Chirped self-similar solitary waves for the generalized nonlinear Schrödinger equation with distributed two-power-law nonlinearities

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(Received 11 July 2019; published 8 October 2019)

We investigate the propagation characteristics of the chirped self-similar solitary waves in non-Kerr nonlinear media within the framework of the generalized nonlinear Schrödinger equation with distributed dispersion, two-power-law nonlinearities, and gain or loss. This model contains many special types of nonlinear equations that appear in various branches of contemporary physics. We extend the self-similar analysis presented for searching chirped self-similar structures of the cubic model to a more general problem involving two nonlinear terms of arbitrary power. A variety of exact linearly chirped localized solutions with interesting properties are derived in the presence of all physical effects. The solutions comprise bright, kink and antikink, and algebraic solitary wave solutions, illustrating the potentially rich set of self-similar pulses of the model. It is shown that these optical pulses possess a linear chirp that leads to efficient compression or amplification, and thus are particularly useful in the design of optical fiber amplifiers, optical pulse compressors, and solitary wave based communication links. Finally, the stability of the self-similar solutions is discussed numerically under finite initial perturbations.

DOI: 10.1103/PhysRevE.100.042208

I. INTRODUCTION

Self-similar pulse propagation in single-mode optical fibers is currently a subject of intensive theoretical and experimental studies [1–7]. These self-similar waves are potentially useful for various applications in optical soliton telecommunications, since they can maintain their overall shapes but allow their amplitudes and widths to change with the modulation of the system parameters such as dispersion, nonlinearity, gain, inhomogeneity, and so on [8,9]. The underlying model frequently exploited is the generalized nonlinear Schrödinger equation (NLSE) with distributed dispersion, nonlinearity, and gain or loss, which possesses a rich variety of linearly chirped self-similar solutions under different parametric conditions (see, e.g., Refs. [1,2]).

However, many practical materials have been shown to display important physical effects, such as saturation, which requires further generalizations of the NLSE to describe the behavior of propagating envelopes. In such non-Kerr materials, the nonlinear refractive index deviates from the Kerr law leading to the appearance of additional higher-order physical effects such as the quintic nonlinearity. Experimentally, the competing cubic-quintic nonlinearity appears in many optical materials, such as chalcogenide glasses [10], organic materials [11], colloids [12], dye solutions [13], and ferroelectrics [14]. For these media, an extension of NLSE including the cubic-quintic nonlinearity is used to model the propagation of light pulses. It is worthy to note that the cubic-quintic NLSE can also be seen as a particular form of the much more generic NLSE with two-power-law nonlinearities, which has attracted much interest in recent years [15–17]. This equation governs the evolution of optical fields in a broad range of nonlinear materials whose nonlinear refractive index presents a generic power-law–type dependence on the electric field amplitude E as [16] $n = n_0 + n_p|E|^p + n_{2p}|E|^{2p}$, where n_0 is the linear index, n_p and n_{2p} are the nonlinear coefficients, and the exponent p is a positive constant [16]. For the case of nonlinear saturation, we must have $n_p n_{2p} < 0$ [16].

Recently, Kruglov *et al.* introduced an interesting self-similar analysis to search for self-similar solutions to the generalized NLSE with distributed dispersion, nonlinearity, and gain or loss [1,2]. This analysis has been extended to study chirped self-similar solutions of the generalized NLSE with additional nonlinear gain or absorption term [5]. More recently, the use of self-similar analysis has also been extended to the case of cubic-quintic optical media, where the chirped bright soliton solutions have been obtained in the anomalous and normal dispersion regimes [18]. Until now, no attempts had been made to find self-similar localized structures in NLS models with two nonlinear terms of arbitrary power. This problem is of prime importance since the governing equation covers all types of single and competing nonlinearities that are of physical interest in applications to various branches of contemporary physics [15–19]. On the basis of this motivation, we adopt the self-similar analysis for investigating the various exact self-similar solutions of the generalized NLSE with distributed arbitrary two-power-law nonlinearities. As is well known, solitary waves in non-Kerr nonlinear media

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differ fundamentally from solitons in a pure Kerr medium, both in shape and interaction. It is interesting to note that having an analytic self-similar solution with arbitrary power has the advantage that we can also consider all particular cases analytically, including the solutions that have been studied in recent works. In addition, exact stable self-similar solutions are of great significance for understanding of the nonlinear dynamics and their relevant applications.

The paper is structured as follows. An extension of the self-similar analysis to investigate chirped self-similar localized solutions of the NLS model with arbitrary power-law nonlinearities is presented in Sec. II. Analytical results for exact self-similar solitary wave solutions are collected in Sec. III. The stability of those self-similar structures is addressed in Sec. IV. The paper is concluded with Sec. V, where we also put forward an extension of the self-similarity problem for the case of propagating optical beams in a cubic-quintic-septimal nonlinear medium which has been studied recently, both experimentally and theoretically (see Refs. [20–25]).

II. MODEL AND SELF-SIMILAR ANALYSIS

We consider the evolution of optical pulses in a nonlinear medium exhibiting two-power-law nonlinearities wherein the pulse propagation is described by the generalized NLSE with distributed coefficients:

$$i\psi_z - \frac{\beta(z)}{2}\psi_{\tau\tau} + \gamma_1(z)|\psi|^p\psi - \gamma_2(z)|\psi|^{2p}\psi - i\frac{g(z)}{2}\psi = 0, \quad (1)$$

where $\psi(z,\tau)$ is the complex envelope of the electric field and z and τ, respectively, represent the propagation distance and retarded time. $\beta(z)$ and $g(z)$ represent the group-velocity dispersion and the distributed gain or loss function, respectively. The functions $\gamma_1(z)$ and $\gamma_2(z)$ stand for the nonlinearity parameters related to the terms $|u|^p u$ and $|u|^{2p}u$, respectively.

As we all know, in the absence of gain, Eq. (1) with constant dispersion and nonlinearity coefficients has been studied in abundant sets [15–17], and different types of soliton solutions have been obtained. The one-soliton solution of the case of dual-power law nonlinearity has also been investigated using the traveling wave technique in Ref. [26]. Here, we focus on self-similar solitary wave solutions of the model with variable dispersion, variable power-law nonlinearities, and variable gain or loss. Such a study is significant especially for practical application in both optical fiber amplifier systems and in fiber compressors.

Equation (1) contains many interesting particular cases such as the generalized cubic NLSE ($p = 2$ and $\gamma_2(z) = 0$) [1,2], and the generalized cubic-quintic NLSE ($p = 2$) [18]. Considering the simplest case $p = 1$, $g(z) = 0$, and γ_2 is a constant, Eq. (1) reduces to the so-called quadratic-cubic NLSE, which has been recently studied as an approximate model of a relatively dense quasi-one-dimensional Bose-Einstein condensate with repulsive contact interactions between atoms and a long-range dipole-dipole attraction between them [27]. Below, we consider the most general case, in which p is arbitrary, and search for various exact self-similar solutions, especially solitary wave solutions for the present model. A variety of exact self-similar localized solutions is found in the presence of all the physical parameters. This allows the construction of exact propagating self-similar waves for a subclass of NLS models with any particular nonlinearity, such as the quadratic-cubic NLSE (for $p = 1$), the cubic-quintic NLSE (for $p = 2$), and so on.

Now, we adopt the self-similar analysis to the cubic NLSE [1,2] to investigate the chirped soliton solutions of the general model (1). We start with the representation of the complex field $\psi(z,\tau)$ in the form

$$\psi(z,\tau) = U(z,\tau)e^{i\phi(z,\tau)}, \quad (2)$$

where U and ϕ are the real amplitude and phase, which are functions of z and τ. Insertion of Eq. (2) into Eq. (1) leads to coupled real equations for U and ϕ that have a rather cumbersome form. To simplify them, and in order to find the linearly chirped self-similar solutions of Eq. (1), we assume a quadratic phase given by

$$\phi(z,\tau) = a(z) + c(z)(\tau - \tau_c)^2, \quad (3)$$

where $a(z)$ and $c(z)$ are functions of z and τ_c is the center of the pulse. Moreover, the amplitude $U(z,\tau)$ of the self-similar solutions can be expressed in the form,

$$U(z,\tau) = \frac{1}{\sqrt{\Gamma(z)}}F(T)\exp\left[\frac{G(z)}{2}\right], \quad (4)$$

where the scaling variable T and the function $G(z)$ are defined by

$$T = \frac{\tau - \tau_c}{\Gamma(z)}, \quad G(z) = \int_0^z g(z')dz'. \quad (5)$$

Here, $G(z)$ and $F(T)$ are some functions to be determined. Without loss of generality, the function $\Gamma(z)$ at $z = 0$ is assumed to take the value $\Gamma(0) = 1$.

Substituting the solution (2) with the phase (3) into Eq. (1) and separating into real and imaginary parts, we obtain the following two equations:

$$U\left[\frac{da}{dz} + \frac{dc}{dz}(\tau - \tau_c)^2\right] = 2\beta(z)c^2(z)(\tau - \tau_c)^2 U - \frac{\beta(z)}{2}U_{\tau\tau} + \gamma_1(z)U^{p+1} - \gamma_2(z)U^{2p+1} \quad (6)$$

and

$$U_z = \beta(z)c(z)U + 2\beta(z)c(z)(\tau - \tau_c)U_\tau + \frac{g(z)}{2}U. \quad (7)$$

Setting the coefficients of independent terms $(\tau - \tau_c)^j$ (with $j = 0, 2$) in Eq. (6) equal to zero, one gets

$$\frac{dc}{dz} = 2\beta(z)c^2(z), \quad (8)$$

$$U\frac{da}{dz} = -\frac{\beta(z)}{2}U_{\tau\tau} + \gamma_1(z)U^{p+1} - \gamma_2(z)U^{2p+1}. \quad (9)$$

On further substitution of Eq. (4) into Eq. (7), we find that Eq. (7) is satisfied if the function $\Gamma(z)$ is expressed as

$$\frac{1}{\Gamma(z)}\frac{d\Gamma}{dz} = -2\beta(z)c(z). \quad (10)$$

Solving Eqs. (8) and (10) gives

$$c(z) = \frac{c_0}{1 - c_0 D(z)}, \quad \Gamma(z) = 1 - c_0 D(z), \quad (11)$$

where $c_0 = c(0)$ is a nonzero integration constant and $D(z)$ represents the accumulated dispersion function given by

$$D(z) = 2\int_0^z \beta(z')dz'. \tag{12}$$

Upon substitution of the amplitude (4) into Eq. (9), we arrive at the following nonlinear differential equation:

$$\frac{d^2F}{dT^2} + \frac{2\Gamma^2}{\beta}\frac{da}{dz}F - \frac{2\gamma_1\Gamma^{2-p/2}}{\beta}\exp\left[\frac{p}{2}G(z)\right]F^{p+1} + \frac{2\gamma_2\Gamma^{2-p}}{\beta}\exp[pG(z)]F^{2p+1} = 0, \tag{13}$$

in terms of the function $F(T)$, which is only dependent on the scaling variable T. Generally, the coefficients in Eq. (13) are functions of variable z and as a result they must be constants in order to find nontrivial solutions $F(T)$, i.e.,

$$-\frac{2\Gamma^2}{\beta}\frac{da}{dz} = \lambda_1, \tag{14}$$

$$\frac{\gamma_1\Gamma^{2-p/2}}{\beta}\exp\left[\frac{p}{2}G(z)\right] = \lambda_2, \tag{15}$$

$$-\frac{\gamma_2\Gamma^{2-p}}{\beta}\exp[pG(z)] = \lambda_3, \tag{16}$$

where λ_i(with $i = 1, 2, 3$) are constants. From Eqs. (14)–(16), we obtain

$$\lambda_1 = -\frac{2}{\beta(0)}\frac{da}{dz}\bigg|_{z=0}, \quad \lambda_2 = \frac{\gamma_1(0)}{\beta(0)}, \quad \lambda_3 = -\frac{\gamma_2(0)}{\beta(0)}, \tag{17}$$

because $\Gamma(0) = 1$ and $G(0) = 0$. In addition, the phase offset $a(z)$ can be determined by using Eq. (14) as

$$a(z) = a_0 - \frac{\lambda_1}{2}\int_0^z \frac{\beta(z')dz'}{[1 - c_0D(z')]^2}, \tag{18}$$

where we used the expression of $\Gamma(z)$ given in Eq. (11) and a_0 is an integration constant. As a consequence, Eqs. (12) and (18) yield an expression for $a(z)$ as

$$a(z) = a_0 - \frac{\lambda_1 D(z)}{4[1 - c_0D(z)]}. \tag{19}$$

Combining Eqs. (3), (11), and (19), we can obtain the expression of the function $\phi(z,\tau)$ as

$$\phi(z,\tau) = a_0 - \frac{\lambda_1 D(z)}{4[1 - c_0D(z)]} + \frac{c_0(\tau - \tau_c)^2}{1 - c_0D(z)}, \tag{20}$$

which reveals that the phase of the complex field can be well controlled by engineering the dispersion profile for given c_0 and τ_c. As concerns the amplitude evolution equation, one can see that, for a nontrivial case, Eq. (13) can be written as

$$\frac{d^2F}{dT^2} - \lambda_1 F - 2\lambda_2 F^{p+1} - 2\lambda_3 F^{2p+1} = 0. \tag{21}$$

Employing the transformation [2] $\theta = T/\tau_0$, one can convert the preceding equation to the following form:

$$\left(\frac{dF}{d\theta}\right)^2 = K\tau_0^2 + \lambda_1\tau_0^2F^2 + \frac{4\lambda_2\tau_0^2}{(p+2)}F^{p+2} + \frac{2\lambda_3\tau_0^2}{(p+1)}F^{2(p+1)}, \tag{22}$$

where τ_0 is the initial pulse width and K is the integration constant.

Now we proceed to find the distributed gain function using Eqs. (15) and (16) and the same is given by

$$g(z) = \frac{2}{p}\frac{1}{\eta(z)}\frac{d\eta(z)}{dz} - \frac{2c_0\beta(z)}{\Gamma(z)}, \tag{23}$$

where we define the function $\eta(z)$ as

$$\eta(z) = \frac{\gamma_1(z)}{\gamma_2(z)}, \quad \eta(0) = \frac{\gamma_1(0)}{\gamma_2(0)}. \tag{24}$$

From Eqs. (15) and (16), the condition for the variation of the nonlinear parameter $\gamma_2(z)$ is

$$\gamma_2(z) = -\frac{\lambda_3}{\lambda_2^2}\frac{\gamma_1^2(z)\Gamma^2(z)}{\beta(z)}. \tag{25}$$

Based on all these findings, we can present the amplitude of the self-similar solution of Eq. (1) as

$$U(z,\tau) = \left(\frac{\rho_2(z)}{\rho_1(z)}\frac{\rho_1(0)}{\rho_2(0)}\right)^{1/p} F\left(\frac{\tau - \tau_c}{\tau_0[1 - c_0D(z)]}\right), \tag{26}$$

where we define the functions $\rho_1(z)$ and $\rho_2(z)$ as

$$\rho_1(z) = \frac{\beta(z)}{\gamma_1(z)}, \quad \rho_2(z) = \frac{\beta(z)}{\gamma_2(z)}, \tag{27}$$

$$\rho_1(0) = \frac{\beta(0)}{\gamma_1(0)}, \quad \rho_2(0) = \frac{\beta(0)}{\gamma_2(0)}. \tag{28}$$

Making use of the quantity $\rho(z) = \rho_2(z)/\rho_1(z)$, we finally write Eq. (26) in the form

$$U(z,\tau) = \left(\frac{\rho(z)}{\rho(0)}\right)^{1/p} F\left(\frac{\tau - \tau_c}{\tau_0[1 - c_0D(z)]}\right). \tag{29}$$

To summarize, the results given in Eqs. (11), (12), (17), (19), and (22)–(25) are the required conditions for the existence of self-similar solutions in Eqs. (2)–(5) of the generalized NLSE with distributed coefficients given in Eq. (1).

III. CHIRPED SELF-SIMILAR SOLITARY WAVE SOLUTIONS

In this section, we analytically solve the nonlinear differential equation (22) and obtain many types of nontrivial self-similar solutions under some parametric conditions. As will be shown below, the functional form of these solutions is dependent on the power of the nonlinearity p which can take any integer value, and therefore enables us to get self-similar solutions for any subclass of the general model (1). In the following subsections, we intend to investigate the three important self-similar solitary waves, namely, bright, kink, and antikink and algebraic.

A. Chirped self-similar bright solitary waves

We first integrate Eq. (22) for the case $K = 0$ and use Eq. (29) to obtain the following amplitude of the solitary wave solution:

$$U(z,\tau) = \left(\frac{\rho(z)}{\rho(0)}\right)^{1/p}\left[\frac{A_b}{\cosh\left[\mu_b\left(\frac{\tau-\tau_c}{\tau_0[1-c_0D(z)]}\right)\right] + B}\right]^{1/p}, \tag{30}$$

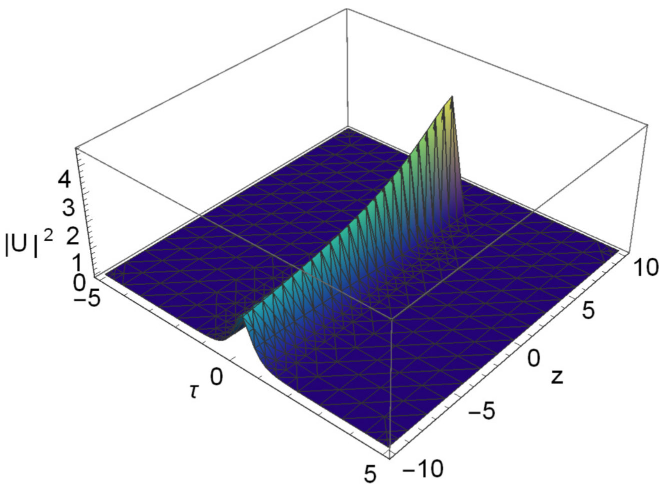

FIG. 1. Pulse compression of chirped self-similar bright soliton when $\rho_1 = 1$, $\rho_2 = 0.4$, $\tau_0 = 0.3$, $\tau_c = -0.3$, $p = 2$, and $c_0 = 0.1$.

where A_b, B, and μ_b are real parameters given by the relations

$$A_b = -\frac{(p+2)\lambda_1 B}{2\lambda_2}, \quad \mu_b = p\tau_0\sqrt{\lambda_1}, \tag{31}$$

with

$$B = \pm\left[1 - \frac{(p+2)^2\lambda_1\lambda_3}{2(p+1)\lambda_2^2}\right]^{-1/2}, \tag{32}$$

provided that the arbitrary constant $\lambda_1 > 0$ in order to ensure the pulse width μ_b to be real, and $\lambda_1 < |\frac{2(p+1)\lambda_2^2}{(p+2)^2\lambda_3}|$.

Expression (30) together with the representation (2) describes a linearly chirped self-similar bright solitary pulse, which propagates in a self-similar manner in a nonlinear medium exhibiting two-power-law nonlinearities. In general, it is well known that the chirping is highly useful in realizing either a compressor or an optical amplifier. However, here, we delineate the compression of chirped self-similar bright solitary pulse under the influence of two-power-law nonlinearities. Figure 1 illustrates the compression of chirped self-similar bright solitary pulse at various stages of propagation.

The above solution is a generalization of a particular bright solution found in Ref. [18] for the case of the cubic-quintic model. Specifically, when $p = 2$ and $g(z) = 0$, one can obtain the relations $\lambda_2 = \frac{\Gamma}{\rho_1}$, $\lambda_3 = -\frac{1}{\rho_2}$, and $\frac{\rho(z)}{\rho(0)} = \frac{1}{\Gamma}$ from Eqs. (15)–(17) and (27). In this case, and for the choice $\lambda_1 = \tau_0^{-2}$ and $\theta = \frac{\tau-\tau_c}{\tau_0[1-c_0D(z)]}$, the solution (30) can be reduced to a simplified form

$$U(z,\tau) = \frac{1}{\tau_0[1-c_0D(z)]} \times \left[\frac{\mp 2\rho_1}{\sqrt{1+\frac{8\rho_1^2}{3\rho_2\tau_0^2[1-c_0D(z)]^2}}\cosh(2\theta) \pm 1}\right]^{\frac{1}{2}}, \tag{33}$$

which when inserted in (2) yields the bright solitary pulse solution of the cubic-quintic model found in Ref. [18].

It is worth noting that many sets of bright solitary waves propagating in different nonlinear non-Kerr media can be obtained from Eq. (30) if the value of the nonlinearity power p is selected.

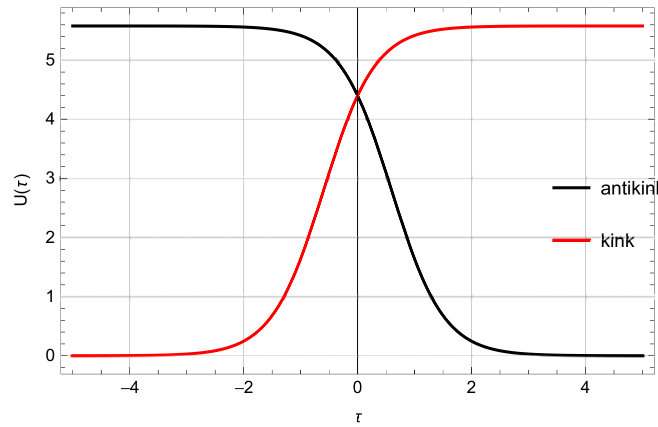

FIG. 2. Propagation of chirped self-similar kink and antikink solitary wave when $\lambda_3 = 0.9$, $\lambda_2 = 0.7$, $p = 2$, $\tau_c = -0.6$, $\tau_0 = 0.6$, and $c_0 = 0.2$.

B. Chirped self-similar kink and antikink solitary waves

Another type of self-similar solution follows from Eqs. (22) and (29) under the conditions $\lambda_1 = \frac{2(p+1)\lambda_2^2}{(p+2)^2\lambda_3}$ and $K = 0$. In this case, the amplitude of the solitary wave solution takes the form

$$U(z,\tau) = \left(\frac{\rho(z)}{\rho(0)}\right)^{1/p} \times \left[A_k\left(1 \pm \tanh\left[\mu_k\left(\frac{\tau-\tau_c}{\tau_0[1-c_0D(z)]}\right)\right]\right)\right]^{1/p}, \tag{34}$$

where

$$A_k = -\frac{\lambda_1(p+2)}{4\lambda_2}, \quad \mu_k = \frac{p\tau_0\sqrt{\lambda_1}}{2}, \tag{35}$$

provided that $\lambda_1 > 0$ to ensure the parameter μ_k to be real. We also assume that $\lambda_2 < 0$ and $\lambda_3 > 0$.

Equation (34) together with the representation (2) describes kink structures, where the upper sign corresponds to the kink-shaped solitary wave, while the lower sign corresponds to the antikink-shaped solution. Figure 2 represents the propagation of chirped self-similar kink and antikink type solitary pulses under the influence of two-power-law nonlinearities.

C. Chirped self-similar algebraic solitary waves

We also report another interesting solitary wave solution for Eq. (22) under the parametric condition $K = \lambda_1 = 0$. In particular, for $\lambda_2 > 0$ and $\lambda_3 < 0$, we find the solution of Eq. (22) with (29) is of the following form:

$$U(z,\tau) = \left(\frac{\rho(z)}{\rho(0)}\right)^{1/p}\left(\frac{2\lambda_2(p+1)(p+2)}{2\lambda_2^2\tau_0^2p^2(p+1)\theta^2 - \lambda_3(p+2)^2}\right)^{1/p}, \tag{36}$$

where $\theta = \frac{\tau-\tau_c}{\tau_0[1-c_0D(z)]}$.

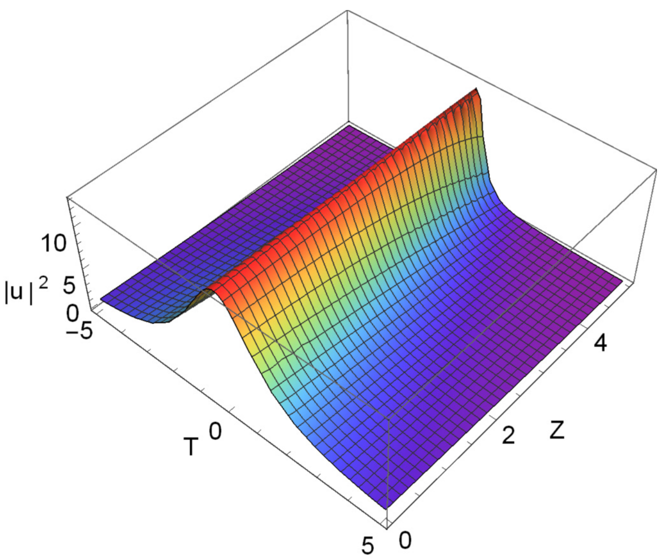

FIG. 3. Propagation of chirped self-similar algebraic solitary wave when $\lambda_3 = -0.2$, $\lambda_2 = 0.5$, $p = 2$, $\tau_c = -0.3$, $\tau_0 = 0.3$, and $c_0 = 0.15$.

Solution (36) together with Eq. (2) describes a linearly chirped self-similar algebraic solitary pulse and its propagation is depicted in Fig. 3 in a nonlinear medium exhibiting two-power-law nonlinearities. It should be noted that Eq. (22) could also admit other types of self-similar solutions such as periodic and singular wave solutions. However, as these solutions are beyond the scope of the paper, they will be reported elsewhere.

Before we leave this section, we would like to mention that Eqs. (30), (34), and (36) together with the representation (2) interestingly describe the exact linearly chirped solitary pulse solutions that can propagate self-similarly in non-Kerr media with varying dispersion, two-power-law nonlinearities, and gain and/or loss. Different from their simplest form given within the constant-coefficient NLSE with two-power-law nonlinear terms framework [15–17,26], the obtained solitary wave solutions would take a different fundamental form; it can exhibit a self-similar nature, which continually compress or expand while propagating with a developing chirp in the presence of gain or loss, respectively. This interesting property has not been singled out so far in the setting of NLS models with two nonlinear terms of arbitrary power.

IV. STABILITY OF THE CHIRPED SOLITARY WAVES

The stability of solitary wave solutions is of prime importance for its physical feasibility. Note that only stable (or weakly unstable) solitary waves can be observed experimentally [28]. It is also interesting to note that the solitary waves propagating in non-Kerr nonlinear media preserve their shape, but their stability is not assured due to the nonintegrability of the underlying generalized NLSE. In fact, their stability under finite perturbation is a crucial issue. It is then essential to analyze the stability of the evolution of exact solutions against small perturbations.

To check the solution stability of the self-similar structures presented earlier, as a representative case, we consider Eq. (30), and perform the stability analysis using a split-step Fourier method. The analysis is twofold: first we study the propagation characteristics through direct numerical simulation based on Eq. (1), followed by the stability check against perturbation such as photon noise. Figure 4 shows the numerical simulation of stable propagation of the self-similar solution. The parameters of choice are $\beta = -0.5$ ps^{2}m^{-1}, $\gamma_1 = 0.2362$ W^{-1}m^{-1}, $\gamma_2 = 0.4724$ W^{-2}m^{-1}, $\tau_0 = 5$ ps, $\sigma = 0.005$ m^{-1}, and $g = 0$. Its worth noting that Fig. 4 attributes to an ideal environment, which is not the case exactly in real world problems. There are numerous effects that can contribute to instability in the propagation of stable structures. Therefore, it becomes necessary to study the stability of the solution in an environment subject to external noise or perturbations. To this end, we generated a photon noise, which corresponds to 0.045% of the average power of the input profile. This is indeed an appreciable noise level, which can potentially perturb propagation characteristics. It is quite evident from Fig. 5 that the present self-similar solution shows a remarkable stability, despite a strong noise perturbation.

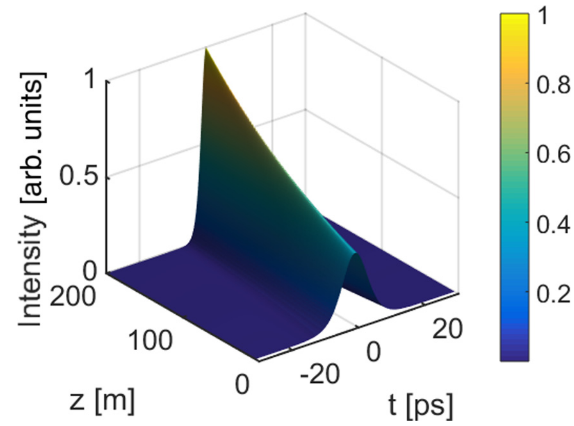

FIG. 4. Simulation of stable self-similar propagation of input profile given by Eq. (30). The parameters of the simulation are $\beta = -0.5$ ps^{2}m^{-1}, $\gamma_1 = 0.2362$ W^{-1}m^{-1}, $\gamma_2 = 0.4724$ W^{-2}m^{-1}, $\tau_0 = 5$ ps, $\sigma = 0.005$ m^{-1}, and $g = 0$.

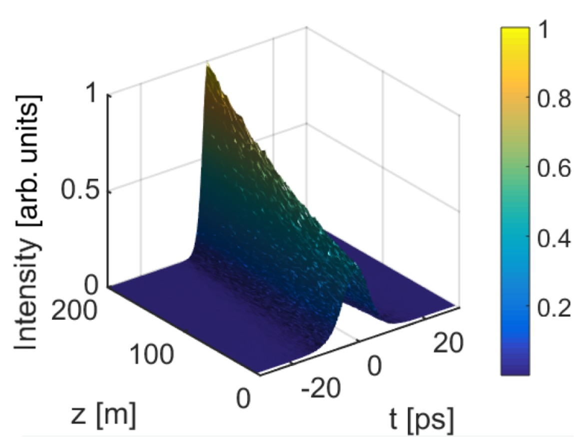

FIG. 5. Stability of the self-similar solution against perturbation for parameters $\beta = -0.5$ ps^{2}m^{-1}, $\gamma_1 = 0.2362$ W^{-1}m^{-1}, $\gamma_2 = 0.4724$ W^{-2}m^{-1}, $\tau_0 = 5$ ps, $\sigma = 0.005$ m^{-1}, and $g = 0$. The relative strength of the noise is 0.045%.

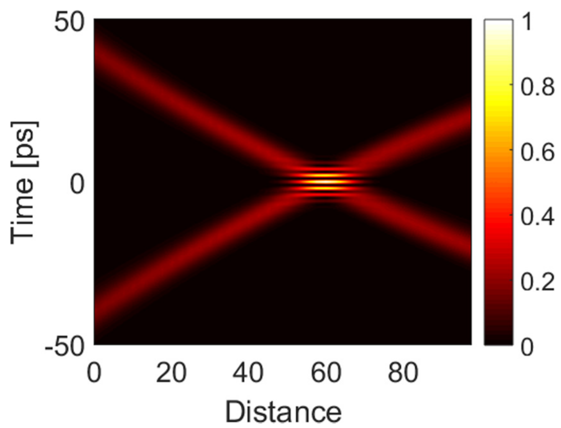

FIG. 6. Evolution of the intensity profile and the colliding dynamics off self-similar structures as a function of propagation distance.

For further insight into the robustness of the self-similar structures, we extend to inspect the stability through a more powerful test than the noise perturbation, namely, collision stability. To perform the stability against collision, we synthesize two similar entities and inject into two channels of the waveguide. The simulation parameters are tuned in such a way that the two entities collide within the simulation window while they propagate down the fiber. Let U_1 and U_2 be the input profiles ($z = 0$) given by Eq. (30). The temporal shift between the two input pulses is set to be 80 ps. The system parameters are the same as defined before (refer to caption). Figure 6 shows the propagation dynamics of the two identical self-similar structures in the opposite channel. The two entities converge during propagation and at a distance $Z_c \approx 60$ m they collide as shown in Fig. 6. It is quite evident from the intensity profile that the two pulse profiles collide elastically and continue their propagation. This further confirms the remarkable stability and robustness of the underlined self-similar structures.

V. CONCLUSION

In conclusion, we would like to point out that the present work is a natural but significant generalization of envelope soliton solutions propagating in non-Kerr media in the presence of two-power-law nonlinearities by considering distributed parameters of group velocity dispersion, power-law nonlinearities, and gain or loss. We have studied ultrashort pulse propagation in such media within the framework of the generalized nonlinear Schrödinger equation with coefficients varying with the propagation distance. A variety of exact linearly chirped solitary pulse solutions that can propagate self-similarly of this model have been found by means of the self-similar scaling analysis. The solutions comprise the chirped self-similar optical bright-, kink-, and antikink-type solitary waves. We have also obtained the chirped self-similar algebraic solitary wave solutions, which are very meaningful in contemporary optics. The conditions on the model parameters for the existence of the derived self-similar structures have also been presented. These conditions show a subtle balance among the distributed dispersion, gain (loss), and two-power-law nonlinearities, which have a profound implication to control the solitary wave dynamics. It is found that the functional form of these propagating envelopes is dependent on the power of the nonlinearity which can take arbitrary integer values, thus allowing us to obtain exact analytic self-similar solutions of special models. These results provide a significant generalization of chirped self-similar localized waves propagating in non-Kerr media in the presence of two-power-law nonlinearities. We are of the opinion that the results reported in this work shall pave the way to realize very compact and highly efficient pulse compressors whose advantages would be the high degree of compression and the high quality of the transform-limited compressed pulses.

It would be particularly relevant to extend the above analysis to study the self-similar propagation of optical beams inside a nonlinear medium exhibiting nonlinearities up to the seventh order. For such a problem, the beam propagation will be governed by the generalized nonlinear Schrödinger equation with distributed dispersion, cubic, quintic, and septimal nonlinearities and gain and/or loss. We note that the investigation of soliton pulse propagation within the framework of the cubic-quintic-septimal model with constant coefficients has attracted much interest in recent years, because such combined nonlinearities have been recently observed in some optical materials (see Refs. [20–25]). Such studies will be deferred to future work.

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