, $\mathbf{M}_1$ and $\mathbf{M}_2$ are the transition probability matrices conditional on the response and nonresponse cases, respectively. $\mathbf{M}_0$ is the convex combination of them due to the Law of Total Probability. The resulting Markov chain is aperiodic and irreducible, provided $x_C x_D \neq 0$. And there is a unique stationary distribution. By solving the linear equation $\vec{\pi}_0 \mathbf{M}_0 = \vec{\pi}_0$, we obtain the stationary distribution $\vec{\pi}_0$ which is given by

$$\vec{\pi}_0 = (\pi_{CC}, \pi_{CD}, \pi_{DD}) = \chi^{-1}(\gamma_3\gamma_4, \gamma_1\gamma_4, \gamma_1\gamma_2), \tag{A4}$$

where

$$\begin{aligned} \gamma_1 &= pkx_D(x_C R_C + x_D R_D) + (1-p)kx_D R_D, \\ \gamma_2 &= \tfrac{1}{2}kx_D p(x_C R_C + x_D R_D) + \tfrac{1}{2}(1-p)kx_D R_D, \\ \gamma_3 &= \tfrac{1}{2}kx_C p(x_C R_C + x_D R_D) + \tfrac{1}{2}(1-p)kx_C R_C, \\ \gamma_4 &= pkx_C(x_C R_C + x_D R_D) + (1-p)kx_C R_C, \end{aligned} \tag{A5}$$

and $\chi = (\gamma_3\gamma_4 + \gamma_1\gamma_4 + \gamma_1\gamma_2)$ is the normalization factor.

$\vec{\pi}_0$ characterizes the relative frequencies of different types of links when the network structure is in the stationary regime.

APPENDIX B: REPLICATOR-LIKE EQUATION OF THE COOPERATION DYNAMICS

Normally it is quite challenging to analyze the entangled dynamics of strategy update and social relationship adjustment [26]. Here we overcome this challenge by assuming that the network update is much faster than the strategy update ($\omega \ll 1$). In this case, the network structure reaches its stationary regime before a strategy update happens. This assumption allows us to obtain a more tractable model approximation. In light of this, the idea of time-scale separation has been frequently used in a variety of complex dynamics [17–19,37,38].

Note that there are three types of social ties. Let N_{CC}, N_{CD}, and N_{DD} be the numbers of *CC*, *CD*, and *DD* links. In the stationary regime we have

$$N_{CC} = L\pi_{CC}, \quad N_{CD} = L\pi_{CD}, \quad N_{DD} = L\pi_{DD}.$$

$L = N\langle d\rangle/2$ is the total number of links, which remains unchanged during the network updates. $\vec{\pi}_0 = (\pi_{CC}, \pi_{CD}, \pi_{DD})$ represents the stationary distribution given by Eq. (A4). We then calculate the average payoffs of cooperators and defectors, respectively, as follows:

$$\begin{aligned} f_C &= (2(b-c)N_{CC} - cN_{CD})/Nx_C \\ &= (2(b-c)\pi_{CC} - c\pi_{CD})L/Nx_C \end{aligned} \tag{B1}$$

and

$$f_D = bN_{CD}/Nx_D = bL\pi_{CD}/Nx_D. \tag{B2}$$

For large populations, the model with Fermi updating rule can be approximately captured by the a stochastic differential equation as follows [11,18]:

$$\dot{x_C} = x_C(1-x_C)\tanh[\beta(f_C - f_D)/2] + \sqrt{\frac{x_C(1-x_C)}{N}}\xi, \tag{B3}$$

where ξ is a Gaussian white noise with variance 1. In the limit of large populations ($N \to +\infty$), the stochastic term vanishes and we obtain

$$\dot{x_C} = x_C(1-x_C)\tanh[\beta(f_C - f_D)/2]. \tag{B4}$$

Let

$$G_0(x_C) = (f_C - f_D)^{-1}\tanh[\beta(f_C - f_D)/2], \tag{B5}$$

then Eq. (B4) can be rewritten as

$$\dot{x_C} = G_0(x_C)x_C(1-x_C)(f_C - f_D). \tag{B6}$$

Note that $G_0(x_C)$ is positive; Eqs. (B6) and (B4) have the same fixed points and stability properties. In other words, they are equivalent to each other in terms of evolutionary stability. Therefore, Eq. (B6) captures the evolution of cooperation in our model. Substituting Eqs. (B1), (B2), (A4), and (A5) into Eq. (B6) leads to

$$\dot{x_C} = Q(x_C)x_C(1-x_C)\{(R_C-R_D)[(1-p)b-c]x_C - R_Dc\}, \tag{B7}$$

where

$$Q(x_C) = \frac{Lk^2}{N\chi}[p(x_CR_C + x_DR_D) + (1-p)R_C] \tag{B8}$$

is a positive rescaling factor. Therefore, we simplify (B7) as

$$\dot{x_C} = x_C(1-x_C)\{(R_C-R_D)[(1-p)b-c]x_C - R_Dc\} \tag{B9}$$

without changing its evolutionary stability.

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