Logical response induced by temperature asymmetry

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It is known that the reliable logical response can be extracted from a noisy bistable system at an intermediate value of noise strength when two random or periodic, two-level, square waveform serve as the inputs. The asymmetry of the potential has a very important role and dictates the type of logical operation, such as OR or AND, exhibited by the system. Here we show that one can construct logic gates with symmetric bistable potential if the two states of the double-well are thermalized with two different heat baths. It has been found that if a given state is kept at a sufficiently low temperature compared to the other, the system shows one kind of logic behavior (say, OR). Interestingly, the system's response turns into the other kind (say, AND) if the temperature of the initial low-temperature well is increased gradually and the quality of the logical response first improves and then weakens after passing through a maximum at a particular value. However, the reliability of the second kind of logical response (AND) is not as good as the first kind (OR) and depends on the amplitude of the inputs. Still one can construct both kinds of logic gates with maximum reliability by properly choosing the initial low-temperature well.

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I. INTRODUCTION

Logic gates are the basis of digital computation [1]. During the logic operation, typically two inputs are converted to a single logical output by following Boolean algebra. For binary logic operations the inputs and outputs possess two states, the "on" state and the "off" state. Therefore, the logic gates are generally used to construct electronic switches. However, the idea of the input-output correspondence of logic operations has been extended to several other areas, such as optical [2], mechanical [3], physical [4], chemical [5–8], biological [9–11], molecular [12,13], and several other domains [14–21].

The logic devices are reduced in size day by day and the speed of the computation increases. In this scenario, the interference of noise with the functioning of the machinery becomes important [22]. Therefore, it is crucial to understand the interplay between the signal, noise, and the dynamics of the system. Much attention has been devoted to understand the novel effect where noise acts constructively, by exploiting the nonlinearity of the system to enhance the strength of a periodic signal significantly. This is well-known as the stochastic resonance [23–26] phenomenon. The presence of noise in the system can also bring about another interesting effect known as stochastic synchronization [27]. In general, synchronization [28,29] occurs where nonlinear self-sustained oscillators are either subject to periodic force or coupled with each other. The effect is exhibited by locking or suppression of the natural frequency by periodic forcing. Noise can induce synchronizationlike phenomenon in stochastic bistable systems which have no natural frequency. In case of this stochastic synchronization process, noise plays a role to create a mean switching frequency between the metastable states which serves as an analog of the natural frequency. These resonance and synchronization phenomena improve the quality of signal during information processing [30–33]. Here, specifically, we focus on understanding the role of the noise present in the system where input-output correspondence can be interpreted as logical response. In spite of some similarities regarding setting up the focus on to understand the response of the system against external inputs in presence of noise, there is a key difference between the present study and the stochastic resonance or stochastic synchronization phenomenon. In the latter cases the system is subject to only one input signal, whereas in the former case system's feedback is studied against two input signals representing two essential binary inputs.

For most of the cases logic operations take place in systems of very small dimension and the role of thermal fluctuations is inevitable. Therefore, to understand the basic idea of the logical response in different systems, the output memory states can be represented by the state of a Brownian particle in a bistable potential, the two wells representing the binary memory states 0 and 1, in general. This is similar in spirit of the consideration of the dynamics of a Brownian particle in a bistable potential subject to periodic signal to understand the process of stochastic resonance [24,26]. The noise term present in the dynamical equation of the system represents thermal fluctuations and the response of the system towards external inputs is captured through the motion of the Brownian particle in the double-well. As the particle jumps between the two wells due to the effect of the noise and the signal, the output is considered to be exemplified by the position variable of the particle. It has been shown that the input-output correlation shows logical behavior at an optimum noise-level and the role of the asymmetry of the potential is

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very important to extract the desired kind of logic operation from the system, considering a simple threshold detector [34]. In the above study, the dynamics is mimicked by the Brownian motion in a double-well subject to two random inputs. Similar effect has been extracted from system where no intrinsic force field derived from potential is present and the nonlinearity in the dynamics is encountered as an effect of the irregular boundary of the system [35,36].

The idea of the enhancement of the logical response at a given range of noise which is also known as "logical stochastic resonance" [34–36] suggests that there are two controlling factors which affect the quality and the type of logical response obtained from a system against external stimuli. One is the noise present in the system and the other is the nonlinearity of the system. So far focus has been set on tuning the noise level and adjusting the nonlinearity, more specifically the asymmetry of the system to improve the reliability of the logical response [34–36]. Here, we address a crucial, yet unexplored, issue concerned with the asymmetry of the noise strength associated with the two states of the system. In contrary to the calibration of the asymmetry of the system, we concentrate on maintaining disparate noise-levels linked to the two output memory states. Different noise strengths alter the stability of the states of the system. Similar effect in other contexts has been studied before, and is known as the Landauer's blow-torch effect [37–47]. Landauer's blow-torch effect is the phenomenon which describes that if a portion of a double-well is heated, mass gets transferred from the hot well to the cold well, i.e., temperature difference changes the stability of the states. In the present paper, the idea is to see whether the temperature-asymmetry-induced modified stability of the two-states can make the system to behave as a logic gate when it is subject to external inputs, and if it is possible, then how to tune the thermalization of the two states to get reliable and desired type of logical response from the system. The aimed construction of the logic circuits would not only answer important theoretical issues related to logical behavior controlled by thermal asymmetry of two binary output states, but might also be very useful for practical purposes. It could be more feasible to control the thermalization of the two states than to adjust the nonlinearity of the system.

The paper is organized as follows. In Sec. II the system and the dynamics will be considered. The general rules of logical input-output correspondence for the basic types of logic gates will be discussed in Sec. III. The numerical results will be analyzed in Sec. IV. The paper will be concluded in Sec. V.

II. THE SYSTEM AND THE DYNAMICS

To understand the underlying idea of the basic logic operations in small systems where the presence of noise is essential, we consider a general model of a bistable potential where the two wells represent two binary memory states 0 and 1. We consider that an overdamped Brownian particle is moving in this double-well potential. The system is subject to two wave-trains of two-level, square-wave, random input signals. Two signals are considered to represent two inputs which are essential components of OR (NOR) or AND (NAND) logic operations. The input signals can also be periodic without the loss of generality. The two levels of the signals represent two binary inputs, 0 and 1; say the upper level represents the memory state 1 and the lower level corresponds to the memory state 0. The inputs can appear in four different combinations; (1,1), (1,0), (0,1), and (0,0). Following the connection of neural network [48], which is the computational circuit designed to perform tasks by "learning" examples, we sum up the inputs to a single input signal. So, (1,0) and (0,1) correspond to the same effective input. Therefore, the resultant input is a three-level, square-wave, random, or periodic drive. Due to the application of the external inputs, the particle switches its position between the two wells. So, the output against the external input can be considered to be the state of the particle in either of the two wells. We consider that the state of the particle in the left well corresponds to the output memory state 0 and that at the right well represents the output memory state 1. In other words, the dynamics of the Brownian particle in the above-mentioned set-up signifies the response of the system towards external inputs, i.e., the output state of the system. This definition of the output in terms of the position of the particle has been considered before to capture the response of the system against external bias in asymmetric noisy bistable potential [34] and bilobal system [35].

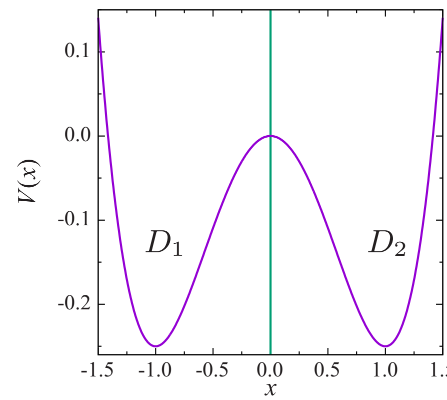

FIG. 1. The bistable system of the form $V(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2$. The noise level associated with the two wells are different. The scaled diffusion coefficient for the left well is D_1 and that for the right well is D_2.

The overdamped dynamics of the Brownian particle in the bistable potential and subject to the external inputs is represented by the following Langevin equation,

$$\Gamma \frac{dx}{dt} = -V'(x) + I_1(t) + I_2(t) + \sqrt{\Gamma k_B T}\xi(t). \qquad (1)$$

In the overdamped limit, the inertial term of the Langevin equation is neglected with respect to the damping force. Therefore, mass does not appear directly in the equation of motion of the overdamped Brownian particle. Here, x represents the position coordinate of the Brownian particle at time t, $-V'(x)$ represents the force field derived from the bistable potential $V(x)$ which has the form, $V(x) = \frac{a}{4}x^4 - \frac{b}{2}x^2$ (Fig. 1). Γ is the friction coefficient. $\xi(t)$ is the zero-mean, Gaussian, white noise and obeys fluctuation-dissipation relation. The properties of $\xi(t)$ are as follows:

$$\langle \xi(t) \rangle = 0,$$
$$\langle \xi(t)\xi(t') \rangle = 2\delta(t - t'). \qquad (2)$$

I_1 and I_2 are the two random or periodic, two-level, square-wave input signals.

In order to make the system and dynamics dimensionless, we scale the position coordinate of the particle with a characteristic length L_x. L_x corresponds to the distance between the two minima of the double-well in absence of any external bias. $\tilde{x} = x/L_x$ represents the dimensionless position coordinate. The time t is divided by the factor $\tau = \Gamma L_x^2/k_B T_R$ to get the dimensionless time, $\tilde{t} = t/\tau$, here k_B is the Boltzmann constant and T_R is a reference temperature. τ is actually twice the time required for the Brownian particle to diffuse the distance L_x. The forces have been made dimensionless by scaling with the factor $\Gamma L_x/\tau$. Equation (1) in dimensionless form becomes,

$$\frac{d\tilde{x}}{d\tilde{t}} = -\tilde{a}\tilde{x}^3 + \tilde{b}\tilde{x} + \tilde{I}_1(\tilde{t}) + \tilde{I}_2(\tilde{t}) + \sqrt{D}\tilde{\xi}(\tilde{t}). \tag{3}$$

$\tilde{a}$ and $\tilde{b}$ are dimensionless coefficients which are obtained by scaling a and b with proper factors ($\tilde{a} = a\frac{\tau L_x^2}{\Gamma}$ and $\tilde{b} = b\frac{\tau}{\Gamma}$). $\tilde{I}_1$ and $\tilde{I}_2$ are the scaled inputs and have the form $\tilde{I}_1 = I_1\frac{\tau}{\Gamma L_x}$ and $\tilde{I}_2 = I_2\frac{\tau}{\Gamma L_x}$. $\tilde{\xi}(\tilde{t})$ is the scaled noise term and D corresponds to the scaled diffusion coefficient and has the form T/T_R, where T_R is a reference temperature. For notational convenience and brevity, we shall use untilde description to represent dimensionless quantities from now on.

Now, for the present purpose, we consider that the two wells are thermalized with two different heat baths; say the left well is connected to a heat bath at temperature T_1 and the right well to the heat bath at temperature T_2. The effective diffusion coefficients for the left and the right well are D_1 and D_2, respectively. Therefore, the equation of motion for the Brownian particle in the two wells differ in terms of the effective noise and can be written as

$$\frac{dx}{dt} = -ax^3 + bx + I_1(t) + I_2(t) + \sqrt{D(x)}\xi(t), \tag{4}$$

where $D(x) = D_1$ for $x < 0$ and $D(x) = D_2$ for $x > 0$. It is possible to maintain different temperatures for the two states by changing the scaled diffusion coefficient, without affecting damping, as the description of the dynamics has been properly scaled with the damping coefficient.

III. RULES OF BASIC LOGIC OPERATIONS

Before going into the discussion of our results, here we revisit the underlying principles of the basic types of the logic operations. In digital computation, both inputs and outputs can have two binary states, the "on" state and the "off" state or the "true" state and the "false" state. Generally, the "on" state or the "true" state is assigned to the logical value 1 and the "off" state or the "false" state is assigned to the logical value 0. Two commonly used logic operations in logic circuits are OR and AND or their negation operation NOR and NAND. For these kinds of logic operations, there are at least two inputs which are converted to a single logical output. In case of the OR logic gate, at least one of the two inputs has to be "true" to get a "true" output, whereas for an AND operation both of the inputs have to be "true" to produce a "true" output signal. The truth table for these logic operations has been presented in Table I.

TABLE I. Truth table for basic logic operations.

Inputs (I_1, I_2)	OR	AND	NOR	NAND
(0,0)	0	0	1	1
(0,1), (1,0)	1	0	0	1
(1,1)	1	1	0	0

Now to understand whether the system under study constitute a particular type of logic gate under a given condition, we need to examine the input-output correspondence. It has been shown previously [34] that logical response of a desired type can be extracted from a system by controlling the nonlinearity (i.e., the asymmetry of the bistable potential) and the noise-level of the system.

Here, in the present work we ask an important question; can we construct logic gates by controlling the asymmetry in the thermalization of the two states of the double-well instead of the asymmetry of the potential? Or in other words; is it possible to build up logic gates with symmetric bistable system by maintaining noise asymmetry? The successful construction of the logic gates by manipulating the noise levels of the two output states, would establish a completely new perspective to set up logic circuits.

IV. NUMERICAL RESULTS AND DISCUSSION

A. Input-output correspondence

For numerical simulations, we consider Eq. (4). The values of the parameter a and b have been taken to be equal to 1. The dynamics of the output [Eq. (4)] is solved using the improved Euler algorithm or Heun's method which is essentially a second-order Runge-Kutta method [49]. This is the technique to solve ordinary differential equations with noise and requires two evaluations of the function at each step. The time step has been taken to be equal to 10^{-3}. The noise has been generated using Box-Muller algorithm. The Langevin dynamics is integrated with both random and periodic input signals I_1 and I_2, separately. As we consider both $I_1(t)$ and $I_2(t)$ can have two discrete levels I_a and $-I_a$ corresponding to the logical inputs 0 and 1 respectively, the combination of the logical input (1,1) produces numerical input signal of strength $2I_a$, (0,1) and (1,0) combination corresponds to 0 and (0,0) combination refers to the $-2I_a$ numerical value of the total input signal. The solution for the position variable x from Eq. (4) signifies the response of the system towards the external stimuli, and is considered as the output signal against the random or the periodic inputs. The output state at the left well is assigned to the logical output value 0 (or 1) and that at the right well to the logical value 1 (or 0). Due to the presence of the noise and external bias in the system, the output state can only be in the left or the right potential well. The system can not rest at the metastable state even if one starts simulating the dynamics keeping the initial condition fixed at the metastable state. To get the output state, we solve the dynamics according to Eq. (4) and measure x. Then depending upon the value of x, we determine whether the output memory sate is at the left or the right well, or in other words, whether the output corresponds to the logical

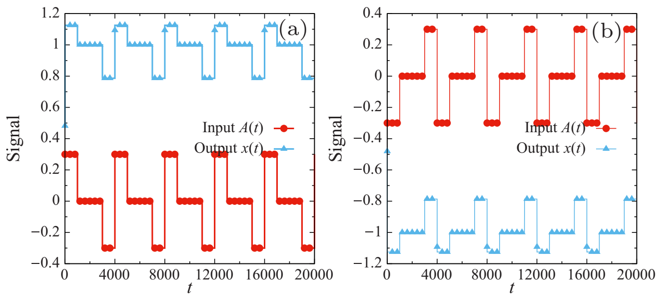

FIG. 2. Input (line with filled circle)-output (line with filled triangle) correspondence extracted from the system in absence of noise. The input signals are periodic. The phases of the signals are different in (a) and (b).

value 0 or 1. The unprocessed numerical value of the position variable x plays the decisive role in case of the determination of the output state.

To do a systematic examination of the input-output correspondence, we start with a noiseless situation. It is observed that the response of the system is not "logical" in absence of noise. This has been shown in Figs. 2(a) and 2(b) for periodic input signals. The observation is similar for random input signals as well.

Next, we introduce noise in the system, however maintaining asymmetry in the noise-level associated with the two output memory states. We keep the temperature of one of the wells fixed and vary temperature of the other well starting from a very low value. Say, first we keep the temperature of the left well fixed which results a constant diffusion coefficient for the left well of value $D_1 = 0.1$ and vary the temperature of the right well with the resulting diffusion coefficient varying within the range between $D_2 = 0.01$ and 1.5. Now, we scrutinize the time series of the input, $I(t) = I_1(t) + I_2(t)$ and the output $x(t)$. It is observed that OR (or NOR) logical response can be extracted from the system when the diffusion coefficient of the right well D_2 is maintained at a very low value. This is true when the system is subject to either random or periodic input signals. It has been demonstrated for three random trajectories in Figs. 3(a)–3(c) for three different values of $I_a = 0.3$, 0.5, and 1.0 at $D_2 = 0.01$. The same has been presented in Figs. 4(a)–4(c) with periodic input signals. With increasing noise strength associated with the right well, the reliability of the OR (or NOR) logical response starts to become weaker and the system does not show definitive logic behavior for a certain range of noise strength. However, the system's response comes within the reliability limit of an AND (or NAND) gate at a much higher value of D_2. This corresponds to a D_2 value which is higher compared to the value of $D_1 = 0.1$. However, the AND (or NAND) response is not as good as the OR (or NOR) response. This fact has been illustrated in Figs. 3(d)–3(f) for three single trajectories with $I_a = 0.3, 0.5$, and 1.0. In this case, the input signals are random. The observations are similar for periodic input signals and have been illustrated in Figs. 4(d)–4(f). Both the logical response start to become weaker if the noise strength D_2 is further increased. Similarly, one can construct AND (or NAND) gate first and then OR (or NOR) gate by keeping D_2 fixed at an intermediate value, say 0.1, and varying D_1 within the range between 0.01 and 1.5. This has been shown in Figs. 5(a)–5(f) and Figs. 4(a)–4(f) for random and periodic input signals, respectively.

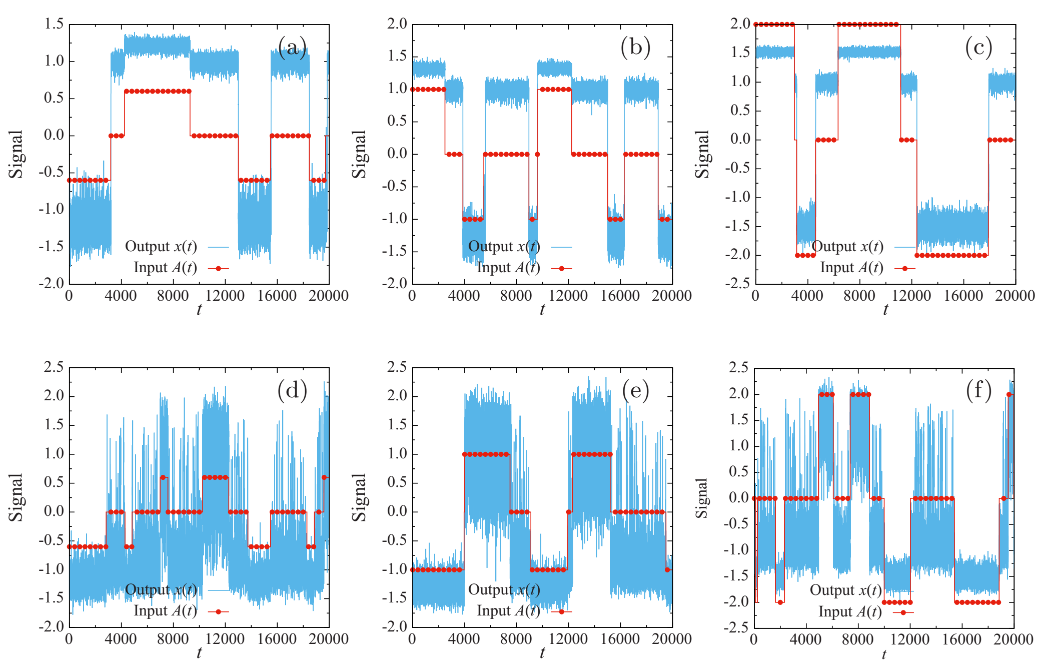

FIG. 3. Input (line with filled circle)-output (solid line) correspondence at $D_2 = 0.01$ for (a) $I_a = 0.3$, (b) $I_a = 0.5$ and (c) $I_a = 1.0$. The system shows OR (or NOR) logical response for these three cases. Input-output correspondence at $D_2 = 0.6$ for (d) $I_a = 0.3$, (e) $I_a = 0.5$, and (f) $I_a = 1.0$. The system tends to show AND (or NAND) logical response for these last three cases. In all of these cases D_1 has been kept fixed at 0.1 and the inputs are considered to be random signals.

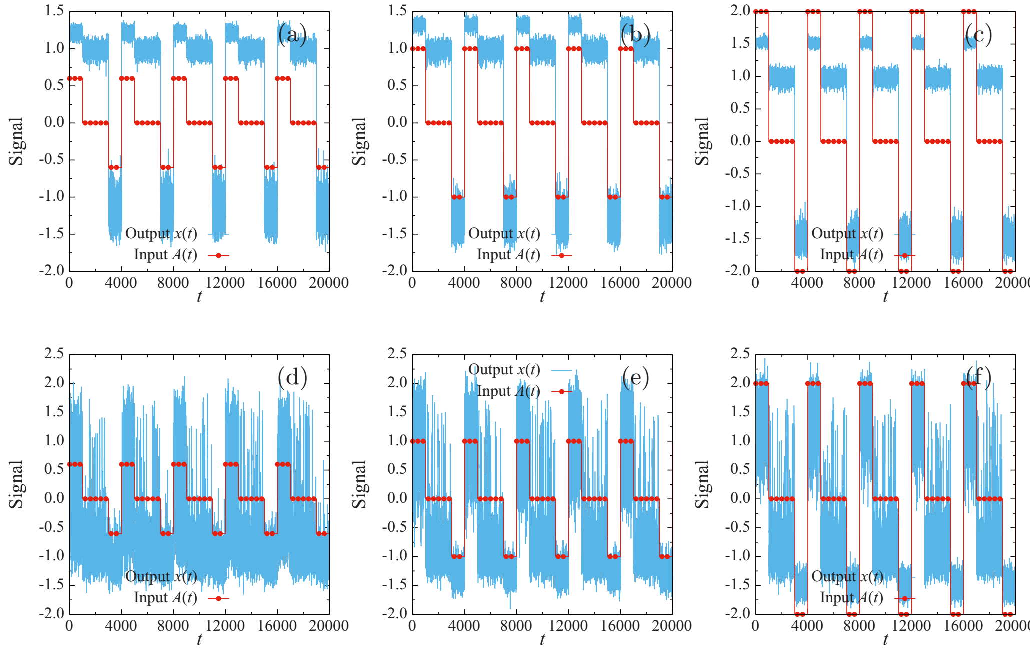

FIG. 4. Input (line with filled circle)-output (solid line) correspondence at $D_2 = 0.01$ for (a) $I_a = 0.3$, (b) $I_a = 0.5$, and (c) $I_a = 1.0$. The system shows OR (or NOR) logical response for these three cases. Input-output correspondence at $D_2 = 0.6$ for (d) $I_a = 0.3$, (e) $I_a = 0.5$, and (f) $I_a = 1.0$. The system tends to show AND (or NAND) logical response for these last three cases. In all of these cases D_1 has been kept fixed at 0.1 and the inputs are considered to be periodic signals.

B. Quantifying the logical response

As noise is present in the system, we cannot comment on the overall behavior of the system just by observing a single trajectory. Therefore, to better understand the logical response of the system quantitatively, we introduce an ensemble-averaged quantifier P_{logic}. P_{logic} is measured in the following way. We define a sum which is initialized to value zero before we start our observation for each trajectory. We check the output corresponding to a given input at every time-step for a given trajectory. If the input-output correspondence

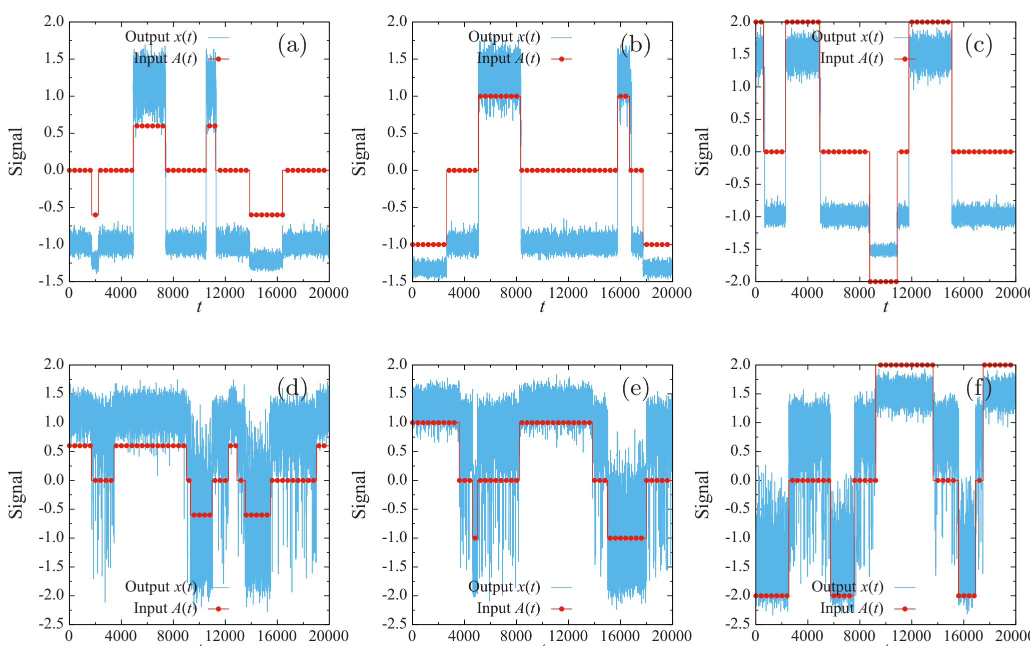

FIG. 5. Input (line with filled circle)-output (solid line) correspondence at $D_1 = 0.01$ for (a) $I_a = 0.3$, (b) $I_a = 0.5$ and (c) $I_a = 1.0$. The system shows AND (or NAND) logical response for these three cases. Input-output correspondence at $D_1 = 0.6$ for (d) $I_a = 0.3$, (e) $I_a = 0.5$ and (f) $I_a = 1.0$. The system tends to show OR (or NOR) logical response for these last three cases. In all of these cases D_2 has been kept fixed at 0.1 and the inputs are considered to be random signals.

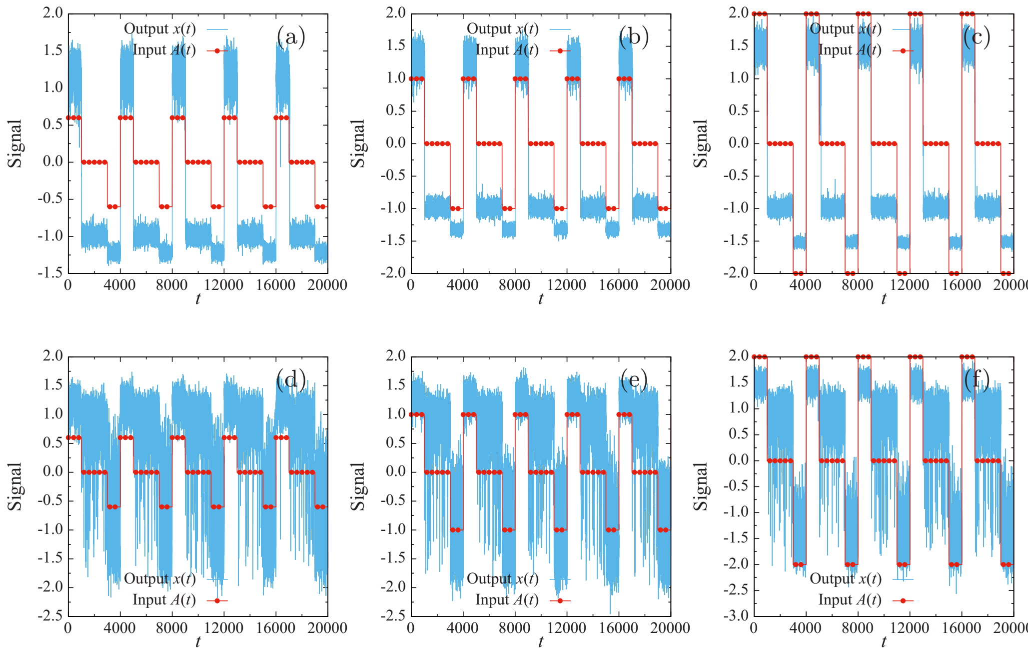

FIG. 6. Input (line with filled circle)-output (solid line) correspondence at $D_1 = 0.01$ for (a) $I_a = 0.3$, (b) $I_a = 0.5$ and (c) $I_a = 1.0$. The system shows AND (or NAND) logical response for these three cases. Input-output correspondence at $D_1 = 0.6$ for (d) $I_a = 0.3$, (e) $I_a = 0.5$, and (f) $I_a = 1.0$. The system tends to show OR (or NOR) logical response for these last three cases. In all of these cases D_2 has been kept fixed at 0.1 and the inputs are considered to be periodic signal.

follows a given kind of logical response, then we add value 1 to the corresponding sum and normalize it at the end of the time series. To mathematically represent the description, let us define the normalized sum corresponding to the jth trajectory as S_j. S_j can be expressed as

$$S_j = \frac{1}{N}\sum_{i=1}^{N}\theta[f_{\rm out}(x) - f_{\rm in}(I_1, I_2)]. \quad (5)$$

Here, N is the total number of time steps for a single trajectory. The function θ has value 1 if $f_{\rm out}$ produces the desired value corresponding to a particular type of logic operation for the given set of inputs (I_1, I_2) and otherwise it has value equal to 0. $f_{\rm out}$ is determined by observing the state of the Brownian particle which is estimated from the position coordinate of the particle:

$$\begin{aligned} f_{{\rm out}(x)} &= 0, \quad \text{if x} < 0,\\ &= 1, \quad \text{if x} > 0. \end{aligned} \quad (6)$$

Say, for example, if at an instant of time, the system is subject to (0,1) input and the particle is at the right well, i.e., at the output memory state 1, $f_{\rm in}(I_1, I_2)$ and $f_{\rm out}(x)$ relate to each other by an OR logic operation and the function θ contributes to the quantifying sum belonging to an OR logical response. Now, we take an average over such sums over an ensemble of size M and the average value corresponds to $P_{\rm logic}$. Therefore, $P_{\rm logic}$ can be expressed as

$$P_{\rm logic} = \frac{1}{M}\sum_{j=1}^{M} S_j. \quad (7)$$

Here, in our present study, $P_{\rm logic}$ has been averaged over 10^6 number of trajectories. We can say, this quantifier has been defined based on a two-state approach as the state of the Brownian particle in the double-well decides the output. However, it is different from the techniques generally adopted to observe synchronization phenomenon [50,51] where average Kramer's rate or signal to noise ratio are calculated to represent the output. In our case, the quantifier $P_{\rm logic}$ that is employed as a measure of the output response has a definition that depends simply on examining the state of the system towards a given input. This consideration works for both random and periodic signals.

As we can see from the time-series of the input-output correspondence of the system [Figs. 3(a)–3(f), Figs. 4(a)–4(f), Figs. 5(a)–5(f), and Figs. 6(a)–6(f)], its response to the inputs is instantaneous. Therefore, there is not much effect of the consideration of the waiting time on the measured quantifier and we have not taken waiting time of the response into account while measuring $P_{\rm logic}$. If the system obeys the basic rules of a certain logic operation at every instant of time for all the trajectories, then $P_{\rm logic}$ has value equal to 1. If the value of $P_{\rm logic}$ is close to 1, then we can say that the system has formed a reliable logic gate. It has been maintained that the input states (0,0), (0,1), (1,0), and (1,1) occur in equal proportion for a given ensemble, so that there appears no bias from the inputs while measuring $P_{\rm logic}$. We have presented the variation of $P_{\rm logic}$ against D_2 and D_1 for three different amplitudes of the input signal I_a in Figs. 7(a) and 7(b). In the first case D_1 is kept constant at 0.1 and D_2 is varied. It is observed that at very low strength of noise D_2, $P_{\rm OR}$ (or $P_{\rm NOR}$) has value almost equal to 1, i.e., OR (or NOR) logic gate is formed with maximum reliability. However, this response becomes weaker with increasing value of D_2. Interestingly, the AND (or NAND) response of the system starts to improve with growing D_2. $P_{\rm OR}$ (or $P_{\rm NOR}$) and $P_{\rm AND}$ (or $P_{\rm NAND}$) cross

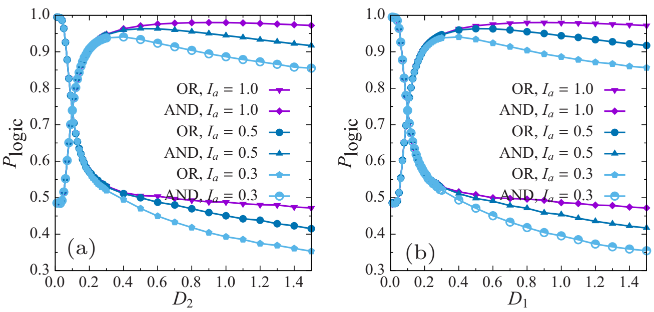

FIG. 7. Variation of P_{logic} against (a) D_2 when D_1 is kept fixed and (b) D_1 when D_2 is kept constant for three different amplitudes of external drive. The system is subject to random input signals.

each other when the values of D_1 and D_2 become equal. The AND (or NAND) logical behavior becomes reliable (within <5% error) at certain strength of noise D_2 and starts to become weaker when D_2 is further increased. Therefore, this second type of logical response shows a maximum at an intermediate strength of noise D_2. This observation is similar to the "logical stochastic resonance" phenomenon [16]. The point to remember here is that the position of the maximum of P_{AND} (or P_{NAND}) and the range through which it remains reliable depends on the amplitude of the external input signifying nonlinear response of the system. Similarly, in the second case we vary D_1 keeping D_2 fixed and first construct AND (or NAND) and then OR (or NOR) logic gate. In the above cases the system is considered to be subject to random input signals. We have performed similar calculations with periodic input signals and retrieved the same effect. We compare the variations of P_{logic} against the noise strengths for random and periodic input signals in Figs. 8(a) and 8(b) for $I_a = 0.5$. The analysis suggests that P_{logic} is quantitatively identical for random and periodic input signals, within the range of the noise strength where the system produces reliable logical response. Therefore, we can say, jittering effect does not affect the numerical simulations when we consider random inputs. This is because the system's response is almost instantaneous to the inputs that can be seen from the time series of the input-output correspondence.

C. The role of thermal asymmetry

To explain the observed behavior of the system, we analyze its response towards individual input sets. In case of

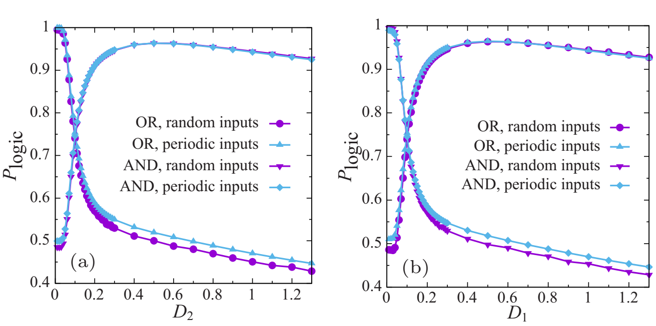

FIG. 8. Variation of P_{logic} against (a) D_2 when D_1 is kept fixed and (b) D_1 when D_2 is kept constant, at $I_a = 0.5$, for random and periodic input signals.

the construction of the logic gate with bistable system, the Brownian particle experiences directional force for the input sets (0,0) and (1,1) because they correspond to $-2I_a$ and $2I_a$ numerical value of the external driving. The negative value of the external force drives the particle to the left well, i.e., 0 (or 1) output state and due to the positive force, the particle moves to the right well, i.e., 1 (or 0) output state. The output for the (0,0) and the (1,1) state for both the OR (or NOR) and AND (or NAND) gate are the same and the external resultant directional force makes the system to satisfy correct logical input-output correspondence. The combination of the inputs (0,1) and (1,0) gives rise to the zero value of the resultant input. In this situation the asymmetry of the potential generally decides whether the left well or the right well would be occupied. If the right well is deeper than the left well, then OR (or NOR) logic gate is formed and in the reverse situation AND (or NAND) gate is formed. Here, we consider that the potential of the system is symmetric and introduce asymmetry in the noise level of the two output states. This difference in the thermalization of the two states now dictates which well would be occupied in the zero-external drive situation. If the temperature of the right well is very low as compared to the left well, then the particle occupies the right well for most of the realizations because the low temperature well freezes particle's motion. This results in an overall OR (or NOR) response of the system towards the external stimuli. Consequently, in the opposite situation, i.e., when the temperature of the left potential well is maintained at a very low value as compared to the right potential well, the system forms an AND (or NAND) gate with very high degree of logical reliability.

If we concentrate on the first case, i.e., when we keep D_1 fixed and vary D_2, then the system's OR (or NOR) response starts to become weaker as D_2 deviates from very small value. This is because now a higher proportion of trajectories would end up in the left potential well in the zero force situation, due to the increased thermal energy of the particle, by making OR (or NOR) response of the system weaker. However, the AND (or NAND) response of the system starts to become stronger with increasing noise strength as the occupancy level of the left well for the zero-external force case increases. The system starts to produce reliable AND (or NAND) response when D_2/D_1 becomes greater than 1 and reaches a certain ratio corresponding to a given I_a. The constructive role of noise remains effective over a range of D_2 value and then becomes weaker which results in a maximum of P_{AND} when plotted against D_2. Similarly, in the second case, one can construct an OR (or NOR) logic gate at an intermediate value of D_1.

One important fact to mention here is that the quantifier to measure the logical response for a single trajectory is a dynamical quantity. However, when we take an ensemble average of the quantifier and calculate P_{logic} it becomes a stationary measure. The stationary state distribution of the two wells gets modified due to the difference in diffusion coefficient of the two wells which is reflected in the value of P_{logic} and this gives rise to the observed phenomena.

The interesting point to note here is that by tuning the noise level, we are able to form basic logic gates at two different range of temperatures and there occurs a transition in the logical behavior depending upon the ratio of the diffusion coefficient associated with the two output states.

V. CONCLUSIONS

To summarize, we generalize the dynamics of the output of logic gates of small scale according to the dynamics of a Brownian particle in a bistable potential subject to external inputs. The state of the particle in either of the two wells represents the binary logical output. In general, the asymmetry of the bistable potential plays the decisive role for the formation of a logic gate of a given type. We focus on a basic issue of understanding the construction of logic gates by keeping the symmetry of the potential intact. We suggest that one can construct logic gates with a symmetric bistable system by thermalizing the two states at two different temperatures. It has been shown that the logic gates of desired type can be constructed by properly choosing the noise levels associated with the two output states. The same system forms different kinds of logic gates depending upon the maintained ratio of the effective diffusion coefficients linked to the two output states. The proposed criteria of the construction of logic gates can be verified experimentally and the present study is supposed to have important practical applications in the build-up of logic circuits of a desired type.

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